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# Deformed Lorentz symmetry and relative locality in a curved/expanding spacetime

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The interest of part of the quantum-gravity community in the possibility of Planck-scale-deformed Lorentz symmetry is also fueled by the opportunities for testing the relevant scenarios with analyses, from a signal-propagation perspective, of observations of bursts of particles from cosmological distances. In this respect the fact that so far the implications of deformed Lorentz symmetry have been investigated only for flat (Minkowskian) spacetimes represents a very significant limitation, since for propagation over cosmological distances the curvature/expansion of spacetime is evidently tangible. We here provide a significant step toward filling this gap by exhibiting an explicit example of Planck-scale-deformed relativistic symmetries of a spacetime with constant rate of expansion (deSitterian). Technically we obtain the first ever example of a relativistic theory of worldlines of particles with 3 nontrivial relativistic invariants: a large speed scale (“speed-of-light scale”), a large distance scale (inverse of the “expansion-rate scale”), and a large momentum scale (“Planck scale”). We address some of the challenges that had obstructed success for previous attempts by exploiting the recent understanding of the connection between deformed Lorentz symmetry and relativity of spacetime locality. We also offer a preliminary analysis of the differences between the scenario we here propose and the most studied scenario for broken (rather than deformed) Lorentz symmetry in expanding spacetimes.

## I. INTRODUCTION

Technically the interest attracted over the last decade by the possibility of Planck-scale-deformed relativistic kinematics, as conceived within the proposal “DSR” (doubly-special, or, for some authors, deformed-special relativity) introduced in Refs. [1, 2], was mainly linked to the following observations:

- it affords us the luxury of introducing as observer-independent laws some of the properties for the Planck scale that have been most popular in the quantum-gravity literature, such as a role for the Planck length (the inverse of the Planck scale) as the minimum allowed value for wavelengths [1, 2];
- it reflects the content of results rigorously established for 3D quantum gravity (see, *e.g.*, Refs. [3–5]);
- it reflects the content of results rigorously established for some 4D noncommutative spacetimes (see, *e.g.*, Refs. [6, 7]);
- it fits the indications emerging from some compelling semiheuristic arguments based on 4D Loop Quantum Gravity [3, 8].

A comparable “motivational list” can be claimed by other scenarios for the fate of relativistic symmetries at the Planck scale, but in the case of these DSR scenarios a crucial additional motivation comes from the opportunities in phenomenology. Some of the novel effects that can be accommodated within a DSR-relativistic kinematics can be tested through observations of bursts of particles

from cosmological distances [9, 10]. The key for these analyses are the implications of DSR deformations for the propagation of signals, and the sought Planck-scale sensitivity is reached thanks to the huge amplification afforded by the cosmological distances.

Our analysis takes off from the realization that this most notable “selling point” for DSR research, based on its phenomenological prospects, has not yet been made truly accessible by the DSR literature developed so far. DSR-deformed relativistic frameworks have been investigated so far only for flat (Minkowskian) spacetimes, and this is not the case relevant for the analysis of signals received from sources at cosmological distances, for which the curvature/expansion of spacetime is very tangible. We here provide a significant step toward filling this gap by exhibiting an explicit example of Planck-scale-deformed relativistic symmetries of a spacetime with constant rate of expansion (deSitterian). Evidently this will lead us to introduce the first ever example<sup>1</sup> of a relativistic theory of worldlines of particles with 3 nontrivial relativistic invariants: a large speed scale (“speed-of-light scale”), a large distance scale (inverse of the “expansion-rate scale”), and a large momentum scale (“Planck scale”).

There had been previous attempts of investigating the interplay between DSR-type deformation scales and spacetime expansion (see, *e.g.*, Refs. [12, 13]), but without ever producing a fully satisfactory picture of how the

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<sup>1</sup> Studies such as those in Refs. [11–13] did contemplate the possibility of 3 invariants, but did not go as far as giving a consistent relativistic picture of worldlines of particles.

worldlines of particles should be formalized and interpreted. In retrospect we can now see that these previous difficulties were due to the fact that the notion of relative locality had not yet been understood, and without that notion the interplay between DSR-deformation scale and expansion-rate scale remains unintelligible.

Relative locality (on which we shall of course return in more detail in later parts of this manuscript) is the space-time counterpart of the DSR-deformation scale  $\ell$  just in the same sense that relative simultaneity is the space-time counterpart of the special-relativistic scale  $c$  (scale of deformation of Galilean Relativity into Special Relativity). This was understood only very recently, in studies such as the ones in Refs. [14–16]. Awareness of the possibility of relative locality is already very important in making sense of the implications of DSR-deformations in a flat/non-expanding spacetime. And, as we shall here show, it plays an even more crucial role in the consistency of the spacetime picture emerging from the interplay between DSR-deformation scale and expansion-rate scale.

While our original (and still here primary) motivation was to address these fascinating, but merely technical, issues concerning the interplay between DSR-deformation scale and expansion-rate scale, as shown in later parts of this manuscript our results appear to carry rather strong significance for the phenomenology. In particular, our constructive analysis leads to a description of the dependence of the novel effects on redshift that is not like anything imagined in the previous DSR literature (relying then only on heuristic arguments for the interplay between DSR-deformation scale and expansion-rate scale).

We work at leading order in the DSR-deformation scale  $\ell$ , since (assuming it is of the order of the Planck length) that is the only realistic target for DSR phenomenology over the next few decades. And in order to keep things simple, without renouncing to any of the most significant conceptual hurdles, we opt to work in a 2D spacetime (one time and one spatial dimension).

The choices of notation we adopt are relatively standard and self-explanatory with the exception of the fact that occasionally we denote with  $X$  the set of spacetime coordinates, i.e.  $X \equiv (t, x)$ , and we denote with  $G$  the set of symmetry generators, i.e.  $G \equiv (E, p, \mathcal{N})$ .

## II. PRELIMINARIES ON DEFORMED LORENTZ SYMMETRY WITHOUT EXPANSION

For our purposes the main starting point is provided by the analysis in Ref. [14] which provided, assuming a Minkowskian spacetime (no expansion), a framework for DSR-deformed relativistic theories of worldlines of particles with two nontrivial relativistic invariants: a large speed scale  $c$  (the “speed-of-light scale”, here mute because of the conventional choice of units  $c = 1$ ) and a large momentum scale  $\ell^{-1}$  (assumed to be roughly of the order of the “Planck scale”). Ref. [14] however has a limitation that for our purposes is significant: the anal-

ysis in Ref. [14] took off from an ansatz, with a single parameter  $\ell$ , for the energy(/momentum) dependence of travel times of massless particles from a given emitter to a given detector, so it did not fully explore the possible issue of a dependence of such travel times on the choice of DSR-deformation of the on-shell relation. Taking as starting point the special-relativistic on-shell relation,  $m^2 = E^2 - p^2$ , one could contemplate, in particular, adding a term of form  $Ep^2$  and/or adding a term of form  $E^3$ . This issue was not much of interest in Ref. [14] since it is easy to see that the difference between  $Ep^2$  deformation and  $E^3$  deformation, if analyzed in a flat/non-expanding spacetime, carries very little significance (it should be inevitably insignificant at least for massless particles). But in our analysis of a first case with spacetime expansion we shall find that there are some significant difference between  $Ep^2$  deformations and  $E^3$  deformations.

So in this section we find appropriate to offer a minor generalization and reformulation of the results reported in Ref. [14], particularly suitable for our generalization to the case of an expanding spacetime, most notably for what concerns the comparison between  $Ep^2$  deformation and  $E^3$  deformation.

We start by assuming that, in a Minkowskian spacetime, the relativistic symmetries leave invariant the following combination of the energy  $E$  and momentum  $p$  of particles:

$$\mathcal{C}_{\alpha,\beta} = E^2 - p^2 + \ell (\alpha E^3 + \beta E p^2) , \quad (1)$$

where  $\alpha$  and  $\beta$  are two numerical parameters.

And we exhibit a corresponding DSR-deformed 1+1D Poincaré algebra of charges compatible with the invariance of  $\mathcal{C}_{\alpha,\beta}$ , describable in terms of the following Poisson brackets:

$$\begin{aligned} \{E, p\} &= 0 , & \{\mathcal{N}, E\} &= p - \ell (\alpha + \beta) p E , \\ \{\mathcal{N}, p\} &= E + \frac{1}{2} \ell (\alpha E^2 + \beta p^2) . \end{aligned} \quad (2)$$

A conveniently intuitive picture of the deformation we are studying is obtained by giving a representation of these symmetry generators in terms of time and space coordinates  $t, x$  and variables  $\Omega, \Pi$  canonically conjugate to them:

$$\begin{aligned} \{\Omega, t\} &= 1 , & \{\Omega, x\} &= 0 , \\ \{\Pi, t\} &= 0 , & \{\Pi, x\} &= -1 , \\ \{t, x\} &= \{\Omega, \Pi\} = 0 , \end{aligned} \quad (3)$$

We find as representation the following:

$$E = \Omega + \frac{1}{2} \ell ((1 - \beta) \Pi^2 - \alpha \Omega^2) , \quad (4)$$

$$p = \Pi , \quad (5)$$

$$\mathcal{N} = -tp + xE + \ell \left( \frac{1}{2} \alpha x E^2 + tpE + \frac{1}{2} \beta x p^2 \right) . \quad (6)$$

As done in Ref. [14], we shall derive the worldlines adopting a covariant formulation of classical relativistic mechanics, also introducing an auxiliary affine parameter labeling points on the worldline of a particle of mass  $m$ . As standard in the covariant formulation of classical relativistic mechanics, evolution is coded in a pure-constraint Hamiltonian; specifically, the evolution in the auxiliary parameter on the worldline is governed by a Hamiltonian constraint:  $\mathcal{C}_{\alpha,\beta} - m^2 = 0$ . This straightforwardly leads [14] to the following worldlines

$$x_{m,p} = x_0 + \left( \frac{p}{\sqrt{p^2 + m^2}} - \ell p \right) (t - t_0) , \quad (7)$$

which in the particularly interesting case of massless particles, restricting our focus on  $p > 0$ , reduces to

$$x_{m=0,p>0}(t) = x_0 + (t - t_0)(1 - \ell|p|) . \quad (8)$$

And let us also stress that evidently the charges  $E, p, \mathcal{N}$  are conserved along the motion, in the sense that  $G = \{\mathcal{C}_{\alpha,\beta}, G\} = 0$ , and they generate respectively deformed time translations, spatial translations, and boosts, by Poisson brackets. By construction we have ensured that these transformations all are relativistic symmetries of the theory, as one can explicitly verify [14] by acting with them on the worldlines (7), finding that the worldlines are covariant.

At this stage of the analysis the physical content of these worldlines is still hidden behind the relativity of locality. A warning that this might be the case is seen in the fact that we are analyzing a case where the on-shell relation, in light of (1), is  $\alpha$  and  $\beta$  dependent,<sup>2</sup>

$$m^2 = E^2 - p^2 + \ell(\alpha E^3 + \beta E p^2) ,$$

whereas our worldlines are independent of  $\alpha$  and  $\beta$ . Most importantly the coordinate velocity one infers from those worldlines is independent of  $\alpha$  and  $\beta$ . But one of the main known manifestations of relative locality is a mismatch [14, 21, 24] between coordinate velocity<sup>3</sup> and physical velocity of particles.

<sup>2</sup> Note that the quantity we denote with  $m$  (and we loosely refer to as the mass) is not the rest energy when  $\alpha \neq 0$ . The rest energy evidently is  $\mu$  such that  $m^2 = \mu^2 + \alpha \ell \mu^3$ . The interested readers can easily verify that none of our most significant observations (most evidently since we mainly focus on massless particles) depends on the difference between  $m^2$  and  $\mu^2$ .

<sup>3</sup> Examples of velocity artifacts were of course known well before the understanding of relative locality for theories with deformed Lorentz symmetry: for example in de Sitter spacetime (and any expanding spacetime) the coordinate velocity of a particle distant from the observer can be described by the observer as a velocity greater than the speed of light, even though in classical de Sitter spacetime the physical velocity measured by an observer close to the particle is of course always no greater than the speed of light. Since these previously known velocity artifacts are connected with spacetime curvature, the recent realization that some of our currently investigated theories formulating (one form or

For what concerns specifically the analysis so far reported in this section, the main challenge resides in the fact that we are used to read velocities off the formulas for worldlines, but this implicitly assumes that translation transformations are trivial. Essentially we take the worldline written by a certain observer Alice to describe both the emission of a particle “at Alice” (in Alice’s origin) and the detection of the particle far away from Alice. The observer/detector Bob that actually detects the particle, since he is distant from Alice, should be properly described by acting with a corresponding translation on Alice’s worldline. And the determination of the “arrival time at Bob” (crucial for determining the physical velocity [21]) should be based on Bob’s description of the worldline, just as much as the “emission time at Alice” should be based on Alice’s description of the worldline. When translations are trivial (translation generators conjugate to the spacetime coordinates) we can go by without worrying about this more careful level of discussion, since the naive argument based solely on Alice’s worldline gives the same result as the more careful analysis using Alice’s worldline for the emission and Bob’s description of that same worldline for the detection. But when translations are nontrivial, and one has associated features of relativity of locality, this luxury is lost.

One way to have “relative locality” is indeed the case here of interest, with the non-trivial translation generators of (4)-(5) acting on spacetime coordinates as follows:

$$\begin{aligned} \{E, t\} &= 1 - \ell \alpha E , & \{E, x\} &= -\ell(1 - \beta)p , \\ \{p, t\} &= 0 , & \{p, x\} &= -1 . \end{aligned} \quad (9)$$

So, following Ref. [21], let us probe the difference between coordinate velocity and physical velocity through the simple exercise of considering the simultaneous emission “at Alice” of two massless particles, one “soft” (with momentum  $p_s$  small enough that  $\ell$ -deformed terms in formulas fall below the experimental sensitivity available) and one “hard” (with momentum  $p_h$  big enough that at least the leading  $\ell$ -deformed terms in formulas fall within the experimental sensitivity available).

We of course describe the relationship between the coordinates of two distant observers in relative rest in terms of the Poisson-bracket action of the translation generators  $E, p$ , *i.e.*  $1 - a_t\{E, \cdot\} + a_x\{p, \cdot\}$ , with  $a_t, a_x$  the translation parameters along  $t$  and  $x$  axes. In the specific case in which Bob detects the soft massless particle in his origin, which restricts us to the possibilities  $a_t = a_x = L$  ( $L$  being the spatial distance between Alice and Bob),

another of “deformed Minkowski spacetime” are subject to relative locality, with some associated velocity artifacts [14, 21, 24], was largely unexpected. Borrowing on terminology already used in some previous works on de Sitter spacetime, we here label as “coordinate velocity” the one obtained with the familiar  $dx/dt$  formula, whereas we reserve the label of a “physical-velocity effect” to effects that determine the correlations between emission times and detection times for the same particle.

one finds that Bob's coordinates are related to Alice's as follows:

$$t_0^B = t_0^A - L + \ell L \alpha E, \quad (10)$$

$$x_0^B = x_0^A - L + \ell L(1 - \beta)p. \quad (11)$$

The case we are considering, with a soft and a hard massless particle simultaneously emitted toward Bob in Alice's origin, is such that the two particles are described by Alice in terms of the worldlines

$$\begin{aligned} x_{p_s}^A(t^A) &= t^A, \\ x_{p_h}^A(t^A) &= t^A(1 - \ell|p_h|). \end{aligned}$$

And these worldlines, in light of (10)-(11), are described by Bob as follows:

$$\begin{aligned} x_{p_s}^B(t^B) &= t^B, \\ x_{p_h}^B(t^B) &= t^B(1 - \ell|p_h|) - \ell L(\alpha + \beta)|p_h|. \end{aligned} \quad (12)$$

This allows us to conclude that Bob, who is at the detector, measures the following difference of times of arrival between the soft photon (detected at Bob at  $t^B = 0$ ) and the hard photon

$$\Delta t^B = \ell L|p_h|(\alpha + \beta). \quad (13)$$

So we see that the nontriviality of the translation transformations does affect the difference between coordinate velocity and physical velocity. And the relativity of locality produced by the nontriviality of the translation transformations, while at first appearing to be counter-intuitive, actually ensures the internal logical consistency of the relativistic framework<sup>4</sup>. Satisfactorily the physical velocity does depend on  $\alpha$  and  $\beta$  just in the way one should expect on the basis of the role of  $\alpha$  and  $\beta$  in the on-shell relation. But the dependence of the physical velocity on  $\alpha$  and  $\beta$  is not very significant, since it comes

<sup>4</sup> As discussed in greater detail in Refs. [14, 16, 21], there is an analogy between the role of relative locality in deformed-Lorentz-invariant theories and the role of relative simultaneity in deformed-Galilean invariant theories (ordinary Lorentz-invariant theories are to be viewed, from this perspective, as deformations of Galilean-invariant theories, which accommodate the invariant scale  $c$  and the relativity of simultaneity, one being the counterpart of the other). The introduction of the invariant momentum scale  $\ell^{-1}$  requires a deformation of Lorentz invariance since ordinary Lorentz transformations change the value of all momentum scales. And for the logical balance of the relativistic theory one finds that having one more thing invariant (the scale  $\ell$ ) requires rendering one more thing relative, which is the role played by relative locality. One can see that the same logics applies to the transition from Galilean Relativity to Einstein's Special Relativity. Taking as starting point Galilean Relativity, the introduction of the invariant velocity scale  $c$  requires a deformation of Galilean boosts, since ordinary Galilean boosts change the value of all velocity scales. And for the logical balance of the relativistic theory one finds that having one more thing invariant (the scale  $c$ ) requires rendering one more thing relative, which is the role played by relative simultaneity.

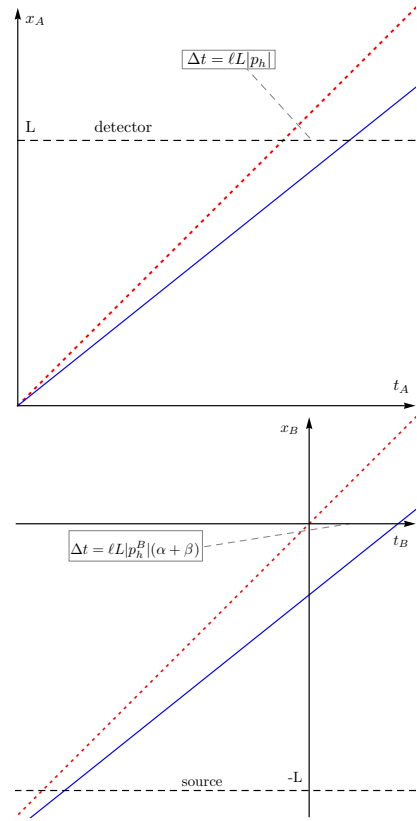


FIG. 1. We illustrate the results for travel times of massless particles derived in this section by considering the case of two distant observers, Alice and Bob, in relative rest. Two massless particles, one soft (dashed red) and one hard (solid blue), are emitted simultaneously at Alice and they reach Bob at different times. Because of the nontriviality of translation transformations, in Bob's coordinatization (bottom panel) the emission of the particles at Alice appears not to be simultaneous. And similarly, for the difference in times of arrival at the distant detector (where Bob is located) Alice finds in her coordinatization (top panel) a value which is not the same as the difference in times of arrival that Bob determines (bottom panel). The case in figure has  $\alpha + \beta = 4/3$ . For visibility we assumed here unrealistic values for the scales involved. For realistic values of the distance between observers and of the energy of the hard particle, taking  $\ell$  as the inverse of the Planck scale, no effect would be visible in figure (the worldlines would coincide).

only in the combination  $\alpha + \beta$ . It is because of this feature that the difference between  $Ep^2$  deformation and  $E^3$  deformation, if analyzed in a flat/non-expanding space-time, carries very little significance. One can get the same physical velocity of massless particles by any mixture of  $Ep^2$  deformation and  $E^3$  deformation (including the cases where one of the two is absent) as long as  $\alpha + \beta$  keeps the same value. There is nothing extraordinary or surprising about this: we are working at leading order in  $\ell$ , so in terms which already have an  $\ell$  factor we can use  $E = |p|$  (for massless particles, and using the fact that at zero-th order in  $\ell$  the massless shell is  $E = |p|$ ).

Evidently within these approximations here of interest a correction term of form  $\ell E p^2$  is indistinguishable from a correction term of form  $\ell E^3$ . While this is not surprising it was still worth devoting to it this section since one of the main findings of the generalization proposed in the next sections is that essentially in presence of spacetime expansion correction terms of form  $\ell E p^2$  are very significantly different from corrections term of form  $\ell E^3$ .

Before moving on to those challenges, in closing this section let us comment briefly on boost transformations. For the purposes we had in this section the relative locality produced by deformed boost transformations did not come into play. Readers interested in those features of relative locality can look in particular at Ref. [14]. The point relevant for us is that, as inferred from Ref. [14], the relative locality produced by boosts does not affect the derivation of the physical velocity, but rather is the way by which the relativistic theory renders the properties of the physical velocity compatible with boost invariance. This all comes down to the fact that  $\ell$ -deformed boosts do not affect the timing of events at the observer: in the simple analysis reported above an observer purely boosted with respect to Alice would also see as simultaneous the emission of the two particles Alice sees as simultaneous; and another observer, this one purely boosted with respect to Bob, would describe the timing of the detections of the two particles at Bob in a way that is completely undeformed. Boosts do play a role in the analysis of co-incidences of events distant from the observers<sup>5</sup>, but play no role in the determination of the physical velocity.

### III. DSR-DEFORMED DE-SITTER-RELATIVISTIC SYMMETRIES AND PHYSICAL VELOCITY

#### A. DSR-deformed de-Sitter-relativistic symmetries and equations of motion

The preliminaries given in the previous section provide a good starting point for the main challenge we intend to face in this manuscript. Those preliminaries summarize some known aspects of  $\ell$ -deformed Lorentz symmetry for Minkowskian (non-expanding) spacetimes, including the features of relativity of locality and the rather

marginal relevance of the differences between  $E p^2$  deformations and  $E^3$  deformations. We now want to produce the first example of theory of classical-particle world-lines with  $\ell$ -deformed de-Sitter-relativistic symmetries. It is interesting that de Sitter relativistic symmetries can themselves be viewed as a deformation of the special-relativistic symmetries of Minkowski spacetime such that the expansion-rate parameter  $H$  is an invariant. And it is known that the invariance of the expansion rate comes at the cost of some velocity artifacts. With the points summarized in the previous section we can easily contrast the two deformations of the special-relativistic symmetries of Minkowski spacetime which provide the starting points for our work. On one side we have  $\ell$ -deformed Lorentz(/Poincaré) symmetries, a deformation by a large momentum scale  $\ell^{-1}$  which produces velocity artifacts connected with the novel feature of relative locality. And on the other side we have de-Sitter-relativistic symmetries, a deformation by a large distance scale  $H^{-1}$  which produces velocity artifacts connected with spacetime expansion. The theory we are seeking must be such that both of these features can be accommodated, while preserving the relativistic nature of the theory. So both  $\ell$  and  $H$  shall be relativistic invariants (for a total of 3, including the speed-of-light scale, here mute because of our choice of units). And we shall have both expansion-induced and relative-locality-induced velocity artifacts.

The strength of these demands appears to confront us with an unsurmountable challenge. But we shall see that the “preliminaries” offered in the previous section draw a very clear path toward our objective. We start by specifying that our  $\ell, H$ -deformed relativistic symmetries shall leave invariant the following combination of the energy  $E$ , momentum  $p$  and boost  $\mathcal{N}$  charges of particles:

$$\mathcal{C}_{H,\alpha,\beta} = E^2 - p^2 + 2HNp + \ell(\alpha E^3 + \beta E p^2) . \quad (14)$$

Evidently for  $\ell \rightarrow 0$  this reproduces the standard invariant of de Sitter symmetries. Something even more general than our correction term  $\ell(\alpha E^3 + \beta E p^2)$  could here be envisaged, but we are not seeking results of maximum generality. On the contrary we want to make the case as convincingly and simply as possible that there are examples of the novel class of relativistic theories we are here proposing. Moreover the correction term  $\ell(\alpha E^3 + \beta E p^2)$  does have enough structure for us to investigate the interplay of  $E p^2$  deformations and  $E^3$  deformations with spacetime expansion, which is the one among our side results that we perceive as most intriguing.

The path drawn in the previous section guides us to observe that the following  $\ell$ -deformed (2D) de Sitter algebra of charges is compatible with the invariance of  $\mathcal{C}_{H,\alpha,\beta}$

$$\begin{aligned} \{E, p\} &= Hp - \ell\alpha H E p , \\ \{\mathcal{N}, E\} &= p + HN - \ell\alpha E(p + HN) - \ell\beta E p , \\ \{\mathcal{N}, p\} &= E + \frac{1}{2}\ell\alpha E^2 + \frac{1}{2}\ell\beta p^2 . \end{aligned} \quad (15)$$

<sup>5</sup> This point emerges most simply and clearly by analyzing, as done in Ref. [14], the illustrative example of an observer Alice which describes two pairs of events, one pair coincident in her origin and one pair coincident far away from her origin. For an observer purely boosted (purely  $\ell$ -deformed boosted) with respect to Alice one finds [14] that the pair of events close to the origin are still coincident, but the pair of events distant from the origin are not coincident. This is the boost counterpart of the fact that, as a result of the properties of deformed translations, and the associated implications for relative locality, the distant pair of events will not be observed as coincident by observers whose origin is close to that pair of events.

One easily sees that for  $\ell \rightarrow 0$  this reproduces the standard properties [25] of the classical de Sitter algebra of charges while for  $H \rightarrow 0$  it reduces to (2).

We give a convenient representation of these symmetry generators in terms of “conformal coordinates”, with conformal time  $\eta$  and spatial coordinate  $x$ , and variables  $\Omega, \Pi$  canonically conjugate to these conformal coordinates:

$$\begin{aligned} \{\Omega, \eta\} &= 1, & \{\Omega, x\} &= 0, \\ \{\Pi, \eta\} &= 0, & \{\Pi, x\} &= -1, \\ \{\eta, x\} &= 0. \end{aligned} \quad (16)$$

We find the following representation

$$E = \Omega - H\eta\Omega + Hx\Pi + \frac{\ell}{2}(1 - \beta)\Pi^2 \quad (17)$$

$$+ \frac{\ell}{2}H\eta\Pi^2 - \frac{\ell}{2}\alpha(\Omega - H\eta\Omega + Hx\Pi)^2, \quad (18)$$

$$p = \Pi, \quad (18)$$

$$\begin{aligned} \mathcal{N} &= x(1 - H\eta)\Omega - \left(\eta - \frac{H}{2}\eta^2 - \frac{H}{2}x^2\right)\Pi \\ &+ \ell\left(\frac{1 + H\eta}{2}x\Pi + (1 - H\eta)\eta\Omega\right)\Pi. \end{aligned} \quad (19)$$

Again we intend to derive the worldlines of particles working within a “covariant formulation”; so we formally introduce an auxiliary affine parameter on the worldline and the formal evolution in the affine parameter is governed by the invariant  $\mathcal{C}_{H,\alpha,\beta}$ .

Of course the fact that  $\mathcal{C}_{H,\alpha,\beta}$  is an invariant of the (deformed-)relativistic symmetries implies that the charges that generate the symmetry transformations are conserved over this evolution:

$$\begin{aligned} \dot{E} &= \{\mathcal{C}_{H,\alpha,\beta}, E\} = 0, & \dot{p} &= \{\mathcal{C}_{H,\alpha,\beta}, p\} = 0, \\ \dot{\mathcal{N}} &= \{\mathcal{C}_{H,\alpha,\beta}, \mathcal{N}\} = 0. \end{aligned}$$

Importantly by consistency with the chosen (deformed) form of the invariant  $\mathcal{C}_{H,\alpha,\beta}$  we have obtained translation transformations which are significantly deformed, specifically for the time direction. In fact, from (17) and (18) one finds

$$\{E, \eta\} = 1 - H\eta - \ell\alpha(1 - H\eta)(\Omega - H\eta\Omega + Hx\Pi), \quad (20)$$

$$\begin{aligned} \{E, x\} &= -Hx - \ell(1 - \beta + H\eta)\Pi \\ &+ \ell\alpha Hx(\Omega - H\eta\Omega + Hx\Pi), \end{aligned} \quad (21)$$

$$\{p, \eta\} = 0, \quad \{p, x\} = -1. \quad (22)$$

This prepares us to deal with relative-locality effects, of the sort described in the previous section but intertwined with the additional complexity of spacetime expansion.

But let us first note down the equations of motion that are obtained in our framework. Following the standard procedures for derivation of worldlines in the covariant formulation of classical mechanics one easily arrives, for

a particle of mass  $m$  (i.e.  $\mathcal{C}_{H,\alpha,\beta} = m^2$ ), at the following result for the worldlines:

$$\begin{aligned} x_{m,p} &= x_0 + \frac{\sqrt{m^2 + (1 - H\eta_0)^2 p^2}}{Hp} \\ &- \frac{\sqrt{m^2 + (1 - H\eta)^2 p^2}}{Hp} - \ell(\eta - \eta_0)p. \end{aligned} \quad (23)$$

For the case of massless particles, specifying  $p > 0$ , this reduces to

$$x_{m=0,p>0}(\eta) = x_0 + (\eta - \eta_0)(1 - \ell|p|). \quad (24)$$

By construction these worldlines (23),(24) are covariant under the (deformed-)relativistic transformations generated by the charges  $E, p, \mathcal{N}$ , as one can also easily verify explicitly.

## B. Travel time of massless particles

As already established in previous studies of the case without spacetime expansion (here summarized in Section II), the DSR-deformed relativistic symmetries introduce (as most significant among many other novel features) a dependence on energy of the travel time of a massless particle from a given source to a given detector. In the analysis we provided so far of our novel proposal of DSR-deformed relativistic symmetries of an expanding spacetime (with constant expansion rate) an indication of dependence on energy of the travel time of a massless particle from a given source to a given detector is found in Eq. (24): the equation governing the properties of the worldline of a massless particle in conformal coordinates is  $\ell$ -corrected and the correction term introduces a dependence on the energy(/momentum) of the particle. However, as also expected on the basis of previous results on the case without spacetime expansion, we are evidently working in a framework where locality is relative (evidence of which has been provided so far within our novel framework implicitly in Eqs. (20)-(21)). So the analysis of the equations of motion written by one observer is inconclusive for what concerns the notion of “travel time”, i.e. the correlation between emission time and detection time. As illustrated for the non-expanding case in the previous section, we must guard against the coordinate artifacts associated to relative locality by analyzing the emission of the particle in terms of the description of an observer near that emission point and we must then analyze the detection of the particle in terms of the description of an observer near the detection point.

This is indeed our next task. We consider the case of two distant observers: Alice at the emitter and Bob at the detector. We keep the analysis in its simplest form, without true loss of generality, by considering the case of simultaneous emission at Alice of only two massless particles, one with “soft” momentum  $p_s$  and one with “hard” momentum  $p_h$ . Evidently, on the basis of the analysis

reported in the previous subsection, Alice describes the two particles according to

$$x_{p_s}^A(\eta^A) = \eta^A, \quad (25)$$

$$x_{p_h}^A(\eta^A) = \eta^A (1 - \ell|p_h^A|), \quad (26)$$

where we specified  $x_0^A = \eta_0^A = 0$ , so that the emission is at  $(0, 0)^A$ . Since translations are a relativistic symmetry of our novel framework, we already know that the same two worldlines will be described by the distant observer Bob in the following way

$$x_{p_s}^B(\eta^B) = x_{0;s}^B + \eta^B - \eta_{0;s}^B, \quad (27)$$

$$x_{p_h}^B(\eta^B) = x_{0;h}^B + (\eta^B - \eta_{0;h}^B) (1 - \ell|p_h^B|), \quad (28)$$

*i.e.* the same type of worldlines but with a difference of parameters here codified in  $x_{0;s}^B, \eta_{0;s}^B, x_{0;h}^B, \eta_{0;h}^B$ . Indeed, Alice's worldlines (25)-(26) and Bob's worldlines (27)-(28) have exactly the same form, but for the ones of Alice we had by construction (by having specified simultaneous emission at Alice) that  $x_{0;h}^A = \eta_{0;h}^A = x_{0;s}^A = \eta_{0;s}^A = 0$  whereas Bob's values of the parameters,  $x_{0;s}^B, \eta_{0;s}^B, x_{0;h}^B, \eta_{0;h}^B$ , should be determined by establishing which ( $\ell$ -deformed) translation transformation connects Alice to Bob.

This is the same task performed in some of the previous studies (such as Ref. [21]) involving relative locality in DSR-deformed relativistic theories without spacetime expansion. But also this aspect of the analysis turns into a rather more challenging exercise for us, having to deal with spacetime expansion. Let us then proceed determining the translation transformation that connects Alice to Bob providing enough details for the reader to appreciate the interplay between the different structures we must handle.

As usual we shall give the action on points  $\eta, x$  of a worldline by a finite transformation  $T_{G;a}$  generated by the generic element  $G$  of the relativistic-symmetry algebra in terms of its exponential representation as

$$T_{G;a} \triangleright X = e^{-aG} \triangleright X \equiv \sum_{n=0}^{\infty} \frac{(-a)^n}{n!} \{G, X\}_n, \quad (29)$$

where  $a$  is the transformation parameter and  $\{G, X\}_n$  is the  $n$ -nested Poisson bracket defined by the relation

$$\{G, X\}_n = \{G, \{G, X\}_{n-1}\}, \quad \{G, X\}_0 = X. \quad (30)$$

And before proceeding we must also implement something else that can be specified about observer Bob. The worldline parameters  $x_{0;h}^A, \eta_{0;h}^A, x_{0;s}^A, \eta_{0;s}^A$  of observer Alice are fully specified ( $x_{0;h}^A = \eta_{0;h}^A = x_{0;s}^A = \eta_{0;s}^A = 0$ ) by its being at the point of simultaneous emission of the two particles. We have introduced observer Bob as one that detects the particles (the particles worldlines should cross Bob spatial origin) but we have so far left completely unspecified its worldline parameters  $x_{0;s}^B, \eta_{0;s}^B, x_{0;h}^B, \eta_{0;h}^B$ . We

evidently can do better than that. And actually we can also insist, without true loss of generality, that the soft particle reaches Bob in his spacetime origin: so we are free to enforce  $x_{0;s}^B = \eta_{0;s}^B = 0$ .

This is particularly convenient because it involves the "soft particle", *i.e.* the one whose momentum  $p_s$  has been chosen to be small enough to render the  $\ell$ -deformed effects inappreciable within the experimental sensitivities available to Alice and Bob. The fact that both  $x_{0;s}^A = \eta_{0;s}^A = 0$  and  $x_{0;s}^B = \eta_{0;s}^B = 0$  for a soft particle leads us to focus on the case of the observer Bob connected to Alice by the following transformation

$$X^B = (e^{a_x p} \triangleright e^{-a_\eta E} \triangleright X)^A, \quad (31)$$

with

$$a_x = \frac{1 - e^{-Ha_\eta}}{H}. \quad (32)$$

Through this we are essentially exploiting the fact that the deformation is ineffective on the soft particle as a way for us to focus on a distant observer Bob whose relationship to Alice (translation parameters connecting Alice to Bob) can be specified using only known results on the undeformed/standard relativistic properties<sup>6</sup>. Indeed in classical (relativistically undeformed) de Sitter spacetime one easily finds that a massless particle emitted in the origin of some observer Alice will cross the origin of all observers connected to Alice by a spatial translation of parameter  $a_x$  combined with a conformal-time translation of parameter  $a_\eta$ , with the request that  $a_x = H^{-1}[1 - e^{-Ha_\eta}]$ .

Using our translation generators (17)-(18) one easily finds that for points on the worldline of the hard particle the map from Alice to Bob is such that

$$\begin{aligned} \eta_h^B &= \frac{1 - e^{Ha_\eta}}{H} + e^{Ha_\eta} \eta_h^A \\ &\quad + \ell \alpha a_\eta e^{Ha_\eta} (1 - H \eta_h^A) (E_h^A - H a_x p_h^A), \\ x_h^B &= e^{Ha_\eta} (x_h^A - a_x) - \ell \beta \frac{\sinh(Ha_\eta)}{H} p_h^A \\ &\quad - \ell \alpha a_\eta e^{Ha_\eta} H (x_h^A - a_x) (E_h^A - H a_x p_h^A) \\ &\quad + \ell \frac{(1 - e^{-Ha_\eta})}{H} (1 + e^{Ha_\eta} H \eta_h^A) p_h^A, \\ p_h^B &= e^{-Ha_\eta} (p_h^A + \ell \alpha H a_\eta p_h^A (E_h^A - H a_x p_h^A)). \end{aligned}$$

Crucial for us is the fact that, in light of this result for the laws of transformation from Alice's  $\eta_h^A, x_h^A, p_h^A$  to Bob's  $\eta_h^B, x_h^B, p_h^B$  (which is of course applicable in particular to the point  $\eta_{0;h}, x_{0;h}, p_h$  on the hard particle worldline) we can deduce that the worldline of the hard particle emitted at Alice is described by Bob as follows

$$x_{p_h}^B(\eta^B) = \eta^B - \ell|p_h^B| \left( \eta^B + \alpha a_\eta + \beta \frac{e^{2Ha_\eta} - 1}{2H} \right), \quad (33)$$

<sup>6</sup> This also implicitly requires [21] that the clocks at Alice and Bob are synchronized by exchanging soft massless particles.



where we made use of all the specifications discussed above, including  $a_x = \frac{1-e^{-H a_\eta}}{H}$ .

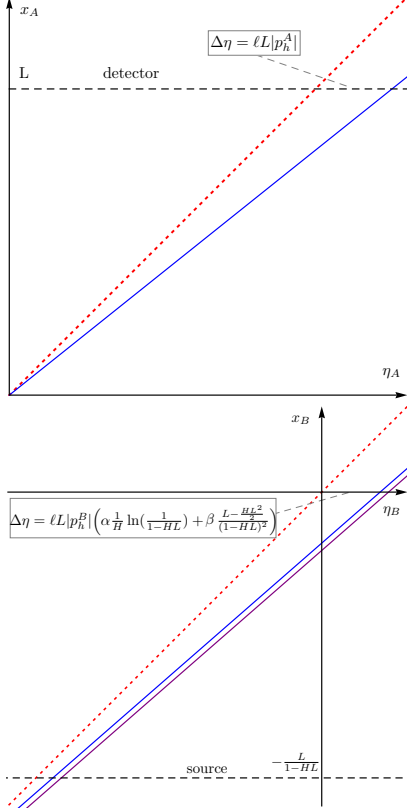


FIG. 2. We illustrate the results for travel times of massless particles derived in this section, adopting conformal coordinates. We consider the case of two distant observers, Alice and Bob, connected by a pure translation, and two massless particles, one soft (dashed red) and one hard (solid blue and solid violet), emitted simultaneously at Alice. And we consider two combinations of values of  $\alpha$  and  $\beta$  producing the same  $\alpha + \beta$ : the case  $\alpha = 2/3, \beta = 2/3$  (Bob's hard worldline in blue) and the case  $\alpha = 1/3, \beta = 1$  (Bob's hard worldline in violet). As in the case without expansion we have here that in Bob's coordinatization (bottom panel) the emission of the particles at Alice appears not to be simultaneous. Similarly for the difference in times of arrival at the distant detector Bob Alice finds in her coordinatization (top panel) not the same value as the difference in times of arrival that Bob determines. Comparison of the blue and violet worldlines shows that in the case with spacetime expansion the travel time does depend individually on  $\alpha$  and  $\beta$  (not just on  $\alpha + \beta$  as in the case without spacetime expansion). For visibility we assumed here unrealistic values for the scales involved.

In turn this allows us to obtain the sought result for the dependence on energy(/momentum) of the travel times of massless particles: by construction of the worldlines and of the Alice→Bob transformation the soft massless particle emitted in Alice's spacetime origin reaches Bob's spacetime origin, whereas from (33) we see that the hard massless particle also emitted in Alice's spacetime origin reaches Bob at a nonzero conformal time. Specifically

the difference in conformal travel times derivable from (33) is

$$\Delta\eta^B = \eta^B \Big|_{x_{p_h}^B=0} = \ell|p^B| \left( \alpha a_\eta + \beta \frac{e^{2Ha_\eta} - 1}{2H} \right). \quad (34)$$

We summarize the relativistic properties of this travel-time analysis, inconformal coordinates, in Fig. 2. By comparison with Fig. 1 one sees that in conformal coordinates the qualitative picture of the energy dependence of travel times is very similar to the one of the case without spacetime expansion. But here, with spacetime expansion, there are tangible differences between  $Ep^2$  deformations and  $E^3$  deformations.

In Fig. 3 we characterize the results of our travel-time analysis in comoving coordinates, which for most studies of spacetime expansion are the most intuitive choice of coordinates. [The differences between Fig. 2 and Fig. 3 all are a straightforward manifestation of the relationship  $\eta = H^{-1}(1 - e^{-Ht})$  between the conformal time  $\eta$  and the comoving time  $t$ .]

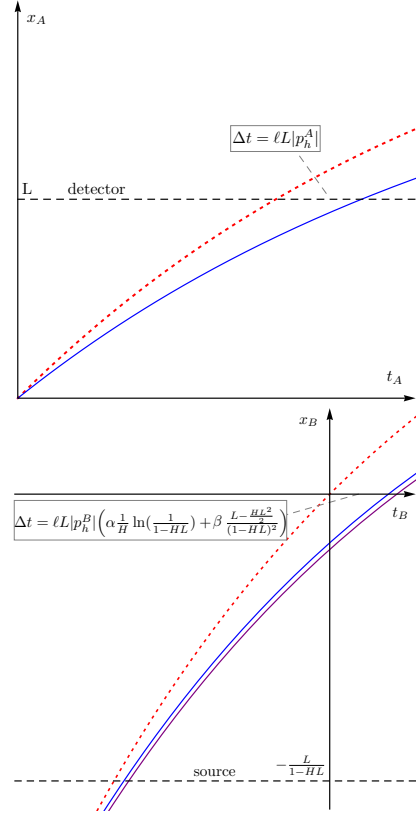


FIG. 3. We here illustrate the results for travel times of massless particles derived in this section, adopting comoving coordinates. The situation here shown is the same situation already shown (there in conformal coordinates) in Fig. 2.

#### IV. ASIDE ON SPACETIME COORDINATES

Having completed the main conceptual tasks of our analysis we find appropriate to pose for a few comments on the role of spacetime coordinates. The notion of relativistic theories with relative locality started to be developed only very recently and it is likely that some of its implications will start to be appreciated at later stages of maturation. This motivates us to offer in this section a few remarks concerning the robustness of our findings, focusing on the setup of the coordinatization of spacetime. Particularly in relation to the steps that took us from Eq. (15) to Eq. (23) our readers might welcome a few additional comments on the role played by the setup for the coordinatization.

Let us start by observing that the coordinatization usually preferred by physicists, the one in which spacetime coordinates are conjugate to the physical energy and momentum, was not available to us. And this is not due to the DSR deformation we are studying. Already the undeformed de Sitter algebra has structure such that the energy and momentum charges, generators of time and spatial translational symmetries, cannot be conjugate to spacetime coordinates. Physically this is due to spacetime expansion: de Sitter spacetime is maximally symmetric, including translational symmetries, but the action of translations must take into account the expansion and cannot be implemented [25] by charges conjugate to the coordinates.

Also our adoption of coordinates with undeformed Poisson brackets may require further comments for some readers. Indeed similar deformations of relativistic symmetries have often adopted (see, *e.g.*, Ref. [17, 18]) spacetime coordinates with Poisson brackets  $\{\tilde{x}, \tilde{t}\} = \ell\tilde{x}$ . This was done in relation to the fact that among the quantum-spacetime pictures that can motivate such studies a notable case is “ $\kappa$ -Minkowski” [19, 20], a non-commutative spacetime with  $[\hat{x}, \hat{t}] = i\ell\hat{x}$  (whose “classical limit” could be described by  $\{\tilde{x}, \tilde{t}\} = \ell\tilde{x}$ ). We opted for standard Poisson brackets on the basis of recent results [21, 22] showing that the type of issues on which we here focused is insensitive to the differences between adopting  $\{\tilde{x}, \tilde{t}\} = \ell\tilde{x}$  and adopting  $\{\tilde{x}, \tilde{t}\} = 0$ .

In the remainder of this section we show explicitly the applicability of those arguments given in Refs. [21, 22] to our study of a case with constant rate of expansion. For this two aspects of complexity of the framework we developed will end up playing a role. On one side we had to face some challenges due to our interest in DSR deformations of de Sitter symmetries, since the correspondingly complex description of the symmetries manifests itself essentially in an extra layer of complexity connecting the variables conjugate to the spacetime coordinates and the translational-symmetry charges. On the other hand we had to deal with relative locality which affects even the interpretation of coordinates, by weakening their role. To see this let us think of some observer Alice and her coordinatization of spacetime points. The main implica-

tion of relative locality is that Alice’s coordinatization is assumed to actually describe physical/observable features only for points close to Alice, while for points distant from Alice one must rely on the description given by some other observer, close to them. This is the main message of previous studies of relative locality, and was here highlighted for example in Fig. 2.

In contemplating these issues it is useful consider the possibility of replacing our coordinates  $x, \eta$  with coordinates  $\tilde{x}, \tilde{\eta}$  obtained from them via the map

$$\begin{aligned}\tilde{x} &= x + \phi_x \ell \eta \Pi + \psi_x \ell x \Omega \\ \tilde{\eta} &= \eta + \phi_\eta \ell x \Pi + \psi_\eta \ell \eta \Omega\end{aligned}\quad (35)$$

where we actually allowed for a 4-parameter family of such coordinate redefinitions, labeled by the map parameters  $\phi_x, \psi_x, \phi_\eta, \psi_\eta$ .

Notice that for appropriate choices of the map parameters the Poisson bracket of the new coordinates may be deformed.

$$\{\tilde{x}, \tilde{\eta}\} = \ell\tilde{x}(\psi_x + \phi_\eta), \quad (36)$$

so we are accommodating, as particular cases of our more general parametrization of the map, just the class of modified Poisson brackets that some intuitions about the classical limit of a quantum-spacetime picture would favour [17, 18]. But also notice that the maps we are considering have sizable consequences only for points distant from the (origin of the reference frame of the) observer: in particular for all choices of  $\phi_x, \psi_x, \phi_\eta, \psi_\eta$  the map takes the point  $x = 0, \eta = 0$  into the point  $\tilde{x} = 0, \tilde{\eta} = 0$ . This may already suggest that our results should be independent of the parameters  $\phi_x, \psi_x, \phi_\eta, \psi_\eta$  that characterize the maps, on the basis of how relative locality affects the analysis of correlations between emission times and detection times (the class of observable correlations on which we focused).

In the interest of clarity we opt for showing explicitly that the observable correlations between emission times and detection times on which we focused do not depend on the choice of coordinates, within the family of possible coordinatizations we are considering (parametrized by  $\phi_x, \psi_x, \phi_\eta, \psi_\eta$ , which include cases of the type (36)).

This verification can be performed by simply repeating, now adopting coordinates  $\tilde{x}, \tilde{\eta}$ , all the steps of the derivations we reported in the previous section for coordinates  $x, \eta$ . The main differences between working with  $x, \eta$  coordinates and working with the  $\tilde{x}, \tilde{\eta}$  are produced by the fact that Eqs. (20)-(22) must be replaced, when working with  $\tilde{x}, \tilde{\eta}$  coordinates, by

$$\begin{aligned}\{E, \tilde{\eta}\} &= 1 - H\tilde{\eta} - \ell\alpha(1 - H\tilde{\eta})(\Omega - H\tilde{\eta}\Omega + H\tilde{x}\Pi) \\ &\quad + \ell\phi_\eta H\tilde{x}\Pi + \ell\psi_\eta(1 + H\tilde{\eta})\Omega,\end{aligned}\quad (37)$$

$$\begin{aligned}\{E, \tilde{x}\} &= -H\tilde{x} - \ell(1 - \beta + H\tilde{\eta})\Pi + \ell\phi_x(1 + H\eta)\Pi \\ &\quad + \ell\alpha H\tilde{x}(\Omega - H\tilde{\eta}\Omega + H\tilde{x}\Pi) + \ell\psi_x H\tilde{x}\Omega,\end{aligned}\quad (38)$$

$$\{p, \tilde{\eta}\} = -\ell\phi_\eta\Pi, \quad \{p, \tilde{x}\} = -1 - \ell\psi_x\Omega. \quad (39)$$

Crucial for the main result we here obtained is the derivation of worldlines giving the description by an observer Alice of two particles, one “hard” and one “soft”, emitted simultaneously at Alice (in the origin of Alice’s reference frame). Of course nothing changes for the soft particle, but the description of the hard-particle worldline obtained on the basis of the (37), by analyzing the flow of the Hamiltonian/constraint (14), is now actually different from the one we obtained in the previous section

$$\begin{aligned}\tilde{x}_{p_s}^A(\tilde{\eta}^A) &= \tilde{\eta}^A, \\ \tilde{x}_{p_h}^A(\tilde{\eta}^A) &= \tilde{\eta}^A (1 - \ell|p_h^A|(1 - \phi_x - \psi_x + \phi_\eta + \psi_\eta)).\end{aligned}$$

A distant observer Bob of the type discussed in the previous section (such that the soft particle crosses the origin of his reference frame) describes the worldlines as follows:

$$\begin{aligned}\tilde{x}_{p_s}^B(\tilde{\eta}^B) &= \tilde{\eta}^B, \\ \tilde{x}_{p_h}^B(\tilde{\eta}^B) &= \tilde{x}_{0;h}^B + (\tilde{\eta}^B - \tilde{\eta}_{0;h}^B)(1 - \ell|p_h^B|(1 - \phi_x - \psi_x + \phi_\eta + \psi_\eta)).\end{aligned}\quad (40)$$

But, as one should have expected on the basis of the observations we offered above, this change in the way worldlines are described by observers is exactly compensated by the fact that the dependence on  $\phi_x, \psi_x, \phi_\eta, \psi_\eta$  found in (37) also affects the description of translation transformations between a given observer Alice, located at the emission point of the particles of interest, and a distant observer Bob, located at the detector. Indeed following the steps discussed in the previous section that allow one to obtain the description of finite translations on the basis of the action of symmetry charges on the coordinates (here given by (37)) one finds the following map between Alice’s coordinates and Bob’s:

$$\begin{aligned}\tilde{\eta}_h^B &= \frac{1 - e^{Ha_\eta}}{H} + e^{Ha_\eta} \tilde{\eta}_h^A + \ell \phi_\eta ((1 - e^{Ha_\eta}) \tilde{x}_h^A - a_x) p_h^A \\ &\quad + \ell \alpha a_\eta e^{Ha_\eta} (1 - H \tilde{\eta}_h^A) (E_h^A - H a_x p_h^A) \\ &\quad - \ell \psi_\eta \left( \frac{1 - e^{Ha_\eta}}{H} + \frac{e^{Ha_\eta} - e^{-Ha_\eta}}{H(1 - H \tilde{\eta}_h^A)} \right) (E_h^A - H \tilde{x}_h^A p_h^A),\end{aligned}\quad (41)$$

$$\begin{aligned}\tilde{x}_h^B &= e^{Ha_\eta} (\tilde{x}_h^A - a_x) - \ell \beta \frac{\sinh(Ha_\eta)}{H} p_h^A \\ &\quad - \ell \alpha a_\eta e^{Ha_\eta} H (\tilde{x}_h^A - a_x) (E_h^A - H a_x p_h^A) \\ &\quad + \ell \frac{(1 - e^{-Ha_\eta}) (1 + e^{Ha_\eta} H \tilde{\eta}_h^A)}{H} p_h^A \\ &\quad + \ell \phi_x \left( \tilde{\eta}_h^A (1 - e^{Ha_\eta}) - \frac{1 - e^{-Ha_\eta}}{H} \right) p_h^A \\ &\quad + \ell \psi_x ((1 - e^{Ha_\eta}) \tilde{x}_h^A - a_x) \frac{E_h^A - H \tilde{x}_h^A p_h^A}{1 - H \tilde{\eta}_h^A},\end{aligned}\quad (42)$$

$$p_h^B = e^{-Ha_\eta} (p_h^A + \ell \alpha H a_\eta p_h^A (E_h^A - H a_x p_h^A)). \quad (43)$$

Combining Eqs. (41)-(43) (particularly for the point  $\tilde{x}_{0;h}^B, \tilde{\eta}_{0;h}^B$  which corresponds to  $\tilde{x}_{0;h}^A = 0, \tilde{\eta}_{0;h}^A = 0$ ) with Eqs. (40) one finds that the observable on which we focused, the difference in arrival times at Bob, is indeed independent on the choice of coordinatization (*i.e.* there

is no dependence on  $\phi_x, \psi_x, \phi_\eta, \psi_\eta$ ):

$$\Delta \tilde{\eta}^B = \tilde{\eta}^B \Big|_{\tilde{x}_{p_h}^B=0} = \ell |p_h^B| \left( \alpha a_\eta + \beta \frac{e^{2Ha_\eta} - 1}{2H} \right), \quad (44)$$

## V. IMPLICATIONS FOR PHENOMENOLOGY

Our main motivation for this analysis was conceptual. There is at this point rather robust evidence that theories in non-expanding spacetime with DSR-deformed relativistic symmetries do have a solid internal logical consistency and are therefore viable candidates for quantum-gravity researchers. Some of their properties, like the relativity of spacetime locality, are “extremely novel” and this will understandably affect how each researcher subjectively gauges the likelihood (or lack thereof) of DSR-deformations of relativistic symmetries for the quantum-gravity realm. But the objective fact is that, if we could confine our considerations to Minkowskian spacetimes, the DSR option is at this point clearly viable. However, it is of course a well established experimental fact that we are not in a Minkowskian spacetime. Even postponing, as we did here, the much reacher collection of experimental facts on gravitational phenomena and the Einsteinian description of spacetime, we felt DSR research should at least start facing the fact that spacetime expansion is very tangible in our data. So we here sought to make the first step away from the extremely confining arena of Minkowskian spacetime, by showing that DSR-deformations of relativistic symmetries are also viable in deSitterian spacetime, expanding at a constant rate.

In facing successfully this challenge, of primarily “academic” interest, we stumbled upon results which are of rather sizable significance for phenomenology, for reasons that were hinted at in earlier parts of this manuscript and we shall now discuss in greater detail.

Our first step toward the phenomenological issues of interest in this section is to reformulate the results for travel times of massless particles derived in the previous section in a way that is in closer correspondence with the type of facts that are established experimentally, when our telescopes observe distant sources of gamma rays. Many of the analyses performed at our telescopes amount to timing the detection of photons emitted from a source at a known redshift  $z$ . Postponing for a moment the fact that the relevant contexts are not such that we could assume a constant expansion rate, let us observe that the results we derived in the previous section are easily reformulated as the following prediction for the differences in detection times of photons of different energies(/momenta) emitted simultaneously by a source at redshift  $z$

$$\Delta t = \ell |p| \left( \alpha \frac{\ln(1+z)}{H} + \beta \frac{z + \frac{z^2}{2}}{H} \right). \quad (45)$$

Here again  $\Delta t$  is the difference in detection times between a hard gamma-ray photon of momentum  $p$  and a

reference ultrasoft photon emitted simultaneously to the hard photon at the distant source. And notice that with  $\Delta t$  we denote the comoving time but in the relevant timing sequences of detections at telescopes the difference between comoving and conformal time is intangible: the telescopes are operated for a range of times within the  $t = 0$  of their resident clock which is relatively small, much smaller than the time scale  $H^{-1}$ , so that conformal time and comoving time essentially coincide (one has  $\eta \equiv H^{-1}(1 - e^{-Ht}) \simeq t$  for  $t \ll H^{-1}$ ). For what concerns redshift we simply relied on the standard formula [23, 26] for the case of constant expansion rate which, in our notation, gives  $z = e^{Ha_\eta} - 1$ .

A first aspect of phenomenological relevance which must be noticed in our result (45) is the dependence on the parameters  $\alpha$  and  $\beta$ , *i.e.* the difference between  $Ep^2$  deformations and  $E^3$  deformations. The fact that for the non-expanding spacetime case this dependence only goes with  $\alpha + \beta$  is of course recovered in our result (45) in the limit of small redshift  $z$ . This is expected since for small redshift the expansion does not manage to have appreciable consequences. What is noteworthy is that already at next-to-leading order in redshift the dependence on  $\alpha$  and  $\beta$  is no longer fully of the form  $\alpha + \beta$ , as seen by expanding our result (45) to second order in  $z$ :

$$\Delta t \Big|_{z \ll 1} \simeq \frac{\ell |p|}{H} \left[ (\alpha + \beta)z + (\beta - \alpha) \frac{z^2}{2} \right]. \quad (46)$$

In a non-expanding spacetime the difference between  $Ep^2$  deformations and  $E^3$  deformations would not have significant implications on travel-time determinations (since it depends on  $\alpha + \beta$  one gets the same result reducing the amount of  $Ep^2$  deformation in favor of an equally sizable increase of the amount of  $E^3$  deformation). The situation in expanding spacetimes is evidently qualitatively different in this respect since  $Ep^2$  deformations and  $E^3$  deformations produce corrections to the travel times that have different functional dependence on redshift. So we are learning that for determinations of travel times from distant astrophysical sources the difference between  $Ep^2$  deformations and  $E^3$  deformations is a phenomenologically viable (phenomenologically determinable) issue.

Related to this observation is also the other point we want to make on the phenomenology side, which concerns the comparison with analogous studies of scenarios where relativistic symmetries are actually “broken” (allowing for a preferred/“aether” frame), rather than DSR-deformed as in the case which was of interest for us in this manuscript. Broken-Lorentz-symmetry theories with a preferred frame are far simpler conceptually than DSR-deformed relativistic theories, and indeed the issue of the interplay between scale of Lorentz-symmetry breakdown and spacetime expansion has been usefully investigated already for several years [27–29], even producing a rather universal consensus on the proper formalization that should be adopted [29, 30, 33]. Again thanks to the simplicity of broken-Lorentz-symmetry theories these results apply to the general case of varying expansion

rate, and for the massless particles they take the form

$$\Delta t = \lambda_{LIV} |p| \int_0^z \frac{dz}{a(z)H(z)}, \quad (47)$$

where  $a$  is the scale factor,  $H$  is the expansion rate, and  $\lambda_{LIV}$  (“LIV” standing for Lorentz Invariance Violation) is the counterpart of our scale  $\ell$ : just like  $\ell$  for us is the inverse-momentum scale characteristic of the onset of the DSR-deformation of relativistic symmetries,  $\lambda_{LIV}$  is the inverse-momentum scale characteristic of the onset of the LIV-breakdown of relativistic symmetries. Of course, there is a crucial difference between the properties of  $\ell$  and the properties of  $\lambda_{LIV}$ :  $\ell$ , as characteristic scale of a deformed-symmetry picture, takes the same value for all observers, while  $\lambda_{LIV}$ , as characteristic scale of a broken-symmetry picture, takes a certain value in the preferred frame and different values in frames boosted with respect to the preferred frame.

Setting momentarily these differences aside we can compare our results for DSR-deformed symmetries with constant rate of expansion with the special case of the broken-symmetry formula (47) obtained for constant rate of expansion:

$$\Delta t = \lambda_{LIV} |p| \frac{z + \frac{z^2}{2}}{H}. \quad (48)$$

Comparing this to our result (45) we find that fixing  $\alpha = 0$  in the deformed-symmetry case one gets a formula (valid in all reference frames) which is the same as the formula of the broken-symmetry case in the preferred frame. So if  $\alpha = 0$  the differences between deformed-symmetry and broken-symmetry cases would be tangible only by comparing studies of travel times of massless particles between two telescopes with a relative boost: the difference there would be indeed that  $\ell$  takes the same value for studies conducted by the two telescopes whereas for  $\lambda_{LIV}$  the two telescopes should give different values.

We must stress however that within our deformed-symmetry analysis we found no reason to focus specifically on the choice  $\alpha = 0$ . And if  $\alpha \neq 0$  in the deformed symmetry case even studies conducted by a single telescope could distinguish between the case of symmetry deformation and the case of symmetry breakdown.

In Fig. 4 we compare the dependence on redshift of our deformed-symmetry effect among two limiting cases of balance between  $\alpha$  and  $\beta$ , and we also compare these results to bounds on travel-time anomalies [31–33] obtained in studies of sources at redshift smaller than 1 (where the assumption of a constant rate of expansion is not completely misleading).

In Fig. 5 we illustrate the “constraining power” of our results and of foreseeable generalizations of our analysis gaining access to cases with non-constant rate of expansion. For better visibility and as a way to offer more intelligible visual messages we restrict our focus to the case in which both  $\alpha$  and  $\beta$  are positive. [There is no reason for making this assumption, but on the other hand it is

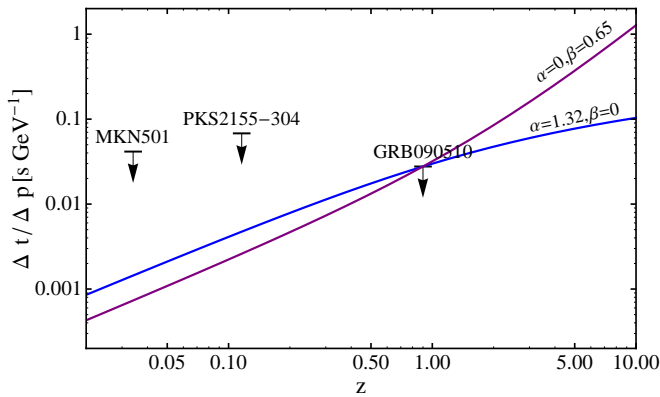


FIG. 4. Here we show the dependence on redshift of the expected time-of-arrival difference divided by the difference of energy of the two massless particles. Two such functions are shown, one for the case  $\alpha = 0, \beta = 0.65$  (violet) and one for the case  $\alpha = 1.32, \beta = 0$  (blue). We also show the upper limits that can be derived from data reported in Refs. [31–33], setting momentarily aside the fact that our analysis adopted the simplifying assumption of a constant rate of expansion (whereas a rigorous analysis of the data reported in Refs. [31–33] should take into account the non-constancy of the expansion rate). The values  $\alpha = 0, \beta = 0.65$  and  $\alpha = 1.32, \beta = 0$  have been chosen so that we have consistency with the tightest upper bound, the one established in Ref. [33]. The main message is coded in the fact that at small values of redshift the blue and the violet lines are rather close, but at large values of redshift they are significantly different (this is a log-log plot). In turn this implies that at high redshift the difference between adding correction terms of form  $E p^2$  and adding correction terms of form  $E^3$  can be very tangible.

easy to see how the constraints established within this assumption can be adapted to the more general situation with  $\alpha$  and  $\beta$  allowed to also be negative.]

In the left panel of Fig. 5 we show how the bound obtained from Ref. [32], at the relatively small redshift of  $z = 0.116$  (where we should really be able to apply our results for constant rate of expansion as a reliable first approximation), provides a constraint on the  $\alpha, \beta$  parameter space. For these purposes we can of course fix the value of  $\ell$  to be exactly the Planck length (the inverse of the Planck scale) since any rescaling of  $\ell$  can be reabsorbed into an overall rescaling of  $\alpha$  and  $\beta$ . Within this choice of conventions the target “Planck-scale sensitivity” would manifest itself as the ability to constrain values of  $\alpha$  and  $\beta$  of order 1. As shown in the left panel of Fig. 5, even restricting our focus on cases with redshift much smaller than 1 (as needed because of the present limitation of applicability of our approach to constant, or approximately constant, rate of expansion) this Planck-scale sensitivity is not far. In the right panel of Fig. 5 we show the much tighter constraint on the  $\alpha, \beta$  parameter space which would be within reach if we could assume our analysis to apply also to redshifts close to 1, as for GRB090510 observed by the Fermi telescope [33]. Ana-

lyzing data from sources at redshift of  $\simeq 1$  assuming a picture with constant rate of expansion cannot produce conservative experimental bounds, but the content of the right panel of Fig. 5 serves the purpose of providing evidence of the fact that full Planck-scale sensitivity will be within reach of improved versions of our analysis, extending our results to the case of expansion at non-constant rate.

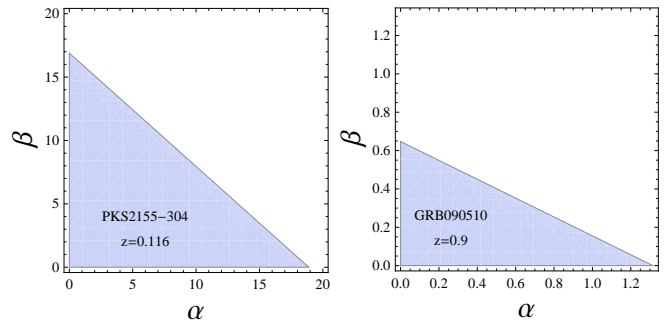


FIG. 5. Here in the left panel we show the constraint on the  $\alpha, \beta$  parameter space that can be obtained from Ref. [32], concerning a source at the relatively small redshift of  $z = 0.116$ , where we can confidently apply our results for constant rate of expansion as a reliable first approximation. Through this we show that even within the confines of our analysis Planck-scale sensitivity (values of  $|\alpha|$  and  $|\beta|$  smaller or comparable to 1) is not far. In the right panel we show the much tighter (indeed “Planckian”) constraint on the  $\alpha, \beta$  parameter space which would be within our reach if we could assume our analysis to apply also to redshifts close to 1, as for GRB090510 observed by the Fermi telescope [33]. By comparing the left and the right panel one also finds additional evidence of how the difference between adding correction terms of form  $E p^2$  and adding correction terms of form  $E^3$  becomes more significant at higher redshifts: in the left panel (data on source at small redshift of  $z = 0.116$ ) the bound on the  $\alpha$  parameter is nearly as strong as on the  $\beta$  parameter, whereas in the right panel (data on source at redshift of  $z \simeq 0.9$ ) the constraint on the alpha parameter is significantly weaker than the constraint on the  $\beta$  parameter.

## VI. SUMMARY AND OUTLOOK

We here addressed a long-standing issue for the study of quantum-gravity-inspired deformations of relativistic symmetries. It led us to propose the first ever example of a relativistic theory of worldlines of particles with 3 nontrivial relativistic invariants: a large speed scale (“speed-of-light scale”), a large distance scale (inverse of the “expansion-rate scale”), and a large momentum scale (“Planck scale”). And on the basis of the observations reported in the previous section it is clear that this is not merely an exercise providing a more rigorous derivation of previous heuristic proposals: our constructive approach produces results that reshape the targets of the relevant phenomenology.

In order to fully empower these phenomenological applications the next step would be to generalize our approach in such a way to render it applicable to cases with varying expansion rate. For what concerns applications in astrophysics this will open the way to exploiting data from the farthest sources, with the associated benefit of even larger distances amplifying the minute Planck-scale effects. And with the ability of handling varying expansion rates this research programme could also gain access to data in cosmology. The main opportunity from this perspective comes from the analyses of cosmology data which were at first inspired by varying-speed-of-light theories (see, *e.g.*, Refs. [34–36] and references therein). In our deformed-relativistic theories the speed of light is not varying in the sense originally intended: enforcing (however deformed) relativistic invariance we did not find room for the speed-of-light scale to have different values between now and earlier epochs of the Universe, but we did find a scenario for having that (with equal strength at all stages of the evolution of the Universe) the speed of photons depends on their energy. This energy-dependence of the speed of photons we found for scenarios with deformed relativistic symmetries may affect cosmology in ways that are similar to (and yet potentially interestingly different from [13]) the varying-

speed-of-light scenarios, simply because of the fact that the Universe was hotter in its earlier stages of evolution, so that the typical energy of particles was higher.

Of course, besides possibly leaping toward these opportunities available when generalizing our results to cases with non-constant expansion rate, a lot more could be done with the constant-expansion-rate case. We here managed, and only at the cost of facing some significant complexity, to obtain a consistent theory of classical-particle worldlines, but of course much more is needed before having established a consistent scheme of relativistic deformation of our current theories. In this respect, a top priority would be to formulate quantum field theories with these deformed relativistic theories.

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