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A Remark on Supersymmetric Bubbles and Spectrum Crossover

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Abstract

Using an exact expression for the domain wall tension in a supersymmetric model we show that a spectrum crossover takes place in passing from weak to strong coupling. In the weak coupling regime elementary excitations are the lightest states, while in the strong coupling regime solitonic objects of a special type – bubbles – assume the role of the lightest states. The crossover occurs at $\lambda^2/(4\pi) \sim 0.4$.

The (possible) recent discovery of the Higgs particle, with the production and decay properties fully consistent with the standard model (SM) implies that the scale of new physics is higher than we hoped. Of special importance is the $\gamma\gamma$ decay of the Higgs particle which agrees, within errors, with the SM prediction [1]. The theoretical number [1] is practically impossible to change without drastic modifications of the electroweak theory.[1] Not only this agreement is remarkable, but we learn, from the fact that $m_H \sim 125\,\mathrm{GeV}$, that the theory, while keeping itself at weak coupling, comes rather close to the boundary of the weak coupling regime, since the Higgs self-interaction coupling $\lambda \sim \frac{1}{2}$.

It is not ruled out that future beyond-SM explorations will uncover a more complicated Higgs sector, with still larger coupling constants. In the strong coupling theories the phenomenon of level crossing is quite common. There is a problem with its detection, because usually it occurs at strong coupling. We are aware of several examples: (i) in two-dimensional models with exact solutions [3], where at weak coupling the lightest state is an elementary excitation, while at strong coupling it is a soliton; (ii) in supersymmetric theories in two and four dimensions in the BPS protected sectors, the so-called curves of marginal stability or domain wall crossings (where this knowledge is essentially algebraic, plus analytic properties) [4]; (iii) in supersymmetric theories with dualities, in the non-BPS sectors, the so-called crossover [5].

In the latter case, the dynamical information needed to detect the level crossing is provided by a weak-strong coupling duality. Here we discuss a simple example in which the necessary dynamical information comes from some general considerations combining supersymmetry and quantum mechanics. The dynamical systems that we keep in mind, that become light at strong coupling are bubbles of an "opposite" vacuum.

Such bubbles were considered in the literature previously, on several occasions [6, 7]. In [6] highly excited bubble states were considered at weak coupling, where they are much heavier than the elementary excitations of the model. High excitation number was crucial for maintaining a well defined bubble (albeit unstable). In [7] pure supersymmetric $\mathcal{N} = 1$ Yang-Mills theory was treated. Needless to say, this is a strongly coupled theory with N degenerate vacua (if the gauge group is $\mathrm{SU}(N)$). However, the domain wall

[1] An example of a dramatic crippling of the theory needed to enhance $\Gamma(H \to \gamma\gamma)$ just by a factor of 2 is presented in [2].

tension T scales as $T \sim N$ [8], hence $T^{1/3}$ is always larger than the glueball mass, and therefore the bubbles under consideration are unstable. Strictly speaking they are absent in the spectrum of the stable states.[2] In both cases [6, 7], the crossover phenomenon is untraceable.

We will focus on a simple supersymmetric set-up with the weak-strong coupling transition. We consider a minimal $\mathcal{N} = 1$ Wess-Zumino model with the superpotential

$$\mathcal{W} = \frac{m^2}{\lambda}\Phi - \frac{\lambda}{3}\Phi^3\,, \tag{1}$$

assuming for simplicity the mass and λ parameters to be real and positive.

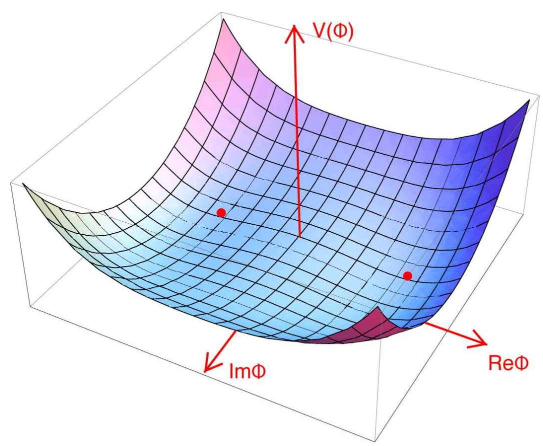

Figure 1: Potential energy in the model (1). Two degenerate isolated minima are marked by dots.

This theory has two isolated degenerate supersymmetric vacua at

$$\phi_{\rm vac} = \pm\frac{m}{\lambda}\,, \tag{2}$$

and a domain wall interpolating between them. The domain wall is BPS saturated and, therefore, its tension T is exactly known (see e.g. [10]),

$$T = \frac{8}{3}\frac{m^3}{\lambda^2}\,. \tag{3}$$

[2]The authors of [7] understand the bubble instability; however, they argue that the bubbles are quasistable at large N. Their argument does not seem persuasive at all given the fact that a typical bubble size scales as $T^{-1/3} \ll \Lambda^{-1}$. It is true, though, that the bubbles in [7] are in the thin wall regime, see [9] for an explanation.

Note that the combination appearing on the right-hand side is renormalization group invariant. We can write this ratio either in terms of the bare m_0 and λ_0, or in terms of the physical mass m and coupling constant λ, the result is the same.

At weak coupling, $\lambda \ll 1$, the parameter m is the mass of the elementary excitation in either of the two vacua. This is the lightest (and the only bosonic) particle in the theory. We claim that if we fix m and analytically continue λ to strong coupling, the would-be "elementary" excitations decay. A number of lighter stable states of a totally different nature appear in the theory.

These states are bubbles of the "opposite" vacuum, analogous to those occurring in the problem of the false vacuum decay [11, 12]. In the latter problem the decaying vacuum has a higher energy density than the genuine vacuum. Therefore, one has to deal with the volume energy of the bubble. In the supersymmetric case at hand both vacua are strictly degenerate at the classical level, as well as at the quantum level. For this reason the energy density in the bulk of the interior of the bubble is exactly the same as in the exterior, with the exception of a layer (with thickness $\sim m^{-1}$, see the second paper in [12]) forming the surface of the wall. As a result, for the large bubble size R ($R \gg m^{-1}$), the bubble dynamics is determined by that of its surface. This dynamics can be readily described as a quantum-mechanical problem in the so-called thin wall approximation.

The wall thickness is determined by m^{-1} while its radius, as we will see shortly, is proportional to $T^{-1/3}$. In this approximation one can neglect the deformation of the wall tension due to a nonvanishing curvature. Therefore, we need $m \gg T^{1/3}$, which, implies, in turn, strong coupling. This domain is amenable to studies due to the exact nature of Eq. (3). Assuming the bubble to be spherical one can write a quantum-mechanical Lagrangian governing its dynamics,

$$\mathcal{L} = -4\pi R^2 T \sqrt{1 - \dot{R}^2}\,, \tag{4}$$

where R is the bubble radius. The corresponding relation for the Hamiltonian $\mathcal{H}$ in terms of R and the conjugate momentum p reads as

$$\mathcal{H}^2 - p^2 = \left(4\pi R^2 T\right)^2 \,. \tag{5}$$

The ground state energy E_0 of a quantized bubble described by this Hamiltonian can be readily found by numerically solving the Schrödinger equation corresponding to Eq.(5),

$$E_0 = c_0\,(4\pi)^{1/3}\,T^{1/3} \approx 3.32\,\frac{m}{\lambda^{2/3}}\,, \tag{6}$$

with $c_0 = 1.027\ldots$.

Generically, the energy of the n-th quantized state of the bubble can be written as $E_n = c_n\,(4\pi)^{1/3}\,T^{1/3}$ with c_n being a dimensionless coefficient. For the first excited bubble the numerical solution gives $c_1 = 1.949\ldots$ and this state is stable with respect to decay into two ground-state bubbles, since $c_1 < 2\,c_0$.

At large n for spherically symmetric bubbles the coefficients c_n can be evaluated by using the Bohr-Sommerfeld type semiclassical quantization applied to Eq. (5). In this way one finds

$$c_n = \left[\frac{3\sqrt{\pi}\,\Gamma(3/4)}{\Gamma(1/4)}\right]^{1/3}\left(n+\frac{1}{2}\right)^{2/3} \qquad (n \gg 1)\,. \tag{7}$$

None of the excited states with $n > 1$ is stable with respect to decay into ground-state bubbles. (It can be noted that although the semiclassical expression (7) is justified at large n, formally setting $n = 0$ and $n = 1$, one finds that it gives $c_0 \approx 0.93$ and $c_1 \approx 1.937$; these values only slightly differ from the exact ones. Thus, the semiclassical formula works reasonably well starting from low n.)

The condition for stability of the ground-state bubble against decay into two elementary bosons, $E_0 < 2m$, implies

$$\lambda^{2/3} > 1.66 \quad \text{or} \quad \alpha \equiv \frac{\lambda^2}{4\pi} > 0.365\,. \tag{8}$$

It should be noted however, that the specific numerical value in the above estimate should be taken with a certain reservation. Indeed, at such value of λ we have $T^{-1/3} \gtrsim m^{-1}$; the condition $T^{-1/3} \gg m^{-1}$ is not met, and literally speaking the bubble cannot be considered in the thin wall approximation, i.e. by virtue of the effective Lagrangian (4). At this point the thin wall approximation is at the boundary of its applicability. It is clear, however, that the ground-state bubble becomes stable and in fact the lowest mass state at a sufficiently large λ, somewhere above the limit (8).

Of course, if the physical coupling constant satisfies Eq. (8) we are not that far from the Landau pole. To make the model under consideration self-consistent one must assume that it has some ultraviolet (UV) completion that embeds it into an asymptotically free field theory (or a non-field-theoretic UV completion).

If the condition (8) is met, the bubbles of the "opposite" vacuum and, perhaps, a number of excitations form a spectrum of stable bosons (supersymmetry implies that there are degenerate in mass fermions too). Needless to say, their masses are not BPS protected. Thus, we deal here with long supersymmetry multiplets. A typical size of the above states is determined by $T^{-1/3}$ rather than by m^{-1}.

The very idea of building various solitonic objects by bending domain walls and stabilizing them appropriately, is not new, of course. We have already mentioned [6, 7]. In addition, in [13] magnetic flux tubes were constructed in this way in supersymmetric non-Abelian Yang-Mills theories. The peculiarity of the example we have considered in this note is that by varying the value of the coupling constant λ we can travel all the way from the weak coupling regime in which the elementary excitations are the lightest states, to the strong coupling regime in which solitonic objects of a special type – bubbles – assume the role of the lightest states.

In conclusion we note that, although nothing can be proven without supersymmetry, it is not ruled out that a similar phenomenon occurs in non-supersymmetric models with spontanepusly broken discrete symmetries in passing from weak to strong coupling.

Acknowledgments

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