Constraining CP violation in neutral B meson mixing with theory input

Marat Freytsis, Zoltan Ligeti, and Sascha Turczyk

Phys. Rev. D **86**, 116004 — Published 3 December 2012

DOI: 10.1103/PhysRevD.86.116004

Constraining CP violation in neutral meson mixing with theory input

Marat Freytsis,[1, 2] Zoltan Ligeti,[1] and Sascha Turczyk[1]

[1]*Ernest Orlando Lawrence Berkeley National Laboratory, University of California, Berkeley, CA 94720*

[2]*Berkeley Center for Theoretical Physics, Department of Physics, University of California, Berkeley, CA 94720*

There has been a lot of recent interest in the experimental hints of CP violation in $B^0_{d,s}$ mixing, which would be a clear signal of beyond the standard model physics (with higher significance). We derive a relation for the mixing parameters, which allows clearer interpretation of the data in models in which new physics enters in M_{12} and/or Γ_{12}. Our results imply that the central value of the DØ measurement of the semileptonic CP asymmetry in $B^0_{d,s}$ decay is not only in conflict with the standard model, but in a stronger tension with data on $\Delta\Gamma_s$ than previously appreciated. This result can be used to improve the constraint on $\Delta\Gamma$ or $A_{\rm SL}$, whichever is less precisely measured.

I. INTRODUCTION

Recently, CP violation in neutral meson mixing received renewed attention due to the DØ hint of CP violation in $B-\overline{B}$ mixing, measured by the CP asymmetry in decays of a $b\bar{b}$ pair to two same-sign muons [1],

$$A^b_{\rm SL} = -[7.87 \pm 1.72\,({\rm stat}) \pm 0.93\,({\rm syst})] \times 10^{-3}\,. \quad (1)$$

At the Tevatron both B^0_d and B^0_s are produced, and hence $A^b_{\rm SL}$ is a linear combination of the two asymmetries [1]

$$A^b_{\rm SL} = (0.594 \pm 0.022)\, A^d_{\rm SL} + (0.406 \pm 0.022)\, A^s_{\rm SL}\,. \quad (2)$$

The central value in Eq. (1) would be a clear sign of new physics (NP) [2, 3]. Measurements at the e^+e^- B factories [4] and at DØ [5] yield

$$A^d_{\rm SL} = -(0.5 \pm 5.6)\times 10^{-3}\,, \qquad A^s_{\rm SL} = -(1.7 \pm 9.2)\times 10^{-3}\,. \quad (3)$$

In B^0_s mixing, a nonzero lifetime difference, $\Delta\Gamma_s \equiv \Gamma_L - \Gamma_H$, was established recently,

$$\begin{aligned} \Delta\Gamma_s &= (0.116 \pm 0.019)\,{\rm ps}^{-1}\,, && \text{LHCb [6]},\\ \Delta\Gamma_s &= (0.068 \pm 0.027)\,{\rm ps}^{-1}\,, && \text{CDF [7]},\\ \Delta\Gamma_s &= (0.163^{+0.065}_{-0.064})\,{\rm ps}^{-1}\,, && \text{DØ [8]}. \end{aligned} \quad (4)$$

In the absence of a world average, we use the most precise measurement from LHCb. For Δm_s the average of the CDF [9] and LHCb [10] measurements is

$$\Delta m_s \equiv m_H - m_L = (17.719 \pm 0.043)\,{\rm ps}^{-1}. \quad (5)$$

One should naturally ask if there are any constraints on the mixing parameters, beyond the obvious one: that the mass and width eigenvalues of the heavy and light mass eigenstates, $m_{H,L}$ and $\Gamma_{H,L}$, must be positive. (We use the notation customary in B physics, but the results apply equally for K^0 and D^0 mixing as well.) The time evolution of the flavor eigenstates is

$$i\,\frac{{\rm d}}{{\rm d}t} \begin{pmatrix} |B^0(t)\rangle \\ |\overline{B}{}^0(t)\rangle \end{pmatrix} = \left(M - \frac{i}{2}\,\Gamma\right) \begin{pmatrix} |B^0(t)\rangle \\ |\overline{B}{}^0(t)\rangle \end{pmatrix}, \quad (6)$$

where M and Γ are 2×2 Hermitian matrices, and CPT invariance implies $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. The physical states are the eigenvectors of the Hamiltonian,

$$|B_{H,L}\rangle = p\,|B^0\rangle \mp q\,|\overline{B}{}^0\rangle\,, \quad (7)$$

where we chose $|p|^2 + |q|^2 = 1$. CP violation in mixing occurs if the mass and CP eigenstates do not coincide,

$$\delta \equiv \langle B_H | B_L\rangle = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2} = \frac{1 - |q/p|^2}{1 + |q/p|^2} \neq 0\,. \quad (8)$$

The solution for the mixing parameters satisfies

$$\frac{q^2}{p^2} = \frac{2M^*_{12} - i\Gamma^*_{12}}{2M_{12} - i\Gamma_{12}}\,, \quad (9)$$

and from this and Eq. (8) it follows that (see, e.g., [11])

$$\delta < \min\left(\frac{|2M_{12}|}{|\Gamma_{12}|}\,,\ \frac{|\Gamma_{12}|}{|2M_{12}|}\right). \quad (10)$$

The measurable CP asymmetry in semileptonic (or any "flavor-specific") decay can be expressed as

$$A_{\rm SL} = \frac{1 - |q/p|^4}{1 + |q/p|^4} = \frac{2\delta}{1+\delta^2} = \frac{{\rm Im}\,(\Gamma_{12}/M_{12})}{1 + |\Gamma_{12}|^2/(4\,|M_{12}|^2)}\,. \quad (11)$$

Thus, in the small δ limit, $A_{\rm SL} = 2\delta + \mathcal{O}(\delta^3)$.

In the $|\Gamma_{12}/M_{12}| \ll 1$ limit, which applies model independently for the $B^0_{d,s}$ systems,

$$\begin{aligned} \Delta m &= 2\,|M_{12}|\, \big[1 + \mathcal{O}(|\Gamma_{12}/M_{12}|^2)\big],\\ \Delta\Gamma &= 2\,|\Gamma_{12}|\, \cos[\arg(-\Gamma_{12}/M_{12})]\, \big[1 + \mathcal{O}(|\Gamma_{12}/M_{12}|^2)\big],\\ A_{\rm SL} &= {\rm Im}\,(\Gamma_{12}/M_{12})\, \big[1 + \mathcal{O}(|\Gamma_{12}/M_{12}|^2)\big]. \end{aligned} \quad (12)$$

In this limit, Eq. (9) implies that q/p is a pure phase to a good approximation, determined by M_{12}, which has good sensitivity to NP. However, if $|\Gamma_{12}/M_{12}| = \mathcal{O}(1)$, relevant for K^0 and D^0 mesons, then q/p depends on both Γ_{12} and M_{12} and the sensitivity to NP in M_{12} (and in $\arg M_{12}$) is diluted [12]. In that case Eq. (12) does not hold, but $A_{\rm SL} = 2\delta$ is a good approximation even in the D^0 system, where the current bound is $|\delta| \lesssim 0.2$ [4].

An additional constraint, the unitarity bound [13, 14], stems from the time-evolution of the normalization of any linear combination of $|B^0\rangle$ and $|\overline{B}{}^0\rangle$ being determined entirely by the Γ matrix. As discussed below, this constrains the eigenvalues of Γ to be positive definite independent of the physical eigenvalues, or equivalently

$$\delta^2 < \frac{\Gamma_H\Gamma_L}{(m_H - m_L)^2 + (\Gamma_H + \Gamma_L)^2/4} = \frac{1-y^2}{1+x^2}\,. \quad (13)$$

Here we define, using $\Gamma = (\Gamma_H + \Gamma_L)/2$,

$$x = \frac{m_H - m_L}{\Gamma}\,, \qquad y = \frac{\Gamma_L - \Gamma_H}{2\Gamma}\,, \tag{14}$$

where x is positive by definition, while $y \in (-1, +1)$.

One may ask if other constraints exist purely from consistency considerations. As we show in a separate paper [15], no additional limit on δ exists, either physical or mathematical, without some knowledge of the Hamiltonian. In particular, the invalidity of the bounds claimed in [16, 17] can be made apparent by introducing new short-distance physics, which reduces $\big|\cos[\arg(\Gamma_{12}/M_{12})]\big|$ while it leaves $|M_{12}|$ and Γ_{12} unchanged.

However, depending on the theoretical status and the experimental measurements of the mixing parameters, further useful and exact relations may arise from these inputs in combination. In this paper, we derive a relation between the mixing parameters of a meson system and only the magnitude $|\Gamma_{12}|$, which is in tension with the DØ measurement in Eq. (1).

It has been known that the data in Eqs. (1) – (3) is not only in tension with the SM, but — assuming the SM calculation of Γ_{12} — also with all models in which NP enters only through M_{12} [18, 3]. This is because Eqs. (10) and (12) imply $\delta < |\Gamma_{12}|/\Delta m$. Our result goes beyond this, because it makes optimal use of data on $\Delta\Gamma$ without theoretical assumptions, and indicates a larger tension independent of the nature of typical new physics.

II. UNITARITY WITH THEORY INPUT

As mentioned above, Eq. (13) was first derived in Refs. [13, 14]. We show that a stronger bound on δ can be obtained using additional input from theoretical calculations. An analogy to the derivation of Ref. [13] will be particularly useful in deriving our results.

We define the complex quantities

$$a_i = \sqrt{2\pi\rho_i}\,\langle f_i|\mathcal{H}|B\rangle\,, \qquad \bar{a}_i = \sqrt{2\pi\rho_i}\,\langle f_i|\mathcal{H}|\overline{B}\rangle\,, \tag{15}$$

with ρ_i denoting the phase space density for final state f_i. If we treat a_i and $\bar{a}_i$ as vectors in a complex N-dimensional vector space, then taking the standard inner product on complex vector spaces, and using the optical theorem [13], amounts to the relations

$$a_i^*\,a_i = \Gamma_{11}\,, \qquad \bar{a}_i^*\,\bar{a}_i = \Gamma_{22}\,, \qquad \bar{a}_i^*\,a_i = \Gamma_{12}\,, \tag{16}$$

where CPT fixes $\Gamma_{11} = \Gamma_{22} = \Gamma$. Applying the Cauchy-Schwarz inequality to the vectors a_i and $\bar{a}_i$ implies [13]

$$|\Gamma_{12}| \le \Gamma_{11}\,. \tag{17}$$

This is equivalent to the statement that the eigenvalues of the Γ matrix must be positive (in addition to $\Gamma_{H,L} > 0$).

To see that this is also equivalent to the unitarity bound of Eq. (13), we use Eq. (7) to define new vectors a_{Hi} and a_{Li} analogously, such that

$$a_i = \frac{1}{2p}\,(a_{Hi} + a_{Li})\,, \qquad \bar{a}_i = \frac{1}{2q}\,(a_{Li} - a_{Hi})\,. \tag{18}$$

For these newly defined vectors we can derive, in a similar manner as for Eq. (16), the relations

$$\begin{aligned} a_{Hi}^*\,a_{Hi} &= \Gamma_H\,, \qquad a_{Li}^*\,a_{Li} = \Gamma_L\,, \\ a_{Hi}^*\,a_{Li} &= -i(m_H - m_L + i\Gamma)\,\delta\,. \end{aligned} \tag{19}$$

Substituting Eq. (18) into Eq. (16), using Eq. (19) and the $|q/p|^2 = (1-\delta)/(1+\delta)$ identity, we obtain Γ_{11} and Γ_{12} in terms of $M_{H,L}$, $\Gamma_{H,L}$ and δ. The unitarity bound in Eq. (13) then arises from using the new expressions for Γ_{11} and Γ_{12} in Eq. (17).

The preceding derivation made no assumptions on the actual values of the matrix elements appearing in the problem. For the kaon system, for which this bound was originally derived, this was a necessity due to the dominance of long-distance physics in the result. For $B_{d,s}$ mesons, the large mass scale $m_b \gg \Lambda_{\rm QCD}$ allows Γ_{11} and Γ_{12} to be calculated in an operator product expansion, and at leading order $|\Gamma_{12}/\Gamma_{11}| = \mathcal{O}[(\Lambda_{\rm QCD}/m_b)^3\,(16\pi^2)]$, where $16\pi^2$ occurs due to a one-loop difference between the two calculations. (In the B_d system there is an additional CKM suppression.) Thus it makes sense to consider some theory input, and we define

$$y_{12} = |\Gamma_{12}|\,/\,\Gamma\,. \tag{20}$$

Using this relation between these matrix elements and proceeding with the same steps as above, we obtain, instead of an inequality as in Eq. (17),

$$\delta^2 = \frac{y_{12}^2 - y^2}{y_{12}^2 + x^2} = \frac{|\Gamma_{12}|^2 - (\Delta\Gamma)^2/4}{|\Gamma_{12}|^2 + (\Delta m)^2}\,. \tag{21}$$

This equation follows from the solution of the eigenvalue problem, and was previously derived in Ref. [11] with the resulting bound on δ noted.[1] (It also appears in related forms in Refs. [19, 20] and follows from Eqs. (9) and (12) in [21].) For fixed x and y, δ^2 is monotonic in y_{12}, so an upper bound on y_{12} gives an upper bound on $|\delta|$. For $y_{12} \le 1$ the usual unitarity bound in Eq. (13) is recovered.

Equation (21) can also be obtained from a scaling argument: As δ only depends on mixing parameters, it is independent of the value of Γ. One can then scale Γ by y_{12}, which cannot affect δ but changes $x \to x/y_{12}$ and $y \to y/y_{12}$. Eq. (21) follows then from this argument and Eq. (13). The derivations above make the physical origin of this relation clear. Even if CPT is violated, the

[1] We were unaware of this in v1 of this paper, and we thank Luis Lavoura and Joao Silva for bringing this to our attention.

scaling argument, and therefore Eq. (21) holds, although Eq. (18) is modified. This applies for $|\delta|^2$, as CPT violation allows δ to be complex.

Even if a precise calculation of Γ_{12} is not possible or one assigns a very conservative uncertainty to it, an upper bound on y_{12} implies an upper bound on $|\delta|$, which is stronger than that in Eq. (13). For small values of y_{12}, as in the B_d system, this bound can be much stronger.

III. COMPARING DATA AND THEORY

We can compare the absolute value of $A^b_{\rm SL}$ measured by DØ, with the result implied by the relation above. At present Eq. (21) only provides an upper bound on $|\delta|$, as the uncertainties of Γ_{12} and $\Delta\Gamma$ allow the numerators for both B_d and B_s to vanish. Denoting this upper bound by $\delta^{d,s}_{\rm max}$ and using the weight factors from Eq. (2),

$$|A^b_{\rm SL}| \le (1.188 \pm 0.044)\, \delta^d_{\rm max} + (0.812 \pm 0.044)\, \delta^s_{\rm max}. \quad (22)$$

Since Eq. (21) only bounds $|\delta|$, the bound on $A^b_{\rm SL}$ is not sensitive to possible cancellations between $A^d_{\rm SL}$ and $A^s_{\rm SL}$ (cf., the opposite signs of $A^{s,d}_{\rm SL}$ in the SM, although $|A^s_{\rm SL}| \ll |A^d_{\rm SL}|$). As $\Delta m_{d,s}$ are precisely known, we plot the bound as a function of $\Delta\Gamma_{d,s}$. If LHCb measures $A^s_{\rm SL} - A^d_{\rm SL}$ [22], then the above bound with modified coefficients apply for that measurement.

In Fig. 1 we set $\Delta\Gamma_d = 0$, which gives the most conservative bound. The darker shaded region shows the upper bound on $A^b_{\rm SL}$ using the 1σ ranges for $|\Gamma^{d,s}_{12}|$ in the SM [23], $2|\Gamma^s_{12}| = (0.087 \pm 0.021)\,{\rm ps}^{-1}$ and $2|\Gamma^d_{12}| = (2.74 \pm 0.51) \times 10^{-3}\,{\rm ps}^{-1}$. The dashed [dotted] curve shows the impact of using the 2σ region for Γ^d_{12} [Γ^s_{12}], and the lighter shaded region includes both 2σ regions. The vertical boundaries of the shaded regions arise because $|\Delta\Gamma_s| > 2\,|\Gamma^s_{12}|$ is unphysical. A tension between the $A^b_{\rm SL}$ measurement and the bound is visible, independent of the discrepancy between the DØ result and the global fit to the latest available experimental data [24].

We derived not an absolute bound in the fashion of the unitarity bound but a relation between calculable and measured quantities. It is thus worth clarifying the relationship of our result to the stated $3.9\,\sigma$ disagreement of $A^b_{\rm SL}$ with the SM reported in [1].

The SM prediction of $A_{\rm SL}$ uses the calculation of Γ_{12}, and $|\Gamma_{12}|$ also enters our bound; thus, the discrepancies are correlated. Although the calculation of $|\Gamma_{12}|$ and ${\rm Im}(\Gamma_{12})$ both rely on the same operator product expansion and perturbation theory, the existence of large cancellations in ${\rm Im}(\Gamma_{12})$ may lead one to think that the uncertainties could be larger in its SM calculation than what is tractable in the behavior of its next-to-leading order calculation [25, 26]. The sensitivity of Γ_{12} to new physics is generally weaker than that of M_{12} (see [27, 28] for other options). Thus, it is interesting to determine δ from Eq. (21), besides its direct calculation.

Of course, independent of the DØ measurement of $A^b_{\rm SL}$, we can also compare the bound implied by our relation to the individual best bounds on the semileptonic asymmetries in Eq. (3). To this end, in Fig. 2 we plot $A^d_{\rm SL}$ vs. $\Delta\Gamma_d$ (and similarly for B_s) allowed by Eq. (21) and the 1σ and 2σ ranges of the SM calculation of $|\Gamma_{12}|$ [23]. Here, there have been no discrepancies claimed between the theory predictions and measurements, but our relation allows us to place a bound tighter than the current experimental constraints which is more robust than the purely theoretical SM calculation as outlined above.

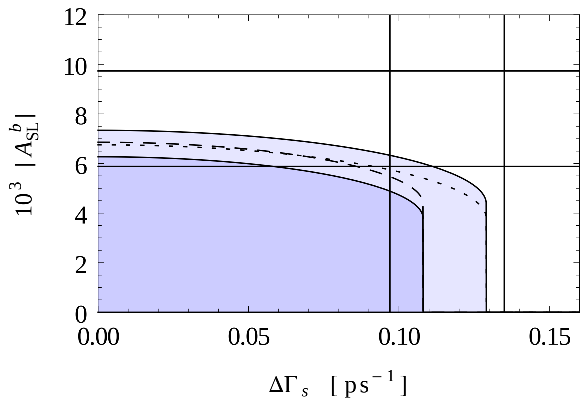

FIG. 1: Upper bounds on $A^b_{\rm SL}$ as a function of $\Delta\Gamma_s$, setting $\Delta\Gamma_d = 0$. The darker [lighter] shaded region is allowed using the 1σ [2σ] range of the theory calculation of $|\Gamma^{d,s}_{12}|$. The pair of horizontal [vertical] lines show the 1σ range of the measured $|A^b_{\rm SL}|$ from DØ [$\Delta\Gamma_s$ from LHCb]. The other curves are described in the text.

Using $\Delta\Gamma_s$ from LHCb in Eq. (4), and neglecting $\Delta\Gamma_d$, we find the 2σ level bounds,

$$|A^d_{\rm SL}| < 7.4 \times 10^{-3}, \qquad |A^s_{\rm SL}| < 4.2 \times 10^{-3}. \qquad (23)$$

While this bound on $A^s_{\rm SL}$ may seem to disagree with Fig. 2, note that in the plot the uncertainties of Γ^s_{12} and $\Delta\Gamma_s$ are not combined. Propagating the uncertainties, $|\Gamma^s_{12}|^2 - (\Delta\Gamma_s)^2/4$ is negative at the 1σ level, an unphysical result, hence the 2σ bounds in Eq. (23). This bound on $A^s_{\rm SL}$ is better than the current best bound in Eq. (3) by more than a factor of 3, while that for $A^d_{\rm SL}$ is comparable. (However, in the case of B_d this is driven primarily by the uncertainty in the lifetime difference. If a non-zero value of $\Delta\Gamma_d$ were observed, a better bound could be derived.) It is worth emphasizing that this implication goes in both directions, given that an observation of $A^d_{\rm SL} \neq 0$ may happen before that of $\Delta\Gamma_d \neq 0$. Due to Eq. (21), as soon as one of the two is measured to be nonzero, the other is constrained to be significantly smaller at worst and given a definite prediction at best.

IV. CONCLUSIONS

We provided a physical derivation of Eq. (21) for neutral meson oscillation parameters, especially relevant for

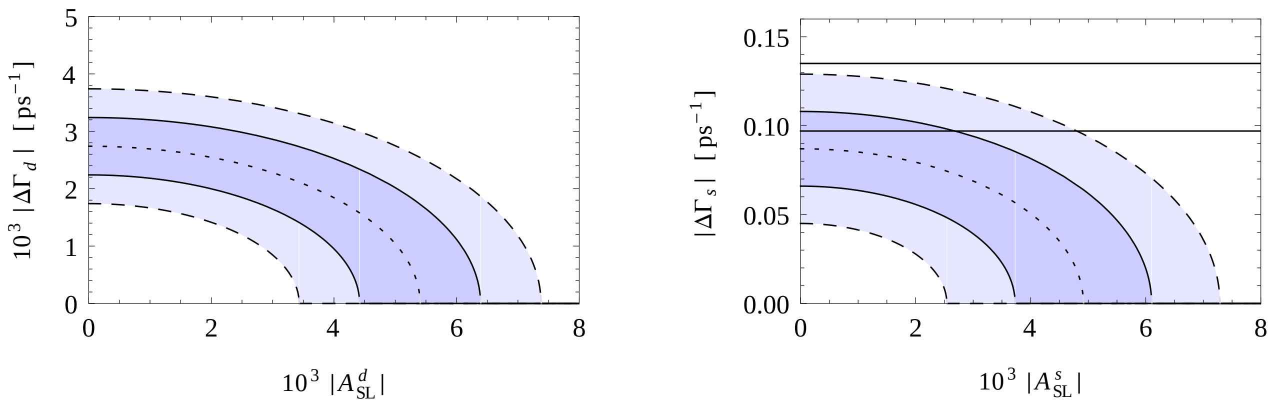

FIG. 2: Left plot: the region allowed by Eq. (21) in the $A^d_{\rm SL} - \Delta\Gamma_d$ plane. The SM calculation of $|\Gamma^{d,s}_{12}|$ at 1σ [2σ] gives the darker [lighter] shaded region. Right plot: same for $A^s_{\rm SL} - \Delta\Gamma_s$; the straight lines show the 1σ range of the LHCb result for $\Delta\Gamma_s$.

the $B^0_{d,s}$ systems, which allows incorporating theoretical input on $|\Gamma_{12}|$ without any approximation, and with or without CPT conservation. This input is typically insensitive to the nature of NP and avoids the largest uncertainties of the direct theoretical calculation of CP violation in mixing. The application to the two neutral B systems, taking into account the recent LHCb measurement [6], leads to bounds on the semileptonic CP asymmetries of both system. These bounds are in tension with the DØ measurement of $A^b_{\rm SL}$, while providing a bound on the individual asymmetries at comparable or better levels than the current experimental bounds. Additionally, once an unambiguous determination of $A_{\rm SL}$ or $\Delta\Gamma$ is made, we can use it to constrain the other observable. Refinements of the $A^{d,s}_{\rm SL}$ measurements are an important part of the future B physics program [29, 30] to search for new physics at both LHCb and the e^+e^- B factories. Future bounds will in particular be helpful to constrain the individual measurements of $A_{\rm SL}$ against the SM as well as consistency checks.

Acknowledgments

We thank Cliff Cheung for not entirely useless discussions, Aneesh Manohar for raising the issue of CPT conservation, and Yuval Grossman and Yossi Nir for helpful conversations when Ref. [17] appeared. We thank Doug Tuttle and Lynn Brantley for organizing the first BCTP summit at Glenbrook, NV, where some of these results were obtained. (Special thanks for the golf carts, for inspiration.) This work was supported in part by the Director, Office of Science, Office of High Energy Physics of the U.S. Department of Energy under contract DE-AC02-05CH11231. ST is supported by a DFG Forschungsstipendium under contract no. TU350/1-1.

[1] V. M. Abazov *et al.* [DØ Collaboration], Phys. Rev. **D84**, 052007 (2011). [arXiv:1106.6308 [hep-ex]].
[2] S. Laplace, Z. Ligeti, Y. Nir, G. Perez, Phys. Rev. **D65**, 094040 (2002). [hep-ph/0202010].
[3] Z. Ligeti, M. Papucci, G. Perez, J. Zupan, Phys. Rev. Lett. **105**, 131601 (2010). [arXiv:1006.0432 [hep-ph]].
[4] D. Asner *et al.* [Heavy Flavor Averaging Group], arXiv:1010.1589 [hep-ex]; and updates at `http://www.slac.stanford.edu/xorg/hfag/`
[5] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **82**, 012003 (2010) [Erratum-ibid. D **83**, 119901 (2011)] [arXiv:0904.3907 [hep-ex]].
[6] LHCb Collaboration, CERN-LHCb-CONF-2012-002, `https://cdsweb.cern.ch/record/1423592`.
[7] M. Dorigo, CDF Collaboration, Talk at Moriond QCD 2012, `http://moriond.in2p3.fr/QCD/2012/qcd.html`.
[8] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **85** (2012) 032006 [arXiv:1109.3166 [hep-ex]].
[9] A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **97**, 242003 (2006). [hep-ex/0609040].
[10] The LHCb Collaboration, LHCb-CONF-2011-050; LHCb-CONF-2011-005.
[11] G. C. Branco, L. Lavoura, J. P. Silva, Int. Ser. Monogr. Phys. **103**, 1-536 (1999). See in particular Ch. 6 and 30.
[12] S. Bergmann, Y. Grossman, Z. Ligeti, Y. Nir and A. A. Petrov, Phys. Lett. B **486**, 418 (2000) [hep-ph/0005181].
[13] J.S. Bell, J. Steinberger, "Weak interactions of kaons", in R. G. Moorhouse et al., Eds., Proceedings of the Oxford Int. Conf. on Elementary Particles, Rutherford Laboratory, Chilton, England, 1965, p. 195.
[14] T. D. Lee, L. Wolfenstein, Phys. Rev. **138**, B1490-B1496 (1965).
[15] M. Freytsis, Z. Ligeti, S. Turczyk, to appear.
[16] C. Berger, L. Sehgal, Phys. Rev. **D76**, 036003 (2007). [arXiv:0704.1232 [hep-ph]].
[17] C. Berger, L. M. Sehgal, Phys. Rev. **D83**, 037901 (2011). [arXiv:1007.2996 [hep-ph]].
[18] B. A. Dobrescu, P. J. Fox and A. Martin, Phys. Rev. Lett. **105**, 041801 (2010) [arXiv:1005.4238 [hep-ph]].

[19] M. Ciuchini, E. Franco, D. Guadagnoli, V. Lubicz, M. Pierini, V. Porretti and L. Silvestrini, Phys. Lett. B **655**, 162 (2007) [hep-ph/0703204].
[20] A. L. Kagan and M. D. Sokoloff, Phys. Rev. D **80**, 076008 (2009) [arXiv:0907.3917 [hep-ph]].
[21] Y. Grossman, Y. Nir and G. Perez, Phys. Rev. Lett. **103**, 071602 (2009) [arXiv:0904.0305 [hep-ph]].
[22] See, e.g., M. Calvi, Talk at FPCP 2011 (p. 29), `http://physics.tau.ac.il/fpcp2011`.
[23] A. Lenz and U. Nierste, arXiv:1102.4274 [hep-ph].
[24] A. Lenz *et al.*, arXiv:1203.0238 [hep-ph].
[25] M. Ciuchini, E. Franco, V. Lubicz, F. Mescia and C. Tarantino, JHEP **0308**, 031 (2003) [hep-ph/0308029].
[26] M. Beneke, G. Buchalla, A. Lenz and U. Nierste, Phys. Lett. B **576**, 173 (2003) [hep-ph/0307344].
[27] Y. Bai and A. E. Nelson, Phys. Rev. D **82**, 114027 (2010) [arXiv:1007.0596 [hep-ph]].
[28] C. Bobeth and U. Haisch, arXiv:1109.1826 [hep-ph].
[29] Z. Ligeti, M. Papucci, G. Perez, Phys. Rev. Lett. **97**, 101801 (2006). [hep-ph/0604112].
[30] Y. Grossman, Z. Ligeti, Y. Nir, Prog. Theor. Phys. **122**, 125-143 (2009). [arXiv:0904.4262 [hep-ph]].