Nonzero θ_{13} signals nonmaximal atmospheric neutrino mixing

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T2K Signals Non-Maximal Atmospheric Neutrino Mixing

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From recent groundbreaking experiments, it is now known that the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing differs significantly from the tribimaximal model in which $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. Flavor symmetry can require that the departures from these two equations are linearly related. $T^{'}$ and A_4, which successfully accommodated the pre-T2K PMNS matrix, predict that $38.07^\circ \leq \theta_{23} \leq 39.52^\circ$ at 95% C.L.. The best fit values, combining the model predictions with T2K, MINOS, Double Chooz, Daya Bay, and RENO data, are $\theta_{23} = 38.7^\circ$ and $\theta_{13} = 8.9^\circ$.

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Of the parameters in the standard model of particle theory, we will focus on the mixing matrices for down-types quarks and for neutrinos, named respectively for Cabibbo, Kobayashi, and Maskawa (CKM) [1, 2], and for Pontecorvo, Maki, Nakagawa, and Sakata (PMNS) [3, 4]. Without losing generality, we choose a basis in which the flavor and mass eigenstates coincide for the three up-type quarks and all three charged leptons.

This investigation will consider one of three mixing angles of CKM quark mixing (Θ_{12}) and two of the three mixing angles of PMNS neutrino mixing (θ_{13} and θ_{23}), ignoring for the moment the CP violating phases in both cases.

We recall the values of the angles θ_{13} and θ_{23} listed in the 2010 Review of Particle Physics[1] [5] since these two are, we suggest, both changed by the T2K measurement [6–11]. The values then were:

$$36.8^\circ \lesssim \theta_{23} \leq 45.0^\circ, \qquad 0.0^\circ \leq \theta_{13} \lesssim 11.4^\circ \tag{1}$$

consistent with vanishing θ_{13} and maximal θ_{23}.

The other angles are not considered to be variables in this analysis, although the superior experimental accuracy of the CKM Gell-Mann-Lévy quark mixing angle [12],

$$\Theta_{12} = (13.03 \pm 0.06)^\circ, \tag{2}$$

played an important role in our investigation of flavor symmetry.

To accommodate the new data, we invoke broken binary tetrahedral ($T^{'}$) flavor symmetry as a promising approach to explaining the mixing angles [13–22].

This flavor symmetry was first used in Ref. [13] solely as a symmetry for quarks, because neutrinos were still believed to be massless. After neutrino masses and mixings were discovered [23], the mixing matrix for neutrinos was measured and found to be very different from the CKM mixing matrix for quarks. A number of theories arose [24–28] to explain this. Eventually a useful approximation to the empirical PMNS mixing was determined to be the tribimaximal (TBM) matrix [29]:

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -1/\sqrt{2} \\ -\sqrt{1/6} & \sqrt{1/3} & 1/\sqrt{2} \end{pmatrix}. \tag{3}$$

Flavor symmetry based on the Tetrahedral Group, $A_4 = T$, was introduced by Ref. [30] to underpin TBM neutrino mixing. Further investigation revealed that this model could not be extended to quarks because a viable CKM matrix could not be obtained [31]. A_4 is not a subgroup of its double cover [20], $T^{'}$, nevertheless from the viewpoint of kronecker products used in model building [14], A_4 behaves *as if* it were a subgroup. This explains why the larger group can act as a successful flavor symmetry for both quarks and leptons.

We shall consider only the projection on the two-dimensional θ_{23} - θ_{13} plane of the three-dimensional θ_{12} - θ_{23} - θ_{13} space. At leading order, requiring $\sin\alpha \sim \alpha^2$ for θ_{13} and $(\frac{\pi}{4} - \theta_{23})$, the calculation of the perturbation of this projection from the TBM matrix in Eq.(3) is independent of the solar neutrino mixing angle θ_{12}. The relevant perturbation away from Eq.(3) was explicitly calculated in Ref. [18, 19].

Before T2K, the neutrino mixing angles were all empirically consistent with the TBM values. However, as the experimental accuracy has now improved in recent data from T2K [6–11], MINOS [32–38], Double Chooz [39–43], Daya Bay [44, 45], and RENO [46, 47], this situation has changed dramatically, as discussed in the global fits of Refs. [48–50]; of these we shall use Fogli et al. [49]. These five remarkable experiments have provided us with a rich new perspective on mixing angles. From flavor symmetry, it is then possible to predict quantitatively how departures from the TBM values,

$$\theta_{12} = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right), \quad \theta_{23} = (\pi/4), \quad \theta_{13} = 0, \tag{4}$$

are related. The model allows one to address this question by relating the perturbations around TBM,

$$\theta_{ij} = (\theta_{ij})_{TBM} + \epsilon_k, \tag{5}$$

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[1] The reader is directed to the references summarized in RPP.

[2] This is a $< 1\%$ approximation for θ_{13} and $(\frac{\pi}{4} - \theta_{23})$ since both angles are less than $\alpha = 12^\circ = 0.2094$ radians with $\sin\alpha = 0.2079$.

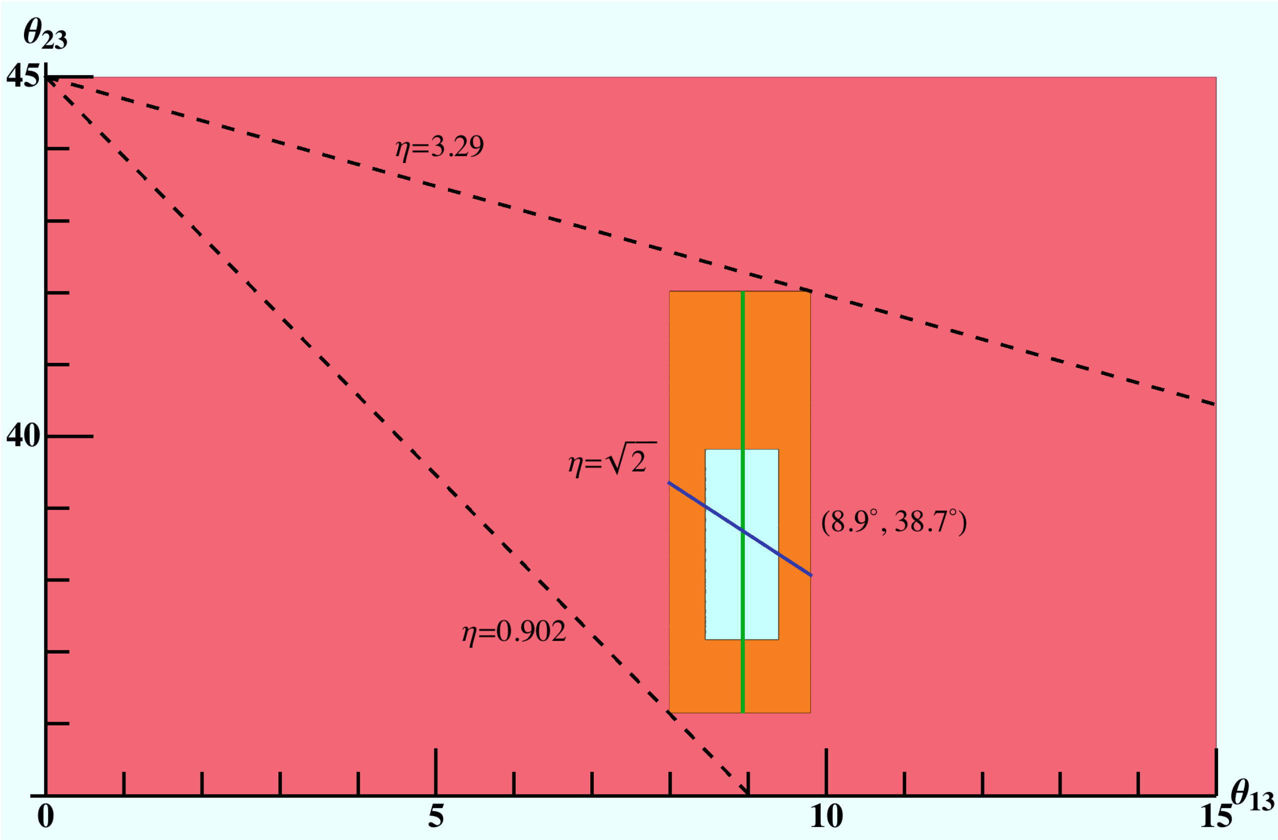

FIG. 1. The global analysis of Ref. [49], incorporating SBL, LBL, Solar, and Atmospheric neutrino observations, excludes the red-shaded region at 2σ. The same assessment excludes the orange-shaded region at 1σ. The best fit value for θ_{13} is indicated by the vertical green line at $\theta_{13} = 8.9^\circ$. Extreme values of the linear correlation coefficient, η, are indicated by dashed lines at $\eta = 0.902$ and $\eta = 3.29$, while our predicted correlation of $\eta = \sqrt{2}$ is indicated by the solid dark blue line. The intersection of our correlation prediction and the θ_{13} best fit occurs at $\theta_{13} = 8.9^\circ$ and $\theta_{23} = 38.7^\circ$, a close match to the current experimental best fit of $\theta_{23} = 38.4^\circ$. (The color plot is in the online version of the paper.)

(where ϵ_3 corresponds to θ_{12}, and so on) to the analogous perturbations around the minimal model's prediction for the CKM Gell-Mann-Lévy quark mixing angle,

$$\tan 2(\Theta_{12}) = \left(\frac{\sqrt{2}}{3}\right). \tag{6}$$

The data from KamLAND, LBL accelerators (like T2K and MINOS), solar experiments, SBL accelerators (such as Double Chooz, Daya Bay, and RENO), and Super-Kamiokande, as combined in Ref. [49] indicate (accounting for CP violation)

$$\sin^2\theta_{13} = 0.0241\ {}^{+0.0049}_{-0.0048} \qquad \text{with 95\% C.L.} \tag{7}$$

for a normal neutrino mass hierarchy, as favored by $T^{'}$.

Because Eq. (6) yields a value of $\Theta_{12} = 12.62^\circ$, which while close, is significantly below the experimental value, Eq. (2), it is possible to perturb to the empirical Θ_{12} and to track the deviations in the PMNS mixing matrix to the linear relationship[3],

$$\theta_{13} = \eta\left(\frac{\pi}{4} - \theta_{23}\right), \tag{8}$$

with the sharp prediction[4] that $\eta = \sqrt{2}$. Thus,

$$\theta_{13} = \sqrt{2}\left(\frac{\pi}{4} - \theta_{23}\right), \tag{9}$$

This prediction is derived in further detail in Ref. [19].

Considering the result, Eq. (8), it requires that if, η is finite as expected, any departure from $\theta_{13} = 0$ signals that $\theta_{23} < \pi/4$. As shown in Fig. (1), the recent experimental data, combined with theory, suggest

[3] A_4 is also capable of producing Eq.(8) with $\eta = \sqrt{2}$, though we give preference in this paper to $T^{'}$ for its capacity to explain CKM mixing.

[4] It is notable that Eq.(8) with $\eta \simeq \sqrt{2}$ appears *en passant* in Ref. [51]; see also Ref. [52] which implied that $\eta \sim 2$. Another, model-independent, correlation was developed in Ref. [53] including the three PMNS mixing angles and the CP-violating phase.

that (θ_{13}, θ_{23}) are respectively closer to (8.9°, 38.7°) than to (0.0°, 45.0°). Before T2K, η was unconstrained, $0 \leq \eta < \infty$. With the current global fit data, we find $0.902 \leq \eta \leq 3.29$.

This is in sharp difference from the previous widespread acceptance of a maximal $\theta_{23} = \pi/4$ which fitted so well with vanishing $\theta_{13} = 0$ in the TBM context.

As the measurement of θ_{13} sharpens experimentally, so will the prediction for θ_{23} from Eq. (9), and measurement of the atmospheric neutrino mixing's departure from maximality will provide an interesting test of the binary tetrahedral flavor symmetry.

Several years ago Super-Kamiokande showed $\theta_{23} > 36.8°$ [54], and current analysis places it at $\theta_{23} \simeq 40.7°$ [55]. Once combined in a global fit of 3ν oscillation, Ref. [49] states the best fit of $\theta_{23} = 38.4°$, tantalizingly close to our central value of $\theta_{23} = 38.7°$.

This suggests to us that the $T^{'}$ flavor symmetry, introduced in Ref. [13], should now be taken much more seriously. As errors in θ_{13} and θ_{23} diminish even further, it will be interesting to see how the prediction of Eq. (9) by $T^{'}$ perseveres, as it would inspire further investigation into other mixing angles for quarks and leptons. This, in turn, may show that $T^{'}$, first mentioned in physics as an example of an $SU(2)$ subgroup [56], is actually a useful approximate symmetry in the physical application of quark and lepton flavors.

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