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Robert D. Pisarski and Vladimir V. Skokov

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Gross-Witten-Wadia transition in a matrix model of deconfinement

Robert D. Pisarski*
Department of Physics, Brookhaven National Laboratory, Upton, NY 11973 and RIKEN/BNL, Brookhaven National Laboratory, Upton, NY 11973

Vladimir V. Skokov†
Department of Physics, Brookhaven National Laboratory, Upton, NY 11973

We study the deconfining phase transition at nonzero temperature in a $SU(N)$ gauge theory, using a matrix model which was analyzed previously at small N. We show that the model is soluble at infinite N, and exhibits a Gross-Witten-Wadia (GWW) transition. In some ways, the deconfining phase transition is of first order: at a temperature T_d, the Polyakov loop jumps discontinuously from 0 to $\frac{1}{2}$, and there is a nonzero latent heat $\sim N^2$. In other ways, the transition is of second order: *e.g.*, the specific heat diverges as $C \sim 1/(T - T_d)^{3/5}$ when $T \to T_d^+$. Other critical exponents satisfy the usual scaling relations of a second order phase transition. In the presence of a nonzero background field h for the Polyakov loop, there is a phase transition at the temperature T_h where the value of the loop $= \frac{1}{2}$, with $T_h < T_d$. Since $\partial C/\partial T \sim 1/(T - T_h)^{1/2}$ as $T \to T_h^+$, this transition is of third order. These properties, closely analogous to those on a femto-sphere at zero coupling, suggest that in infinite volume, the GWW transition may be an infrared stable fixed point of a $SU(\infty)$ gauge theory.

The properties of the deconfining phase transition for a $SU(N)$ gauge theory at nonzero temperature are of fundamental interest. At small N, this transition can only be understood through numerical simulations on the lattice [1]. Large N can be studied through numerical simulations [2] and in reduced models [3]. In the pure glue theory, this transition can be modeled through an effective model, such as a matrix model [4–10].

One limit in which the theory can be solved analytically is by putting it on a sphere of femto-scale dimensions [11–15]. An effective theory is constructed directly by integrating out all modes with nonzero momentum, and gives a matrix model which is soluble at large N [16–19]. As a function of temperature, it exhibits a Gross-Witten-Wadia (GWW) transition [20]. That is, it exhibits aspects of both first order *and* second order phase transitions; thus it can be termed "critical first order" [15]. Since the theory has finite spatial volume, however, there is only a true phase transition at infinite N. Thus on a femtosphere, the GWW transition appears to be mere curiosity.

Matrix models have been developed as an effective theory for deconfinement in four spacetime dimensions (and infinite volume). These models, which involve zero [6], one [7], and two parameters [8, 9], are soluble analytically for two and three colors, and numerically for four or more colors. In this paper we show that these models are also soluble analytically for infinite N. Most unexpectedly, we find that the model exhibits a GWW transition, very similar to that on a femtosphere. This is surprising because on a femtosphere, the matrix model is dominated by the Vandermonde determinant, and looks nothing like the matrix models of Refs. [6–9].

This leads us to speculate that the GWW transition may not be an artifact of a femtosphere, but might be an infrared stable fixed point for $SU(\infty)$ gauge theories in infinite volume. In Sec. (IV) we estimate how large N must be to see signs of the GWW transition at infinite N.

I. ZERO BACKGROUND FIELD

We expand about a constant background field for the vector potential, $A_0^{ij} = (2\pi T/g)\, \delta^{ij}\, q_i$, where $i, j = 1 \ldots N$. This A_0 field is a diagonal $SU(N)$ matrix, and so $\sum_{i=1}^{N} q_i = 0$. The thermal Wilson line is the matrix $\mathbf{L} = \exp(2\pi i \mathbf{q})$; its trace is the Polyakov loop in the fundamental representation, $\ell_1 = \mathrm{tr}\, \mathbf{L}/N$. At any N, this represents a possible ansatz for the region where the expectation value of the Polyakov loop is less than unity. This region has been termed the "semi" quark gluon plasma (QGP) [5]. At infinite N, this ansatz is the simplest possible for the master field in the semi-QGP.

The potential we take is a sum of two terms,

$$\widetilde{V}_{\rm eff}(q) \;=\; -\, d_1(T)\, \widetilde{V}_1(q) \;+\; d_2(T)\, \widetilde{V}_2(q)\;, \qquad (1)$$

where

$$\widetilde{V}_n(q) = \sum_{i,j=1}^{N_c} |q_i - q_j|^n (1 - |q_i - q_j|)^n\;. \qquad (2)$$

The term $\sim \widetilde{V}_2(q)$ is generated perturbatively at one loop order; that $\sim \widetilde{V}_1(q)$ is added to drive the transition to the confined phase. Previously, the functions d_1 and d_2 were chosen as $d_1(T) = (2\pi^2/15)\, c_1\, T^2 T_d^2$ and $d_2(T) = 2\pi^2/3\, (T^4 - c_2\, T^2 T_d^2)$, where T_d is the temperature for deconfinement [7–9]. These matrix models also included terms independent of the q's, $\sim c_3\, T^2\, T_d^2$ and $\sim B\, T_d^4$.

* pisarski@bnl.gov
† vskokov@quark.phy.bnl.gov

The values of these parameters were chosen to agree with results from numerical simulations on the lattice [7–9]. As we show, however, when N is infinite, at the transition temperature the nature of the solution is independent not only of the values of these parameters, but even of the choice of the functions $d_1(T)$ and $d_2(T)$ (modulo modest assumptions, given later).

The matrix model in Eqs. (1) and (2) is rather different from that on a femtosphere [11–15]. On a femtosphere the dominant term driving confinement is the Vandermonde determinant, $\sim \Pi_{i,j} \log|\exp(2\pi i q_i) - \exp(2\pi i q_j)|$; in the present model it is the terms $\sim \widetilde{V}_n(q)$. The logarithmic singularities of the Vandermonde determinant are stronger than those of the from the absolute values $\sim |q_i - q_j|^n$ in the $\sim \widetilde{V}_n(q)$.

To treat infinite N, we introduce the variable $x = i/N$, so that $q_i \to q(x)$, and the potential is an integral over x. It is useful to introduce the eigenvalue density, $\rho(q) = dx/dq$ [16]. The integrals over x then become integrals over q, weighted by $\rho(q)$. The eigenvalue density must be positive, and by definition is normalized to

$$\int_{-q_0}^{q_0} dq\ \rho(q) = 1\ . \quad (3)$$

Polyakov loops are traces of powers of the thermal Wilson line,

$$\ell_j = \frac{1}{N}\,\mathrm{tr}\,\mathbf{L}^j = \int_{-q_0}^{q_0} dq\ \rho(q)\ \cos(2\pi j q)\ . \quad (4)$$

As noted before, the first Polyakov loop, ℓ_1, is that in the fundamental representation. For $j \geq 2$, the relationship of the ℓ_j to Polyakov loops in irreducible representations is more involved [5], but all ℓ_j are gauge invariant, and so physical quantities.

By a global $O(2)$ rotation we can assume that the expectation value of ℓ_1 is real. Consequently, we take $\rho(q)$ to be even in q, $\rho(q) = \rho(-q)$. Anticipating the results, we also assume that the integral over q does not run the full range from $-\frac{1}{2}$ to $\frac{1}{2}$, but only over a limited range, from $-q_0$ to $+q_0$.

Going to integrals over q, we can take out overall factors of N^2 from the potentials, with $\widetilde{V}_n(q) = N^2\, V_n(q)$, where

$$V_n(q) = \int dq \int dq'\,\rho(q)\,\rho(q')|q-q'|^n(1-|q-q'|)^n\ . \quad (5)$$

In this expression and henceforth, all integrals over q run from $-q_0$ to $+q_0$, as in Eqs. (3) and (4).

We then define $\widetilde{V}_{\rm eff}(q) = N^2 V_{\rm eff}(q)$, where $V_{\rm eff} = -d_1 V_1 + d_2 V_2$. Solving the model at infinite N, then, is just a matter of finding the (minimal) stationary point of $V_{\rm eff}(q)$ with respect to the q_i's.

The equations of motion follow by differentiating the potential in Eq. (1) with respect to q_i, and then taking the large N limit. Doing so, we find

$$0 = [d_1 + d_2]\ q\ - \frac{d_1}{2}\int dq'\rho(q')\,\mathrm{sign}(q-q')$$
$$+ d_2 \int dq'\rho(q')\left[-3(q-q')|q-q'| + 2(q-q')^3\right]\ , \quad (6)$$

where $\mathrm{sign}(x) = \pm 1$ for $x \gtrless 0$. For simplicity we write $d_1(T)$ and $d_2(T)$ just as d_1 and d_2.

To solve the equation of motion in Eq. (6), we follow Jurkiewicz and Zalewski [19] and use the following trick. What is difficult is that Eq. (6) is an integral equation for $\rho(q)$. To reduce this to a differential equation, take $\partial/\partial q$ of Eq. (6),

$$0 = d_1 + d_2 - d_1\,\rho(q) \quad (7)$$
$$+ 6\,d_2 \int dq'\rho(q')\left[-(q-q')\mathrm{sign}(q-q') + (q-q')^2\right]\ .$$

Notice that this does not give us the second variation of the potential with respect to an arbitrary variation of q, which is related to the mass squared. Instead, we take the derivative of the equation of motion, with respect to a solution of the same.

We then continue until we eliminate any integral over q'. Taking $\partial/\partial q$ of Eq. (7) gives

$$d_1\,\frac{d\rho(q)}{dq} = 6\,d_2 \int dq'\rho(q')\left[-\mathrm{sign}(q-q') + 2(q-q')\right]\ . \quad (8)$$

Lastly, by taking one final derivative, we obtain

$$\frac{d^2}{dq^2}\rho(q) + d^2\left[\rho(q) - 1\right]\ = 0\ . \quad (9)$$

In this expression we introduce the ratio $d^2(T) = 12\,d_2(T)/d_1(T)$. We assume that like the solution at small N [7–9], that $d(T)$ increases with T, and $d(T) \to \infty$ as $T \to \infty$. We note that the only detailed property of $d(T)$ which we require is that its expansion about T_d is linear in $T - T_d$. This is a minimal assumption which is standard in mean field theory.

We thus need to solve Eqs. (6) - (9), subject to the condition of Eq. (3). The solution of Eq. (9) is trivial,

$$\rho(q) = 1 + b\cos(d\,q)\quad ,\quad q : -q_0 \to q_0\ , \quad (10)$$

where b is a constant to be determined. We assume that $\rho(q) = 0$ for $|q| > q_0$. We have checked numerically that a multi gap solution [19], where $\rho(q) \neq 0$ over a set of gaps in q, does not minimize the potential; see the discussion at the end of Sec. (III).

When $q_0 < \frac{1}{2}$, $\rho(q_0) \neq 0$, and the solution drops discontinuously to zero at the endpoints. This stepwise discontinuity is charactertistic of the model, and presumably reflects the singularities from the absolute values in the potential.

The eigenvalue density in Eq. (10) is simpler than that in the GWW model [11–15, 17–19], where

$$\rho_{GW}(q) = \frac{1}{2}\cos(\pi q)\left[1 - \frac{\sin^2(\pi q)}{\sin^2(\pi q_0)}\right]^{1/2}\ . \quad (11)$$

For any q_0, this vanishes at the endpoints, $\rho_{GW}(\pm q_0) = 0$, while at the transition, $q_0 = \frac{1}{2}$. Due to the Vandermonde determinant in the potential, the density $\rho_{GW}(q)$ has a nontrivial analytic structure in the complex q-plane, while $\rho(q)$ does not. Since the Vandermonde potential is so different from $V_{\rm eff}$, though, it is natural to find that $\rho_{GW}(q)$ is unlike $\rho(q)$ in Eq. (10).

Eq. (10) solves Eq. (8) without further constraint. To solve the remaining equations, remember that all integrals run from $-q_0 \to q_0$. The normalization condition of Eq. (3) gives $b \sin(d\, q_0) = d(\frac{1}{2} - q_0)$. After some algebra, one can show that Eqs. (6) and (7) are equivalent, with the solution

$$\cot(d\, q_0) = \frac{d}{3}\left(\frac{1}{2} - q_0\right) - \frac{1}{d\,(1/2 - q_0)} \;, \qquad (12)$$

and

$$b^2 = \frac{d^4}{9}\left(\frac{1}{2} - q_0\right)^4 + \frac{d^2}{3}\left(\frac{1}{2} - q_0\right)^2 + 1 \;. \qquad (13)$$

Thus in the end, we only have to solve two coupled algebraic equations, Eqs. (12) and (13), for q_0 and b as functions of $d = d(T)$.

At low temperature, d is small, and the theory is in the confined phase, where $b = 0$ and $q_0 = \frac{1}{2}$. The eigenvalue density is constant, $\rho(q) = 1$, and all Polyakov loops vanish, $\ell_j = 0$. Thus the confined phase is characterized by the maximal repulsion of eigenvalues. The GWW model also has a constant eigenvalue density in the confined phase, which is expected, as only a constant eigenvalue density gives $\ell_j = 0$ for all loops.

In the limit of high temperature $d \to \infty$. The solution is $q_0 = 6/d^2$ and $b = d^2/12$. The eigenvalue density is $\rho \approx d^2/12$, which becomes a delta-function $\delta(q)$ for infinite d. That is, at high temperatures all eigenvalues coalesce into the origin, and all Polyakov loops equal one, $\ell_j = 1$.

As the temperature and so $d(T)$ is lowered, the transition occurs when $q_0 = \frac{1}{2}$, for which $d(T_d) = 2\pi$. At the transition point, the eigenvalue density is

$$\rho(q) = 1 + \cos(2\pi q) \quad ; \quad T = T_d \;. \qquad (14)$$

From Eq. (4),

$$\ell_1(T_d^+) = \frac{1}{2} \quad , \quad \ell_j(T_d) = 0 \;, j \geq 2 \;. \qquad (15)$$

Thus at the transition, only the Polyakov loop in the fundamental representation is nonzero, equal to $\frac{1}{2}$.

What is unforeseen is that at T_d^+, the eigenvalue density in the present model, Eq. (14), coincides *identically* with that in the GWW model, Eq. (11). Consequently, properties exactly at T_d^+, such as the expectation values of the ℓ_j, are the same in the two models. Since they differ away from T_d, other properties are similar, but not necessarily identical.

Consider the behavior in the deconfined phase just above the transition point, taking $d = 2\pi(1 + \delta d)$. Solving for δq and b in the limit of small δd, one finds that $\ell_1 - 1/2 \sim \delta d^{2/5}$; for $j \geq 2$, $\ell_j \sim \delta d$. Assuming that $\delta d \sim T_d - T$,

$$\ell_1(T) - \frac{1}{2} \sim (T_d - T)^\beta \quad , \quad \beta = \frac{2}{5} \;. \qquad (16)$$

That is, near the transition $\ell_1(T)$ exhibits a power like behavior which is characteristic of a second order phase transition — although $\ell_1(T_d^+) \neq 0$.

For arbitrary d, after some algebra one finds that at q_0^s, the solution of Eqs. (12) and (13), the potential equals

$$V_{\rm eff}(q_0^s) - V_{\rm eff}^{\rm conf} = -\, d_2\, \frac{16}{15}\left(\frac{1}{2} - q_0^s\right)^5 \;. \qquad (17)$$

The potential in the confined phase is $V_{\rm eff}^{\rm conf} = V_{\rm eff}(\frac{1}{2}) = -d_1/6 + d_2/30$. In these matrix models, the pressure is $p(T) = -V_{\rm eff}(q_0^s) + V_{\rm eff}^{\rm conf}$. This subtraction ensures that the pressure, and the associated energy density, are suppressed by $\sim 1/N^2$ in the confined phase. In the models of Ref. [7, 8], $V_{\rm eff}^{\rm conf}(q)$ is given by a constant independent of q, the term $\sim c_3$. Expanding about T_d,

$$V_{\rm eff}(q_0) - V_{\rm eff}^{\rm conf} + \frac{48 d_2}{\pi^4}\, \delta d \;\sim \delta d^{7/5} + \ldots \qquad (18)$$

Assuming that $\delta d \sim T - T_d$, as is true of the functions in Refs. [7–9], the leading term $\sim \delta d$ shows that the first derivative of the pressure with respect to temperature, which is related to the energy density $e(T)$, is nonzero at T_d^+. Since the pressure and the energy density are suppressed by $\sim 1/N^2$ in the confined phase, the latent heat is nonzero and $\sim N^2$, $\sim e(T_d^+)$.

Using the explicit forms for $d_1(T)$ and $d_2(T)$, we find that the latent heat is $e(T_d^+)/(N^2 T_d^4) = 1/\pi^2 \sim .10...$ This is about four times smaller than the lattice results of Ref. [2] who find ~ 0.39 for the same quantity. The lattice results can be accomodated by adding a term like a MIT bag constant to the model [8]. Such a term is $\sim T_d^4$ but independent of the q's, and so only changes the latent heat, but does not affect any other result.

The second term in Eq. (18) shows that the second derivative of the pressure with respect to temperature diverges as $T \to T_d^+$,

$$\frac{\partial^2}{\partial T^2}\, p(T) \sim \frac{1}{(T - T_d)^\alpha} \quad , \quad \alpha = \frac{3}{5} \;. \qquad (19)$$

This is the usual divergence of the specific heat for a second order phase transition.

II. NONZERO BACKGROUND FIELD, $T = T_d$

Background fields can be added for each loop ℓ_j. In this paper we just consider a background field for the simplest loop, ℓ_1, since only that is nonzero at T_d, Eq. (15). We add

$$V_h(q) = -\,\frac{d_1}{(2\pi)^2}\; h\; \ell_1 \qquad (20)$$

to the potential $V_{\rm eff}(q)$, and find the solution as before. After taking three derivatives of the equation of motion, with respect to a solution, we obtain the analogy of Eq. (9),

$$\frac{d^2}{dq^2}\rho(q) + d^2\,[\rho(q)-1] + (2\pi)^2\; h\; \cos(2\pi q) \; = 0\ . \qquad (21)$$

This equation is valid for any d. It is necessary to treat the case of T_d, where $d = 2\pi$, seperately from $T \neq T_d$.

In this section we consider the point of phase transition, where $d = 2\pi$. The solution of Eq. (21) is

$$\rho(q) = 1 + b\,\cos(2\pi\, q) - \pi\, h\; q\; \sin(2\pi q)\ , \qquad (22)$$

where $q : -q_0 \to q_0$. Notice that the h-dependent term $q\,\sin(2\pi q)$ arises because when $T = T_d$, Eq. (21) represents a driven oscillator at the resonance frequency. The value of the constants b and q_0 now depend upon both $d(T)$ and the background field, h.

The analogy of Eq. (8) is solved by Eq. (22). The normalization condition, Eq. (3), plus the analogy of Eq. (7), gives two equations for b and q_0; as before, Eq. (6) does not give a new condition.

When $h \neq 0$, the explicit form of the analogy of Eq. (3) is elementary, but that of Eq. (7) is rather ungainly. We thus present the results of the solution in the limit of small background field, $h \ll 1$. In this limit, at the minimum the h-dependence of the potential is

$$V_{\rm eff}(q_0^s, h) + \frac{d_1}{8\pi^2} h \sim h^{7/5} + \ldots\ . \qquad (23)$$

The expectation value of the loop ℓ_1 is $\ell_1 - \frac{1}{2} \sim h^{2/5}$. so that the critical exponent $\delta = 5/2$. This shows that the critical exponents satisfy the usual Griffths scaling relation, $2 - \alpha = \beta(1+\delta)$.

The effective potential, as a function of ℓ_1, is computed by taking the Legendre transform,

$$\Gamma(\ell_1) = V_{\rm eff}(h) + \frac{d_1}{(2\pi)^2} h_1 \ell_1\ . \qquad (24)$$

Expanding the potential at T_d^+, $\Gamma(\ell_1) \sim (\ell_1 - \frac{1}{2})^{7/2}$. This is in contrast to the femtosphere, where the potential behaves as $\sim (\ell_1 - \frac{1}{2})^3$ about the similar point [12, 15].

Expanding at T_d^- gives the expansion of the potential about $\ell_1 = 0$. One can show, and we verify in the next section, that this potential vanishes. This implies that the potential has an unusual form: it is zero from $\ell_1 : 0 \to \frac{1}{2}$, and then turns on as $\sim (\ell_1 - \frac{1}{2})^{7/2}$. Graphically, this potential is like that on the femtosphere; see, *e.g.*, Fig. (1) of Ref. [15].

III. NONZERO BACKGROUND FIELD, $T \neq T_d$

Consider now the theory in a nonzero background field for ℓ_1, Eq. (20), away from the transition, so $d \neq 2\pi$. The eigenvalue density again solves Eq. (21). The solution is simpler when $d \neq 2\pi$, and is just the sum of the solution for $h = 0$ and an h-dependent term,

$$\rho(q) = 1 + b\,\cos(d\,q) + \frac{1}{1-(d/2\pi)^2}\; h\; \cos(2\pi q)\ . \qquad (25)$$

The solution follows as previously, and we simply summarize the results.

We first consider the confined phase, defined to be the solution for which $q_0 = \frac{1}{2}$ and $b = 0$. After Legendre transformation, the effective potential is

$$\Gamma(\ell_1) = \left(1 - \frac{d^2}{4\pi^2}\right)\frac{1}{\pi^2}\; \ell_1^2\ . \qquad (26)$$

This shows that in the confined phase, when $d < 2\pi$ the mass squared of the ℓ_1 loop is positive, as expected. It also shows that this mass vanishes at T_d when $h = 0$; this justifies the statements about the potential at the end of the previous section.

Consider a special value of d, $d_h^2 = 4\pi^2(1-h)$; the corresponding temperature is defined to be T_h, $d(T_h) = d_h$. At this temperature, the eigenvalue density of Eq. (25) coincides exactly with that at the transition in zero background field, Eq. (14). Notably, the values of the loop at $h \neq 0$ and $T = T_h$ are the same as for $h = 0$ and $T = T_d$: $\ell_1(T_h) = \frac{1}{2}$, with $\ell_j = 0$ for $j \geq 2$, Eq. (15). Thus we may suspect that something special happens at $T = T_h$. For example, the confined phase is only an acceptable solution when $T < T_h$, as only then is the eigenvalue density positive definite.

This suggests that a phase transition occurs at d_h. To show this, we compute for about this value of d, taking $d^2 = d_h^2 + 4\pi^2\, h\,\delta d$. Solving the model as before in the limit of small δd, we find $V_{\rm eff}(h) - V_{\rm eff}^{\rm conf}(h) \sim \delta d^{5/2}$. Taking $\delta d \sim T_h - T$, we find that the *third* derivative of the pressure, with respect to temperature, diverges at T_h,

$$\frac{\partial^3}{\partial T^3}\, p(T) \sim \frac{1}{(T-T_h)^{1/2}} \quad , \quad T \to T_h^+\ . \qquad (27)$$

In zero background field, then, there is a critical first order transition at a temperature T_d. Turning on a background field $\sim h\,\ell_1$, the first order transition is immediately wiped out for any $h \neq 0$. Even so, there remains a third order phase transition, at a temperature $T_h < T_d$, where the expectation value of the loop $\ell_1 = \frac{1}{2}$. This behavior is the same as on a femtosphere [12, 14, 15].

In principle one can also add a background field for any loop, ℓ_j for $j \geq 2$. It is direct to derive the equations of motion and obtain a solution for the eigenvalue density. Obtaining the minimum of the potential is not elementary, though. The original model of Gross and Witten

[17] involves the Vandermonde determinant plus a term $\sim |\mathrm{tr}\mathbf{L}|^2$. The solution for the eigenvalue density is a function which is nonzero on one interval, between $-q_0$ and q_0. Jurkiewicz and Zalewski [19] showed that when terms such as $|\mathrm{tr}\,\mathbf{L}^2|^2$ are added to the GWW model, that in general it involves functions which are nonzero on more than one interval. We have checked numerically that when only $h_1 \neq 0$, that such multi-gap solutions do not minimize the potential. We do find, however, that multi-gap solutions do minimize the potential in the presence of background fields for ℓ_j when $j \geq 2$. Since only $\ell_1 \neq 0$ at T_d and T_h, we defer the problem of background ℓ_j for $j \geq 2$.

IV. FINITE N

The model can be solved numerically at finite N. This confirms, as expected on general grounds [8], that the deconfining transition is of first order for any $N \geq 3$. It also shows that the critical behavior found at infinite N is smoothed out at large but finite N.

Using the numerical solution of the model, in the Figure we show the behavior of the specific heat, divided by N^2-1, for different values of N. To see the putative divergence of the specific heat at infinite N, rather large values of N are necessary, $N \geq 40$.

This Figure also shows that the increase in the specific heat only manifests itself very close to the transition, within $\sim 0.2\%$ of T_d. At present, direct numerical simulations on the lattice treat moderate values of $N \sim 4-10$ [2]. For most quantities there seems to be a weak variation with N.

The present matrix model suggests that *very* near T_d, a novel phase transition may occur at large N. The values of N at which critical first order behavior arise can presumably be studied only in reduced models [3].

This begs the important question: in infinite volume, does the GWW transition occur for $SU(\infty)$ [21]? On the femtosphere, there is a GWW transition at zero coupling [12], which becomes an ordinary first order transition at small but nonzero coupling [13]. Similarly, in the presence of additional couplings, such as in the presence of additional couplings, such as $(|\mathrm{tr}\,\mathbf{L}|^2)^2$, the GWW transition becomes an ordinary first order transition [15]. Unfortunately, we have not been able to solve the present model in the presence of additional couplings. Thus it is possible that as on a femtosphere, the presence of additional couplings washes out the GWW transition.

On a femtosphere, though, correlation lengths cannot be larger than the radius of the sphere. In infinite volume, however, as a second order phase transition is approached correlation lengths diverge and coupling constants flow under the action of the renormalization group.

This leads us to conclude with a conjecture. We find it remarkable that two very different theories, the present model and that with a Vandermonde determinant, both exhibit a GWW transition at T_d. Perhaps in infinite spatial volume, the GWW transition is an infrared stable fixed point of a $SU(\infty)$ gauge theory at nonzero temperature. Since the GWW transition is not a standard second order transition, the analysis of the renormalization group is not trivial. Nevertheless, the study of reduced models [3], such as of the specific heat in the Figure, can directly test this conjecture. After all, gauge theories are remarkable things, and can surprise us.

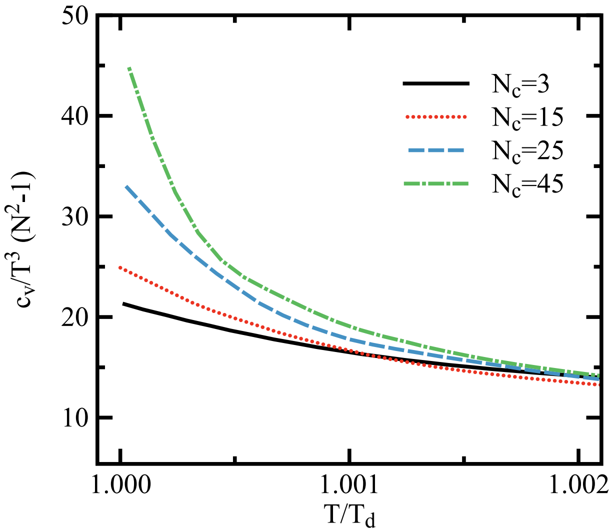

FIG. 1. Plot of the specific heat, divided by $(N^2-1)T^3$, for different values of N.

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[21] It is also possible to develop a matrix model to study deconfinement in three spacetime dimensions [22]. This involves different functions of q than those in four dimensions, Eq. (2). It is not clear if this model is soluble analytically at infinite N, or if it exhibits a GWW transition in that limit.

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