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A Heavy SM-like Higgs and a Light Stop from Yukawa-Deflected Gauge Mediation

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Abstract

To obtain a SM-like Higgs boson around 125 GeV in the Minimal Supersymmetric Standard Model with minimal gauge mediation of supersymmetry breaking (GMSB), a heavy stop at multi-TeV level is needed and incurs severe fine-tuning, which can be ameliorated in the framework of the deformed GMSB with visible-hidden direct Yukawa interactions (YGMSB). We examine some general features of the YGMSB and focus on the scenario with Higgs-messenger couplings \( H-YGMSB \) which can automatically maintain the minimal flavor violation (MFV). It turns out that such a Yukawa mediation scenario can give a large \(-A_t\) and \(-m_{\tilde{t}_{L,R}}^2\), leading to a maximal stop mixing, and thus can readily give a light stop \( \tilde{t}_1 \) below the TeV scale. However, we find that in the minimal \( H-YGMSB \) scenario, \( m_{H_u}^2 \) is too large and then the electroweak symmetry breaking is inconsistent with the large stop mixing. To solve this problem, we modify the hidden sectors in two ways, adding a new strong gauge dynamics or introducing the \((10, \overline{10})\) messengers. For each case we present some numerical study.

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I. INTRODUCTION

Supersymmetry (SUSY) elegantly stabilizes the electroweak scale. However, SUSY must be broken and the SUSY-breaking must happen in some hidden sector and then mediated to the visible sector. In order to avoid the catastrophic flavor-changing neutral currents (FCNCs), the mediation mechanism must be rather special. Since gauge interaction is flavor blind, the gauge mediated SUSY-breaking (GMSB) [1, 2] can generate a flavor-universal soft spectrum and suppress FCNCs. In addition to realize the minimal flavor violation (MFV) [3], the GMSB has some other virtues, e.g., it has only a few parameters and hence very predictive. Furthermore, it may accommodate the natural SUSY [4, 5] since the stop/gluino renormalization group equation (RGE) effect can be reduced considerably by lowering the messenger scale.

The present experimental results also indirectly support the GMSB. Firstly, the LHC SUSY search [6, 7] did not find any colored sparticles. Such null search results can be naturally understood in the GMSB where the squarks and gluino lie at the top of the hierarchical soft spectrum. Secondly, the dark matter (DM) detection experiments (like XENON100 [8]) have so far yielded null results. These results can be also naturally interpreted in the GMSB where the DM is the superweakly interacting gravitino.

However, the LHC hints of a SM-like Higgs near 125 GeV [9] place the minimal GMSB in an uncomfortable situation [10]. In the MSSM, the SM-like Higgs mass \(m_h\) at the tree-level is upper bounded by \(m_Z\), so a large stop radiative correction is required to lift up \(m_h\):

\[
m_h^2 = m_Z^2 \cos^2 2\beta + \frac{3m_t^4}{4\pi^2 v^2} \left[ \log \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} + \frac{X_t^2}{m_{\tilde{t}_2}^2} \left( 1 - \frac{X_t^2}{12m_{\tilde{t}_2}^2} \right) \right],
\]

with the average stop mass \(m_{\tilde{t}} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}\) and the stop mixing \(X_t = A_t - \mu \cot \beta\). To obtain a Higgs mass \(m_h \approx 125\) GeV without multi-TeV stops (heavy stops cause severe fine-tuning and lead to null results for the future LHC search), we should go to the maximal mixing scenario with \(|X_t| \approx \sqrt{6} m_{\tilde{t}}\) [11]. Even in this ideal case \(m_{\tilde{t}}\) should be close to the TeV scale [12]. The maximal mixing scenario is hard to realize in the minimal GMSB where \(A_t\) is only generated from the RGE running (mainly from the effects of the gluino mass) which, at the same time, also increases stop masses. So it is urgent to explore some deformed GMSB which can give a large \(A_t\) and/or decreased stop soft mass at the boundary.

In order to obtain a large stop mixing, we in this work turn to the deformed GMSB with
direct visible-hidden Yukawa couplings (YGMSB) (note that the YGMSB considered here is different from the framework proposed in [13], which focuses on the interactions between the messengers and another hidden sector). Actually, the Higgs-messenger couplings have been studied in the early days of the GMSB [14] and more recently are studied for various purposes, e.g., dynamically solving the $\mu/B\mu$–problem [15, 16], making the next-to-minimal supersymmetric model (NMSSM) with GMSB viable [17], breaking a dark $U(1)_X$ gauge symmetry [19] or generating the seesaw scale in neutrino physics [18]. In this work we will first investigate some general features of the YGMSB and then focus on the models with Higgs-messenger coupling, where the MFV is automatically maintained. We find that in such models the Yukawa interactions can give large $-A_t$ and $-m^2_{\tilde{t}_L,R}$, driving the stop sector towards maximal mixing. However, this will lead to a large $m^2_{H_u}$, rendering the electroweak symmetry breaking (EWSB) inconsistent with the large stop mixing. To tackle this problem, we explore two realistic hidden sectors by introducing a new strong gauge dynamics or using $(10, \overline{10})$ messengers.

The paper is organized as follows. In Section II we present some general insights into the SUSY breaking soft spectrum of the YGMSB and discuss the application to the MSSM. In Section III we focus on the YGMSB with the Higgs bridge. The discussion and conclusion are given in Section IV. In appendices A and B we present some details of our calculations.

II. VISIBLE-HIDDEN YUKAWA COUPLINGS

In this section we first present a brief review on the basic technique used in this work and then give a general analysis for the features of the soft spectrum in the YGMSB.

A. The Wave Function Renormalization Method

The soft SUSY-breaking effect can be treated in a supersymmetric way [20] and the renormalized spurion superfields (e.g., the wave function $Z$), which encode the SUSY-breaking information, can be utilized to extract the soft terms [21]. Here the crucial observation [20] is that, after crossing the messenger threshold $M$, the wave function $Z$ develops the $\theta$–dependence through the replacement $M \to \sqrt{XX^\dagger}$, where $X = M + F\theta^2$ is the spurion field parameterizing the typical scales of the hidden sector and $\sqrt{F}(\ll M)$ characterizes the
To illustrate the method, we consider a visible field $Q$ with renormalized wave function $Z_Q$. The Kahler potential of $Q$ is

$$\mathcal{L} = \int d^4\theta Z_Q(X, X^\dagger, \mu)Q^\dagger Q,$$

where $\mu$ is the renormalization scale. We expand $Z_Q$ in $\theta$ and $\bar{\theta}$ and employ the field redefinition

$$Q' = Z_Q^{\frac{1}{2}} \left(1 + \frac{\partial \ln Z_Q}{\partial X} F \theta^2\right)|_{X=M} Q,$$

with $Z_Q$ being the scalar component of $Z_Q$. Now $Q'$ has a canonically normalized kinetic term and its soft mass square can be read from the coefficient of $\bar{\theta}^2\theta^2$:

$$\tilde{m}_Q^2(\mu) = -\frac{\partial^2 \ln Z_Q(X, X^\dagger, \mu)}{\partial \ln X \partial \ln X^\dagger} \bigg|_{X=M} \frac{FF^\dagger}{M M^\dagger} \equiv -Z_Q''|_{X=M} \frac{FF^\dagger}{M M^\dagger}. \tag{4}$$

If $Q$ interacts with the visible matters via an operator $\lambda QQ_1Q_2$, through the same manipulation we get a corresponding trilinear soft term $\lambda A\lambda QQ_1Q_2$ with

$$A_\lambda = \frac{\partial \ln Z_Q(X, X^\dagger, \mu)}{\partial \ln X} \bigg|_{X=M} \frac{F}{M} = Z_Q'|_{X=M} \frac{F}{M}. \tag{5}$$

Hereafter we define $F/M \equiv \Lambda$.

The derivatives $Z_Q'$ and $Z_Q''$ can be explicitly expressed in terms of the anomalous dimensions, the beta-functions of the couplings and their discontinuities. We formally integrate the one-loop RGE

$$\frac{\ln Z_Q(\mu)}{\ln Z_Q(\mu_0)} = \frac{1}{2} \int_{\ln \frac{\mu}{\Lambda_{UV}}}^{\ln \frac{\mu_0}{\Lambda_{UV}}} dt' \gamma^+_Q + \int_{\ln \frac{\mu}{\Lambda_{UV}}}^{\ln \frac{\mu_0}{\Lambda_{UV}}} dt' \gamma^-_Q,$$

where $\mu < M < \mu_0$. The above quantities denoted with superscripts $+$ and $-$ are respectively defined above and below the messenger mass scale. Then we obtain

$$\tilde{m}_Q^2|_{\mu=M} = \frac{1}{2} \sum_\lambda \left[ \beta^+_\lambda \frac{\partial (\Delta \gamma_Q)}{\partial \lambda} - \Delta \beta_\lambda \frac{\partial (\gamma_Q)}{\partial \lambda} \right]_{\mu=M} \Lambda^2, \tag{7}$$

$$A_\lambda(\mu)|_{\mu=M} = -\sum_Q \Delta \gamma_Q|_{\mu=M} \Lambda, \tag{8}$$

with $\Delta X = [X^+ - X^-]|_{\mu=M}$ and $\beta_\lambda = d\lambda/dt$. From the simple loop-factor counting one can find that the soft mass square and trilinear term respectively arise at the two-loop and one-loop level.
In the above derivations we have assumed that $\gamma^j_i$ is a diagonal matrix in the $Q_i$–flavor space. In this case, it is more convenient to rewrite the derivatives in Eq. (7) with respect to $\lambda^2$, and redefine the beta function as

$$\beta_\lambda = \frac{d\lambda^2}{dt} = 2\lambda^2 \sum_{Q_\lambda} \gamma_{Q_\lambda},$$

(9)

where $Q_\lambda$ runs over all fields participating the interactions involving $\lambda$. We will use this convention in the following. The previous discussions can be directly generalized to a more general situation where $\gamma^j_i$ develops non-diagonal elements [23].

**B. Some General Insights into Visible-Hidden Yukawa Couplings**

SUSY should be as natural as possible and thus the MSSM with light stops and gluino is preferred. However, the presence of a relatively heavy Higgs around 125 GeV requires rather heavy stops, which renders the fine-tuning worse than $\sim 1\%$ [4] (the fine-tuning can be alleviated in the NMSSM [5]). For the GMSB model, such a heavy Higgs boson is even more problematic, owing to the fact that no stop trilinear soft term is generated at the boundary. So, the stop sector should be properly modified, which at the boundary should have the following features:

- A large negative $A_t$. The negative sign is important and the reason can be explicitly found from the following discussions (see Eq. 25), i.e., if the initial $M_3$ and $A_t$ have opposite sign, at the weak scale $|A_t|$ will receive an enhancement from $M_3$.

- Reduced stop soft mass squares relative to the first and second family squarks. This helps to achieve the maximal stop mixing scenario with a relatively light stop sector.

In the following we will show that they can be elegantly realized in the framework of YGMSB.

1. **Basic Features of the Soft Spectrum in the YGMSB**

As mentioned in the introduction, the YGSMB has been used in different circumstances. The basic features of its soft spectrum are of crucial importance, especially the signs of the soft terms which are relevant to the discussion in this work. We simplify the discussion by ignoring the gauge interaction at the moment, which is valid in the large Yukawa coupling
limit. In fact, the gauge interaction contribution only appears in the beta functions, taking the form of \( \beta_\lambda = -\lambda^2 g^2 / 16 \pi^2 + \ldots \), and thus it can be easily traced back when necessary.

Through Yukawa interactions, the visible fields \( \phi_i \) can couple to the messengers \( \Phi_i \) in two ways: \( \phi_1 \Phi_2 \) and \( \phi_1 \phi_2 \Phi \). The field which directly couples to the messengers is dubbed as the bridge field, denoted by \( B \). Then the general YGMSB takes a form of the Wess-Zumino model:

\[
W = \left( \frac{\lambda_{ij}}{2} B_i B_j \Phi_a + \frac{\lambda'_{ab}}{2} B_i \Phi_a \Phi_b \right) + \frac{\kappa_{ijk}}{6} B_i B_j B_k + \frac{y_{ij}}{2} B_i B_j \phi_l + \frac{y'_{ilm}}{2} B_i \phi_l \phi_m. \tag{10}
\]

Here we use \( i/j/k \) for the bridge field indices, \( a/b/c \) for messenger indices while \( l/m/n \) for the light field indices (the light fields are the visible fields which couples to \( B \) unless specified otherwise). Moreover, each letter used to label the Yukawa coupling type is specified, e.g., \( \lambda \) is used to label the type with two-bridges and one-messenger. The light fields’ soft terms are given by

\[
-\mathcal{L}_{\text{soft}} = \frac{\kappa_{ijk} A_{kij}}{6} B_i B_j B_k + \frac{y_{ij}}{2} A_{ij} B_i B_j \phi_l + \frac{y'_{ilm}}{2} A_{ilm} B_i \phi_l \phi_m, \tag{11}
\]

where we have omitted the soft mass terms.

In Eq. (10), the bridge field \( B \) first encodes the SUSY-breaking information in its one-loop wave function. Then through Yukawa interactions, the information is mediated to the light field \( \phi \). In this picture, the chiral field \( B \) essentially plays the role of a force mediator, while in the pure GMSB the vector superfield is the mediator. This difference will lead to a remarkable difference in the soft terms between the GMSB and YGMSB.

Based on Eqs. (7) and (8), now we present an analysis for the structure of the soft terms from the Yukawa mediation. We will emphasize the signs of various terms as well as the possible cancelations between them. First of all, it is noticed that the Yukawa interactions contribute positively to the anomalous dimension. As a result, after the decoupling of the bridge-messenger interactions, we get \( \Delta \gamma_B > 0 \) for bridges and \( \Delta \gamma_\phi = 0 \) for the light fields. Using these properties, some inferences can be obtained:

- In light of Eq. (8), the \( A- \)term, which only depends on the discontinuities of the bridge fields \( \Delta \gamma_B \propto \lambda^2(\lambda'^2) \), always takes a negative sign.

- The anomalous dimension of the light field is smooth when it crosses the messenger threshold. Then, in terms of Eq. (7), only the second term which comes from the
discontinuities of $\beta_{y\phi_i}$ contributes to $m^2_{\phi_i}$:

$$m^2_{\phi_i} \sim -\frac{1}{(16\pi^2)^2} \lambda^2 y^2_{\phi_i}.$$  \hfill (12)

So it is definitely negative.

These two features are the main guidelines for the model building in this work.

The soft mass square of the bridge field is much more involved due to its dual identities: it is not only the force mediator but also a light field. Therefore, its soft mass square $m^2_B$ receives two kinds of contributions, as shown from Eq. (7). The subtle points come from the potential cancellations which will be discussed later. But since our primary interest is the general structure of $m^2_B$, we can explicitly find its expression, with details presented in Appendix A. From the general expression, we can decompose it into the following three parts (with a common factor $\Lambda^2/(16\pi^2)^2$ factored out):

1. The quartic terms of the visible-hidden coupling constants: $\lambda^4$, $\lambda^2\lambda'^2$, $\lambda'^4$. They are definitely positive and generically dominant in $m^2_B$ in the large $\lambda/\lambda'$ limit.

2. The cross terms $\lambda^2\kappa^2$ and $\lambda'^2\kappa^2$ (repeating index will be summed in the following unless specified otherwise):

$$\hat{\lambda}_{ij}\hat{\kappa}_j - 2\hat{\kappa}_{ij}\hat{\lambda}_j - \hat{\kappa}_{ij}\hat{\lambda}'_j,$$  \hfill (13)

where $\hat{\lambda}_{ij} \equiv \lambda_{ija}\lambda^{ija}$ with only the index $a$ summed over, and other quantities are defined similarly. Remarkably, the term $\lambda'^2\kappa^2$ always takes the negative sign, implying that if we work in a model with a proper structure, the dominant term given in the first item can be reduced. As a case in point, in the NMSSM with the singlet coupling to messengers, such a cancelation is important to trigger the EWSB [17].

3. The cross terms $\lambda^2y^2$, $\lambda^2y'^2$ and $\lambda'^2y^2$:

$$2\left(\hat{\lambda}_{ij}\hat{y}_j - \hat{y}_{ij}\hat{\lambda}_j\right) + \hat{\lambda}_{ij}\hat{y}'_j - \hat{y}_{ij}\hat{\lambda}'_j.$$  \hfill (14)

Whether or not the terms in the bracket can cancel is model dependent, but the third and last terms take definite signs. Anyway, using the general formula given in Appendix A it is easy to get the soft mass squares in a given model. Note that the term $\propto \lambda'^2y'^2$ vanishes as a result of cancellation.
In concrete models some Yukawa couplings will be turned off and thus the expressions can be greatly simplified. In the following the first and third item will be the focuses of our discussion.

Before ending this section, we remind that the wave function renormalization method cannot be used to extract the one-loop contribution for the soft mass square of $B$. Actually, it is model-dependent [16] and usually vanishes at the leading order of SUSY-breaking, say suppressed by $F^2/M^4$ [14]. In the following discussions we will ignore such a contribution.

2. Model Classification

Restricting our discussions within the MSSM and considering the phenomenological requirements, we classify the models into two basic types. One type contains matter bridges, especially the $q_3$ field, and the other type contains Higgs fields as the only bridges.

Here we consider the first type. The minimal messengers under consideration are $n$ pairs of vector-like particles, $(\bar{\Phi}_D, \Phi_D)$ and $(\bar{\Phi}_L, \Phi_L)$, where $\Phi_D$ and $\bar{\Phi}_L$ carry the same SM gauge group charges as $D_i^c$ and $L_i$, respectively. $\Phi = (\Phi_D, \Phi_L) \sim 5$ forms a complete multiplet of the $SU(5)$ grand unification theory (GUT). The SM gauge invariance allows for the following general superpotential

$$W = \sum_{i=1}^{n} \eta_i X \Phi_i \bar{\Phi}_i + W_{1,2m} + W_{MSSM},$$

where the first term denotes the ordinary hidden sector and $W_{MSSM}$ consists of the MSSM Yukawa interactions $Y^u Q U^c H_u + Y^d Q D^c H_d + Y^e L E^c H_d$. The visible-hidden Yukawa couplings take the form of

$$W_{1m} = \lambda_{u,ij} Q_i \Phi_L U_j^c + \lambda_{d,ij} Q_i \Phi_L D_j^c + ...,$$
$$W_{2m} = \lambda_i Q_i H_d \bar{\Phi}_D + \lambda'_i Q_i \Phi_L \bar{\Phi}_D + ...,$$

with the dots being the couplings involving leptons. The terms in $W_{1m}$ are similar to the models studied in [23, 26], where $W_{1m}$ is due to the (large) Higgs-messenger mixings. $W_{2m}$ is a generalization of the Higgs-messenger mixing to the matter-messenger mixing. In such kind of models the dangerous high-dimensional operators, which may induce fast proton decay, could be forbidden with the help of additional symmetries.
The direct visible-hidden couplings may incur large flavor violations and undermine the original motivation of the GMSB. However, according to our above general analysis, the dangerous FCNCs can be avoided if the flavor structure in $W_{1,2m}$ is such that the same set of messengers only significantly couple to one single family of matters. For example, in the context of messenger-Higgs mixing [23], the flavor structure in $W_{1m}$ is identical to the MSSM counterpart, i.e., $\lambda_{f,ij} \sim Y_{ij}'$. Therefore, effectively only the third family couples to the messengers due to the family hierarchy of the SM Yukawa couplings. Actually, the flavor violations in this kind of models respect the MFV.

We would like to point out that the YGMSB potentially is able to provide a natural SUSY spectrum [27] without FCNC problems. This is realized by taking the first two families of matters as bridges which couple to the messengers:

$$W_{\text{VH}} \supset \lambda_{10,a} 10_a \Phi_a \Phi_a + \lambda_{5,a} 5_a \Phi_a (a = 1, 2),$$

where $10_a$ and $5_a$ are the matter superfields in the $SU(5)$ model. In a complete model, a flavor symmetry should naturally account for the above Yukawa coupling structure. Provided that $\lambda_{10,a}, \lambda_{5,a} \sim 1$, then according to the analysis in Section II B 1, the sfermion masses of the first two generations obtain large and dominant positive contributions from the Yukawa mediation. But the third generation sfermion masses still originate from the ordinary GMSB, and can be much lighter than the first two generations of sfermions. This kind of realization of natural SUSY may be easier than those in [24, 25].

In the following we turn our attention to the main point of this work, namely the second type in which the Higgs bridges the visible and hidden sectors. One of the main features of this type is that the resulted soft terms automatically satisfy the MFV since here the small sfermion flavor violations originate from the flavor violations in the SM. For example, the up-type squark mass squares take the form of $m_{\tilde{u}_{ij}}^2 \propto \lambda_u^2 (y_u)^{ij}$ with $\lambda_u$ being the $H_u$-messenger Yukawa couplings. In the proceeding section we discuss in depth this type of models and study their phenomenological features. We will start from a toy model and then propose two simple modifications on the hidden sector to obtain the realistic YGMSB with Higgs bridge.
III. THE YGMSB WITH HIGGS BRIDGE (H-YGMSB)

A. A Toy Model with Higgs Bridge

To show the main features of the $H-$YGMSB, we start from a toy model. First of all, in order to couple the Higgs fields with the messengers, the minimal messenger content must be extended. For this purpose, two SM singlets ($S, \bar{S}$) are introduced and they couple to the goldstino superfield via $\eta_S X S \bar{S}$. Then the Higgs-messenger couplings are

$$W_H = \lambda_u S \Phi_L H_u + \lambda_d \bar{S} \Phi_L H_d.$$  \hspace{1cm} (18)

Such a structure originally is motivated by the possible solution to the $\mu/B\mu-$problem [15, 16]. But here we do not try to solve this problem, and instead evade it by turning off $\lambda_d$ (which is not an important parameter in this work) and treat $\mu/B\mu$ as free parameters. Alternatively, one can just introduce one singlet and get the coupling

$$W'_H = \lambda S H_u H_d.$$  \hspace{1cm} (19)

The basic features of these two models in Eq.(18) and Eq.(19) are quite similar, as shown in Appendix B. However, we find that $W_H$ is more preferred for building realistic models. Therefore, in the following we focus on $W_H$ (recently some aspects of $W'_H$ were studied in [35]).

We would like to make a comment. A proper symmetry should be introduced to guarantee that the messengers only couple to the Higgs rather than the matters. It does not give rise to a new problem, since it amounts to how to distinguish the Higgs and matters, e.g., $H_d$ and $L_i$, which also should be addressed in the MSSM. The well-known solution is imposing some symmetry such as the $R-$parity, $U(1)_{PQ}$ or $U(1)_R$, etc., on the model.

To calculate the soft spectrum in the YGMSB, we should work out the discontinuities of the anomalous dimensions of the relevant fields. Above the messenger scale $M$, they are
given by (despite our assumption $\lambda_d = 0$, we still list the relevant quantities for completeness)

\[
\begin{align*}
\gamma_{\bar{H}_u}^{+} &= \frac{1}{16\pi^2} \left[ \lambda_u^2 + 3h_t^2 - 2C_2 g_2^2 - 2(1/2)^2(3/5)g_1^2 \right], \\
\gamma_{\bar{H}_d}^{+} &= \frac{1}{16\pi^2} \left[ \lambda_d^2 + 3h_b^2 + h_\tau^2 - 2C_2 g_2^2 - 2(1/2)^2(3/5)g_1^2 \right], \\
\gamma_{\Phi_L}^{+} &= \frac{1}{16\pi^2} \left[ \lambda_u^2 - 2C_2 g_2^2 - 2(1/2)^2(3/5)g_1^2 \right], \\
\gamma_{\bar{\Phi}_L}^{+} &= \frac{1}{16\pi^2} \left[ \lambda_d^2 - 2C_2 g_2^2 - 2(1/2)^2(3/5)g_1^2 \right], \\
\gamma_{\bar{S}}^{+} &= \frac{1}{16\pi^2} 2\lambda_u^2, \quad \gamma_{\bar{S}}^{+} = \frac{1}{16\pi^2} 2\lambda_d^2,
\end{align*}
\]

where $C_2 = 3/4$ and $C_3 = 4/3$ are the quartic Casimirs for $SU(2)_L$ and $SU(3)_C$, respectively. Below the scale $M$ the messengers decouple, and hence $\gamma^\phi_\bar{\phi}$ of the bridges and light fields are obtained from the corresponding $\gamma^\phi_\phi$ by setting $\lambda_{u,d} \rightarrow 0$. Then, with Eq. (20), the Yukawa-mediated SUSY-breaking soft terms can be calculated in light of Eq. (7). In the following we present them and analyze their implications.

1. The Maximal Mixing Stop Sector with a Light Stop

We look at the stop sector which is of our main interest. Compared to the situation in the pure GMSB, it is modified towards the desired form outlined at the beginning of this section even if we work in the $H - YGMSB$ with a single term $\lambda_u S \bar{\Phi}_L H_u$. First, at the one-loop level a large negative $A_t$ is generated at the boundary

\[
A_t = -\frac{\Lambda}{16\pi^2} \lambda_u^2, \quad A_b = A_\tau = -\frac{\Lambda}{16\pi^2} \lambda_d^2.
\]

Note that they are universal to three generations. Next, the stops, together with other third family sfermions, obtain sizable negative contributions:

\[
\begin{align*}
\Delta m_{\tilde{Q}_3}^2 &= -\frac{\Lambda^2}{(16\pi^2)^2} \left( h_t^2 \lambda_u^2 + h_b^2 \lambda_d^2 \right), \\
\Delta m_{\tilde{U}_3}^2 &= -\frac{2\Lambda^2}{(16\pi^2)^2} h_t^2 \lambda_u^2, \quad \Delta m_{\tilde{D}_3}^2 = -\frac{2\Lambda^2}{(16\pi^2)^2} h_b^2 \lambda_d^2.
\end{align*}
\]

By contrast, the Yukawa-mediation contributions to the first two families of sfermions are negligible since they couple to the Higgs very weakly.

Given the above modifications to the stop sector, the maximal mixing scenario can be realized in this toy model. Taking into account the RGE effect, the weak-scale stop parameters
can be parameterized as [5]

\[ m_{\tilde{Q}_3}^2 \approx C_{g1} M_3^2 + C_{L1} \tilde{m}_{\tilde{Q}_3}^2 - C_{R1} \tilde{m}_{U_3}^2, \]
\[ m_{\tilde{U}_3}^2 \approx C_{g2} M_3^2 - C_{L2} \tilde{m}_{\tilde{Q}_3}^2 + C_{R2} \tilde{m}_{\tilde{U}_3}^2, \]  

(24)

where the quantities in the right side are defined at the scale \( M \) (hereafter we will use this convention for the RGE effect estimations). In addition, the stop sector trilinear term takes the form of

\[ A_t \approx C_A \tilde{A}_t - C_{g1} M_3. \]  

(25)

In the above equations, \( C_i \) are positive numbers, determined by the MSSM Yukawa and gauge couplings as well as \( M \). There are hierarchies \( C_{R1} \ll C_{L1} \), \( C_{R2} \ll C_{L2} \), \( C_A \rightarrow 1 \) while others are negligible; as \( M \) increases (say to \( \gtrsim 10^{12} \) GeV), \( C_{L1} \sim C_{R2} \) are reduced no more than half, but \( C_{R1}, C_{L2}, C_A \) are generated at \( O(0.1) \).

Note that for a high scale \( M \) the gluino effect is significant and roughly \( C_{g1} \approx C_{g2} \gg C_{gA} \approx 1 \). With these approximate features we simplify Eq. (24) as

\[ m_{\tilde{Q}_3}^2 \approx C_{L1} (\delta_g - 1) |\Delta m_{\tilde{Q}_3}^2|, \quad m_{\tilde{U}_3}^2 \approx C_{R2} (\delta_g - 2) |\Delta m_{\tilde{Q}_3}^2| > 0, \]

(26)

where the \( \delta_g \) terms approximately measure the \( SU(3)_C - GMSB \) and gluino contributions.

We now can see how the \( H-YGMSB \) accommodates the maximal stop mixing. It is noticed that in the stop mass square matrix the difference between the diagonal entries is \( \sim |\Delta m_{\tilde{Q}_3}^2| \), which is much larger than the non-diagonal entries \( m_t |A_t| \). Consequently, its heavier and lighter eigenvalues can be approximated to be \( m_{\tilde{Q}_3}^2 \) and \( m_{\tilde{U}_3}^2 \), respectively. And then the degree of mixing is estimated as

\[ x_t^2 \equiv \frac{X_t^2}{m_t^2} \sim \frac{A_t^2}{|\Delta m_{\tilde{Q}_3}^2|} \left[ C_{L1} C_{R2} (\delta_g - 2) \right]^{-1/2}, \]

(27)

where the term \( (\delta_g - 1)^{-1/2} \) has been neglected. Considering a quite low scale \( M \), we then get \( x_t^2 \approx (\lambda_u^2 / h_t^2) (\delta_g - 2)^{-1/2} \) with good approximation. Since \( \lambda_u \) is only allowed to be mildly larger than the gauge coupling of \( SU(3)_C \) due to the bound \( m_{\tilde{U}_3}^2 > 0 \), the maximal mixing \( x_t^2 \approx 6 \) requires an enhancement from \( (\delta_g - 2)^{-1/2} \sim O(3) \). This enhancement comes from the negative stop soft mass square contributions from the \( H-YGMSB \). So, our scenario is quite different from the one proposed in the top-bridge models [23, 26] where the stop soft mass squares are increased and thus one needs a rather large \( |A_t| \) (then a rather large \( \lambda_u \)), to lift up \( x_t^2 \). In our Higgs-bridge model the condition \( x_t^2 \approx 6 \) can be realized while a light stop
is maintained, which is favored by naturalness. Note that we introduce a new fine-tuning at the boundary, namely the cancellation between the gauge- and Yukawa-contributions to the stop soft mass squares. But this tuning is quite mild, estimated to be \( \delta_g - 2 \sim 0.1 \).

Some comments are in orders. In the MSSM \( h_t \sim 1 \) correlates the naturalness of the weak scale with \( m_h \) [5], and \( m_h \simeq 125 \) GeV means a large fine-tuning of the MSSM, especially in the GMSB case. Interestingly, in the \( H - Y \)GMSB this large \( h_t \) helps to relax the correlation and thus may alleviate the naturalness. But we still suffer a rather large fine-tuning. The weak scale \( m_Z \) is affected by the stops via the RGE:

\[
\frac{m_Z^2}{2} \sim C_G m^2_{\tilde{t},G} - C_Y m^2_{\tilde{t},Y} + \ldots, \tag{28}
\]

where the subscript \( Y \) and \( G \) denote the boundary soft terms from the gauge and Yukawa mediations respectively. We have \( C_G \sim 0.5 \) even if \( M \) is as low as 100 TeV. Furthermore, \( m_{\tilde{t},G} \) should be around the TeV scale (in order to lift up \( m_h \) and satisfy the LHC bounds on the squarks and gluino). Therefore, tuning at a level of 1\% is unavoidable and we need further exploration on a sufficiently natural model.

2. The Problematic Radiative EWSB

If the \( H - Y \)GMSB is required to give a relatively heavy SM-like Higgs with relatively light stops, it will be difficult to realize the radiative EWSB. As is well known, the successful EWSB should satisfy the following two equations:

\[
\frac{m_Z^2}{2} \approx m^2_{H_d} - \frac{\tan^2 \beta m^2_{H_u}}{\tan^2 \beta - 1} - \mu^2 \approx -m^2_{H_u} - \mu^2, \tag{29}
\]

\[\sin 2\beta = \frac{2B\mu}{m^2_{H_u} + m^2_{H_d} + 2\mu^2}. \tag{30}\]

Here the Higgs parameters are defined at the electroweak scale, and the soft mass squares can be expressed as (similar to Eq. 24)

\[
m^2_{H_u} \approx 0.62 \tilde{m}^2_{H_u} - 1.10 M^2_3 - 0.10 \tilde{A}^2_t - 0.37 \tilde{m}^2_Q - 0.32 \tilde{m}^2_{U^c} \quad (\text{for } M = 10^{12} \text{ GeV}),
\]

\[
m^2_{H_u} \approx 0.80 \tilde{m}^2_{H_u} - 0.15 M^2_3 - 0.12 \tilde{A}^2_t - 0.20 \tilde{m}^2_Q - 0.18 \tilde{m}^2_{U^c} \quad (\text{for } M = 10^6 \text{ GeV}). \tag{31}\]

The parameter \( m^2_{H_d} \) is approximated as its boundary value. Since \( B\mu \) is regarded as a free parameter, Eq. (30) can always be satisfied. In the ordinary GMSB, Eq. (29) is also satisfied
since the significant RGE effects from the heavy colored spartiles drive $m_{H_u}^2$ to be negative at the low energy, as shown in Eq. (31).

However, in the $H$−YGMSB the soft mass squares of the Higgs bridges receive large positive contributions from Yukawa mediations:

$$\Delta m_{H_u}^2 = \frac{\Lambda^2}{(16\pi^2)^2} \left[ \lambda_u^2 \left( 4\lambda_u^2 - 3(g_2^2 + \frac{1}{5}g_1^2) \right) \right],$$

$$\Delta m_{H_d}^2 = \frac{\Lambda^2}{(16\pi^2)^2} \left[ \lambda_d^2 \left( 4\lambda_d^2 - 3(g_2^2 + \frac{1}{5}g_1^2) \right) \right].$$

Compared to $\Delta m_{\tilde{U}_3}^2$ shown in Eq. (23), $\Delta m_{H_u}^2$ takes an opposite sign and additionally is enhanced by the factor $2\lambda_u^2/h_t^2$. As a consequence, the realization of stop maximal mixing scenario is inconsistent with the radiative EWSB. To see this clearly, using Eq. (22) we explicitly rewrite Eq. (31) as

$$m_{H_u}^2 \sim 2.48(\lambda_u^2/h_t^2)|\Delta m_{Q_3}^2| - 1.01M_3^2 \quad \text{(for } M = 10^{12}\text{ GeV}),$$

$$m_{H_u}^2 \sim 3.20(\lambda_u^2/h_t^2)|\Delta m_{Q_3}^2| - 0.15M_3^2 \quad \text{(for } M = 10^6\text{ GeV}).$$

Here the dependence on $M_3^2$ arises at two-loop, and therefore its coefficient is expected to be smaller than the coefficients $C_{g_1} \sim C_{g_2}$ in $m_{\tilde{q}_3}^2$, which arise at one-loop. This fact, combined with the stop maximal mixing condition, allows us to find a bound on $m_{H_u}^2$:

$$m_{H_u}^2 > (2.48\lambda_u^2/h_t^2 - 2C_{R2}) |\Delta m_{Q_3}^2| > 0 \quad \text{(for } M = 10^{12}\text{ GeV}),$$

where $C_{R2} < 1$ and $\lambda_u > h_t$ are used. This bound becomes stronger as the messenger scale lowers and thus the EWSB is not consistent with the maximal stop mixing in the toy model of $H$−YGMSB. It is noticed that a higher scale $M$ helps to lower $m_{H_u}^2$ and hence benefits the radiative EWSB.

3. The muon anomalous magnetic moment from the light smuon

Before presenting realistic models, we introduce another potential merit of the spectrum of the $H$−YGMSB. It may account for the muon anomalous magnetic moment $a_\mu \equiv (g_\mu - 2)/2$, which can be regarded as a harbinger of new physics. Its experimental value [28] and the SM prediction [29] are given by

$$a_\mu^{\text{exp}} = 11659208.9(6.3) \times 10^{-10}, \quad a_\mu^{\text{SM}} = 11659182.8(4.9) \times 10^{-10}.$$
Their discrepancy implies that the new physics contribution should be

\[ \delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (26.1 \pm 8.0) \times 10^{-10}. \]  

(37)

Within the MSSM the chargino and neutralino loops give the dominant contributions [30]

\[ \delta a_{\mu}^{\text{MSSM}} \simeq \frac{g_2^2 m_\mu^2 M_2 \mu \tan \beta}{8 \pi^2 m_\mu^4}. \]  

(38)

In the MSSM with GMSB, since a SM-like Higgs boson around 125 GeV significantly pushes up the sparticle masses (including the left-handed smuon mass), it is hard to give the required contribution.

In the \( H - \text{YGMSB} \) the smuon mass can be lowered considerably. Then with a properly large \( \mu \) and \( M_2 \), \( \delta a_{\mu}^{\text{MSSM}} \) might be able to reach the required value in Eq. (37). However, we note that the trace \( S \equiv \text{Tr}(Y_f \bar{m}_f^2) \), which vanishes in the pure GMSB due to the anomaly-free of \( U(1)_Y \), is now given by

\[ S \simeq \bar{m}_{H_u}^2 - \bar{m}_{H_d}^2 + \bar{m}_{Q_3}^2 - 2 \bar{m}_{U_e}^2 + m_{D_e}^2 - \bar{m}_{\ell_3}^2 + \bar{m}_{E_e}^2 \]

\[ \simeq \frac{\Lambda^2}{(16 \pi^2)^2} \left[ \lambda_u^2 (4 \lambda_u^2 + 3 h_t^2) - \lambda_d^2 (4 \lambda_d^2 + 3 h_b^2) \right]. \]  

(39)

It takes a large and positive value by virtue of the contribution \( \Delta m_{H_u}^2 \). Therefore, by means of the RGE effect it will family-universally increase the masses of the sparticles with negative \( U(1)_Y \)–charge (including \( \bar{\mu}_L \)). So in this toy model of \( H - \text{YGMSB} \) it is also hard to give the required contribution to muon \( g - 2 \). Note that this difficulty arises from the large positive \( \Delta m_{H_u}^2 \) and thus has the same origin as the problem of radiative EWSB. By contrast, in the top-bridge models \( \Delta m_{H_u}^2 \) is negative and the contribution to muon \( g - 2 \) can be enhanced more readily [26].

**B. Realistic Hidden Sectors for the \( H - \text{YGMSB} \)**

To solve the EWSB problem in the simplest Higgs bridge model, we modify the messenger structure. In the following we present some simple and realistic modifications for the toy model given above, based on the crucial observation that the gauge interaction of the messengers can decrease \( m_{H_u}^2 \).
1. Introducing a Hidden (Strong) Gauge Group

As the first modification, we assume that the messengers \((S, \bar{S})\) and \((\Phi, \bar{\Phi})\) introduced in the toy model are charged under a hidden gauge group \(G_h\) with gauge coupling \(g_h\) (they form vector-like representations under \(G_h\) for the sake of anomaly cancelation) while visible fields are neutral. Although the model has the same superpotential as \(W_H\) in Eq. (18), the presence of \(G_h\), say \(SU(N)\), brings great difference. Now the anomalous dimensions above the messenger scale are modified to be

\[
\gamma_{H_u}^+ = \frac{1}{16\pi^2} \left[ N\lambda_u^2 + 3h_i^2 - 2C_2g_2^2 - 2(1/2)^2(3/5)g_1^2 \right],
\]

\[
\gamma_{\Phi^L}^+ = \frac{1}{16\pi^2} \left[ \lambda_u^2 - 2C_h\lambda_h^2 - 2C_2g_2^2 - 2(1/2)^2(3/5)g_1^2 \right],
\]

\[
\gamma_{S_\bar{S}}^+ = \frac{1}{16\pi^2} \left[ 2\lambda_u^2 - 2C_h\lambda_h^2 \right],
\]

(40)

where \(C_h = (N^2 - 1)/2N\) is the quadratic Casimir group invariant for the superfield in the (anti-)fundamental representation under \(G_h = SU(N)\). For the Abelian \(G_h\), \(C_h = Q_\phi^2\) with \(Q_\phi\) being the \(G_h\) charge of \(\phi\). The messengers’ anomalous dimensions decrease due to their hidden gauge interactions, but for the Higgs bridges, which are neutral under \(G_h\), their anomalous dimensions are not affected. Note that in \(\gamma_{H_u}^+\) the term \(\propto \lambda_u^2\) is enhanced by the messenger number \(N\).

By virtue of \(G_h\), the Higgs bridges get the desired negative soft mass squares (for comparison, see Eq. 32):

\[
\Delta m_{H_u}^2 = \frac{NA^2}{(16\pi^2)^2} \left[ \lambda_u^2 \left( (N + 3)\lambda_u^2 - 4C_h\lambda_h^2 - 3\left(g_2^2 + \frac{1}{5}g_1^2\right) \right) \right],
\]

\[
\Delta m_{H_d}^2 = \frac{NA^2}{(16\pi^2)^2} \left[ \lambda_d^2 \left( (N + 3)\lambda_d^2 - 4C_h\lambda_h^2 - 3\left(g_2^2 + \frac{1}{5}g_1^2\right) \right) \right].
\]

(41)

(42)

\(G_h\) does not affect \(\beta_{g_{\phi_1}}\) and \(\gamma_{\phi_1}\), and thus the soft terms of the light fields, especially the terms in the stop sector, are the same as in the toy model except for an overall factor \(N\). Note that all the above discussions are valid only when \(G_h\) is broken below the messenger scale, which can be realized easily and will not be discussed further in this work.

We now look at the consistency of introducing \(G_h\) and check the constraints. Generically, \(\lambda_u \gtrsim 1\) is needed to get the maximal stop mixing, but such a large Yukawa coupling at the low scale potentially spoils the perturbativity of the theory up to the GUT scale. The presence of the hidden strong gauge group can greatly improve the situation. This can be
explicitly seen from
\[ \beta_{\lambda_u} \approx \frac{\lambda_u^2}{8\pi^2} \left[ (N + 3)\lambda_u^2 + 3h_i^2 - 4C_h g_h^2 \right] . \]

(43)

Here a large \( g_h \) can cancel a large part of the Yukawa term contribution and hence prevent \( \lambda_u \) from the Landau pole below the GUT scale. To realize the substantial cancelation, we may need \( g_h \gtrsim 1 \). But this does not mean that \( G_h \) will quickly run into the strong coupling region. Actually, the beta-function of \( g_h \) is

\[ b_h > (1 + 5) \times 2/2 - 3 \times N = 3(2 - N) , \]

(44)

where the factor 5 is due to the fact that \( \Phi \) is in the fundamental representation of \( SU(5) \). Thus for \( N \geq 2 \) we obtain \( b_h \leq 0 \) and consequently the \( G_h \) gauge dynamics is asymptotic free or conformal. In addition, \( G_h \) distinguishes the messengers from the visible fields with identical SM gauge group charges and thus forbids their dangerous mixings. In a word, the \( H-YGMSB \) equipped with a hidden gauge group is an attractive framework.

In the following we present some numerical analysis for the above model, using the code SuSpect [32]. We take the top quark pole mass as 174.1 GeV, and choose \( N = 2 \) and a relatively low messenger scale \( M = 10^6 \) GeV for the sake of naturalness [5].

As shown in the left panel of Fig. 1, a relatively heavy Higgs boson requires a relatively large \( \Lambda \), which is expected. The considerable cancelation between the contributions from the hidden gauge interaction and Yukawa interaction in Eq. (41) is reflected in the right panel in Fig. 1. From it one can see that the allowed parameter space for \( \lambda_u \) and \( g_h \) is rather small, and moreover it shrinks as the Higgs mass increases.

FIG. 1: Scatter plots of viable parameter space projected on the planes of \( \lambda_u \) versus \( \Lambda \) (left panel) and \( g_h \) (right panel). Here we choose \( M = 10^6 \) GeV, \( \lambda_d = 0 \), and \( \tan \beta = 25 \).
In Fig. 2 we project the parameter space on the planes of the stop mass versus $x_t$ and the Higgs mass. This figure shows that both properly heavy stops and sizable stop mixing are required to lift up $m_h$. For example, when $m_h > 126$ GeV, the light stop mass needs to be at least 700 GeV even in the maximal mixing scenario $x_t \simeq -2.5$. But for a moderately heavy Higgs $m_{\tilde{t}_1}$ typically is far below 1 TeV provided significant stop mixing, and such a light stop may be accessible at the LHC [31]. This is contrary to the ordinary GMSB where very heavy stops are needed [10]. In addition, the lightest slepton (in this model it is the right-handed stau with mass varying in the region 100-300 GeV), typically the next-to-lightest supersymmetric particle, may also be accessible at the LHC. The other colored sparticles are rather heavy, say 2 TeV, and can satisfy the present LHC search bounds.

FIG. 3: Same as Fig.1, but showing $\delta a_{\mu}$ versus the Higgs mass for $\tan \beta = 25$ (left panel) and $\tan \beta = 35$ (right panel). The solid line in each panel denotes the 1σ lower limit of $\delta a_{\mu}$.

Fig 3 shows the prediction of $\delta a_{\mu}$ versus the Higgs mass. From the figure we clearly see
the trend that $\delta a_\mu$ becomes smaller as $m_h$ gets heavier, and the reason has been explained in Section III A 3. With a sufficiently large $\tan \beta$ and for $m_h \lesssim 123$ GeV the model can reach the $1\sigma$ lower limit.

2. Variant Messenger Representation

Instead of introducing extra strong gauge dynamics, we can implement the idea of gauge-Yukawa cancelation by varying the messenger representation. We consider the variant hidden sector with messengers forming the $SU(5)$ representation $(10, \overline{10})$, which are decomposed to the SM components as $10 = (Q_\Phi, E_\Phi, U_\Phi)$. The Higgs-messengers couplings now are given by

$$W_{hid} \supset \lambda_u Q_\Phi H_u + \lambda_d \overline{Q}_\Phi H_d \overline{U}_\Phi.$$  \hspace{1cm} (45)

First, with such a messenger content, the pure gauge mediated contributions to the soft mass terms are

$$m_f^2 = 2 \times 3 \left[ C_3 \left( \frac{\alpha_3}{4\pi} \right)^2 + C_2 \left( \frac{\alpha_2}{4\pi} \right)^2 + 2 \times \frac{5}{3} \left( \frac{Y}{2} \right)^2 \left( \frac{\alpha_Y}{4\pi} \right)^2 \right] \Lambda^2,$$  \hspace{1cm} (46)

$$M_3 = \frac{\alpha_3}{4\pi} 3\Lambda, \quad M_2 = \frac{\alpha_2}{4\pi} 3\Lambda, \quad M_1 = \frac{\alpha_Y}{4\pi} 5\Lambda.$$  \hspace{1cm} (47)

Roughly speaking, the messenger number is 3 in this model. Next, the Higgs bridges receive extra contributions which are proportional to the $SU(3)_C$ gauge coupling $g_3$:

$$\Delta m_{H_u}^2 = \frac{3\lambda_u^2\Lambda^2}{(16\pi^2)^2} \left[ 6\lambda_u^2 - 4C_3g_3^2 - 4C_2g_2^2 - (13/15)g_1^2 \right],$$  \hspace{1cm} (48)

$$\Delta m_{H_d}^2 = \frac{3\lambda_d^2\Lambda^2}{(16\pi^2)^2} \left[ 6\lambda_d^2 - 4C_3g_3^2 - 4C_2g_2^2 - (13/15)g_1^2 \right].$$  \hspace{1cm} (49)

As expected, these results can be reproduced from Eq. (41) by taking $N = 3$. The $SU(3)_C$ contributed term can typically reduce $6\lambda_u^2$ by about 90% if $\lambda_u \lesssim 1$ and thus make the EWSB viable. From Fig. 4 we see that most samples are constrained to lie around $\lambda_u \sim 1$.

Numerically this model is more attractive for its single new parameter (we have set $\lambda_d = 0$ as before). But here the messenger mass scale $M$ is an important parameter for the sake of radiative EWSB (see the relevant discussion in Section III A 2). Thus for comparison we take two cases $M = 5 \times 10^8$ GeV and $M = 5 \times 10^{12}$ GeV. $\tan \beta = 25$ is fixed. Then some observations are obtained:
FIG. 4: Scatter plots of viable parameter space projected on the planes of \( \lambda_u \) versus \( \Lambda \). The messenger mass scale is fixed to be \( 5 \times 10^8 \) GeV for the left panel and \( 5 \times 10^{12} \) GeV for the right.

- Practically, Fig. 4 is a contour plot of \( m_h \) on the \( \lambda_u - \Lambda \) plane. For a given \( m_h \), there is a corresponding curve, e.g., the borderline between the green and red region labeling the \( m_h = 123 \) GeV curve. In each curve, the case with a smaller \( \Lambda \) but larger \( \lambda_u \) reflects that the maximal mixing scenario works. But the degree of mixing is clearly competing with the EWSB, and a higher messenger scale helps to relieve their tension, as is shown in Fig. 5. Note that Fig. 4 has revealed this tension: in case of \( M = 5 \times 10^8 \) GeV we need a large \( \Lambda \) (heavy stops) and \( \lambda_u \) (significant stop mixing) to give \( m_h = 126 \) GeV, which makes the EWSB very difficult. We find only a few points have \( m_h \gtrsim 126 \) GeV. By contrast, for \( M = 5 \times 10^{12} \) GeV case \( m_h \gtrsim 126 \) GeV can be accommodated more readily.

FIG. 5: Same as Fig.4, but projected on on the planes of the light stop mass \( m_{\tilde{t}_1} \) versus \( x_t \equiv X_t/m_{\tilde{t}} \).

- From the muon \( g - 2 \), this model is not so attractive, as shown by Fig. 6. In this model the smuon generically is heavier than in the previous model. Also, we usually have a
FIG. 6: Same as Fig. 4, but showing $\delta a_\mu$ versus the Higgs mass. The solid line in each panel denotes the 1$\sigma$ lower limit of $\delta a_\mu$.

smaller $\mu$ ($\lesssim$ 1 TeV) since it is determined by $|m^2_{H_u}|$ which typically is relatively small due to the difficulty in triggering EWSB. Thus the muon $g - 2$ is hard to explain in this model.

Thus, the $H$–YGSM with $(10, \bar{10})$ messenger content is viable given a sufficiently high messenger scale. However, compared to the previous model, the degree of stop mixing is limited due to the EWSB constraint. Additionally, the muon $g - 2$ can not be accommodated. Overall, the model with a new gauge dynamic is favored.

IV. CONCLUSION

If the SM-like Higgs mass is indeed around 125 GeV, then the MSSM with pure GMSB must have very heavy stops, which can be improved in the framework of YGSM. In this work we first investigated some general features of the soft spectrum of the YGSM, and then focused on the YGSM with Higgs-messenger interactions. We found that such models are attractive from several aspects: (i) They automatically maintain the MFV; (ii) The Yukawa mediation generates a large $-A_t$ and a large $-m^2_{\tilde{t}_{L,R}}$ simultaneously, driving the stop sector towards the maximal mixing region; (iii) Stop can be light and thus may be accessible at the LHC. However, generically $m^2_{H_u}$ is too large and makes the EWSB inconsistent with a large stop mixing. So we further explored two kinds of realistic hidden sectors: one with a new strong gauge dynamics and the other has a variant messenger representation $(10, \bar{10})$. Some numerical studies were presented for these models.
Finally, we make some remarks:

- Although our YGMSB models have attractive phenomenology and can simply accommodate a more natural SUSY, they challenge the conventional secluded hidden sector dynamics and may not be compatible with the popular dynamical SUSY-breaking models like the simple ISS model. Basically, this incompatibility is owing to the fact that the hidden sector fields (usually) are composite degree of freedoms while the SM gauge dynamics is only a spectator to the hidden sector dynamics. To circumvent the problem, one may turn to the composite third family [25].

- In this work we focused on the Higgs mass in the MSSM, but the Higgs mass alone is not enough to distinguish the MSSM from other supersymmetric models such as the NMSSM. Then we need other observables, for example, the di-photon signal rate from the Higgs boson decays [33].

- We note that very recently there are some discussions on the vacuum stability problem in extended GMSB models [34], but in our work we did not take this bound into account.

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Appendix A: General Formulas for Soft Terms

A general formula for the soft terms in the YGMSB can be obtained. The model and notation conventions are given in Eqs. (10) and (11). First, the anomalous dimensions above the messenger scale are

\[
\gamma_{B_i}^+ = \tilde{\lambda}_i + \frac{1}{2} \tilde{\lambda}'_i + \frac{1}{2} \tilde{\kappa}_i + \tilde{y}_i + \frac{1}{2} \tilde{y}'_i, \\
\gamma_{\Phi^a}^+ = \frac{1}{2} \tilde{\lambda}_a + \tilde{\lambda}'_a, \quad \gamma_{\phi^m}^+ = \frac{1}{2} \tilde{y}_m + \tilde{y}'_m, \tag{A1}
\]
where the contactor \( \hat{\lambda}_{ij} \equiv \lambda_{ija}\lambda_{ija} \), with only “a” summed over. Similarly, the omitted indices should be summed and the quadratic symbols \( \hat{\kappa} \) and \( \hat{y} \) are used in the following. Below the messenger scale, the anomalous dimensions for light fields are obtained by turning off \( \lambda \) and \( \lambda' \).

Using the wave function renormalization method mentioned before, the soft mass square could be obtained. First, we give the corrections for bridge field divided by three parts explicitly, i.e., \( m_{B_i}^2 = m_1^2 + m_2^2 + m_3^2 \) where \( m_1^2 \), \( m_2^2 \), and \( m_3^2 \) are the terms proportional to \( \lambda^4 \), \( \lambda^2\kappa^2 \), and \( \lambda^2y^2 \) respectively (we neglect the kinetic mixing for simplicity)

\[
\begin{align*}
m_1^2 & = \frac{\Lambda^2}{512\pi^4} \left[ 2\hat{\lambda}_{ija}(\Delta\gamma_{B_i} + \Delta\gamma_{B_j} + \Delta\gamma_{\Phi_a}) + \hat{\lambda}'_{aib}(\Delta\gamma_{B_i} + \Delta\gamma_{\Phi_a} + \Delta\gamma_{\Phi_b}) \right], \\
m_2^2 & = \frac{\Lambda^2}{512\pi^4} \left[ \hat{\lambda}_{ij}\hat{\kappa}_j - 2\hat{\kappa}_{ij}\hat{\lambda}_j - \hat{\kappa}_{ij}\hat{\lambda}'_j \right], \\
m_3^2 & = \frac{\Lambda^2}{512\pi^4} \left[ 2(\hat{\lambda}_{ij}\hat{y}_j - \hat{y}_{ij}\hat{\lambda}_j) + \hat{\lambda}_{ij}\hat{y}'_j - \hat{y}_{ij}\hat{\lambda}'_j \right],
\end{align*}
\]

\( \Delta\gamma \) is same as the one defined in Section II. The corrections to the light field \( \phi_i \) are

\[
m_{\phi_i}^2 = -\frac{\Lambda^2}{512\pi^4} \left[ 2\hat{y}_{ij}(\Delta\gamma_{B_i} + \Delta\gamma_{B_j}) - \hat{y}'_i(\Delta\gamma_{B_i}) \right].
\]

**Appendix B: Soft Spectra of the Second Model**

We give the soft spectra of the second model \( W'_H = \lambda SH_uH_d \). The trilinear terms are given by

\[
A_t = A_b = A_r = -\frac{1}{16\pi^2}\lambda^2\Lambda.
\]

The stop soft mass squares are

\[
\begin{align*}
\Delta m_{Q_t}^2 &= -\frac{1}{(16\pi^2)^2} \left( h_t^2\lambda^2 + h_b^2\lambda^2 \right) \Lambda^2, \\
\Delta m_{U_t}^2 &= -\frac{2}{(16\pi^2)^2} h_t^2\lambda^2\Lambda^2, \\
\Delta m_D^2 &= -\frac{2}{16\pi^2} h_b^2\lambda^2\Lambda^2.
\end{align*}
\]

The Higgs mass squares are given by

\[
\begin{align*}
\Delta m_{H_u}^2 &= \frac{3}{(16\pi^2)^2} \left[ \lambda^4 + h_b^2\lambda^2 - \lambda^2(g_s^2 + \frac{1}{5}g_1^2) \right] \Lambda^2, \\
\Delta m_{H_d}^2 &= \frac{3}{(16\pi^2)^2} \left[ \lambda^4 + h_t^2\lambda^2 - \lambda^2(g_s^2 + \frac{1}{5}g_1^2) \right] \Lambda^2.
\end{align*}
\]
The mainly concerned part of the soft spectrum is quite similar to the first model, after the mapping $\lambda^2 \rightarrow \lambda_u^2$.


[13] Z. Kang, T. Li, T. Liu and J. M. Yang, JHEP 1103, 078 (2011); F. Galli and A. Mariotti,
[31] Y. Kats and D. Shih, JHEP 1108, 049 (2011); Y. Kats, P. Meade, M. Reece and D. Shih,
Note our definition of the anomalous dimensions $\gamma$ is same as used in [22] but different from [21] by a factor $-2$. 