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REVISITING D^0–$\overline{D}^0$ MIXING USING U-SPIN

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We prove that D^0-$\bar{D}^0$ mixing in the standard model occurs only at second order in U-spin breaking. The U-spin subgroup of SU(3) is found to be a powerful tool for analyzing the cancellation of intermediate-state contributions to the D^0–$\overline{D}^0$ mixing parameter $y = \Delta\Gamma/(2\Gamma)$. Cancellations due to states *within a single U-spin triplet* are shown to be valid to first order in U-spin breaking. Illustrations are given for triplets consisting of (a) pairs of charged pions and kaons; (b) pairs of neutral pseudoscalar members of the meson octet; (c) charged vector-pseudoscalar pairs, and (d) states of four charged kaons and pions.

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I Introduction

The parameters $x = \Delta m/\Gamma$ and $y = \Delta\Gamma/2\Gamma$ describing mixing between D^0 and $\overline{D}^0$ have been established at levels of an appreciable fraction of a percent [1], $x = (0.63^{+0.19}_{-0.20})\%, y = (0.75 \pm 0.12)\%$. A key question is whether such levels can be attained in the standard model or require new physics.

In the SU(3) limit, the contributions to y of classes of intermediate states shared by D^0 and $\bar{D}^0$ cancel one another in the standard model. Previous investigations have examined the degree to which this cancellation holds inclusively [2]. Applying an exclusive approach, Ref. [3] finds that for multiparticle states near threshold, SU(3) is broken enough by phase space effects that values of y (and, generically via dispersion relations [4], x) of order a percent are conceivable. This is despite the fact, proved using the full machinery of SU(3) in Ref. [3], that neutral D meson mixing occurs only at second order in SU(3) breaking.

Contributions to y of intermediate states with zero strangeness typically cancel with those of states with strangeness ± 1. For example, a contribution from the singly-Cabibbo-suppressed (SCS) transitions $D^0 \to (\pi^+\pi^-, K^+K^-) \to \overline{D}^0$ is canceled by a contribution from the doubly-Cabibbo-suppressed (DCS) transition $D^0 \to K^+\pi^-$ followed by the Cabibbo-favored (CF) transition $K^+\pi^- \to \overline{D}^0$, plus a contribution from the CF transition $D^0 \to K^-\pi^+$ followed by the DCS transition $K^-\pi^+ \to \overline{D}^0$. The intermediate states in this case comprise a single triplet of the SU(3) subgroup known as U-spin [5]. Just as the fundamental representation of I-spin (isospin) is composed of (u, d), that of U-spin is (d, s).

U-spin symmetry has been known for a long time to provide useful relations among amplitudes of hadronic D decays [6, 7]. Typical U-spin breaking, described by quantities such as $(m_s - m_d)/\Lambda_{\rm QCD}$ or $f_K/f_\pi - 1$, is of order $0.2 - 0.3$ and may be treated perturbatively in hadronic matrix elements. A very early suggestion was made in Ref. [8] that SU(3) breaking at this level in a penguin amplitude may account for the somewhat unexpected large value of the ratio of branching ratios $\mathcal{B}(D^0 \to K^+K^-)/\mathcal{B}(D^0 \to \pi^+\pi^-) = 2.8$ [9]. A recent study of D decays into two pseudoscalars [10, 11] has shown that U-spin breaking at a level between 10 to 20 percent in an enhanced nonperturbative penguin amplitude may account well for this ratio and for the unexpected large difference between CP asymmetries measured recently in these two processes [12, 13]. Two other studies discussing these two effects of U-spin breaking have been presented recently in Refs. [14] and [15].

In this paper we shall show that a cancellation, to first order in U-spin breaking, of contributions to D^0–$\overline{D}^0$ mixing within single U-spin triplets is a very general result. In Sec. II we apply U-spin and its first-order breaking to a D^0–$\overline{D}^0$ mixing amplitude, $A_{D\bar D} \equiv \langle \bar D^0 | H_W\, H_W | D^0\rangle$. In Sec. III we express $\Delta\Gamma$ as a sum of contributions from U-spin triplet states, deriving in Sec. IV a general sum rule corresponding to the cancellation of triplet state contributions to $\Delta\Gamma$. Examples of these sum rules for pairs of charged pseudoscalar mesons, pairs of neutral pseudoscalar mesons, and charged vector-pseudoscalar pairs, are given in Sec. V. Some results involving large U-spin breaking are noted in Sec. VI for states of four charged pions and kaons and for states involving $K^0\bar K^0$ and a pair of charged pions or kaons, while Sec. VII concludes.

II U-spin breaking in a D^0–$\bar D^0$ mixing amplitude

Let us consider an amplitude which connects D^0 and $\bar D^0$ through second order weak interactions,

$$A_{D\bar D} \equiv \langle \bar D^0 | H_W^{\Delta C=-1}\, H_W^{\Delta C=-1} | D^0 \rangle \ . \tag{1}$$

We will now show that this amplitude vanishes in the U-spin symmetry limit, and that it vanishes also when including first-order U-spin corrections. Keeping the flavor structure of the $\Delta C = -1$ weak Hamiltonian but suppressing its Lorentz structure and denoting $C \equiv \cos\theta_c, S \equiv \sin\theta_c$, one has

$$H_W^{\Delta C=-1} = \frac{G_F}{\sqrt{2}}[C(\bar s c) - S(\bar d c)][C(\bar u d) + S(\bar u s)] \ . \tag{2}$$

Only six out of the sixteen terms in $H_W H_W$ obey $\Delta S = 0$ and contribute to $A_{D\bar D}$:

$$\begin{aligned} A_{D\bar D} &= \frac{G_F^2\, C^2 S^2}{2} \langle \bar D^0 | - [(\bar d c)(\bar u s)][(\bar s c)(\bar u d) - [(\bar s c)(\bar u d)][(\bar d c)(\bar u s)] + [(\bar d c)(\bar u d)][(\bar d c)(\bar u d)] \\ &\quad + [(\bar s c)(\bar u s)][(\bar s c)(\bar u s)] - [(\bar d c)(\bar u d)][(\bar s c)(\bar u s)] - [(\bar s c)(\bar u s)][(\bar d c)(\bar u d)] | D^0 \rangle \end{aligned} \tag{3}$$

We will now show that the operator contributing to $A_{D\bar D}$ in Eq.(3) transforms like $U = 2, U_3 = 0$. Neglecting $c\bar u$ terms, which are singlets under U-spin, this operator reduces to

$$\mathcal{O}_{D\bar{D}} = -(\bar{d}s)(\bar{s}d) - (\bar{s}d)(\bar{d}s) + [(\bar{s}s) - (\bar{d}d)][(\bar{s}s) - (\bar{d}d)] \ . \tag{4}$$

Taking the quark and antiquark pairs of states, $(|d\rangle, |s\rangle)$ and $(|\bar{s}\rangle, -|\bar{d}\rangle)$, to be U-spin-doublets, we find the following behavior of $\bar{q}q'$ operators under U-spin,

$$(\bar{s}d) = (1,-1) \ , \qquad -(\bar{d}s) = (1,1) \ , \qquad (\bar{s}s) - (\bar{d}d) = \sqrt{2}(1,0) \ , \tag{5}$$

implying

$$\mathcal{O}_{D\bar{D}} = (1,1) \otimes (1,-1) + (1,-1) \otimes (1,1) + 2(1,0) \otimes (1,0) \ . \tag{6}$$

Using Clebsch-Gordan coefficients for $1 \otimes 1$, the operator $\mathcal{O}_{D\bar{D}}$ is seen to transform as pure $U = 2, U_3 = 0$.

D^0 and $\bar{D}^0$ are U-spin singlets. Therefore $A_{D\bar{D}}$ vanishes in the U-spin symmetry limit. Assuming that U-spin breaking may be treated perturbatively, a U-spin breaking mass term ($\propto \bar{s}s - \bar{d}d$) behaves like $U = 1, U_3 = 0$. Thus $A_{D\bar{D}}$ vanishes also in the presence of first-order U-spin-breaking corrections, and may obtain a nonzero value only when including second-order U-spin breaking. For a short notation, we will refer to this behavior of vanishing in the U-spin symmetry limit including first-order U-spin breaking corrections as *vanishing in USFB*.

Ref. [3] presented a lengthy SU(3) group theoretical argument involving high representations of this group showing that $A_{D\bar{D}}$ vanishes in the limit of flavor SU(3) symmetry and when including first-order SU(3) breaking corrections. We have shown that this behavior is actually due to only U-spin, a particular SU(2) subgroup of SU(3).

III $\Delta\Gamma$ as a sum over U-spin triplet states

Let us consider the width-difference between the two neutral D mass eigenstates, neglecting CP violation in D^0-$\bar{D}^0$ mixing [16, 17, 18]

$$\Delta\Gamma = \sum_{f^D} \rho(f^D) \left(\langle \bar{D}^0 | H_W^{\Delta C=-1} | f^D \rangle \langle f^D | H_W^{\Delta C=-1} | D^0 \rangle + c.c. \right) \ . \tag{7}$$

Here $|f^D\rangle$ are normalized states into which both D^0 and $\bar{D}^0$ may decay, while $\rho(f^D)$ are corresponding densities of states, i.e., phase space factors.

The $\Delta C = -1$ weak Hamiltonian involves three parts corresponding to $\Delta S = -1, 0, 1$, and behaving like three components of a U-spin vector,

$$\begin{aligned}
H_W^{\Delta C=-1} &= H_{U_3=-1} + H_{U_3=0} + H_{U_3=+1} \ , \\
H_{U_3=-1} &= \frac{G_F\, C^2}{\sqrt{2}} (\bar{s}c)(\bar{u}d) \ , \\
H_{U_3=0} &= \frac{G_F\, CS}{\sqrt{2}} [(\bar{s}c)(\bar{u}s) - (\bar{d}c)(\bar{u}d)] \ , \\
H_{U_3=+1} &= -\frac{G_F\, S^2}{\sqrt{2}} (\bar{d}c)(\bar{u}s) \ .
\end{aligned} \tag{8}$$

As D^0 and $\bar{D}^0$ are U-spin singlets, the sum in (7) obtains contributions only from intermediate U-spin vector states, $|f^{U=1}_{U_3=0,\pm 1}\rangle$,

$$\Delta\Gamma = \sum_{f^{U=1}} \sum_{U_3=0,\pm1} \rho(f^{U=1}_{U_3}) \left(\langle \bar{D}^0|H_{-U_3}|f^{U=1}_{U_3}\rangle\langle f^{U=1}_{U_3}|H_{U_3}|D^0\rangle + c.c.\right) \ . \tag{9}$$

The sum over $f^{U=1}$ involves all possible U spin triplet states to which D^0 decays.

IV Sum rule for a single U-spin triplet

Let us now consider the contribution to $\Delta\Gamma$ of a single U-spin triplet $|f^{U=1}\rangle$ accessible to D^0 decay,

$$\Delta\Gamma(\{f^{U=1}\}) = \sum_{U_3=0,\pm1} \rho(f^{U=1}_{U_3}) \left(\langle \bar{D}^0|H_{-U_3}|f^{U=1}_{U_3}\rangle\langle f^{U=1}_{U_3}|H_{U_3}|D^0\rangle + c.c.\right) \ . \tag{10}$$

The triplet $|f^{U=1}_{U_3}\rangle$ may consist, or instance, of three states involving pairs of charged pseudoscalar mesons, as studied below in Sec. V.A. These states have a common phase space factor in the U-spin symmetry limit. All other triplets states in the sum (9) are orthogonal to $|f^{U=1}_{U_3}\rangle$.

As mentioned, in the limit of U-spin symmetry $\rho(f^{U=1}_{U_3})$ is independent of U_3,

$$\Delta\Gamma(\{f^{U=1}\}) = \rho(f^{U=1})\, \langle \bar{D}^0|H_{-U_3} \sum_{U_3=0,\pm1} |f^{U=1}_{U_3}\rangle\langle f^{U=1}_{U_3}|H_{U_3}|D^0\rangle + c.c. \tag{11}$$

The operator $\sum |f^{U=1}_{U_3}\rangle\langle f^{U=1}_{U_3}|$ acts as a unit operator in the triplet $|f^{U=1}_{U_3}\rangle$ space and as a zero operator on all other triplet states in (9) which are orthogonal to $|f^{U=1}_{U_3}\rangle$. Thus our argument in Sec. II implies $\Delta\Gamma(\{f^{U=1}\}) = 0$. This result holds also in the presence of first-order U-spin breaking in $\langle f^{U=1}|H_W|D^0\rangle$ and $\langle \bar{D}^0|H_W|f^{U=1}\rangle$ as such corrections behaving like $U = 1, U_3 = 0$ cancel in $\Delta\Gamma(\{f^{U=1}\})$. First-order U-spin breaking in phase space factors $\rho(f^{U=1}_{U_3})$, which have been neglected in (11), may be made to cancel by a judicious choice of low mass final states. This will be demonstrated in Section V through several specific examples.

Neglecting CP violation in D^0 decays and denoting $|\bar{f}\rangle \equiv CP|f\rangle$, one has

$$\langle \bar{D}^0|H_W|f\rangle = \langle \bar{f}|H_W|D^0\rangle^* \ , \tag{12}$$

implying

$$\langle \bar{D}^0|H_W|f\rangle\langle f|H_W|D^0\rangle + c.c. = 2\mathrm{Re}(\langle \bar{f}|H_W|D^0\rangle^*\langle f|H_W|D^0\rangle) \ . \tag{13}$$

The generic form of a U-spin sum rule which holds in USFB by a judicious choice of final states is thus

$$\mathrm{Re}\,[\sum_{U_3=\pm1,0} \langle \bar{f}^1_{U_3}|H_W|D^0\rangle^*\langle f^1_{U_3}|H_W|D^0\rangle] = 0 \ . \tag{14}$$

We note that the states $|f\rangle$ and $|\bar{f}\rangle$ do not necessarily belong to the same $U = 1$ representation. For instance $|K^{*+}\pi^-\rangle$ and $|K^{*-}\pi^+\rangle$, which are each other's CP-conjugates, are $U_3 = 1$ and $U_3 = -1$ states in two different $U = 1$ triplets.

In the special case that $|f_0^1\rangle$ is a CP eigenstate with eigenvalue η_{CP} we will denote $CP|f_{U_3}^1\rangle = \eta_{CP}|\bar{f}_{U_3}^1\rangle$ for all three triplet states. [Note that while $|\bar{f}_{U_3}^1\rangle$ is a state transforming as $|1, -U_3\rangle$, the two states $|\bar{f}_1^1\rangle$ and $|f_{-1}^1\rangle$ may differ by a sign. See Eq. (18) below.] Using this convention one finds

$$\begin{aligned}
\langle \bar{D}^0|H_W|f_0^1\rangle \langle f_0^1|H_W|D^0\rangle + c.c. &= 2\eta_{CP}|\langle f_0^1|H_W|D^0\rangle|^2 \,, & (15)\\
\langle \bar{D}^0|H_W|f_{\pm 1}^1\rangle \langle f_{\pm 1}^1|H_W|D^0\rangle + c.c. &= 2\eta_{CP}\mathrm{Re}(\langle \bar{f}_1^1|H_W|D^0\rangle^* \langle f_1^1|H_W|D^0\rangle) \,. & (16)
\end{aligned}$$

Thus we have derived the following generic form for a U-spin sum rule in USFB for triplet states of which $|f_0^1\rangle$ is a CP eigenstate,

$$\frac{\eta_{CP}}{2}\Delta\Gamma(\{f^{U=1}\}) = |\langle f_0^1|H_W|D^0\rangle|^2 + 2\mathrm{Re}(\langle \bar{f}_1^1|H_W|D^0\rangle^* \langle f_1^1|H_W|D^0\rangle) = 0 \,. \tag{17}$$

Note that the triplet states $|f_{U_3}^1\rangle$ may be admixtures of low mass physical states. We will now demonstrate the sum rule (17) and corresponding expressions for $y(\{f^{U=1}\}) \equiv \Delta\Gamma(\{f^{U=1}\}/2\Gamma$ in several examples.

V Examples of $U = 1$ sum rules

A D^0 decays to pairs of charged pseudoscalar mesons, $\pi^\pm, K^\pm$

The pairs (π^-, K^-) and $(K^+, -\pi^+)$ are U-spin doublets. The four possible two-particle states can be written in the form of U-spin states:

$$|\pi^- K^+\rangle = |1, 1\rangle \,, \qquad |K^- \pi^+\rangle = -|1, -1\rangle \,, \tag{18}$$

$$\frac{1}{\sqrt{2}}|K^-K^+ - \pi^-\pi^+\rangle = |1, 0\rangle \qquad \frac{1}{\sqrt{2}}|K^-K^+ + \pi^-\pi^+\rangle = |0, 0\rangle \,. \tag{19}$$

Using $\langle 0, 0|H_W|D^0\rangle = 0$ one may write

$$\begin{aligned}
|\langle 1, 0|H_W|D^0\rangle|^2 &= |\langle 1, 0|H_W|D^0\rangle|^2 + |\langle 0, 0|H_W|D^0\rangle|^2 \\
&= |\langle K^-K^+|H_W|D^0\rangle|^2 + |\langle \pi^-\pi^+|H_W|D^0\rangle|^2 \,. & (20)
\end{aligned}$$

Consequently the sum rule (14) reads

$$\begin{aligned}
\frac{1}{2}\Delta\Gamma(\pi^\pm, K^\pm) &= |\langle K^-K^+|H_W|D^0\rangle|^2 + |\langle \pi^-\pi^+|H_W|D^0\rangle|^2 \\
&+ 2\mathrm{Re}(\langle \pi^-K^+|H_W|D^0\rangle^* \langle K^-\pi^+|H_W|D^0\rangle) = 0 \,. & (21)
\end{aligned}$$

A corresponding expression for $y(\pi^\pm, K^\pm) = \Delta\Gamma(\pi^\pm, K^\pm)/2\Gamma$ is obtained in terms of branching ratios and the strong phase difference δ between amplitudes for $D^0 \to \pi^-K^+$ and $D^0 \to K^-\pi^+$,

$$y(\pi^\pm, K^\pm) = \mathcal{B}(D^0 \to \pi^-\pi^+) + \mathcal{B}(D^0 \to K^-K^+) - 2\cos\delta\sqrt{\mathcal{B}(D^0 \to K^-\pi^+)\mathcal{B}(D^0 \to \pi^-K^+)} \,. \tag{22}$$

The minus sign of the last term on the right-hand-side may be traced back to three minus signs appearing in the second operator equation (5), the third operator equation (8) and the second state equation (18). The strong phase difference δ vanishes in the U-spin symmetry limit [19], and $\cos\delta = 1$ holds up to a first-order U-spin breaking correction. Thus the quantity $y(\pi^\pm, K^\pm)$ which vanishes in USFB is given by

$$y(\pi^\pm, K^\pm) = \mathcal{B}(D^0 \to \pi^-\pi^+) + \mathcal{B}(D^0 \to K^-K^+) - 2\sqrt{\mathcal{B}(D^0 \to K^-\pi^+)\mathcal{B}(D^0 \to \pi^-K^+)} = 0\,. \tag{23}$$

Using updated branching fractions [9]

$$\begin{aligned}
\mathcal{B}(D^0 \to \pi^+\pi^-) &= (1.401 \pm 0.027) \times 10^{-3}\,, && (24)\\
\mathcal{B}(D^0 \to K^+K^-) &= (3.96 \pm 0.08) \times 10^{-3}\,, && (25)\\
\mathcal{B}(D^0 \to K^-\pi^+) &= (3.88 \pm 0.05)\%\,, && (26)\\
\mathcal{B}(D^0 \to K^+\pi^-) &= (1.31 \pm 0.08) \times 10^{-4}\,, && (27)
\end{aligned}$$

we find $y(\pi^\pm, K^\pm) = (0.85 \pm 0.17) \times 10^{-3}$, substantial cancellation between positive and negative terms in (23) (corresponding to almost an order of magnitude suppression), and an order of magnitude below the observed value of $y = (0.75 \pm 0.12)\%$ (no CP violation assumed [1]).

One can see that U-spin breaking cancels to first order in phase space factors in Eq. (23). Expand the ratios of phase space factors $\rho(K^+K^-)/\rho(\pi^+\pi^-)$ and $\rho(\pi^\pm K^\mp)/\rho(\pi^+\pi^-)$ to first order in $\Delta \equiv (m_K^2 - m_\pi^2)/m_D^2$ [equivalently, to first order in $(m_s - m_d)/m_c$]. The coefficients of Δ in the three terms on the right-hand side of Eq. (23) are in the ratio $0 : -2 : 2$ and hence cancel one another.

Ref. [3] discussed contributions to y from two pseudoscalar states belonging to a common SU(3) representation. The expression (22) corresponding to states which are members of a U-spin triplet has been considered as an arbitrary partial contribution to this value of y, without motivating that choice and without noticing that $y(\pi^\pm, K^\pm)$ in Eq. (23) vanishes to first-order U-spin breaking.

B Decays to pairs of neutral pseudoscalar mesons, $\pi^0, \eta, K^0, \bar{K}^0$

When considering final states involving two neutral pseudoscalar mesons we will neglect $\eta - \eta'$ mixing by taking $\eta = \eta_8$. This approximation does not spoil the derived USFB sum rule because $\eta - \eta'$ mixing is due to first-order U-spin breaking transforming as $U = 1$.

The following superpositions of single-particle states belong to a U-spin triplet:

$$|K^0\rangle = |1,1\rangle\,, \qquad |\bar{K}^0\rangle = -|1,-1\rangle\,, \qquad \frac{1}{2}(\sqrt{3}\,|\eta\rangle - |\pi^0\rangle) = |1,0\rangle\,, \tag{28}$$

while the orthogonal U-spin singlet is

$$\frac{1}{2}(|\eta\rangle + \sqrt{3}\,|\pi^0\rangle) = |0,0\rangle\,. \tag{29}$$

Here we have used the convention $\pi^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\eta = (2s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{6}$, and all states are labeled by $|U, U_3\rangle$.

We now form U-spin multiplets out of pairs of the above states. Consider first the states with $U_3 = 1$:

$$
\begin{aligned}
|2,1\rangle &= (|1,1\rangle \otimes |1,0\rangle + |1,0\rangle \otimes |1,1\rangle)/\sqrt{2} \ , & (30)\\
|1,1\rangle &= (|1,1\rangle \otimes |1,0\rangle - |1,0\rangle \otimes |1,1\rangle)/\sqrt{2} \ , & (31)\\
|1',1\rangle &= |1,1\rangle \otimes |0,0\rangle \ , & (32)
\end{aligned}
$$

where the states on the left are two-particle states, while those on the right are one-particle states given in Eqs. (28) and (29) in terms of neutral pseudoscalar mesons. By Bose statistics we need not consider the state (31) as it is made of an antisymmetric product. We shall also need the two-particle states with $U_3 = 0$. There are two $U = 0$ states

$$
\begin{aligned}
|0',0\rangle &= |0,0\rangle \otimes |0,0\rangle \ , & (33)\\
|0,0\rangle &= (|1,1\rangle \otimes |1,-1\rangle + |1,-1\rangle \otimes |1,1\rangle - |1,0\rangle \otimes |1,0\rangle)/\sqrt{3} \ , & (34)
\end{aligned}
$$

two $U = 1$ states

$$
\begin{aligned}
|1',0\rangle &= |1,0\rangle \otimes |0,0\rangle \ , & (35)\\
|1,0\rangle &= (|1,1\rangle \otimes |1,-1\rangle - |1,-1\rangle \otimes |1,1\rangle)/\sqrt{2} \ , & (36)
\end{aligned}
$$

and one $U = 2$ state

$$
|2,0\rangle = (|1,1\rangle \otimes |1,-1\rangle + |1,-1\rangle \otimes |1,1\rangle + 2|1,0\rangle \otimes |1,0\rangle)/\sqrt{6} \ . \tag{37}
$$

Again, by Bose statistics, we need not consider the state (36) further. Now we calculate the contribution to y of decay amplitudes participating in the transition $D^0 \to \overline{D}^0$ due to pairs of neutral mesons belonging to the $U = 1$ multiplet. We first discuss the contributions of the $S = \pm 1$ states $K^0\pi^0, K^0\eta^0, \bar{K}^0\pi^0$, and $\bar{K}^0\eta$.

As H_W transforms according to $\Delta U = 1$, and the initial D^0 has $U = 0$, the transition amplitude $\langle 2,1|H_W|D^0\rangle$ vanishes. Expressed in terms of physical mesons, this means

$$
[\sqrt{3}A(D^0 \to K^0\eta) - A(D^0 \to K^0\pi^0)]/2 = 0 \ . \tag{38}
$$

We also have the transition of interest,

$$
\langle 1',1|H_W|D^0\rangle = [A(D^0 \to K^0\eta) + \sqrt{3}A(D^0 \to K^0\pi^0)]/2 = (2/\sqrt{3})A(D^0 \to K^0\pi^0) \ , \tag{39}
$$

where (38) was used in the second equality. Thus, in analogy with the last term in Eq. (22), one gets a contribution to y of the form

$$
y(|\Delta S| = 1) = -(8/3)\sqrt{\mathcal{B}(D^0 \to K^0\pi^0)\mathcal{B}(D^0 \to \bar{K}^0\pi^0)} \ . \tag{40}
$$

Using (38) one obtains a contribution to y from the $\Delta S = \pm 1$ transitions involving all pairs of neutral octet members,

$$
y(|\Delta S| = 1) = -2\sqrt{[\mathcal{B}(D^0 \to K^0\eta) + \mathcal{B}(D^0 \to K^0\pi^0)][\mathcal{B}(D^0 \to \bar{K}^0\eta) + \mathcal{B}(D^0 \to \bar{K}^0\pi^0)]} \ . \tag{41}
$$

Here, as in the case of charged pions and kaons, one may neglect the cosine of a strong phase difference which is second order in U-spin breaking.

Now we turn to the SCS ($\Delta S = 0$) transitions $\langle 1', 0|H_W|D^0\rangle$. We have a number of relations between amplitudes for D^0 decays to $\eta\eta$, $\eta\pi^0$, and $\pi^0\pi^0$, and will find the usual SU(3) result $A(D^0 \to K^0\bar{K}^0) = 0$, stemming from the vanishing of the transitions $\langle 0', 0|H_W|D^0\rangle$, $\langle 0, 0|H_W|D^0\rangle$, and $\langle 2, 0|H_W|D^0\rangle$. The first of these implies

$$A(D^0 \to \eta\eta) + 2\sqrt{3}A(D^0 \to \eta\pi^0) + 3A(D^0 \to \pi^0\pi^0) = 0 \; . \tag{42}$$

Linear combinations of the second and third imply $A(D^0 \to K^0\bar{K}^0) = 0$ and the relation

$$3A(D^0 \to \eta\eta) - 2\sqrt{3}A(D^0 \to \eta\pi^0) + A(D^0 \to \pi^0\pi^0) = 0 \; . \tag{43}$$

The transition of interest is

$$\langle 1', 0|H_W|D^0\rangle = \frac{\sqrt{3}}{4}A(D^0 \to \eta\eta) + \frac{1}{2}A(D^0 \to \eta\pi^0) - \frac{\sqrt{3}}{4}A(D^0 \to \pi^0\pi^0) \; . \tag{44}$$

The absolute square of this equation contains three interference terms. However, adding to that expression a suitable linear combination of the absolute square of the previous two equations (the coefficients each turn out to be 1/32), one finds an expression without interference terms:

$$|\langle 1', 0|H_W|D^0\rangle|^2 = \frac{1}{2}|A(D^0 \to \eta\eta)|^2 + |A(D^0 \to \eta\pi^0)|^2 + \frac{1}{2}|A(D^0 \to \pi^0\pi^0)|^2 \; . \tag{45}$$

When calculating decay rates involving identical particles, one must multiply the first and last terms by 2, leading to the result

$$y(\Delta S = 0) = \mathcal{B}(D^0 \to \eta\eta) + \mathcal{B}(D^0 \to \eta\pi^0) + \mathcal{B}(D^0 \to \pi^0\pi^0) \; . \tag{46}$$

The final result for $y(\pi^0, \eta, K^0, \bar{K}^0)$ is obtained by adding this contribution to that from the $\Delta S = \pm 1$ transitions to pairs of neutral mesons:

$$\begin{aligned} y(\pi^0, \eta, K^0, \bar{K}^0) &= \mathcal{B}(D^0 \to \eta\eta) + \mathcal{B}(D^0 \to \eta\pi^0) + \mathcal{B}(D^0 \to \pi^0\pi^0) \\ &- 2\sqrt{[\mathcal{B}(D^0 \to K^0\eta) + \mathcal{B}(D^0 \to K^0\pi^0)][\mathcal{B}(D^0 \to \bar{K}^0\eta) + \mathcal{B}(D^0 \to \bar{K}^0\pi^0)]} \; . \end{aligned} \tag{47}$$

The relation $y(\pi^0, \eta, K^0, \bar{K}^0) = 0$ which holds in USFB is, of course, satisfied by less precise SU(3) rate relations summarized in Table I. Early examples of SU(3) analyses may be found in Refs. [20] and [21]. The results in Table I follow from the behavior of the $\Delta S = \pm 1, 0$ pieces in H_W as three components $U_3 = \pm 1, 0$ of a U-spin triplet operator.

As in the example of charged pions and kaons, first-order SU(3)-breaking contributions from phase space cancel in $\Delta y(\pi^0, \eta, K^0, \bar{K}^0)$. Here we use the rate relations of Table I. An η in the final state counts for 4/3 of a strange quark, as η_8 is an $s\bar{s}$ pair 2/3 of the time. [This is equivalent to using a Gell-Mann – Okubo mass formula (either linear or quadratic) for M_η in terms of M_K and M_π.] The contributions to the sum rule (47) are then (neglecting common factors)

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{2} - \frac{4}{3} = 0 \; , \tag{48}$$

Table I: Absolute squares of amplitudes $A(D^0 \to f)$ for final states consisting of two neutral pseudoscalar mesons. A factor of two has been included for final states with two identical particles. An overall common factor has been omitted. The η is taken as a pure octet member.

Final state f	$\|A\|^2$
$\bar{K}^0\pi^0$	$(1/2)C^4$
$\bar{K}^0\eta$	$(1/6)C^4$
$\pi^0\pi^0$	$(1/2)C^2S^2$
$\pi^0\eta$	$(1/3)C^2S^2$
$\eta\eta$	$(1/2)C^2S^2$
$K^0\pi^0$	$(1/2)S^4$
$K^0\eta$	$(1/6)S^4$

while the coefficients of Δ from these corresponding terms are

$$\frac{1}{2}\cdot\frac{8}{3}+\frac{1}{3}\cdot\frac{4}{3}+\frac{1}{2}\cdot 0-2\cdot\left\{\frac{1}{2}+\frac{1}{6}\cdot\left[1+\frac{4}{3}\right]\right\}=\frac{16}{9}-\frac{16}{9}=0\ . \tag{49}$$

No information is available for the decays $D^0 \to K^0\eta^0$ and $D^0 \to K^0\pi^0$, so we can't tell how well Eq. (47) cancels. Since we expect it vanishes to first order in U-spin breaking, we have a sum rule that may be used to predict the sum of these two DCS branching fractions:

$$\mathcal{B}(D^0 \to K^0\eta)+\mathcal{B}(D^0 \to K^0\pi^0)=(7.4\pm 1.2)\times 10^{-5}\ . \tag{50}$$

This is for a pure octet η, but we have argued that η–η' mixing is second order in SU(3) breaking. When data become available it will be interesting to compare this prediction with the data and with the central value obtained from an SU(3) fit with an 11.7° η–η' mixing angle [22],

$$\mathcal{B}(D^0 \to K^0\eta)+\mathcal{B}(D^0 \to K^0\pi^0)=(2.8+6.9)\times 10^{-5}=9.7\times 10^{-5}\ . \tag{51}$$

This value involves however an uncertainty from first-order SU(3) breaking corrections which do not affect the prediction (50).

C Decays to charged PV states

When one of the final state mesons is a pseudoscalar meson P and the other a vector meson V there are more U-spin [or SU(3)] amplitudes as the final-state particles do not belong to the same multiplet. The U-spin doublets are:

$$\text{Pseudoscalar mesons}:\begin{pmatrix} K^+ \\ -\pi^+ \end{pmatrix}\ ;\ \begin{pmatrix} \pi^- \\ K^- \end{pmatrix}\ ; \tag{52}$$

$$\text{Vector mesons}:\begin{pmatrix} K^{*+} \\ -\rho^+ \end{pmatrix}\ ;\ \begin{pmatrix} \rho^- \\ K^{*-} \end{pmatrix}\ . \tag{53}$$

One can then form U-spin triplet PV states of charge zero out of the above doublets in two different ways, using the two pairs (π^-, K^-) and (K^{*+}, ρ^+) on the one hand and their charge-conjugates on the other. A test of the very generic sum rule (14) is quite challenging, as it requires measuring relative phases between D^0 decay amplitudes for a PV state and its charge-conjugate. In principle, this may be achieved by a Dalitz plot analysis for decays to a common three-body final state.

Facing this experimental difficulty, we will now study testable U-spin sum rules similar to (23), in which first-order U-spin breaking corrections cancel in phase space factors but may occur in hadronic amplitudes. In the U-spin symmetry limit there are two classes of amplitude relations, depending on which pair of U-spin doublets we consider:

$$A(D^0 \to \pi^- K^{*+}) = -\lambda A(D^0 \to K^- K^{*+}) = \lambda A(D^0 \to \pi^- \rho^+) = -\lambda^2 A(D^0 \to K^- \rho^+) \ ; \tag{54}$$

$$A(D^0 \to \rho^- K^+) = -\lambda A(D^0 \to K^{*-} K^+) = \lambda A(D^0 \to \rho^- \pi^+) = -\lambda^2 A(D^0 \to K^{*-} \pi^+) \ , \tag{55}$$

where $\lambda \equiv \tan\theta_c$. One can form sets of contributions to y out of either set, but in neither case do we have assurance that first-order U-spin-breaking effects in hadronic amplitudes cancel one another.

C.1 $(K^{*+}, \rho^+)(\pi^-, K^-)$ final states

Using one pair of U-spin doublets, a set of contributions to y for which branching fractions are known for all four processes is

$$y_1 \equiv \mathcal{B}(D^0 \to \pi^- \rho^+) + \mathcal{B}(D^0 \to K^- K^{*+}) - 2\sqrt{\mathcal{B}(D^0 \to K^- \rho^+)\mathcal{B}(D^0 \to \pi^- K^{*+})} = 0 \ . \tag{56}$$

Substituting the known branching fractions [9]

$$\begin{aligned} \mathcal{B}(D^0 \to \pi^- \rho^+) &= (9.8 \pm 0.4) \times 10^{-3} \ , && (57)\\ \mathcal{B}(D^0 \to K^- K^{*+}) &= (4.38 \pm 0.21) \times 10^{-3} \ , && (58)\\ \mathcal{B}(D^0 \to K^- \rho^+) &= (10.8 \pm 0.7)\% \ , && (59)\\ \mathcal{B}(D^0 \to \pi^- K^{*+}) &= (3.39^{+1.80}_{-1.02}) \times 10^{-4} \ , && (60) \end{aligned}$$

one finds $y_1 = (2.1^{+1.9}_{-3.3}) \times 10^{-3}$, with the error dominated by the uncertainty in the last branching fraction. Some cancellation occurs, but it is not as well-determined as for charged pions and kaons (Sec. VI A).

The effects of U-spin breaking in phase space factors lead to first-order corrections proportional to $M_K^2 - M_\pi^2$ or $M_{K^*}^2 - M_\rho^2$, both of which can be seen to cancel one another in Eq. (56).

C.2 $(K^+, \pi^+)(\rho^-, K^{*-})$ final states

Using the other combination of P and V U-spin doublets, one can write their contribution to y as

$$y_2 \equiv \mathcal{B}(D^0 \to \rho^- \pi^+) + \mathcal{B}(D^0 \to K^{*-} K^+) - 2\sqrt{\mathcal{B}(D^0 \to K^{*-} \pi^+)\mathcal{B}(D^0 \to \rho^- K^+)} = 0 \ . \tag{61}$$

We have almost enough information to check this sum rule:

$$\mathcal{B}(D^0 \to \rho^- \pi^+) = (4.96 \pm 0.24) \times 10^{-3} \ , \quad (62)$$

$$\mathcal{B}(D^0 \to K^{*-} K^+) = (1.56 \pm 0.12) \times 10^{-3} \ , \quad (63)$$

$$\mathcal{B}(D^0 \to K^{*-} \pi^+) = (5.63 \pm 0.35)\% \ , \quad (64)$$

$$\mathcal{B}(D^0 \to \pi^- \pi^0 K^+) = (3.04 \pm 0.17) \times 10^{-4} \ . \quad (65)$$

The value $\mathcal{B}(D^0 \to K^{*-}\pi^+) = (5.65 \pm 0.35)\%$ quoted above is three times the average of the values [9] $\mathcal{B}(D^0 \to K^{*-}\pi^+ \to K^-\pi^0\pi^+) = (2.22^{+0.40}_{-0.19})\%$ and $\mathcal{B}(D^0 \to K^{*-}\pi^+ \to K_S\pi^-\pi^+) = (1.66^{+0.15}_{-0.17})\%$, using the lower error bar for the first and the upper error bar for the second (because the average lies between them).

The most recent data contributing to this last branching fraction are from Belle [24] and BaBar [25]. The former makes no statement about how much of the $\pi^-\pi^0$ state corresponds to a ρ^-, but a ρ^- is clearly visible in the Dalitz plot of the latter. Assuming that all the $\pi^-\pi^0$ is in a ρ^-, one obtains a value of $y_2 = (-1.75 \pm 0.44) \times 10^{-3}$, but one may be oversubtracting. It would be very useful if an analysis of the decay $D^0 \to \pi^-\pi^0 K^+$ could extract $\mathcal{B}(D^0 \to \rho^- K^+)$.

As in the case of y_1, the contributions of SU(3) breaking in the phase space factors of y_2 cancel one another to first order.

C.3 Using both pairs of U-spin multiplets

One can write a sum rule involving all eight PV modes which involves only the products of decay amplitudes and their charge conjugates. In the absence of strong phase differences one then finds a contribution to y of the form

$$\begin{aligned} y_3 &= \sqrt{\mathcal{B}(D^0 \to \rho^-\pi^+)\mathcal{B}(D^0 \to \rho^+\pi^-)} + \sqrt{\mathcal{B}(D^0 \to K^{*-}K^+)\mathcal{B}(D^0 \to K^{*+}K^-)} \\ &- \sqrt{\mathcal{B}(D^0 \to K^{*-}\pi^+)\mathcal{B}(D^0 \to K^{*+}\pi^-)} - \sqrt{\mathcal{B}(D^0 \to \rho^+K^-)\mathcal{B}(D^0 \to \rho^-K^+)} \quad (66) \end{aligned}$$

Evaluation yields $y_3 = (-0.5^{+0.8}_{-1.2}) \times 10^{-3}$. This is reassuringly small, but we have not justified the neglect of the strong phase differences between the amplitudes for charge-conjugate final states, contributing to y_1 (proportional to $T_P + E_V$ in the notation of Ref. [23]) and to y_2 (proportional to $T_V + E_P$ in that notation). An analysis of related charm decays to PV final states finds a small but non-negligible strong phase difference between the two [23, 26].

VI Four-body states of pions and kaons

The states of four pions and/or kaons were identified in Ref. [3] as likely candidates for substantial SU(3) breaking in D^0–$\overline{D}^0$ mixing. The four-kaon channel is closed to D^0 decays so arguments based on the cancellation of first-order SU(3) breaking or U-spin-breaking effects will fail.

A full analysis of cancellations in four-body final states would require comparison of similar kinematic regions for individual U-spin multiplets. This is beyond the scope of

Table II: U-spin triplets of four charged pions and kaons.

Multiplet	Norm	Meson states
$\|1,1\rangle_1$	$1/2$	$-\|\pi^+K^+\pi^-\pi^-\rangle + \|K^+\pi^+\pi^-\pi^-\rangle + \|K^+K^+K^-\pi^-\rangle - \|K^+K^+\pi^-K^-\rangle$
$\|1,0\rangle_1$	$1/\sqrt{2}$	$-\|\pi^+K^+K^-\pi^-\rangle + \|K^+\pi^+\pi^-K^-\rangle$
$\|1,-1\rangle_1$	$1/2$	$\|\pi^+\pi^+K^-\pi^-\rangle - \|\pi^+K^+K^-K^-\rangle - \|\pi^+\pi^+\pi^-K^-\rangle + \|K^+\pi^+K^-K^-\rangle$
$\|1,1\rangle_2$	$1/2$	$-\|\pi^+K^+\pi^-\pi^-\rangle - \|K^+\pi^+\pi^-\pi^-\rangle - \|K^+K^+K^-\pi^-\rangle - \|K^+K^+\pi^-K^-\rangle$
$\|1,0\rangle_2$	$1/\sqrt{2}$	$\|\pi^+\pi^+\pi^-\pi^-\rangle - \|K^+K^+K^-K^-\rangle$
$\|1,-1\rangle_2$	$1/2$	$\|\pi^+\pi^+K^-\pi^-\rangle + \|\pi^+\pi^+\pi^-K^-\rangle + \|\pi^+K^+K^-K^-\rangle + \|K^+\pi^+K^-K^-\rangle$
$\|1,1\rangle_3$	$1/2$	$-\|\pi^+K^+\pi^-\pi^-\rangle + \|K^+\pi^+\pi^-\pi^-\rangle - \|K^+K^+K^-\pi^-\rangle + \|K^+K^+\pi^-K^-\rangle$
$\|1,0\rangle_3$	$1/\sqrt{2}$	$-\|\pi^+K^+\pi^-K^-\rangle + \|K^+\pi^+K^-\pi^-\rangle$
$\|1,-1\rangle_3$	$1/2$	$\|\pi^+\pi^+\pi^-K^-\rangle - \|\pi^+K^+K^-K^-\rangle - \|\pi^+\pi^+K^-\pi^-\rangle + \|K^+\pi^+K^-K^-\rangle$

the present article, but we can identify some useful groupings of charged pions and kaons. These belong to U-spin doublets, as mentioned earlier, so the U-spin multiplets containing them are those in the product

$$\left[U = \frac{1}{2}\right]^4 = 1(U=2) + 3(U=1) + 2(U=0) \ . \tag{67}$$

It is the $U = 1$ multiplets which interest us as they are the only ones reached from D^0 via $H_W^{\Delta C=-1}$. Three mutually orthogonal $U = 1$ multiplets are summarized in Table II.

If they obey the pattern of previous examples, the sum rules will involve cancellations of $U_3 = 0$ contributions against ones of $U_3 = \pm 1$. By counting kaons one can see that, at least formally, first order U-spin-breaking corrections in phase space seem to cancel one another. However, the $|1,0\rangle_2$ state is particularly susceptible to U-spin-breaking because it is the only one which contains the state of four charged kaons. Hence if a source of a significant contribution to y is to be sought in the states of four kaons, the sum rule associated with the triplet $|1, U_3\rangle_2$ would be a good place to look. As we will show now this contribution is expected to be negative, while the measured value of y is positive [1].

One may assume that nonresonant four-body decays are dominated by states in which relative angular momenta for all particle pairs are zero, so that the state with four charged pions is CP-even. The branching fraction for a nonresonant state involving three charged kaons and a charged pion [9], $\mathcal{B}(K^+K^-K^-\pi^+)_{\rm nonres} = (3.3 \pm 1.5) \times 10^{-5}$, is three orders of magnitude smaller than the branching fraction for a single kaon and three pions [9], $\mathcal{B}(K^-\pi^-\pi^+\pi^+)_{\rm nonres} = (1.88 \pm 0.26)\%$, and may be neglected. Thus the contribution to y from the triplet $|1, U_3\rangle_2$ is given by an expression similar to (23), but a term $\mathcal{B}(K^+K^+K^-K^-)$ is missing on the right-hand-side,

$$y(|1, U_3\rangle_2) = \mathcal{B}(\pi^+\pi^+\pi^-\pi^-) - 2\sqrt{\mathcal{B}(K^-\pi^-\pi^+\pi^+)\mathcal{B}(K^+\pi^+\pi^-\pi^-)} \ . \tag{68}$$

All three branching ratios correspond to nonresonant four-body final states. Assuming the usual U-spin hierarchy similar to (54),

$$\mathcal{B}(K^+\pi^+\pi^-\pi^-) : \mathcal{B}(\pi^+\pi^+\pi^-\pi^-) : \mathcal{B}(K^-\pi^-\pi^+\pi^+) \simeq \lambda^4 : \lambda^2 : 1 \ , \tag{69}$$

one finds $y(|1, U_3\rangle_2)$ to be negative. Using the measured value of $\mathcal{B}(K^-\pi^-\pi^+\pi^+)_{\rm nonres}$ to normalize the other two branching fractions one obtains $y(|1, U_3\rangle_2) \simeq -1.0 \times 10^{-3}$.

To conclude this section we discuss briefly a U-spin sum rule for four-body D^0 decays involving $K^0\bar{K}^0$ and a pair of charged pions or kaons. A U-spin relation following from a symmetry under a $d \leftrightarrow s$ reflection,

$$\langle K^0\bar{K}^0\pi^+\pi^-|H_W|D^0\rangle = -\langle \bar{K}^0K^0K^+K^-|H_W|D^0\rangle \ , \tag{70}$$

is strongly broken by phase space which forbids decays into four kaons [9],

$$\mathcal{B}(D^0 \to K^0\bar{K}^0\pi^+\pi^-) = (4.92 \pm 0.92) \times 10^{-3} \ , \qquad \mathcal{B}(D^0 \to \bar{K}^0K^0K^+K^-) = 0 \ . \tag{71}$$

The first branching ratio would explain the measured value of y, if a sum rule including the difference

$$\mathcal{B}(K^0\bar{K}^0\pi^+\pi^-) - 2\sqrt{\mathcal{B}(K^0\bar{K}^0\pi^+K^-)\mathcal{B}(K^0\bar{K}^0K^+\pi^-)} \ , \tag{72}$$

(in which branching fractions involving three kaons are highly suppressed) could be obtained for a U-spin triplet state. K^0 and $\bar{K}^0$ are two members of a U-spin triplet to which also $(\sqrt{3}\eta - \pi^0)/\sqrt{2}$ belongs. [See Eq. (28)]. One may show that in fact there exists a U-spin triplet sum rule including the difference (72) which, however, involves also unmeasured branching fractions and interference terms with amplitudes involving π^0 and η in addition to a pair of charged pions or kaons.

VII Conclusions

In the limit of U-spin symmetry, contributions of on-shell intermediate states to the parameter $y = \Delta\Gamma/(2\Gamma)$ describing D^0–$\overline{D}^0$ mixing cancel one another. This has been shown to be a consequence of the fact that the mixing amplitude transforms as a U-spin operator with $U = 2$, $U_3 = 0$, while the states $|D^0\rangle$ and $|\overline{D}^0\rangle$ have $U = 0$ because they contain no s or d quarks or antiquarks.

The cancellation of first-order U-spin-breaking effects then follows from the fact that first-order U-spin-breaking (equivalent to insertion of a term $m_s - m_d$) transforms as $U = 1, U_3 = 0$ and therefore cannot contribute to the mixing.

This result implies that sum rules may be written for contributions to y each involving a distinct U-spin triplet, explaining, for example, why the cancellation of contributions from the intermediate states $K^-\pi^+, K^-K^+, \pi^-\pi^+$, and $K^+\pi^-$ occurs. These states belong to a $U = 1$ multiplet all of whose members are sufficiently far below M_D that U-spin-breaking effects in phase space factors may be treated to first order in perturbation theory, and indeed – as expected from the general theorem – they cancel one another to first order in U-spin-breaking.

Examples of multiplets for which cancellation of contributions to y cancel one another have been given. In addition to the above case of pairs of charged kaons or pions, sum rules are seen to hold for pairs of neutral members of the pseudoscalar octet, pairs of charged pseudoscalar and vector mesons, and specific groupings of four charged pions and kaons.

When looking for standard model culprits which could induce large values of y, Ref. [3] identified final states consisting of four particles, noting that four-kaon final states lie above M_D and hence are inaccessible. Perturbative U-spin-breaking is thus a very poor approximation for sum rules involving such states. We have identified a grouping of amplitudes which includes the state $|K^+K^+K^-K^-\rangle$ and thus is a good candidate to participate in strong U-spin-breaking contributions to y. We have shown that this contribution is most likely negative of order -10^{-3}. In contrast, we have shown that four-body D^0 decays involving $K^0\bar{K}^0$ and a pair of charged pions or kaons may lead to a positive contribution to y at the level observed experimentally. Thus, summing over U-spin triplet contributions provides an order of magnitude estimate for y, but is short of being a precise method for calculating this parameter.

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