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We consider the origins of the gluon chain model. The model serves as a realization of the dynamics of the chromoelectric flux between static quark-antiquark sources. The derivation is based on the large-N_C limit of the Coulomb gauge Hamiltonian in the presence of a background field introduced to model magnetic charge condensation inducing electric confinement.

I. INTRODUCTION

The gluon chain model of Greensite and Thorn [1–4] identifies the chromoelectric flux tube that exists between static quark charges with a string of quasi particles, constituent gluons. Through lattice simulations and phenomenological analyses it is well established that the instantaneous, Coulomb potential between static charges is confining [5–8]. Even though it does not correspond to a physical observable, the static potential does provide physical insight into the possible origins of the confinement mechanism as illustrated by the Gribov-Zwanzinger model [9, 10] and other, *e.g.* variational models [11–15]. Lattice simulations indicate that the corresponding string tension is larger (by a factor of 2 to 3) as compared to the string tension extracted from large, time-dependent Wilson loops. This is consistent with expectations of variational analysis. At fixed quark-antiquark separation the Coulomb potential corresponds to the energy of a quark-antiquark pair in a vacuum state that is unmodified by the presence of the pair while the energy extracted from the Wilson loop corresponds to the energy of the exact QCD eigenstate in which the quark-antiquark ($Q\bar{Q}$) pair polarizes the vacuum [16]. The gluon chain model is a particular realization of the latter, *i.e.*, the exact pair state. Confinement originates from the condensation of chromomagnetic charges [17–20]. Formation of the gluon chain should therefore also provide insights into the interplay between constituent gluons and magnetic domains in the vacuum.

In the Hamiltonian formulation the true $Q\bar{Q}$ state is generated by the evolution operator $\lim_{\beta\to\infty}\exp(-\beta H)$ from the unperturbed vacuum. This is because in a physical gauge the Hamiltonian H contains all gluon interactions which also couple to the classical, external quark-antiquark color source. In this paper we investigate if/how the gluon chain emerges from the evolution operator. We follow a canonical formulation of QCD in the Coulomb gauge since it contains only physical degrees of freedom and these can be directly related to quasi particles. The gluon field is decomposed into normal modes representing particle excitations, and a physical state is represented as a superposition of multi-gluon states. Furthermore the normal mode expansion is performed with respect to a non-vanishing classical background. Such a background is introduced to (phenomenologically) parametrize topologically disconnected sectors of the vacuum. In terms of the path integral representation these sectors correspond to large field configurations, *i.e.*, field domains that cannot be smoothly connected to the null field configuration [21].

The paper is organized as follows. In the next section we review the structure of the Hamiltonian, introduce the particle basis, and discuss the role of the individual interaction terms in formation of the chain. In Sec. III we propose a simplified computational scheme for studying formation of the chain state and discuss numerical results. A summary and outlook are given in Section IV.

II. QCD HAMILTONIAN AND GLUONS

In the Coulomb gauge [22] the gluon field is described by the vector potential, $\mathbf{A}^a(\mathbf{x})$ that, for each color component $a = 1\cdots N_C^2-1$, satisfies the transversality condition, $\boldsymbol{\nabla}\cdot\mathbf{A}^a = 0$. In the Schrödinger representation the conjugate momenta, which are proportional to the electric field, are given by $\boldsymbol{\Pi}^a(\mathbf{x}) = -i\delta/\delta\mathbf{A}^a(\mathbf{x})$. The temporal component of the gluon field is eliminated using Gauss's law. This leads to an instantaneous interaction between color charges. The total color charge density has two components, $\rho(\mathbf{x},a) = \rho_g(\mathbf{x},a)+\rho_q(\mathbf{x},a)$, corresponding to gluons and quarks, respectively. In the following we ignore dynamical quarks, and the only quark charge we consider is that of a static quark-antiquark pair placed along the z-axis a distance R apart. The corresponding density is therefore given by

$$\rho_q(\mathbf{x},a) = Q_i^\dagger(\mathbf{x})T^a_{ij}Q_j(\mathbf{x}) - \bar{Q}_i^\dagger(\mathbf{x})T^a_{ji}\bar{Q}_j(\mathbf{x}). \qquad (1)$$

Here $Q_i^\dagger(\mathbf{x})(Q_i(\mathbf{x}))$ represents an operator that creates (annihilates) a quark at $\mathbf{x}$ in a state with color $i = 1\cdots N_C$, and T^a are the $SU(N_C)$ color matrices in the fundamental representation. We suppress the (irrelevant) spin indices. Similarly $\bar{Q}_i^\dagger(\mathbf{x})$ and $\bar{Q}_i(\mathbf{x})$ are the creation and annihilation operators for antiquarks. A state with a static $Q\bar{Q}$ pair is created by the operator $Q_i^\dagger(\hat{z}R/2)\bar{Q}_j^\dagger(-\hat{z}R/2)$. The gluon charge density is given by

$$\rho_g(\mathbf{x},a) = f_{abc}\mathbf{A}^b(\mathbf{x})\cdot\boldsymbol{\Pi}^c(\mathbf{x}), \qquad (2)$$

and the Hamiltonian takes the form

$$H = H_K + H_B + H_C \tag{3}$$

where the kinetic plus magnetic terms are given by

$$H_K + H_B = \frac{1}{2}\int d\mathbf{x}(\mathscr{J}^{-1}[\mathbf{A}]\mathbf{\Pi}\mathscr{J}^{-1}[\mathbf{A}]\mathbf{\Pi} + \mathbf{B}^2), \tag{4}$$

and

$$H_C = \frac{1}{2}\int d\mathbf{x}d\mathbf{y}\,\mathscr{J}^{-1}[\mathbf{A}]\rho(\mathbf{x},a)\mathscr{J}[\mathbf{A}]K^{ab}(\mathbf{x},\mathbf{y},[\mathbf{A}])\rho(\mathbf{y},a) \tag{5}$$

represents the instantaneous Coulomb interaction between color charges. Here $\mathscr{J}[\mathbf{A}] = \mathrm{Det}(-\mathbf{D}\cdot\mathbf{\nabla})$ is the Faddeev-Popov determinant, $\mathbf{D} = \mathbf{D}_{ab} = \mathbf{\nabla}\delta_{ab} + g f_{acb}\mathbf{A}^c$ is the covariant derivative, and $\mathbf{B} = \mathbf{B}^a = \nabla\times\mathbf{A}^a + g f_{abc}\mathbf{A}^b\times\mathbf{A}^c/2$ is the magnetic field. The non-abelian Coulomb kernel is formally given by

$$K(\mathbf{x},\mathbf{y},[\mathbf{A}]) = (\mathbf{D}\cdot\mathbf{\nabla})^{-1}(-g^2\mathbf{\nabla}^2)(\mathbf{D}\cdot\mathbf{\nabla})^{-1}. \tag{6}$$

The above describes the Hamiltonian in the Schrödinger representation. The particle basis representation is obtained via a canonical transformation from $\mathbf{A},\mathbf{\Pi}$ to a set of operators $\alpha^\dagger(\mathbf{k},\lambda,a),\alpha(\mathbf{k},\lambda,a)$ representing creation and annihilation of gluons with three-momentum $\mathbf{k}$ ($k=|\mathbf{k}|$, $[d\mathbf{k}] = d\mathbf{k}/(2\pi)^3$), helicity λ, and color a

$$\mathbf{A}^a(\mathbf{x}) = \int[d\mathbf{k}]\frac{1}{\sqrt{2\omega(k)}}[\sum_{\lambda=\pm}\mathbf{e}(\mathbf{k},\lambda)\alpha(\mathbf{k},\lambda,a)e^{i\mathbf{k}\cdot\mathbf{x}} + h.c]$$
$$\mathbf{\Pi}^a(\mathbf{x}) = \frac{1}{i}\int[d\mathbf{k}]\sqrt{\frac{\omega(k)}{2}}[\sum_{\lambda=\pm}\mathbf{e}(\mathbf{k},\lambda)\alpha(\mathbf{k},\lambda,a)e^{i\mathbf{k}\cdot\mathbf{x}} - h.c]. \tag{7}$$

Particle operators satisfy ladder algebra and generate a Fock space labeled by the number of gluons, n_i, occupying a state of a given momentum, helicity and color, $i = (\mathbf{k},\lambda,a)$

$$|n_1,n_2,\cdots n_i\cdots\rangle = (\alpha_1^\dagger)^{n_1}(\alpha_2^\dagger)^{n_2}\cdots(\alpha_i^\dagger)^{n_i}\cdots|0\rangle. \tag{8}$$

The state with no gluons, $|0\rangle \equiv |0,0,\cdots\rangle$ is annihilated by all annihilation operators α_i.

A. The vacuum state

In the absence of quark sources, after normal-ordering the gluon operators, the Hamiltonian

$$H = \langle 0|H|0\rangle + :H: \tag{9}$$

contains an infinite number of terms that connect states with any numbers of gluons [11]. The ground state, $|\Omega\rangle$, can therefore be formally written as

$$|\Omega\rangle = \left[\sum_{n_1}\sum_{n_2}\cdots\right]\Psi_{n_1,n_2,\cdots}|n_1,n_2,\cdots\rangle. \tag{10}$$

The non-uniqueness associated with the definition of a gluon state, and thus the Hamiltonian in Eq. (9), arises from the arbitrariness in the choice of the function $\omega(k)$ in Eq. (7). For example the choice $\omega(k) = k$ corresponds to a basis of non-interacting particles which diagonalizes the free Hamiltonian (*i.e.*, for $g = 0$). Other proposals, based on the variational principle, have been analyzed in [11–15]. These studies considered an optimal choice for the basis of states obtained with $\omega(k)$ that approaches the free particle limit for large k and is large and possibly divergent in the infrared (IR), *i.e.*, for $k\to 0$. This is because an IR enhanced $\omega(k)$ suppresses contributions to vacuum expectation values (*vev*) from fields near the Gribov horizon [23] and removes the Landau pole from the Coulomb kernel (*cf.* Eq. (6)). With such an optimal choice the vacuum in Eq. (10) is approximated by the state with a vanishing number of gluons[1], *i.e.*, $|\Omega\rangle = |0\rangle$, and the ground state energy is therefore given by the first term in Eq. (9).

B. The variational $Q\bar{Q}$ state

We next consider a state containing the $Q\bar{Q}$ pair. A variational state, $|R\rangle$, which does not take into account the back reaction of quarks on the vacuum can be defined as (in the volume $\mathscr{V}$)

$$|R\rangle = \frac{1}{\mathscr{V}\sqrt{N_C}}Q_i^\dagger(\frac{R}{2}\hat{z})\bar{Q}_i^\dagger(-\frac{R}{2}\hat{z})|0\rangle, \tag{11}$$

and it is normalized, $\langle R|R\rangle = 1$. Even if $|0\rangle$ was the exact ground state, this state would only be an approximation to the exact QCD eigenstate containing the $Q\bar{Q}$ pair. This is because with $\rho_q \neq 0$ the term in H_C proportional to $\rho_q\times\rho_g$ does not conserve the gluon number. The expectation value of the Hamiltonian in the variational $Q\bar{Q}$ state defines the Coulomb potential, $V_c(R)$, which is proportional to the expectation value of the Coulomb kernel in the variational vacuum,

$$V_c(R)\delta_{ab} = -\langle 0|K^{ab}(R)|0\rangle. \tag{12}$$

Here $K^{ab}(R)$ is given by Eq. (6) evaluated at the positions of the quark and the antiquark. The vacuum expectation value may be computed by expanding the covariant derivatives in powers of $\mathbf{A}$ (*cf.* Eq. (5)) and noticing that in the variational vacuum,

$$\langle 0|\mathbf{A}^a(\mathbf{x})\mathbf{A}^b(0)|0\rangle = \delta_{ab}\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{\delta_T(\mathbf{k})}{2\omega(k)}e^{i\mathbf{k}\cdot\mathbf{x}} \tag{13}$$

where $\delta_T(\mathbf{k}) = \sum_{\lambda=\pm}e_i(\mathbf{k},\lambda)e_j^*(\mathbf{k},\lambda) = \delta_{ij} - k_ik_j/k^2$. The behavior of $V_c(R)$ at large-R is correlated with the IR behavior of $\omega(k)$. While early variational studies indicated that with a

[1] More accurate approximations, which take into account residual correlations among the "optimal gluons," can be constructed using the standard many-body techniques of cluster expansion [24, 25].

proper choice of $\omega(k)$ it would be possible to obtain a confining potential, more detailed analyses showed that all solutions are massive, *i.e.*, when transformed to momentum space $V_c(k)$ is always finite in the limit $k \to 0$, *a.k.a.* non-confining [15]. We now believe this is consistent with lattice results. As shown in [5] the large-R strength of the Coulomb potential originates from magnetic charges in the vacuum. These are absent in the variational model calculation of Eq. (12) that is driven by fields in the neighborhood of the $\mathbf{A} = 0$ configuration. Magnetic charges are topologically disconnected from the first Gribov region where the expansion applies. Thus it is likely that the string tension, σ_c, of the variational model $V_c(r) \sim \sigma_c r$ should at most only be a fraction of the Coulomb string tension and, more likely, V_c of the variational model ought not to be confining. In the following we further explore these scenarios.

It is straightforward to show that the expectation value of the Hamiltonian in the variational $Q\bar{Q}$ state is given in terms of V_c by

$$\langle R|H|R\rangle - \langle 0|H|0\rangle = C_F V_c(R) - C_F V_c(0) \qquad (14)$$

where the last term arises from self-energies of the two static quarks ($C_F = (N_C^2-1)/2N_C$ is the $SU(N_C)$ color Casimir in the fundamental representation). As already mentioned above the Coulomb term, H_C, involves coupling between quark and gluon charges. It seems reasonable to expect that this interaction might be responsible for generating the gluon chain. In the particle basis the gluon charge density is given by

$$\rho_g(\mathbf{x},a) = \sum_i \rho_i^1(\mathbf{x},a)\alpha_i^\dagger\alpha_i + \sum_{ij}(\rho_{ij}^2(\mathbf{x},a)\alpha_i^\dagger\alpha_j^\dagger + h.c.). \quad (15)$$

The first term is diagonal in the particle basis and because $|0\rangle$ contains no gluons it vanishes when applied to the $Q\bar{Q}$ state defined by Eq. (11). The second term, however, changes the number of gluons by two and thus could be generating the chain. We will return to this possibility below. There are other, more complicated interactions involving the quark charge and gluon operators that change the number of gluons. They originate from the $\mathbf{A}$-dependence of the Coulomb kernel. In the particle basis, the Coulomb kernel can be written as

$$K^{ab}(R) = -V_c(R) + :K^{ab}(R): \qquad (16)$$

where the normal-ordered part is schematically given by

$$:K(R): \;= \sum_{\{n\},\{m\}} K_{n_1,n_2,\cdots;m_1,m_2\cdots}(\alpha_1^\dagger)^{n_1}(\alpha_2^\dagger)^{n_2}\cdots\alpha_1^{m_1}\alpha_2^{m_2}\cdots. \qquad (17)$$

Here $K_{\{n\},\{m\}}$ are the matrix elements of the full kernel evaluated between states containing $\{n\}$ and $\{m\}$ gluons, respectively. Thus, when multiplied by ρ_q the normal-ordered Coulomb kernel mixes the variational $Q\bar{Q}$ state with states containing an arbitrary number of gluons. As shown in [26], however, in the large-R limit the matrix elements $K_{\{n\},\{m\}}$ for $\{m\},\{n\} \neq 0$ are expected to be smaller than those for $\{m\} = \{n\} = 0$. Therefore we expect that at large-R the dominant interaction between quark sources and dynamical gluons originates from the off-diagonal gluon charge density (*c.f.* Eq. (15)) coupled to the quark charge via V_c, and is therefore given by

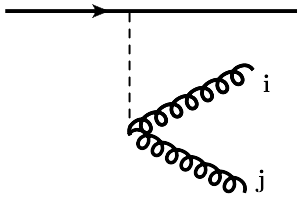

FIG. 1. Interaction between quark charge (upper line) and the off-diagonal gluon charge ρ_{ij}^2 which creates two gluons. The dashed line represents the Coulomb potential given by the *v.e.v* of the fully dressed Coulomb kernel.

$$-\int d\mathbf{x}d\mathbf{y}\rho_q^a(\mathbf{x})V_c(|\mathbf{x}-\mathbf{y}|)\sum_{ij}(\rho_{ij}^2(\mathbf{y},a)\alpha_i^\dagger\alpha_j^\dagger + h.c), \quad (18)$$

and shown in Fig. (1). In Eq. (18) the gluon charge density creates (annihilates) two constituent gluons in a color antisymmetric state. Thus the combined spin and spatial wave function of the gluon pair also has to be antisymmetric. However, since ρ_g is a scalar under rotations, the matrix element, ρ_{ij}^2 is symmetric in spin and relative momentum. Thus the above candidate operator for the gluon chain actually vanishes identically.

The variational basis based on the mode expansion in Eq. (7) seems incompatible with the gluon chain picture. There is further evidence that a model in which the vacuum is described solely in terms of fluctuations around the $\mathbf{A} = 0$ configuration, as implied by Eq. (7), is inadequate. If V_c is confining then the expectation value of H in a single gluon state is infinite [27] at all temperatures, and the model fails to predict the deconfinement phase transition [28]. It is well established that confinement is related to the presence of magnetic domains in the vacuum, and these are absent in the variational vacuum state. One would expect that the magnetic term $\mathbf{B}^2$ plays an important role in confinement since even the classical Yang-Mills field equations have monopole solutions [29].

In presence of QCD instantons (*a.k.a.* monopoles) quantization has to be performed in each topological sector. In our phenomenological approach we approximate this by generalizing the mode expansion of Eq. (7) to describe field fluctuations, $\mathbf{A}_f$, with respect to a classical background field, $\mathbf{A}_B$.

$$\mathbf{A}^a(\mathbf{x}) \to \mathbf{A}_f^a(\mathbf{x}) + \mathbf{A}_B^a(\mathbf{x}). \qquad (19)$$

This classical field mocks the nontrivial topological vacuum and will be specified later. Thus Eq. (7) now applies to $\mathbf{A}_f \equiv \mathbf{A} - \mathbf{A}_B$ and $\mathbf{\Pi_f} = \mathbf{\Pi}$. Since Eq. (19) is a canonical transformation the Hamiltonian can be obtained by substitution. Thus in the background field, at large-R, the dominant contribution to the Coulomb interaction between quark and gluon charges becomes,

$$H_C \to H_{qq} + H_{qg}^D + H_{gg}^D + H_{gb}^D + H_{gb}^M. \qquad (20)$$

Here H_{qq} is the interaction between quark charges mediated by the Coulomb potential,

$$H_{qq} = -\int d\mathbf{x}d\mathbf{y}\rho_q(\mathbf{x},a)V_c(|\mathbf{x}-\mathbf{y}|)\rho_q(\mathbf{y},a), \qquad (21)$$

H^D_{qg} is the quark-gluon charge density interaction diagonal with respect to the gluon number,

$$H^D_{qg} = -\int d\mathbf{x}d\mathbf{y}\rho_q(\mathbf{x},a)V_c(|\mathbf{x}-\mathbf{y}|)\rho^D_g(\mathbf{y},a) \qquad (22)$$

with $\rho^D_g = \sum_i \rho^1_i(\mathbf{x},a)\alpha^\dagger_i\alpha_i$, and H^D_{gg} is the normal-ordered, diagonal interaction between gluon charge densities

$$H^D_{gg} = -:\int d\mathbf{x}d\mathbf{y}\rho^D_g(\mathbf{x},a)V_c(|\mathbf{x}-\mathbf{y}|)\rho^D_g(\mathbf{y},a):. \qquad (23)$$

Finally the two terms proportional to $\mathbf{A}_B$, H^D_{gb} and H^M_{gb}, are given by

$$H^D_{gb}+H^M_{gb} = -\int d\mathbf{x}d\mathbf{y}\rho^B_g(\mathbf{x},a)V_c(|\mathbf{x}-\mathbf{y}|)\rho^B_g(\mathbf{y},a), \qquad (24)$$

with

$$\rho^B_g(\mathbf{x},a) = f_{abc}\mathbf{A}^b_B(\mathbf{x})\mathbf{\Pi}^c(\mathbf{x}) \qquad (25)$$

and describe the interaction of physical gluons with the background field and the gluon pair creation in the presence of the background, respectively. Physical states should be color neutral, thus creation or annihilation of a single gluon can be neglected. In the presence of the background, the expectation value of the charge operator

$$Q^a_B = f_{abc}\int d\mathbf{x}\mathbf{A}^b_B(\mathbf{x})\mathbf{\Pi}^c(\mathbf{x}) \qquad (26)$$

in physical states vanishes. However, in a simple classical model for the distribution of background fields, as described in Appendix A, quantum charge fluctuations do not vanish, *i.e.*, $Q^aQ^a \neq 0$ even for color singlet states. We thus modify the right hand side of Eq. (24) in such a way that these fluctuations do not contribute to the energy, yielding

$$H^D_{gb}+H^M_{gb} = -\int d\mathbf{x}d\mathbf{y}\rho^B_g(\mathbf{x},a)V_c(|\mathbf{x}-\mathbf{y}|)\rho^B_g(\mathbf{y},a) + V_c(0)Q^a_BQ^a_B. \qquad (27)$$

After normal-ordering, the term in Eq. (24) proportional to $\alpha^\dagger\alpha$ defines H^B_{gb}, and the term proportional to $\alpha^\dagger\alpha^\dagger + h.c.$ gives H^M_{gb}. The difference between the gluon density-density interaction and the normal ordered Hamiltonian of Eq. (23) is proportional to either $\alpha^\dagger\alpha$ or $\alpha\alpha + h.c.$ These, together with the kinetic and magnetic terms combine to [11] *i)* renormalize ω via a gap equation which eliminates terms proportional to $\alpha\alpha + h.c$, and *ii)* modify the single gluon energy. Thus the final Hamiltonian can be expressed in the form

$$H \to H_g + H_C = \sum_i E_i\alpha^\dagger_i\alpha_i + H_{qq} + H^D_{qg} + H^D_{gg} + H^D_{gb} + H^M_{gb} \qquad (28)$$

where $E_i = E(k)$ is the single gluon energy in the presence of the background field. The action of these operators on gluon chain states is shown in Figs. (2), (3), (4), and (5).

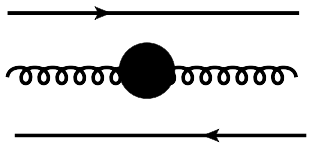

FIG. 2. Illustration of the matrix element of the one-body operator Hamiltonian, $E_i\alpha^i\alpha_i$, in the gluon chain state with $N = 1$.

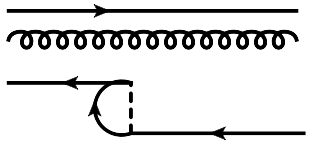

FIG. 3. Part of the leading contribution at large-N_C from H_{qq} which corresponds to quark or antiquark self energy, shown here for the $N = 1$ gluon chain state.

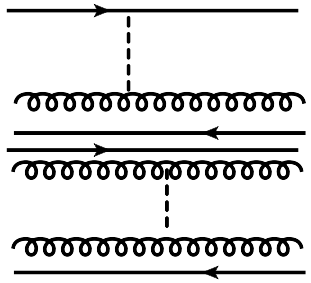

FIG. 4. Quark-gluon, H^D_{qg}, and gluon-gluon interaction, H^D_{gg}, matrix elements, for $N = 1$ and $N = 2$ chain states, respectively

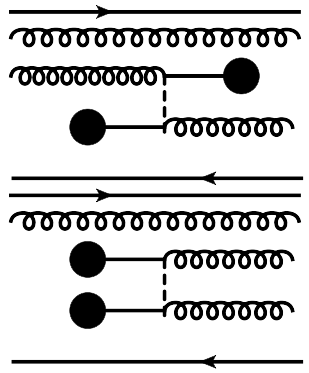

FIG. 5. Diagonal H^D_{gb} and off-diagonal H^M_{gb} interactions between quasi-gluons and the background field (shown as blobs), for $N = 3$ gluon chain states.

C. The basis for the gluon chain

We define the chain in a large-N_C limit by a model in which the gluon chain state is a superposition of multi-gluon states,

$$|Q\bar{Q},R\rangle = \sum_N a_N|N\rangle \qquad (29)$$

with each state in the sum describing a product of N single gluons ordered in color and space along a straight line be-

tween the quark-antiquark sources,

$$|N\rangle = \frac{Z_N}{\mathscr{V}\sqrt{N_C}} \int_{-R/2}^{R/2} dx_N \int_{-R/2}^{x_N} dx_{N-1} \cdots \int_{-R/2}^{x_2} dx_1 \times Q_i^\dagger\left(\frac{R}{2}\hat{z}\right)[G^\dagger(x_N)\cdots G^\dagger(x_1)]_{ij}\bar{Q}_j^\dagger\left(-\frac{R}{2}\hat{z}\right)|0\rangle. \tag{30}$$

In the large-R limit the longitudinal, *i.e.*, along the $Q\bar{Q}$ axis, and perpendicular motions of gluons factorize. The spatial distribution of gluons in the plane perpendicular to the $Q\bar{Q}$ axis is given by a single-particle wave function, $\psi(\mathbf{k},\lambda) = e^{*i}(\mathbf{k},\lambda)\psi^i(\mathbf{k}_\perp)$, which defines the gluon operators, G, in the chain ($\mathbf{x} = (0_\perp, x)$)

$$G_{ij}^\dagger(x) = \sum_\lambda \int [d\mathbf{k}]\alpha^\dagger(\mathbf{k},\lambda,a)T_{ij}^a\psi(\mathbf{k},\lambda)e^{-i\mathbf{k}\cdot\hat{z}x}. \tag{31}$$

The normalization constant Z_N is obtained from $\langle N|N\rangle = Z_N^2(C_F I R)^N/\Gamma(N+1) = 1$ where I is the normalization integral for the spatial wave function, ψ, ($[d\mathbf{k}_\perp] \equiv d^2\mathbf{k}_\perp/(2\pi)^2$)

$$I = \langle\psi|\psi\rangle = \int [d\mathbf{k}_\perp]\psi^i(\mathbf{k}_\perp)\delta_T^{ij}(\mathbf{k}_\perp)\psi^j(\mathbf{k}_\perp). \tag{32}$$

In the large-N_C limit, computation of the leading contributions to the matrix elements of the effective Hamiltonian of Eq. (28) in the basis of the gluon chain states, Eq. (30), is straightforward. The details and numerical results are presented in the following section.

III. FORMATION OF THE GLUON CHAIN AT LARGE $Q\bar{Q}$ SEPARATION

As discussed in Sec. II B one could consider two models for $V_c(R)$. In what we refer to as model-I $V_c(r)$ will be linearly confining and of the form

$$V_c^I(r) = \sigma_c r + V_c(0), \tag{33}$$

and in model-II V_c is asymptotically flat,

$$\lim_{r\to\infty} V_c^{II}(r) = V_c(\infty) < \infty. \tag{34}$$

We concentrate on the interactions induced by the effective Hamiltonian in the limit of large quark-antiquark separation.

A. Matrix elements of the effective Hamiltonian in the chain basis space

The one body term, H_g, in Eq. (28) acts independently on individual gluons in the chain created by the operators $G^\dagger$ (*cf.* Eq. (31)). Using

$$[G(x), G^\dagger(y)]_{ij} = C_F\langle\psi|\psi\rangle\delta(x-y) = C_F I\delta(x-y), \tag{35}$$

we find

$$\begin{aligned}\langle N|H_g|N\rangle &= Z_N^2(C_F\langle\psi|\psi\rangle)^{N-1}\sum_{i=1}^N\int_{-R/2}^{R/2}dx_N\cdots\int_{-R/2}^{x_{i+1}}dx_i \\ &\quad\times C_F\langle\psi|E|\psi\rangle\int_{-R/2}^{x_i}dx_{i-1}\cdots\int_{-R/2}^{x_2}dx_1 \\ &= N\frac{\langle\psi|E|\psi\rangle}{\langle\psi|\psi\rangle} \equiv N\frac{N_C}{2}[e - V_c(0)]\end{aligned} \tag{36}$$

where

$$\langle\psi|E|\psi\rangle = \int [d\mathbf{k}_\perp]E(|\mathbf{k}_\perp|)\psi^i(\mathbf{k}_\perp)\delta_T^{ij}(\mathbf{k}_\perp)\psi^j(\mathbf{k}_\perp). \tag{37}$$

and to define e we subtracted from the single gluon energy in Eq. (37) a constant proportional to the potential at the origin. In color singlet states the total energy of the system should be invariant under a constant shift [30, 31], which we now demonstrate. The single gluon energy, $E(k)$, contains self energies. In the variational approximation the component of the self energy due to the Coulomb interaction is given by [11]

$$\Sigma_C(k) = -\frac{N_c}{2}\int [d\mathbf{q}]\tilde{V}_c(\mathbf{k}-\mathbf{q})\frac{1+\hat{\mathbf{k}}\cdot\hat{\mathbf{q}}}{2}\frac{\omega(k)}{\omega(q)} \tag{38}$$

where $\tilde{V}_c$ is the Fourier transform of the Coulomb potential. For a linearly rising, confining potential, *e.g.*, model-I, the low momentum singularity of $\tilde{V}(k)$ is not integrable and the resulting infinite self energy can be interpreted as a manifestation of confinement of color charges. A finite self energy is obtained by subtracting the IR singularity which leads to

$$\Sigma_C(k) = \Sigma_C'(k) - \frac{N_c}{2}V_c(0) \tag{39}$$

with $\Sigma_C'(k)$ finite and given by

$$\Sigma_C'(k) - \frac{N_c}{2}\int [d\mathbf{q}]\tilde{V}_c(\mathbf{k}-\mathbf{q})\left[\frac{1+\hat{\mathbf{k}}\cdot\hat{\mathbf{q}}}{2}\frac{\omega(k)}{\omega(q)} - 1\right] \tag{40}$$

that follows from

$$V_c(0) = \int [d\mathbf{q}]\tilde{V}_c(\mathbf{q}). \tag{41}$$

Even though for a confining potential the Fourier transform is defined modulo a constant, it is expected that when all, self and mutual, interactions between color charges are accounted for the dependence on $V_c(0)$ disappears from color singlet matrix elements. This will also be the case for the matrix elements of the effective Hamiltonian in the chain basis considered here. In anticipation of this result, in Eq. (36) we defined an IR finite single particle energy $e(k)$ by separating the Coulomb self energy equal to $-N_c V_c(0)/2$. Thus, in the last line of Eq. (36), e is finite, and the IR singularity of the confining Coulomb potential is explicit in the term proportional to $V_c(0)$. In the case of model-II with non-confining interactions, self-energies are IR finite but we can perform the subtractions nevertheless.

In absence of chained gluons, $N = 0$, the interaction between quark densities produces the Coulomb potential between quark charges (*cf.* Eq. (14)). With N gluons separating

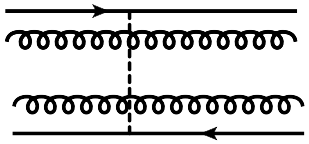

FIG. 6. A non-planar diagram, which we do not take into account, representing direct Coulomb interaction between the quark and the antiquark in the presence of 2 gluons.

the quark from the antiquark, the direct interaction between quark charges is nonplanar (*cf.* Fig. (6)) and suppressed by a power of N_C compared to a successive Coulomb interaction between the quark and the nearest gluon or the interaction between any two nearest-neighbor gluons in the chain. To leading order in N_C the H_{qq} contribution thus reduces to the quark self energies,

$$\langle N|H_{qq}|N\rangle = -C_F V_c(0) \to -\frac{N_C}{2}V_c(0). \tag{42}$$

The quark gluon interaction, to leading order in N_C, couples the quark (or the antiquark) to the nearest gluon in the chain. For example, for the antiquark-gluon interaction we find

$$\begin{aligned}\langle N|H^D_{qg}|N\rangle &\to Z_N^2(C_F\langle\psi|\psi\rangle)^{N-1}\\ &\times\int_{-R/2}^{R/2}dx_N\cdots\int_{-R/2}^{x_2}dx_1 C_F\langle g\bar{Q}|H_{qg}(x_1)|g\bar{Q}\rangle\\ &= \frac{N}{R^N}\int_{-R/2}^{R/2}dx_1\left(x_1+\frac{R}{2}\right)^{N-1}\frac{\langle g\bar{Q}|H_{qg}(x_1)|g\bar{Q}\rangle}{\langle\psi|\psi\rangle}\end{aligned} \tag{43}$$

where

$$\begin{aligned}&\langle g\bar{Q}|H_{qg}(x_1)|g\bar{Q}\rangle = \frac{N_C}{2}\int d^2\mathbf{x}_\perp[d\mathbf{k}_\perp][d\mathbf{q}_\perp]e^{i(\mathbf{k}_\perp-\mathbf{q}_\perp)\cdot\mathbf{x}_\perp}\\ &\times\frac{1}{2}\left[\sqrt{\frac{\omega(|\mathbf{k}_\perp|)}{\omega(|\mathbf{q}_\perp|)}}+\sqrt{\frac{\omega(|\mathbf{q}_\perp|)}{\omega(|\mathbf{k}_\perp|)}}\right]V_c\left(\sqrt{|\mathbf{x}_\perp|^2+|\frac{R}{2}-x_1|^2}\right)\\ &\times\psi^i(\mathbf{k}_\perp)[\delta_T(\mathbf{k}_\perp)\delta_T(\mathbf{q}_\perp)]^{ij}\psi^j(\mathbf{q}_\perp).\end{aligned} \tag{44}$$

In the limit $R\to\infty$ where $x_1/R = O(1)$ this reduces to

$$\langle g\bar{Q}|H_{qg}(x_1)|g\bar{Q}\rangle = \frac{N_C}{2}V_c\left(|\frac{R}{2}-x_1|\right)\langle\psi|\psi\rangle. \tag{45}$$

Taking into account both quark and antiquark contributions, for the H^D_{qg} matrix element we obtain

$$\langle N|H^D_{qg}|N\rangle = NN_C\int_{-1/2}^{1/2}dz\left(z+\frac{1}{2}\right)^{N-1}V_c\left(R\left(z-\frac{1}{2}\right)\right). \tag{46}$$

For the linearly rising potential of model-*I* Eq. (46) yields

$$\langle N|H^D_{qg}|N\rangle^I = N_C\frac{\sigma_c R}{N+1}+N_C V_c(0). \tag{47}$$

while in the case of model-*II* we find

$$\langle N|H^D_{qg}|N\rangle^{II} = N_C V_c(\infty). \tag{48}$$

The interaction between two nearby gluons in the chain given by H^D_{gg} is also straightforward to compute, and passing directly to the $R\to\infty$ limit we find,

$$\begin{aligned}&\langle N|H^D_{gg}|N\rangle = Z_N^2(C_F\langle\psi|\psi\rangle)^{N-2}\sum_i^{N-1}\int_{-R/2}^{R/2}dx_N\cdots\\ &\times\int_{-R/2}^{x_{i+2}}dx_{i+1}\int_{-R/2}^{x_{i+1}}dx_i C_F\left(\frac{N_C}{2}\right)^2V_c(x_{i+1}-x_i)\langle\psi|\psi\rangle^2\\ &\times\int_{-R/2}^{x_i}dx_{i-1}\cdots\int_{-R/2}^{x_2}dx_1\\ &= \frac{N!}{C_F}\int_{-1/2}^{1/2}dz\int_{-1/2}^{z}dw\left(\frac{N_C}{2}\right)^2V_c(R(z-w))\frac{(1+w-z)^{N-2}}{(N-2)!}.\end{aligned} \tag{49}$$

For the linear potential of Eq. (33), to leading order in N_C this yields

$$\langle N|H^D_{gg}|N\rangle^I = \frac{N_C}{2}\frac{N-1}{N+1}\sigma_C R+\frac{N_C}{2}(N-1)V_c(0) \tag{50}$$

and for the asymptotically constant potential

$$\langle N|H^D_{gg}|N\rangle^{II} = \frac{N_C}{2}(N-1)V_c(\infty). \tag{51}$$

Since all terms in the effective Hamiltonian (including the self energies) are $O(g^2)$, and $\lim_{N_C\to\infty}N_C g^2 = O(1)$, at large N_C all matrix elements are finite when expressed in terms of $\bar{e} = N_C e/3$, $\bar{\sigma}_c = N_C\sigma_c/3$ for model-*I* and $\bar{V}_c(\infty) = N_C V_c(\infty)/3$, $\bar{V}_c(0)\equiv N_C V_c(0)/3$ for model-*II*, respectively.

Adding all diagonal contributions of the effective Hamiltonian matrix that are independent of the background field, we thus find,

$$\langle N|H|N\rangle^I = \frac{3}{2}N\bar{e}+\frac{3}{2}\bar{\sigma}_C R \tag{52}$$

and

$$\begin{aligned}\langle N|H|N\rangle^{II} &= \frac{3}{2}N\bar{e}+\frac{3}{2}(N+1)(\bar{V}_c(\infty)-\bar{V}_c(0))\\ &\equiv \frac{3}{2}Nm_g+c\end{aligned} \tag{53}$$

for model-*I* and model-*II*, respectively. For $N=0$ this agrees with Eq. (14), while, for $N\geq 1$, eigenstates of Eq. (52) or (53) represent a tower of chain states with energies proportional to the number of gluons in the chain. Clearly the lowest energy state of the diagonal part of the Hamiltonian is the variational $Q\bar{Q}$ state, with $N=0$ gluons. The genuine chain contribution to the lowest energy state must therefore originate from the terms in the Hamiltonian which couple the constituent gluons to the background field. The interaction of physical gluons with the background is given by

$$\langle N|H^D_{gb}|N\rangle = Z_N^2 (C_F\langle\psi|\psi\rangle)^N \sum_{i=1}^{N-1}\int_{-R/2}^{R/2} dx_N \cdots \int_{-R/2}^{x_{i+2}} dx_{i+1} \times \int_{-R/2}^{x_{i+1}} dy_i \int_{-R/2}^{y_i} dx_i F_B(|y_i - x_i|) \int_{-R/2}^{x_i} dx_{i-1} \cdots \int_{-R/2}^{x_2} dx_1 \tag{54}$$

where

$$F_B(|y-x|) = \frac{N_C}{2}\gamma(|y-z|)(V_c(|y-z|) - V_c(0)), \tag{55}$$

and

$$\gamma = \int d\mathbf{x}_\perp d\mathbf{y}_\perp [d\mathbf{k}_\perp][d\mathbf{q}_\perp]\sqrt{\omega(\mathbf{k}_\perp)\omega(\mathbf{q}_\perp)}e^{-i\mathbf{q}_\perp\mathbf{x}_\perp + i\mathbf{k}_\perp\mathbf{y}_\perp} \times \frac{[\psi(\mathbf{q}_\perp)\delta_T(\mathbf{q})]^i G_c^{ij}(\mathbf{x}_\perp - \mathbf{y}_\perp, y-z)[\psi(\mathbf{k}_\perp)\delta_T(\mathbf{k})]^j}{\langle\psi|\psi\rangle}. \tag{56}$$

The correlation function G_c is given by the density of the vacuum fields

$$G^{ij}(\mathbf{x}_\perp - \mathbf{y}_\perp, x-y) = \frac{\langle A_c^{ia}(\mathbf{x}_\perp, x) A_c^{ja}(\mathbf{y}_\perp, y)\rangle}{N_C^2 - 1}. \tag{57}$$

Here the expectation value is taken with respect to the distribution of sources of the background field. These might effectively describe monopole-antimonopole pairs in 3D, vortex surfaces in 4D, merons, *etc.*. A simple model is considered in the Appendix. Since it is these background fields that are responsible for confinement in the first place, *i.e.* generation of the Coulomb potential $V_c(R)$, we assume that the density of the underlying magnetic sources is approximately uniform over the quark-antiquark separation. So for $|x-y| \lesssim R$ we expect that in general

$$G^{ij}(\mathbf{x}_\perp - \mathbf{y}_\perp, x-y) \sim G^{ij}(\mathbf{x}_\perp - \mathbf{y}_\perp), \tag{58}$$

and therefore γ in Eq. (55) reduces to a constant of $O(\Lambda_{QCD})$, *i.e.* it is independent of the longitudinal distribution of gluons along the chain.

For model-I evaluation of the integrals in Eq. (56) then yields

$$\langle N|H^D_{gb}|N\rangle^I = N_C N! \frac{N-1}{\Gamma(N+3)}\gamma\sigma_C R^2 = 3\frac{N-1}{(N+1)(N+2)}\gamma\bar{\sigma}_C R^2. \tag{59}$$

while for model-II we find

$$\langle N|H^D_{gb}|N\rangle^{II} = 3\gamma R\frac{N-1}{N+1}(\bar{V}_c(\infty) - \bar{V}_c(0)). \tag{60}$$

Finally we consider the components of the interaction between physical gluons and the background that changes the gluon number. From Eq. (24) we find (for $N \geq 2$)

$$\langle N-2|H^M_{gb}|N\rangle = \langle N|H^M_{gg}|N-2\rangle = Z_N Z_{N-2}(C_F\langle\psi|\psi\rangle)^{N-1} \times \sum_{i=1}^{N-1}\int_{-R/2}^{R/2} dx_N \cdots \int_{-R/2}^{x_{i+1}} dx_i F_B(x_{i+1} - x_i) \cdots \int_{-R/2}^{x_2} dx_1 \tag{61}$$

which gives

$$\langle N-2|H^M_{gb}|N\rangle^I = \langle N|H^M_{gg}|N-2\rangle^I = \frac{3\sqrt{N(N-1)}}{2N(N+1)}\gamma\bar{\sigma}_c R^2. \tag{62}$$

and

$$\langle N-2|H^M_{gb}|N\rangle^{II} = \langle N|H^M_{gg}|N-2\rangle^{II} = = \frac{3}{2}\gamma R\sqrt{\frac{N-1}{N}}(\bar{V}_c(\infty) - \bar{V}_c(0)). \tag{63}$$

for the two models, respectively. Collecting all the terms, Eqs. (52),(59),(62) for model-I and Eqs. (53),(60),(63) for model-II, we find the following expression for the matrix elements of the Hamiltonian in the gluon chain basis for large-N,

$$\langle N'|H|N\rangle^I = \frac{3}{2}N\bar{e}\delta_{N'N} + \frac{3}{2}R\bar{\sigma}_C\left(1 + r\frac{R}{N}\gamma\right)\delta_{N'N} + \frac{3}{2}\gamma\bar{\sigma}_C\frac{R}{N}R\delta_{N',N-2} + \frac{3}{2}\gamma\bar{\sigma}_C\frac{R}{N'}R\delta_{N'-2,N} \tag{64}$$

$$\langle N'|H|N\rangle^{II} = \frac{3}{2}N\bar{m}_g\delta_{N'N} + \frac{3}{2}r\gamma R(\bar{V}_c(\infty) - \bar{V}_c(0))\delta_{N'N} + \frac{3}{2}\gamma R(\bar{V}_c(\infty) - \bar{V}_c(0))(\delta_{N',N-2} + \delta_{N'-2,N}) \tag{65}$$

Here r is the ratio of the diagonal to off-diagonal matrix elements in the limit of large-R. The specific value $r = 2$ follows from the fact that in the two models both terms originate from the same interaction *cf.* Eq. (27). Below, while presenting numerical result, we also discuss the dependence of the lowest eigenvalues on this ratio.

B. Numerical Results

Before analyzing the spectra of the effective chain model Hamiltonians we consider the large-R limit of the matrix

$$\langle N'|H|N\rangle = R(\delta_{N',N-2} + \delta_{N'-2,N}). \tag{66}$$

It is straightforward to show that the ground state energy of H for large-R approaches $-2R$. For the Hamiltonian of model-I this implies that if the kinetic term (proportional to $N\bar{e}$) was ignored, the lowest eigenvalue of H^I for large-R would behave as

$$\frac{3}{2}\gamma\bar{\sigma}_C\left(1 + r\frac{R}{\langle N\rangle}\right) - 2\frac{3}{2}\gamma\bar{\sigma}_C\frac{R}{\langle N\rangle} = \frac{3}{2}\gamma\bar{\sigma}_C R + \frac{3}{2}\gamma\bar{\sigma}_C(r-2)\frac{R^2}{\langle N\rangle}. \tag{67}$$

The special role of the value $r = 2$ is now transparent. At $r = 2$ the quadratic term vanishes and the lowest chain state energy grows linearly with R. If $r >> 2$ the lowest eigenvalue

is dominated by the diagonal term. In this case the expectation value of N,

$$\langle N\rangle = \frac{\sum_N N|\psi_0(N)|^2}{\sum_N |\psi_0(N)|^2}, \tag{68}$$

where ψ_0 is the wave function of the lowest energy chain state, can be determined by minimizing the diagonal part with respect to N. This gives

$$\langle N\rangle = \sqrt{\frac{2\gamma}{\bar{e}}}R \tag{69}$$

and the ground state energy approaches

$$E_0^I = 3R\sqrt{r\gamma\bar{\sigma}_C\bar{e}} + \frac{3}{2}R\bar{\sigma}_C. \tag{70}$$

Thus for $r > 2$ the energy of the chain is higher than the energy of the bare state, $|0\rangle$. If $r < 2$ the off-diagonal term dominates and the ground state energy becomes negative and proportional to $-R^2$ while the average number of gluons in the chain $\langle N\rangle = O(1)$. However, when the kinetic term is included in the critical case $r = 2$ the lowest energy of the chain state no longer increases linearly with R. After numerical diagonalization we find

$$\langle N\rangle^I \propto (R\ \mathrm{GeV})^{0.623\pm0.004}, \frac{E_0^I}{\mathrm{GeV}} \propto (R\ \mathrm{GeV})^{0.787\pm0.006} \tag{71}$$

for a reasonable set of parameters $\bar{e} = 600$ MeV, $\gamma = 1$ GeV and $\bar{\sigma}_C = 0.1$ GeV^2. We also find weak dependence of the exponents on these parameters. That is, for the chain model-I, we find that the lowest energy chain state has higher energy than the bare state. In the critical case the energy increases less rapidly than the length of the chain, R, and is proportional to R^2 for $r > 2$. The average number of gluons grows weakly with R. The results are summarized in Figs. (7), (8) .

In the case of model-II for $r > 2$, one easily finds,

$$E_0^{II} = \frac{3}{2}(r-1)\gamma R[\tilde{V}_c(\infty) - \tilde{V}_c(0)]$$
$$\langle N\rangle^{II} \propto R^{1/3} \tag{72}$$

while for $r < 2$ with the off-diagonal term dominating,

$$E_0^{II} = -\frac{3}{2}(r-1)\gamma R[\tilde{V}_c(\infty) - \tilde{V}_c(0)]. \tag{73}$$

Finally for the critical choice $r = 2$ numerical digitalization yields

$$\langle N\rangle^{II} \propto (R\ \mathrm{GeV})^{0.338\pm0.005}, \frac{E_0^{II}}{\mathrm{GeV}} \propto (R\ \mathrm{GeV})^{0.379\pm0.005} \tag{74}$$

for the set of parameters, $m_g = 600$ MeV, $\bar{\sigma}_C = 0.1$ GeV^2, and $\gamma = 1$ GeV, $\hat{V}_c(\infty) - \hat{V}_c(0) = 1$ GeV, with the results shown in Figs. (8), (9).

In model-II as R-increases at some point the energy of the ground state chain increases less than the Coulomb potential. The chain state, however, has energy which is higher than that of the bare state, with the latter approaching a constant at large-R. Thus in both models interactions among the chain increase the energy of the $Q\bar{Q}$ pair as compared to the state with no gluons.

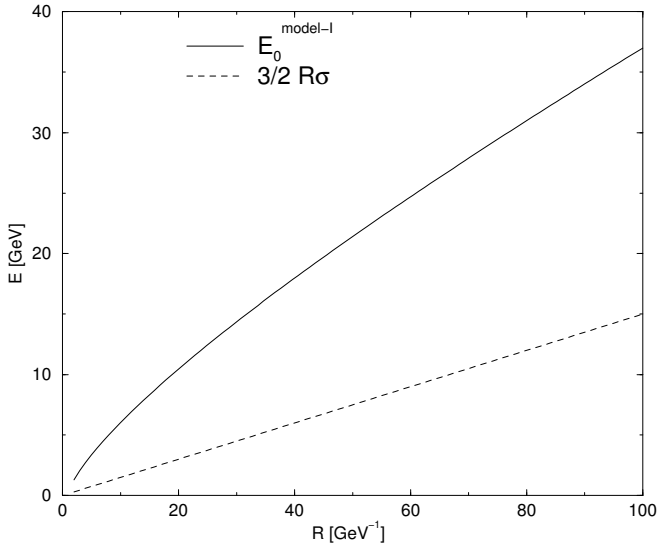

FIG. 7. Ground state energy (solid line) of the chain Hamiltonian in model-I. A power law fit yields $E_0^I = 0.984(RGeV)^{0.787}GeV$. The dashed line gives the energy of the bare state using for the string tension $\bar{\sigma}_C = 0.1$ GeV^2.

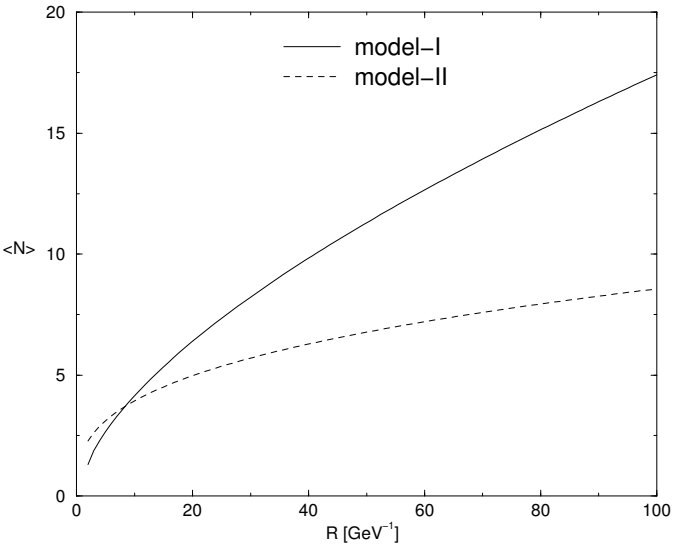

FIG. 8. Expectation value of the the number of gluons in the chain as a function of $Q\bar{Q}$ separation, R. A power law fit gives $\langle N\rangle^I = 0.985(R\ \mathrm{GeV})^{0.623}$ and $\langle N\rangle^{II} = 0.984(R\ \mathrm{GeV})^{0.787}$ for model-I (solid line) and model-II (dashed line), respectively.

IV. SUMMARY AND OUTLOOK

We investigated microscopic origins of the gluon chain model. By analyzing the physical gauge interactions among constituent gluons, we found a scenario for generating a chain. In this scenario a state with a number of gluons in the chain that is increasing with the separation between the $Q\bar{Q}$ source emerges from interactions of dynamical gluons with the background field. The background field is necessary in a phenomenological model of confinement if the latter is to orig-

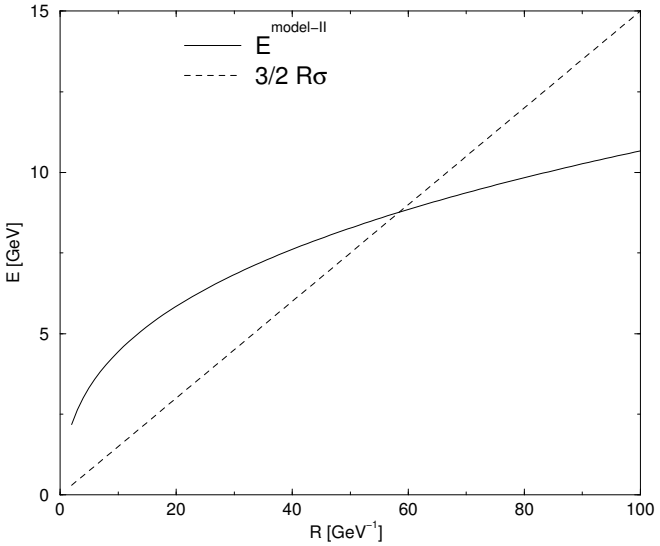

FIG. 9. Ground state energy (solid line) of the chain Hamiltonian in model-*II*. A power law fit yields $E_0^I = 1.867(RGeV)^{0.379}GeV$. The dashed line gives the energy of the bare state using for the string tension $\bar{\sigma}_C = 0.1$ GeV^2.

inate from condensation of chromomagnetic charges. These interactions introduce off-diagonal elements into the effective Hamiltonian, which is one of the main differences between this and the chain model where the pair-production is absent. We have shown that the resulting ground state energy is convex [32] but the two models considered are still too simplistic to generate the linearly rising potential. While this deficiency can potentially be improved by considering more sophisticated models for the background field we found it difficult to reproduce the Zwanziger conjecture of "no-confinement without Coulomb confinement" [16]. The result of [16] states that the expectation value of the Coulomb kernel in the QCD vacuum state is confining (if the exact $Q\bar{Q}$ potential is confining) with a string tension which is not less than that of the exact potential. Our results in the model considered (*c.f.* Fig. 7) indicate the opposite. We find the energy of the chain state, which is supposed to represent the exact $Q\bar{Q}$ state to be higher then that of the bare one, defined as the expectation value of the Coulomb kernel in a state with no-backward reaction from the sources on the vacuum.

We find it interesting that a such a reasonable and simple model does not comply with the exact expectations from QCD. There clearly must exist other non-diagonal interactions which are important. It was proposed that triple-gluon interactions could play an important role [33]. Such interactions may originate from the magnetic, $\mathbf{B}^2$ term, but since it does not depend explicitly on the separation between sources it most likely is not significant at large separations. Other three-gluon interactions could originate from the Coulomb kernel, if the background field in the true (chain) state is polarized. This would lead to a non-vanishing three-gluon coupling mediated by the Coulomb interaction and thus R-dependent Finally it is possible that a resolution of this problem requires renormalization for the single-gluon energies in the presence of the chain so that effectively $\bar{e}$ decreases with the number of gluons. We leave these questions for future investigations.

ACKNOWLEDGMENTS

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Appendix A: Background field model

The correlation function G defined in Eq. (57) is computed using a classical distribution of sources of the background field. For example if these are monopole- antimonopole pairs the density $\rho_N(\mathbf{c}_i\bar{\mathbf{c}}_i)$ depends on the locations of the N pairs. The expectation value of a function of $\mathbf{A}_c$ is computed from

$$\langle \mathbf{A}_c^a(\mathbf{x})\mathbf{A}_c^a(\mathbf{y})\rangle = \frac{Z[\mathbf{A}_c\mathbf{A}_c]}{Z[1]} \tag{A1}$$

where

$$Z[\mathscr{O}[\mathbf{A}_c]] = \sum_N^\infty \int dn^a \int \Pi_i^N d\mathbf{c}_i d\bar{\mathbf{c}}_i \rho(\mathbf{c}_i\bar{\mathbf{c}}_i)\mathscr{O}[\mathbf{A}_c] \tag{A2}$$

and the background field is given by

$$\mathbf{A}_c^a(\mathbf{x}) = n^a \sum_{i=1}^N [\mathbf{A}_m(\mathbf{x}_\perp - \mathbf{c}_{\perp,i}) - \mathbf{A}_m(\mathbf{x}_\perp - \bar{\mathbf{c}}_{\perp,i})] \tag{A3}$$

where $\mathbf{A}_m$ is the abelian monopole field, and n^a represents the (common) orientation of monopoles in the $SU(N_C)$ algebra. For a uniform distribution of monopole-antimonopole pairs along the $Q\bar{Q}$-axis ($\hat{z}$-axis) with the density given by

$$\rho(\mathbf{c}_i,\bar{\mathbf{c}}_i) = \frac{\rho^N}{(\mathscr{V}_\perp R)^N N!}\Pi_{i=1}^N \theta(\frac{R}{2} - |\mathbf{c}_i\hat{z}|)\theta(\frac{R}{2} - |\bar{\mathbf{c}}_i\hat{z}|) \tag{A4}$$

the background field is approximately constant along the $Q\bar{Q}$ axis. In Eq. (A4) ρ is the density of monopoles which is equal to the density of antimonopoles

$$\rho = \frac{\rho_\perp(N_C^2-1)}{R}. \tag{A5}$$

For the correlation function $G(\mathbf{x}_\perp, x)$ we then obtain

$$G(\mathbf{x}_\perp, x) = G(\mathbf{x}_\perp) = \rho_\perp \left[\int d\mathbf{c}_\perp \mathbf{A}_m(\mathbf{x}_\perp - \mathbf{c}_\perp)\mathbf{A}_m(\mathbf{c}_\perp) - \frac{1}{\mathscr{V}_\perp}\int d\mathbf{c}_\perp d\mathbf{c}'_\perp \mathbf{A}_m(\mathbf{c}_\perp)\mathbf{A}_m(\mathbf{c}'_\perp)\right]. \tag{A6}$$

The last term originates from the charge neutrality of the monopole-antimonopole distribution. If the core of the

monopole field is smoothed out over a distance scale $a = O(\Lambda_{QCD}^{-1})$ then

$$G(\mathbf{x}_\perp) \sim \rho_\perp \log \frac{|\frac{R}{2} - |\mathbf{x}_\perp||}{a} \tag{A7}$$

where the $\log R$ dependence comes from cutting off the long range integral over the transverse plane. This is the standard expression for the correlation function of a pair of 2D vortices separated by a distance R. The 2D reduction originates from the assumption the monopoles are uniformly distributed, Eq. (A5), along the direction of the $Q\bar{Q}$ separation.

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