Self-organizing neutrino mixing matrix

Ernest Ma

Phys. Rev. D **86**, 117301 — Published 19 December 2012
DOI: 10.1103/PhysRevD.86.117301

UCRHEP-T523

Self-Organizing Neutrino Mixing Matrix

Ernest Ma

Department of Physics and Astronomy, University of California, Riverside, California 92521, USA

Centre for Theoretical Physics (CFTP), Technical University of Lisbon, 1049-001 Lisbon, Portugal

Abstract

A new and novel idea for a predictive neutrino mass matrix is presented, using the non-Abelian discrete symmetry A_4 and the seesaw mechanism with only two heavy neutral fermion singlets. Given the components of the one necessarily massless neutrino eigenstate, the other two massive states are automatically generated. A realistic example is discussed with predictions of a normal hierarchy of neutrino masses and maximal CP violation.

To understand the observed neutrino mixing pattern in terms of a symmetry, the charged-lepton mass matrix and the neutrino mass matrix must be considered at the same time. Given that $m_{e,\mu,\tau}$ are all different, it is by no means trivial to find a symmetry which predicts a leptonic mixing matrix as the mismatch between the unitary matrices which diagonalize the respective mass matrices in the two different sectors. This was successfully done using the non-Abelian discrete symmetry A_4 [1, 2, 3] and applied [4] to the case of tribimaximal mixing. Whereas the specific prediction of $\theta_{13} = 0$ is now refuted by data [5, 6]. it does not mean that A_4 itself is not valid, only those additional assumptions beyond A_4 which are used to enforce the tribimaximal hypothesis. Two variations [7, 8] of the original A_4 model [4] are in fact completely consistent with $\sin^2 2\theta_{13} = 0.1$.

In this paper, an entirely different application of A_4 is presented for a predictive neutrino mass matrix. It is based on an earlier proposal [9] which works very well if $\sin^2 2\theta_{13}$ is small [10] but not with present data [5, 6]. The new and novel idea is to combine the A_4 texture with the seesaw mechanism using only two heavy neutral fermion singlets. As a result, a massless neutrino eigenstate must appear. If it is identified with ν_1, then ν_2 and ν_3 are generated with $m_2 = \sqrt{\Delta m^2_{21}}$ and $m_3 = \sqrt{\Delta m^2_{31}}$. The tribimaximal case may in fact be derived this way in a certain symmetry limit. Here it will be shown how a realistic pattern of masses and angles emerges, with predictions of the Dirac phase δ_{CP} for leptonic CP violation and the effective mass m_{ee} in neutrinoless double beta decay.

Before showing how A_4 allows this to happen, consider the end result, i.e.

$$\mathcal{M}_\nu = \begin{pmatrix} (2A+2B)u_1^2 & (-A-B+iC)u_1u_2 & (-A-B-iC)u_1u_3 \\ (-A-B+iC)u_1u_2 & (2A-B-iC)u_2^2 & (-A+2B)u_2u_3 \\ (-A-B-iC)u_1u_3 & (-A+2B)u_2u_3 & (2A-B+iC)u_3^2 \end{pmatrix}. \quad (1)$$

Note that in this basis, the charged-lepton mass matrix is diagonal, which is not an assumption but a consequence of the A_4 symmetry. It is clear from the above that there is one massless eigenstate, i.e.

$$\nu_1 = (u_1^{-1}, u_2^{-1}, u_3^{-1})/\sqrt{|u_1|^{-2} + |u_2|^{-2} + |u_3|^{-2}} \quad (2)$$

for any A, B, C. Let $\nu_{1,2,3}$ be defined by the tribimaximal basis, i.e.

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} \sqrt{2/3} & -\sqrt{1/6} & -\sqrt{1/6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}, \quad (3)$$

then for $u_1 = 1/2$, $u_2 = u_3 = -1$,

$$\mathcal{M}_\nu^{(1,2,3)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3(B+A)/2 & i\sqrt{3/2}C \\ 0 & i\sqrt{3/2}C & -3(B-A) \end{pmatrix}. \quad (4)$$

This shows that for $C = 0$, tribimaximal mixing is obtained with

$$m_1 = 0, \quad m_2 = 3(B+A)/2, \quad m_3 = -3(B-A). \tag{5}$$

Of course, the choice $u_1 = 1/2, u_2 = u_3 = -1$ is arbitrary. It is not preferred in any way within this framework. The assertion is only that if this choice is made, then what follows is automatic. Since C is in general not zero, deviation from tribimaximal mixing will occur, as shown below.

The form of the neutrino mass matrix of Eq. (1) is diagonalized by the unitary matrix U, i.e.

$$U \mathcal{M}_\nu U^T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \tag{6}$$

where

$$\nu_1 = \left(\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \right), \tag{7}$$

$$\nu_2 = \frac{1}{\sqrt{1+3\zeta^2}} \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} + i\sqrt{\frac{3}{2}}\zeta, \frac{1}{\sqrt{3}} - i\sqrt{\frac{3}{2}}\zeta \right), \tag{8}$$

$$\nu_3 = \frac{1}{\sqrt{1+3\zeta^2}} \left(-i\zeta, -\frac{1}{\sqrt{2}} - i\zeta, \frac{1}{\sqrt{2}} - i\zeta \right). \tag{9}$$

This solution is obtained with

$$B + A = \frac{2}{1+3\zeta^2} \left(\frac{m_2}{3} - \zeta^2 m_3 \right), \tag{10}$$

$$B - A = -\frac{1}{1+3\zeta^2} \left(\frac{m_3}{3} - \zeta^2 m_2 \right), \tag{11}$$

$$C = -\frac{\sqrt{2}\zeta}{1+3\zeta^2} (m_3 + m_2). \tag{12}$$

Using Eqs. (7),(8),(9), the mixing angles in the conventional definition are given by

$$\sin\theta_{13} = \frac{\zeta}{\sqrt{1+3\zeta^2}}, \quad \sin\theta_{12} = \frac{1}{\sqrt{3}\sqrt{1+2\zeta^2}}, \quad \sin\theta_{23} = -\frac{1}{\sqrt{2}}. \tag{13}$$

As for CP violation, using the Jarlskog invariant, it is easily shown that

$$\sin\delta_{CP} = 1, \tag{14}$$

i.e. maximal CP violation. Note that Eqs. (10),(11),(12) allow complex values of m_2 and m_3, but Eqs. (7),(8),(9) remain the same, because A, B, C are in general complex as well.

Hence Eqs. (13) and (14) reamin valid. In other words, this model's three mixing angles and one Dirac phase are independent of the Majorana phases of $\nu_{2,3}$. Note that $\sin\delta_{CP} = 1$ depends on the choice $u_1 = 1/2, u_2 = u_3 = -1$. If these are changed, in magnitude or in phase, $\sin\delta_{CP}$ will change, but its value will be fixed accordingly by the model.

Using the experimental constraints [11]

$$|m_2|^2 = 7.50 \pm 0.20 \times 10^{-5} \text{ eV}^2, \tag{15}$$

$$|m_3|^2 = 2.32 + 0.12(-0.08) \times 10^{-3} \text{ eV}^2, \tag{16}$$

and assuming $m_{2,3}$ to be real, the two cases of $m_2 = \pm 0.00866$ eV (with $m_3 = 0.04817$ eV) are considered, as well as $\zeta = 0.165$ from $\sin^2 2\theta_{13} = 0.098$. The parameter values of this model are then determined to be

$$A = 0.00877 \text{ eV}, \quad B = -0.00586 \text{ eV}, \quad C = -0.01226 \text{ eV}, \tag{17}$$

$$A = 0.00365 \text{ eV}, \quad B = -0.01141 \text{ eV}, \quad C = -0.00852 \text{ eV}, \tag{18}$$

respectively. The effective neutrino mass in neutrinoless double beta decay is $m_{ee} = |A + B|/2 = 0.0015$ or 0.0039 eV. They represent the minimum and maximum values of m_{ee} in the presence of arbitrary Majorana phases. Note also that θ_{13} is related to θ_{12} by

$$\tan^2\theta_{12} = \frac{1 - 3\sin^2\theta_{13}}{2} < 1/2. \tag{19}$$

This is a generic consequence of any model which has $\nu_1 \sim (2, -1, -1)$ and is favored by data. In another class of models where $\nu_2 \sim (1, 1, 1)$, the relationship becomes

$$\tan^2\theta_{12} = \frac{1}{2 - 3\sin^2\theta_{13}} > 1/2, \tag{20}$$

which is disfavored by data.

It has been shown that the neutrino mass matrix of Eq. (1) allows it to generate $\nu_{2,3}$ once the massless state $\nu_1 \sim (u_1^{-1}, u_2^{-1}, u_3^{-1})$ is decided. It has the simple and verifiable predictions of Eqs. (13) and (14), if $\nu_1 \sim (2, -1, -1)$. To derive Eq. (1), the symmetry A_4 is used following Ref. [9]. The lepton and Higgs representations are listed in Table 1. The six Higgs doublets may be replaced by just one standard-model Higgs doublet and six scalar singlet flavons. In that case, the singlet flavons are not observable at the electroweak energy scale. The important departure from Ref. [9] is that $N_1^c \sim 1$ is now missing. The two Z_2 symmetries are used to distinguish the two different sets of Higgs doublets. Because of the A_4 multiplication rules [1], the charged-lepton mass matrix is diagonal with

$$\begin{pmatrix} m_e \\ m_\mu \\ m_\tau \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} h_1 v_1 \\ h_2 v_2 \\ h_3 v_3 \end{pmatrix}, \tag{21}$$

Particle	$SU(2)_L \times U(1)_Y$	A_4	$Z_2^{(1)}$	$Z_2^{(2)}$
$(\nu, l)_{1,2,3}$	(2,–1/2)	3	+	+
$l^c_{1,2,3}$	(1,1)	3	–	+
$N^c_{2,3}$	(1,0)	$1'$, $1''$	+	–
$(\phi^0, \phi^-)_{1,2,3}$	(2,–1/2)	1, $1'$, $1''$	–	+
$(\eta^+, \eta^0)_{1,2,3}$	(2,1/2)	3	+	–

Table 1: Particle content of proposed A_4 model of neutrino mass.

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$ and $v_{1,2,3}$ are the vacuum expectation values of $\phi^0_{1,2,3}$. The Dirac mass matrix linking $\nu_{e,\mu,\tau}$ to $N^c_{2,3}$ is now

$$\mathcal{M}_D = \begin{pmatrix} f_2 u_1 & f_3 u_1 \\ f_2 \omega u_2 & f_3 \omega^2 u_2 \\ f_2 \omega^2 u_3 & f_3 \omega u_3 \end{pmatrix}, \tag{22}$$

where $u_{1,2,3}$ are the vacuum expectation values of $\eta^0_{1,2,3}$. The most general Majorana mass matrix for $N^c_{2,3}$ is given by

$$\mathcal{M}_N = \begin{pmatrix} M_2 & M_1 \\ M_1 & M_3 \end{pmatrix}. \tag{23}$$

Note that M_1 is an invariant mass under A_4, but $M_{2,3}$ are soft terms which break A_4. In this model, A_4 is broken completely by $v_{1,2,3}$ and $u_{1,2,3}$ as well as in the soft terms of the Lagrangian. This means that the special vacuum alignment for tribimaximal mixing (A_4 breaking to residual Z_3 in the charged-lepton sector and A_4 breaking to residual Z_2 in the neutrino sector, which is technically very difficult to implement) is not needed or desired. After inverting $\mathcal{M}_N$ and using the seesaw formula $\mathcal{M}_\nu = -\mathcal{M}_D(\mathcal{M}_N)^{-1}\mathcal{M}_D^T$, Eq. (1) is obtained with

$$A = -\frac{f_2 f_3 M_1}{M_1^2 - M_2 M_3}, \quad B = \frac{f_2^2 M_3 + f_3^2 M_2}{2(M_1^2 - M_2 M_3)}, \quad C = \frac{\sqrt{3}(f_2^2 M_3 - f_3^2 M_2)}{2(M_1^2 - M_2 M_3)}. \tag{24}$$

The next step is to choose $\nu_1 \sim (u_1^{-1}, u_2^{-1}, u_3^{-1})$. Since ν_1 is guaranteed to be massless, Eq. (1) is reduced to a 2×2 mass matrix in the basis $\nu_2' \sim [-u_1(u_2^{-2} + u_3^{-2}), u_2^{-1}, u_3^{-1}]$ and $\nu_3' \sim (0, u_2, -u_3)$. Diagonalizing this then yields the two mass eigenstates $\nu_{2,3}$ with $m_{2,3}$ and the corresponding mixing angle and Dirac phase. The special case of $\nu_1 \sim (2, -1, -1)$ has been studied in this paper, but the method may be adapted to any ν_1.

In conclusion, a remarkable form of the neutrino mass matrix has been derived using A_4 and a reduced seesaw mechanism. It has a simple solution as shown by Eqs. (13) and

(14). It is numerically consistent with all present data and predicts the exciting possibilty of maximal CP violation in the neutrino sector.

This work is supported in part by the U. S. Department of Energy under Grant No. DE-AC02-06CH11357.

References

[1] E. Ma and G. Rajasekaran, Phys. Rev. **D64**, 113012 (2001).

[2] E. Ma, Mod. Phys. Lett. **A17**, 2361 (2002).

[3] K. S. Babu, E. Ma, and J. W. F. Valle, Phys. Lett. **B552**, 207 (2003).

[4] E. Ma, Phys. Rev. **D70**, 031901 (2004).

[5] Daya Bay Collaboration: F. P. An *et al.*, Phys. Rev. Lett. **108**, 171803 (2012).

[6] RENO Collaboration: J. K. Ahn *et al.*, Phys. Rev. Lett. **108**, 191802 (2012).

[7] E. Ma and D. Wegman, Phys. Rev. Lett. **107**, 061803 (2011).

[8] H. Ishimori and E. Ma, Phys. Rev. **D86**, 045030 (2012).

[9] E. Ma, Mod. Phys. Lett. **A20**, 2601 (2005).

[10] L. Lavoura and H. Kuhbock, Mod. Phys. Lett. **A22**, 181 (2007).

[11] Particle Data Group: J. Beringer *et al.*, Phys. Rev. **D86**, 010001 (2012).