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Weyl-Weyl Correlator in de Donder Gauge on de Sitter

P. J. Mora[1*], N. C. Tsamis[2†] and R. P. Woodard[1,‡]

[1] *Department of Physics, University of Florida*
Gainesville, FL 32611, UNITED STATES

[2] *Institute of Theoretical Physics & Computational Physics*
Department of Physics University of Crete
GR-710 03 Heraklion, HELLAS

ABSTRACT

We compute the linearized Weyl-Weyl correlator using a new solution for the graviton propagator on de Sitter background in de Donder gauge. The result agrees exactly with a previous computation in a noncovariant gauge. We also use dimensional regularization to compute the one loop expectation value of the square of the Weyl tensor.

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* e-mail: pmora@phys.ufl.edu
† e-mail: tsamis@physics.uoc.gr
‡ e-mail: woodard@phys.ufl.edu

1 Introduction

De Sitter space holds great phenomenological interest as a paradigm for the background geometry of primordial inflation. From the perspective of an inflationary cosmologist, there is no point to working on the full de Sitter manifold or paying any special account to the full de Sitter group. The cosmological patch of de Sitter is merely the special case of a spatially flat, Friedman-Robertson-Walker geometry whose Hubble parameter happens to be exactly constant, and the important symmetries are homogeneity and isotropy. In stark contrast, mathematical physicists accord the de Sitter geometry a special status as the unique maximally symmetric solution to the Einstein equations with a positive cosmological constant. They believe strongly that the full de Sitter group should play the same role in organizing quantum field theory on de Sitter as the Poincaré group does for flat space.

It should be emphasized that quantum field theories do what they please, without regard for the prejudices of those who study them. So de Sitter invariance is recovered, when it is present, whether or not explicit account is taken of it. This is exactly what happens if one constructs the Bunch-Davies mode sums (in D spacetime dimensions with Hubble parameter H) for the propagators of a minimally coupled scalar with $M_S^2 > 0$ [1], a spin one half fermion with $M_F^2 > -\frac{D}{2}H^2$ [2], or a transverse vector with $M_V^2 > -2(D-1)H^2$ [3]. However, infrared divergences break de Sitter invariance if the mass-squared drops below these bounds [3, 4].[1]

The case of gravitons has long been recognized as dynamically equivalent to that of massless, minimally coupled scalars [7]. Hence there can be no de Sitter invariant graviton propagator. For this reason a noninvariant gauge fixing functional was employed to construct the only graviton propagator [8, 9] for which any loop computations have been performed [10, 11, 12, 13, 14]. Although this propagator breaks de Sitter invariance because of its gauge fixing functional, the method of the compensating gauge transformation reveals a physical violation of de Sitter invariance as well [15].

Mathematical physicists disputed this conclusion for many years because adding de Sitter invariant gauge fixing terms to the action results in a propagator equation with de Sitter invariant solutions [16]. However, it has recently been shown that there is a topological obstacle to adding invariant gauge fixing functionals on any manifold, such as de Sitter, which possesses a linearization instability [17]. Ignoring this problem for scalar quantum electrodynamics on de Sitter leads to on shell singularities in the one loop scalar self-mass-squared [18], and would cause similar problems were it done in quantum gravity. It is still

[1]The reason for this is obvious: each mode is an independent harmonic oscillator, and making the mass-squared drop below the stated bounds results in a potential which curves downwards. So the mode tends to roll down its potential. If the quantum state is released centered about the origin, it will spread, and how far it spreads depends upon when it was released. Mathematical physicists sometimes deny this by resorting to analytic regularization schemes which automatically subtract off power law infrared divergences. This results in the curious claim that tachyonic scalars have de Sitter invariant propagators, except for the discrete values $M^2 = -N(N+D-1)H^2$ [5]. The special thing about these masses is that they make a formerly power law infrared divergence logarithmic, and hence visible to the analytic regularization [3]. In fact, it is never correct to subtract off infrared divergences, and the subtracted mode sums which result from this bogus procedure solve the propagator equation without being true propagators [6].

possible to impose invariant gauges which are "exact" in the sense that the field obeys some strong operator equation. One would naively think that exact gauge conditions could be obtained by taking singular limits of gauge fixing terms [5], but this step involves analytic continuation in a gauge parameter, which is highly suspect when infrared divergences are present [3]. The more reliable technique is simply to construct the propagator directly in the exact gauge. This has been done recently for de Donder gauge [19], and the resulting propagator shows de Sitter breaking in both the spin two and spin zero sectors [19, 20].

The current argument against de Sitter breaking in quantum gravity seems to be based on three points:

- That de Sitter invariant propagators result from taking certain limits of gauge fixing terms which naively enforce exact gauge conditions [21];
- That the infrared divergence which precludes a de Sitter invariant propagator for dynamical gravitons is a gauge artifact [22]; and
- That previous de Sitter breaking solutions for the graviton propagator [8, 9, 19, 20] are in a different sector of the Hilbert space [22].

We have already explained that the first argument involves a dubious analytic continuation. (It would be interesting to check if acting the graviton kinetic operator on the claimed propagators produces the appropriate functional projection operators.) The second argument has been rebutted by demonstrating that the putative gauge transformation amounts to an alternate quantization scheme which changes physical quantities such as the tensor power spectrum [23]. The final argument was partially answered by computing the linearized Weyl-Weyl correlator for the first of the two de Sitter breaking propagators [24]. The result for this that had previously been accepted by mathematical physicists [25] turns out to contain some mistakes [24], but correcting these leads to complete agreement with the de Sitter breaking propagator [26]. We do not believe that the Weyl-Weyl correlator completely checks the graviton propagator, but mathematical physicists ascribe great significance to it [22], and the result certainly undermines their skepticism about the first of the two de Sitter breaking propagators.

The purpose of this paper is to make the same partial check of the de Sitter breaking propagator recently derived in exact de Donder gauge [19, 20]. Section 2 reviews a number of technical results we shall need from previous work. The actual computation is performed in section 3. Our conclusions comprise section 4.

2 Notation and Previous Work

The purpose of this section is to explain notation and introduce certain key results from previous work which facilitate the present study. We begin with results from the recent computation of the Weyl-Weyl correlator for the older of the two de Sitter breaking propagators [24]. Then the de Donder gauge propagator is described [19, 20].

2.1 For the Weyl-Weyl Correlator

We work on the "cosmological patch" of D-dimensional de Sitter, which can be covered using conformal coordinates $x^\mu = (\eta, \vec{x})$ with,

$$-\infty < \eta < 0 \qquad , \qquad -\infty < x^i < +\infty \qquad \text{for} \qquad i = 1, \dots, D{-}1 \; . \tag{1}$$

The metric in these coordinates is conformal to that of flat space,

$$ds^2 = a^2 \Bigl(-d\eta^2 + d\vec{x} \cdot d\vec{x}\Bigr) \qquad \text{where} \qquad a \equiv -\frac{1}{H\eta} \; . \tag{2}$$

The parameter H is known as the Hubble constant, and is related to the cosmological constant by $\Lambda = (D-1)H^2$.

It is convenient to represent the propagator between points x^μ and x'^μ using the de Sitter length function $y(x; x')$,

$$y(x; x') \equiv a a' H^2 \Biggl[\Bigl\| \vec{x} - \vec{x}' \Bigr\|^2 - \Bigl(|\eta - \eta'| - i\varepsilon\Bigr)^2\Biggr] \; . \tag{3}$$

Except for the factor of $i\varepsilon$ (whose purpose is to enforce Feynman boundary conditions) the de Sitter length function can be expressed as follow in terms of the geodesic length $\ell(x; x')$ from x^μ to x'^μ,

$$y(x; x') = 4 \sin^2\Bigl(\frac{1}{2} H \ell(x; x')\Bigr) \; . \tag{4}$$

When de Sitter invariance cannot be maintained, we elect to preserve homogeneity and isotropy — this is known as the "E(3)" vacuum [27]. This means the propagator depends upon $y(x; x')$ and the scale factors at x^μ and x'^μ. An important example is the propagator of the massless, minimally coupled scalar [28],

$$i\Delta_A(x; x') = A\Bigl(y(x; x')\Bigr) + k \ln(a a') \; , \tag{5}$$

where the constant k is,

$$k \equiv \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D{-}1)}{\Gamma(\frac{D}{2})} \; . \tag{6}$$

The function $A(y)$ is,

$$\begin{aligned} A(y) = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \Biggl\{ & \Gamma\Bigl(\frac{D}{2} {-} 1\Bigr) \Bigl(\frac{4}{y}\Bigr)^{\frac{D}{2}-1} + \frac{\Gamma(\frac{D}{2}{+}1)}{\frac{D}{2}{-}2} \Bigl(\frac{4}{y}\Bigr)^{\frac{D}{2}-2} + A_1 \\ & - \sum_{n=1}^{\infty} \Biggl[\frac{\Gamma(n{+}\frac{D}{2}{+}1)}{(n{-}\frac{D}{2}{+}2)(n{+}1)!} \Bigl(\frac{y}{4}\Bigr)^{n-\frac{D}{2}+2} - \frac{\Gamma(n{+}D{-}1)}{n\Gamma(n{+}\frac{D}{2})} \Bigl(\frac{y}{4}\Bigr)^{n} \Biggr] \Biggr\}, \end{aligned} \tag{7}$$

where the constant A_1 is,

$$A_1 = \frac{\Gamma(D{-}1)}{\Gamma(\frac{D}{2})} \Biggl\{ -\psi\Bigl(1 {-} \frac{D}{2}\Bigr) + \psi\Bigl(\frac{D{-}1}{2}\Bigr) + \psi(D{-}1) + \psi(1) \Biggr\}. \tag{8}$$

Two permutations have great importance for us: *Riemannization* and *Weylization*. The first was originally introduced as "the standard permutation" in a study of one loop corrections to an invariant correlator of two Riemann tensors on flat space background [29]. It operates on any 8-index bi-tensor "seed" $S_{\alpha\beta\gamma\delta\mu\nu\rho\sigma}$ with the algebraic symmetries of a graviton propagator with two ordinary derivatives at each point:

$$S_{\alpha\beta\gamma\delta\mu\nu\rho\sigma} = S_{\beta\alpha\gamma\delta\mu\nu\rho\sigma} = S_{\alpha\beta\delta\gamma\mu\nu\rho\sigma} = S_{\alpha\beta\gamma\delta\nu\mu\rho\sigma} = S_{\alpha\beta\gamma\delta\mu\nu\sigma\rho} \; . \tag{9}$$

Riemannization permutes the seed tensor so that it has the algebraic symmetries of the product of two linearized Riemann tensors,

$$\text{Riem}\Big[S_{\alpha\beta\gamma\delta\mu\nu\rho\sigma}\Big] \equiv \mathcal{R}_{\alpha\beta\gamma\delta}{}^{\epsilon\zeta\kappa\lambda} \times \mathcal{R}_{\mu\nu\rho\sigma}{}^{\theta\phi\psi\omega} \times S_{\epsilon\zeta\kappa\lambda\theta\phi\psi\omega} \; , \tag{10}$$

where,

$$\mathcal{R}_{\alpha\beta\gamma\delta}{}^{\epsilon\zeta\kappa\lambda} \equiv \delta^{\epsilon}_{\alpha}\delta^{\kappa}_{\gamma}\delta^{\zeta}_{\beta}\delta^{\lambda}_{\delta} - \delta^{\epsilon}_{\gamma}\delta^{\kappa}_{\beta}\delta^{\zeta}_{\delta}\delta^{\lambda}_{\alpha} + \delta^{\epsilon}_{\beta}\delta^{\kappa}_{\delta}\delta^{\zeta}_{\alpha}\delta^{\lambda}_{\gamma} - \delta^{\epsilon}_{\delta}\delta^{\kappa}_{\alpha}\delta^{\zeta}_{\gamma}\delta^{\lambda}_{\beta} \; . \tag{11}$$

Weylization operates on any bi-tensor seed with the algebraic symmetries of the product of two Riemann tensors,

$$S_{\alpha\beta\gamma\delta\mu\nu\rho\sigma}(x;x') \;=\; S_{\gamma\delta\alpha\beta\mu\nu\rho\sigma}(x;x') = S_{\mu\nu\rho\sigma\alpha\beta\gamma\delta}(x';x) \; , \tag{12}$$

$$S_{(\alpha\beta)\gamma\delta\mu\nu\rho\sigma}(x;x') \;=\; 0 = S_{\alpha[\beta\gamma\delta]\mu\nu\rho\sigma}(x;x') \; . \tag{13}$$

Weylization subtracts off the traces within each index group to produce something with the algebraic symmetries of the product of two Weyl tensors,

$$\text{Weyl}\Big[S_{\alpha\beta\gamma\delta\mu\nu\rho\sigma}(x;x')\Big] \equiv \mathcal{C}_{\alpha\beta\gamma\delta}{}^{\epsilon\zeta\kappa\lambda}(x) \times \mathcal{C}_{\mu\nu\rho\sigma}{}^{\theta\phi\psi\omega}(x') \times S_{\epsilon\zeta\kappa\lambda\theta\phi\psi\omega}(x;x') \; . \tag{14}$$

where,

$$\mathcal{C}_{\alpha\beta\gamma\delta}{}^{\epsilon\zeta\kappa\lambda} \equiv \delta^{\epsilon}_{\alpha}\delta^{\zeta}_{\beta}\delta^{\kappa}_{\gamma}\delta^{\lambda}_{\delta} - \Big[g_{\alpha\gamma}\delta^{\zeta}_{\beta}\delta^{\lambda}_{\delta} - g_{\gamma\beta}\delta^{\zeta}_{\delta}\delta^{\lambda}_{\alpha} + g_{\beta\delta}\delta^{\zeta}_{\alpha}\delta^{\lambda}_{\gamma} - g_{\delta\alpha}\delta^{\zeta}_{\gamma}\delta^{\lambda}_{\beta}\Big]\frac{g^{\epsilon\kappa}}{D-2}$$
$$+\Big[g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\gamma}g_{\beta\delta}\Big]\frac{g^{\epsilon\kappa}g^{\zeta\lambda}}{(D-2)(D-1)} \; . \tag{15}$$

The older of the two de Sitter breaking propagators [8, 9] leads to the following result for the Weyl-Weyl correlator [24],

$$\Big\langle\Omega\Big|C_{\alpha\beta\gamma\delta}(x) \times C_{\mu\nu\rho\sigma}(x')\Big|\Omega\Big\rangle = 4\pi G\,\text{Weyl}\Bigg(\text{Riem}\Bigg[D_\alpha D_\gamma D'_\mu D'_\rho\, i\Delta_A(x;x')$$
$$\times\Big[\mathcal{R}_{\beta\nu}(x;x')\mathcal{R}_{\delta\sigma}(x;x') + \mathcal{R}_{\beta\sigma}(x;x')\mathcal{R}_{\delta\nu}(x;x')\Big]\Bigg]\Bigg) + O(G^2) \; . \tag{16}$$

Here and henceforth, G is Newton's constant, D_μ stands for the covariant derivative (in the de Sitter background) with respect to x^μ, D'_ρ denotes the covariant derivative with respect to

x'^ρ, and $\mathcal{R}_{\mu\nu}(x;x')$ is the de Sitter invariant, mixed partial derivative of $y(x;x')$, normalized so that its flat space limit gives $\eta_{\mu\nu}$,

$$\mathcal{R}_{\mu\nu}(x;x') \equiv -\frac{1}{2H^2}\frac{\partial^2 y(x;x')}{\partial x^\mu \partial x'^\nu} \; . \tag{17}$$

Because the computation was done in D spacetime dimensions, it is a simple matter to take the coincidence limit using dimensional regularization and contract the indices together [24],

$$\Bigl\langle \Omega \Bigl| C^{\alpha\beta\gamma\delta}(x) C_{\alpha\beta\gamma\delta}(x) \Bigr| \Omega \Bigr\rangle = 64\pi (D-3) D (D+1)(D+2) A''(0) G H^4 + O(G^2 H^8). \tag{18}$$

The coincidence limit of the second derivative of $A(y)$ is,

$$A''(0) = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \times \frac{1}{16}\frac{\Gamma(D+1)}{\Gamma(\frac{D}{2}+2)} \; . \tag{19}$$

2.2 For the de Donder Gauge Propagator

The graviton propagator in de Donder gauge can be expressed as the sum of a spin zero part and a spin two part,

$$i\Bigl[{}_{\alpha\beta}\Delta_{\rho\sigma}\Bigr](x;x') = i\Bigl[{}_{\alpha\beta}\Delta^0_{\rho\sigma}\Bigr](x;x') + i\Bigl[{}_{\alpha\beta}\Delta^2_{\rho\sigma}\Bigr](x;x') \; . \tag{20}$$

Each part is represented as the product of differential projectors that enforce the de Donder condition on each coordinate, acting on a scalar structure function. For the spin zero part this form is,

$$i\Bigl[{}_{\mu\nu}\Delta^0_{\rho\sigma}\Bigr](x;x') = \mathcal{P}_{\mu\nu}(x) \times \mathcal{P}_{\rho\sigma}(x')\Bigl[\mathcal{S}_0(x;x')\Bigr] \; . \tag{21}$$

The spin zero projector $\mathcal{P}_{\mu\nu}$ is a sum of longitudinal and trace terms,

$$\mathcal{P}_{\mu\nu} \equiv D_\mu D_\nu + \frac{g_{\mu\nu}}{D-2}\Bigl[\square + 2(D-1)H^2\Bigr] \; . \tag{22}$$

The spin two part takes the form,

$$i\Bigl[{}_{\mu\nu}\Delta^2_{\rho\sigma}\Bigr](x;z) = \frac{1}{4H^4}\mathbf{P}_{\mu\nu}{}^{\alpha\beta}(x) \times \mathbf{P}_{\rho\sigma}{}^{\kappa\lambda}(x')\Bigl[\mathcal{R}_{\alpha\kappa}(x;x')\mathcal{R}_{\beta\lambda}(x;x')\mathcal{S}_2(x;x')\Bigr] \; . \tag{23}$$

The spin two projector $\mathbf{P}_{\mu\nu}{}^{\alpha\beta}$ is,

$$\begin{aligned}
\mathbf{P}_{\mu\nu}{}^{\alpha\beta} = \frac{1}{2}\Bigl(\frac{D-3}{D-2}\Bigr)\Biggl\{ & -\delta^\alpha_{(\mu}\delta^\beta_{\nu)}\Bigl[\square - DH^2\Bigr]\Bigl[\square - 2H^2\Bigr] + 2D_{(\mu}\Bigl[\square + H^2\Bigr]\delta^{(\alpha}_{\nu)}D^{\beta)} \\
& -\Bigl(\frac{D-2}{D-1}\Bigr)D_{(\mu}D_{\nu)}D^{(\alpha}D^{\beta)} + g_{\mu\nu}g^{\alpha\beta}\Bigl[\frac{\square^2}{D-1} - H^2\square + 2H^4\Bigr] \\
& -\frac{D_{(\mu}D_{\nu)}}{D-1}\Bigl[\square + 2(D-1)H^2\Bigr]g^{\alpha\beta} - \frac{g_{\mu\nu}}{D-1}\Bigl[\square + 2(D-1)H^2\Bigr]D^{(\alpha}D^{\beta)}\Biggr\}.
\end{aligned} \tag{24}$$

It is transverse and traceless on each index group,

$$g^{\mu\nu} \mathbf{P}_{\mu\nu}{}^{\alpha\beta} = 0 = \mathbf{P}_{\mu\nu}{}^{\alpha\beta} g_{\alpha\beta} \; , \tag{25}$$
$$D^{\mu} \mathbf{P}_{\mu\nu}{}^{\alpha\beta} = 0 = \mathbf{P}_{\mu\nu}{}^{\alpha\beta} D_{\alpha} \; . \tag{26}$$

A key identity concerns the result of acting either $\mathbf{P}_{\mu\nu}{}^{\alpha\beta}(x)$ or $\mathbf{P}_{\rho\sigma}{}^{\kappa\lambda}(x')$ on $\mathcal{R}_{\alpha\kappa}(x;x')\mathcal{R}_{\beta\lambda}(x;x')$ times a de Sitter invariant structure function. If the operator is acted by itself then the result is complicated, as expression (24) indicates. However, *if the longitudinal and trace parts are projected out on the other index group,* then a very simple result follows [20],

$$\mathbf{P}_{\mu\nu}{}^{\alpha\beta}(x) \times \mathbf{P}_{\rho\sigma}{}^{\kappa\lambda}(x') \Bigl[\mathcal{R}_{\alpha\kappa}\mathcal{R}_{\beta\lambda} F(y)\Bigr]$$
$$= -\frac{1}{2}\Bigl(\frac{D\!-\!3}{D\!-\!2}\Bigr) \mathbf{P}_{\rho\sigma}{}^{\kappa\lambda}(x') \Biggl[\mathcal{R}_{\mu\kappa}\mathcal{R}_{\nu\lambda} \square \Bigl[\square \!-\! (D\!-\!2)H^2\Bigr] F(y)\Biggr] \; . \tag{27}$$

In deriving an explicit form for the propagator, it was not possible to use this identity a second time, to simplify the action of $\mathbf{P}_{\rho\sigma}{}^{\kappa\lambda}(x')$, because no operator remains to project out longitudinal and trace parts on the index group at x^{μ}. However, we will see that Riemannization and Weylization provide the crucial projections, which allows the identity to be used twice in computing the Weyl-Weyl correlator.

The identity (27) is so crucial because the spin two structure function obeys the relation,

$$\square^2 \Bigl[\square \!-\! (D\!-\!2)H^2\Bigr]^2 \mathcal{S}_2(x;x') = 32 H^4 \Bigl(\frac{D\!-\!2}{D\!-\!3}\Bigr)^2 i\Delta_A(x;x') \; . \tag{28}$$

While the spin two structure function is, by itself, very complicated [19, 20], the action of precisely the derivatives in (28) reduces it to the propagator of a massless, minimally coupled scalar. We shall not require the spin zero structure function but its form is known as well [19, 20].

From relations (5) and (28) it is apparent that the spin two structure function consists of a de Sitter invariant part plus a de Sitter breaking part which is cubic in $u \equiv \ln(aa')$ [19, 20],

$$\mathcal{S}_2(x;x') = S_2(y) + \delta S_2(u) \; . \tag{29}$$

We shall not require the explicit result of acting the projectors on the de Sitter invariant part. The de Sitter breaking part gives [20],

$$i\Bigl[{}_{\mu\nu}\Delta^{\rm br,2}_{\rho\sigma}\Bigr](x;x') \!=\! \frac{1}{4H^4} \mathbf{P}_{\mu\nu}{}^{\alpha\beta}(x) \!\times\! \mathbf{P}_{\rho\sigma}{}^{\kappa\lambda}(x') \Bigl[\mathcal{R}_{\alpha\kappa}(x;x')\mathcal{R}_{\beta\lambda}(x;x') \delta S_2(u)\Bigr] \; , \tag{30}$$
$$= k\Biggl[\ln(4aa') \!+\! 2\psi\Bigl(\frac{D\!-\!1}{2}\Bigr) \!-\! 4 \!+\! \frac{1}{D\!-\!1}\Biggr] (aa')^2 \Biggl\{2\overline{\eta}_{\mu(\rho}\overline{\eta}_{\sigma)\nu} \!-\! \frac{2}{D\!-\!1}\, \overline{\eta}_{\mu\nu}\overline{\eta}_{\rho\sigma}\Biggr\}, \tag{31}$$

where $\eta_{\mu\nu}$ is the spacelike Lorentz metric and an overbar denotes the suppression of its temporal components,

$$\overline{\eta}_{\mu\nu} \equiv \eta_{\mu\nu} + \delta^0_{\mu}\delta^0_{\nu} \; . \tag{32}$$

3 The Computation

In this section we assemble the results which have just been presented to compute the linearized Weyl-Weyl correlator for the de Donder gauge propagator. The argument consists of five steps. One paragraph is devoted to each step.

The first step consists of expressing the linearized Weyl-Weyl correlator in terms of the graviton propagator. We define the graviton field by expanding the full metric around de Sitter,

$$\Big(\text{fullmetric}\Big)_{\mu\nu}(x) \equiv g_{\mu\nu}(x) + \sqrt{16\pi G}\, h_{\mu\nu}(x) \; . \tag{33}$$

Because the Weyl tensor of de Sitter vanishes, the Weyl tensor of the full metric is linear in the graviton field. It can be given a very simple form using the tensors defined in expressions (11) and (15),

$$C_{\alpha\beta\gamma\delta}(x) = \mathcal{C}_{\alpha\beta\gamma\delta}{}^{\epsilon\zeta\kappa\lambda} \times \mathcal{R}_{\epsilon\zeta\kappa\lambda}{}^{\theta\phi\psi\omega} \times -\frac{1}{2} D_\phi D_\omega \sqrt{16\pi G}\, h_{\theta\psi}(x) + O(G) \; . \tag{34}$$

The graviton propagator is the expectation value (the time-ordered product) of two graviton fields. Hence the linearized Weyl-Weyl correlator can be expressed using the operations of Riemannization and Weylization that were defined in expressions (10) and (14),

$$\begin{aligned} &\Big\langle \Omega \Big| C_{\alpha\beta\gamma\delta}(x) \times C_{\mu\nu\rho\sigma}(x') \big| \Omega \Big\rangle \\ &\qquad = 4\pi G \,\text{Weyl}\Bigg(\text{Riem}\Big[D_\beta D_\delta D'_\nu D'_\sigma \, i\Big[{}_{\alpha\gamma}\Delta_{\mu\rho}\Big](x;x')\Big]\Bigg) + O(G^2) \; . \end{aligned} \tag{35}$$

The second step is to note that the spin zero part of the propagator drops out of the Weyl-Weyl correlator. To see this, first write (35) as,

$$\begin{aligned} &\Big\langle \Omega \Big| C_{\alpha\beta\gamma\delta}(x) \times C_{\mu\nu\rho\sigma}(x') \big| \Omega \Big\rangle \\ &\qquad = 4\pi G\, \mathcal{P}_{\alpha\beta\gamma\delta}{}^{\kappa\lambda}(x) \times \mathcal{P}_{\mu\nu\rho\sigma}{}^{\theta\phi}(x') \times i\Big[{}_{\kappa\lambda}\Delta_{\theta\phi}\Big](x;x') + O(G^2) \; , \end{aligned} \tag{36}$$

where $\mathcal{P}_{\alpha\beta\gamma\delta}{}^{\kappa\lambda}$ is the contraction of (15) and (11) into two covariant derivatives,

$$\mathcal{P}_{\alpha\beta\gamma\delta}{}^{\theta\psi} \equiv \mathcal{C}_{\alpha\beta\gamma\delta}{}^{\epsilon\zeta\kappa\lambda} \times \mathcal{R}_{\epsilon\zeta\kappa\lambda}{}^{\theta\phi\psi\omega} \times D_\phi D_\omega \; . \tag{37}$$

Note that the differential operator (37) projects out both longitudinal and trace terms,

$$\mathcal{P}_{\alpha\beta\gamma\delta}{}^{\kappa\lambda} D_\kappa = 0 = \mathcal{P}_{\alpha\beta\gamma\delta}{}^{\kappa\lambda} g_{\kappa\lambda} \; . \tag{38}$$

Of course this means it annihilates the spin zero projector (22). Hence the linearized Weyl-Weyl correlator derives entirely from the spin two part of the propagator (23),

$$\begin{aligned} &\Big\langle \Omega \Big| C_{\alpha\beta\gamma\delta}(x) \times C_{\mu\nu\rho\sigma}(x') \big| \Omega \Big\rangle \\ &\qquad = 4\pi G\, \mathcal{P}_{\alpha\beta\gamma\delta}{}^{\kappa\lambda}(x) \times \mathcal{P}_{\mu\nu\rho\sigma}{}^{\theta\phi}(x') \times i\Big[{}_{\kappa\lambda}\Delta^2_{\theta\phi}\Big](x;x') + O(G^2) \; . \end{aligned} \tag{39}$$

The next step is to note that the de Sitter breaking contribution to the spin two part drops out. From the conformal invariance of the Weyl tensor we have,

$$\text{Weyl}\Biggl(\text{Riem}\Bigl[D_\beta D_\delta D'_\nu D'_\sigma \, i\Bigl[{}_{\alpha\gamma}\Delta^{\text{br},2}_{\mu\rho}\Bigr](x;x')\Bigr]\Biggr) = (a'a)^2 \text{Weyl}\Biggl(\text{Riem}\Bigl[\partial_\beta \partial_\delta \partial'_\nu \partial'_\sigma \Bigl\{(aa')^{-2} \, i\Bigl[{}_{\alpha\gamma}\Delta^{\text{br},2}_{\mu\rho}\Bigr](x;x')\Bigr\}\Bigr]\Biggr) \; . \tag{40}$$

Now use expression (31) for the de Sitter breaking contribution to the spin two part of the propagator to conclude,

$$\partial_\beta \partial_\delta \partial'_\nu \partial'_\sigma \Bigl\{(aa')^{-2} \, i\Bigl[{}_{\alpha\gamma}\Delta^{\text{br},2}_{\mu\rho}\Bigr](x;x')\Bigr\} = \Bigl\{2\overline{\eta}_{\alpha(\mu}\overline{\eta}_{\rho)\gamma} - \frac{2\overline{\eta}_{\alpha\gamma}\overline{\eta}_{\mu\rho}}{D-1}\Bigr\} \times \partial_\beta \partial_\delta \partial'_\nu \partial'_\sigma \Bigl[\ln(4aa') + \text{Constant}\Bigr] = 0 \; . \tag{41}$$

Hence the linearized Weyl-Weyl correlator derives entirely from the de Sitter invariant contribution to the spin two part,

$$\Bigl\langle \Omega \Bigl| C_{\alpha\beta\gamma\delta}(x) \times C_{\mu\nu\rho\sigma}(x') |\Omega\Bigr\rangle = 4\pi G \, \mathcal{P}_{\alpha\beta\gamma\delta}{}^{\epsilon\zeta}(x) \times \mathcal{P}_{\mu\nu\rho\sigma}{}^{\theta\phi}(x') \times \frac{1}{4H^4} \mathbf{P}_{\epsilon\zeta}{}^{\kappa\lambda}(x) \times \mathbf{P}_{\theta\phi}{}^{\psi\omega}(x') \Bigl[\mathcal{R}_{\kappa\psi}(x;x') \mathcal{R}_{\lambda\omega}(x;x') S_2(y)\Bigr] + O(G^2) \; . \tag{42}$$

In step four we take advantage of the fact that $\mathcal{P}_{\alpha\beta\gamma\delta}{}^{\epsilon\zeta}(x)$ projects out longitudinal and trace parts to apply identity (27) twice in expression (42),

$$\Bigl\langle \Omega \Bigl| C_{\alpha\beta\gamma\delta}(x) \times C_{\mu\nu\rho\sigma}(x') |\Omega\Bigr\rangle = \frac{\pi G}{4H^4} \Bigl(\frac{D-3}{D-2}\Bigr)^2 \mathcal{P}_{\alpha\beta\gamma\delta}{}^{\epsilon\zeta}(x) \times \mathcal{P}_{\mu\nu\rho\sigma}{}^{\theta\phi}(x') \times \Bigl[\mathcal{R}_{\epsilon\theta} \mathcal{R}_{\zeta\phi} \square \Bigl[\square - (D-2)H^2\Bigr] \square' \Bigl[\square' - (D-2)H^2\Bigr] S_2(y)\Bigr] + O(G^2) \; . \tag{43}$$

Now note that $\square' F(y) = \square F(y)$ [28] and use expression (28) to conclude,

$$\square \Bigl[\square - (D-2)H^2\Bigr] \square' \Bigl[\square' - (D-2)H^2\Bigr] S_2(y) = 32 H^4 \Bigl(\frac{D-2}{D-3}\Bigr)^2 A(y) \; , \tag{44}$$

where $A(y)$ is the de Sitter invariant part (7) of the scalar propagator. Substituting this in (43) implies,

$$\Bigl\langle \Omega \Bigl| C_{\alpha\beta\gamma\delta}(x) \times C_{\mu\nu\rho\sigma}(x') |\Omega\Bigr\rangle = 8\pi G \, \mathcal{P}_{\alpha\beta\gamma\delta}{}^{\epsilon\zeta}(x) \times \mathcal{P}_{\mu\nu\rho\sigma}{}^{\theta\phi}(x') \times \Bigl[\mathcal{R}_{\epsilon\theta}(x;x') \mathcal{R}_{\zeta\phi}(x;x') A(y)\Bigr] + O(G^2) \; . \tag{45}$$

The final step begins by expressing (45) in terms of Riemannization and Weylization,

$$\Bigl\langle \Omega \Bigl| C_{\alpha\beta\gamma\delta}(x) \times C_{\mu\nu\rho\sigma}(x') |\Omega\Bigr\rangle = 4\pi G \, \text{Weyl}\Biggl(\text{Riem}\Bigl[D_\beta D_\delta D'_\nu D'_\sigma \Bigl[\Bigl(\mathcal{R}_{\alpha\mu}\mathcal{R}_{\gamma\rho} + \mathcal{R}_{\alpha\rho}\mathcal{R}_{\gamma\mu}\Bigr) A(y)\Bigr]\Bigr]\Biggr) + O(G^2) \; . \tag{46}$$

Note that acting any of the covariant derivatives on the intermediate tensor factors produces a metric [18, 20],

$$D_\beta \mathcal{R}_{\alpha\mu}(x;x') = \frac{1}{2} g_{\alpha\beta}(x) \frac{\partial y(x;x')}{\partial x'^\mu} \; . \tag{47}$$

Because any terms of this form are annihilated by Weylization, we can move the four covariant derivatives through to act on $A(y)$. Even acting two mixed derivatives erases the difference between $A(y)$ and the full scalar propagator,

$$D_\beta D'_\nu A(y) = D_\beta D'_\nu i\Delta_A(x;x') \; . \tag{48}$$

(This is why the de Sitter breaking contribution (41) vanished.) Hence we conclude,

$$\begin{aligned} &\Big\langle \Omega \Big| C_{\alpha\beta\gamma\delta}(x) \times C_{\mu\nu\rho\sigma}(x') | \Omega \Big\rangle \\ &\quad = 4\pi G \text{Weyl}\Bigg(\text{Riem}\Bigg[\Big(\mathcal{R}_{\alpha\mu} \mathcal{R}_{\gamma\rho} + \mathcal{R}_{\alpha\rho} \mathcal{R}_{\gamma\mu} \Big) D_\beta D_\delta D'_\nu D'_\sigma \, i\Delta_A \Bigg] \Bigg) + O(G^2) \; . \end{aligned} \tag{49}$$

Reshuffling some indices gives the same form (16) that was derived for the older of the two de Sitter breaking propagators [8, 9].

Because our result (49) for the linearized Weyl-Weyl correlator is the same as for the other de Sitter breaking propagator, taking the coincidence limit and contracting the indices must reproduce expression (18) as well. It is worth pointing out that this is the first time the de Donder gauge propagator has been used in a loop computation.

4 Discussion

We have computed the linearized Weyl-Weyl correlator for the recently constructed graviton propagator in de Donder gauge [19, 20]. Our result (49) is identical to expression (16), which was found using the graviton propagator constructed with a noninvariant gauge fixing term [8, 9]. Mathematical physicists obtain the same result [25], after correcting some mistakes [26].

We do not accept that the Weyl-Weyl correlator provides a complete check of the graviton propagator. It is sensitive to neither the spin zero part nor to the infrared divergent, de Sitter breaking part. However, it does offer a partial check, and both de Sitter breaking propagators pass this check. Perhaps that fact may ease doubts that have been expressed about these propagators accessing a different sector of the graviton Hilbert space [22].

Turnabout is fair play, so let us suggest an interesting test of the exact gauge, de Sitter invariant propagators which are claimed to result from singular limits of the provably false [3] procedure of adding de Sitter invariant gauge fixing terms [21]. This is to act the graviton kinetic operator on them and then integrate the result onto itself to check that it is a functional projection operator. We predict that the alleged propagators will fail this test. It is worth noting that the de Sitter breaking, de Donder gauge propagator [19] was constructed to pass it.

Our result provides support for the suspicion that the new de Donder gauge propagator may be simple to use, in spite of its cumbersome tensor form (20-24) and complicated structure functions. The same sort of cumbersome tensors and complicated structure function appear in the Lorentz gauge photon propagator [30]. However, all known loop computations [31] result in the tensors contracting to simple forms, and in precisely the right differential operators being acted to simplify the structure function. We so far have only this one result for the new propagator, but the same sort of simplifications took place in (43). It will be interesting to see what happens with other computations.

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