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Realistic Fermion Masses and Nucleon Decay Rates in SUSY $SU(5)$ with Vector-Like Matter

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Abstract

We show that by adding a vector-like $5 + \bar{5}$ pair of matter fields to the spectrum of the minimal renormalizable SUSY $SU(5)$ theory the wrong relations for fermion masses can be corrected, while being predictive and consistent with proton lifetime limits. Threshold correction from the vector-like fields improves unification of gauge couplings compared to the minimal model. It is found that for supersymmetric spectra lighter than 3 TeV, which would be testable at the LHC, at least some of the nucleon decay modes should have partial lifetimes shorter than about 2×10^{34} yrs., which is within reach of ongoing and proposed experiments.

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1 Introduction

While elegant and simple, the minimal renormalizable supersymmetric $SU(5)$ model [1, 2, 3] suffers from two main drawbacks. The first is the wrong predictions it makes for the light fermion masses. This theory predicts the asymptotic relations $m_d^0 = m_e^0$, $m_s^0 = m_\mu^0$ and $m_b^0 = m_\tau^0$ connecting the charge $-1/3$ quark masses and charged lepton masses, valid at the grand unification scale of 2×10^{16} GeV. Such relations would enable one to calculate the down-type quark masses in terms of the charged lepton masses by evolving the mass parameters via the renormalization group equations (RGE). The relation $m_b^0 = m_\tau^0$ is generally considered a successful prediction of minimal SUSY $SU(5)$, since the b -quark mass computed in terms of τ -lepton mass is typically within about 20% of its experimental value. The relations involving the lighter families, however, lead to wrong predictions. For example, the RGE-invariant relation $m_d/m_s = m_e/m_\mu$, which follows from the asymptotic relations of the minimal model, differs from experimental values by about a factor of 10 ($m_d/m_s \simeq 1/20$ while $m_e/m_\mu \simeq 1/200$ at low energy scale [4]).

The second drawback of the minimal SUSY $SU(5)$ model is its prediction for proton lifetime for the mode $p \rightarrow \bar{\nu} K^+$ which arises via the exchange of colored Higgsinos. The lifetime is generically too fast compared to the present experimental limits. This prediction follows mainly from the requirement of gauge coupling unification. The spectrum of the minimal supersymmetric standard model (MSSM) at low energies does not lead to a precise unification of the three gauge couplings when the full two-loop RGE are used, and therefore requires some threshold correction from the GUT scale. The only possibility in the minimal renormalizable $SU(5)$ set-up is to make the color triplets from the $5_H + \bar{5}_H$ Higgs fields (which transforms as $(3, 1, -1/3) + h.c.$ under $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group) somewhat lighter compared to the vector supermultiplets (the X and Y gauge bosons of $SU(5)$). Since the same color triplets mediate $d = 5$ proton decay [5, 6], making it lighter than the GUT scale results in a considerably shorter proton lifetime [7, 8, 9, 10, 11, 12], typically in conflict with experimental limits. Notice that this outcome is due to the minimal particle content: the same color triplet that corrects the RGE running of the gauge couplings is coupled to the Standard Model (SM) fermions with fixed Yukawa couplings. (The color triplet Yukawa couplings are unified with the Yukawa couplings of the $SU(2)_L$ doublets also contained in $5_H + \bar{5}_H$ that generate quark and lepton masses and mixings.) There is no other choice in the minimal model for correcting the RGE running of the gauge couplings.

There are various well known ways out of these two problems. The most commonly

used solution is the inclusion of higher dimensional operators. Due to the vicinity of M_{GUT} to M_{Planck} such operators may not be negligible numerically, especially for the lighter fermion masses [13]. For example, they can easily improve the calculated masses of the first two generations. Their influence for proton decay is even bigger. They make the Yukawa couplings to the color triplet Higgs different from those to the weak doublet Higgs, so that there is some freedom which can be used to somewhat suppress the $d = 5$ proton decay amplitudes. Alternatively, these higher dimensional operators can allow for a lighter color octet and weak triplet (remnants of $SU(5)$ symmetry breaking via a 24_H) which can increase both the GUT scale and the color triplet masses [14, 15, 16], alleviating the $d = 5$ proton decay problem significantly.

The problem with this natural solution is that it automatically introduces a large number of new parameters into the game, thus precluding any quantitative prediction. So, although the model can be made consistent and realistic, it is difficult to test it. There is also some questions about the strengths of these higher dimensional operators being of the right magnitude if they are induced by quantum gravity effects. In this paper we take a different approach. We assume that our supersymmetric $SU(5)$ GUT is renormalizable. After all, we really do not know how gravity influences our particle physics world, and a conservative approach would be to not rely heavily on gravity-induced corrections. This approach of using only renormalizable couplings has brought great success in the electroweak sector of the Standard Model. The renormalizability of the theory would greatly reduce possible couplings in the theory resulting in enhanced predictivity. With this in mind we shall add to the minimal supersymmetric $SU(5)$ as little as possible: a vector-like $5 + \bar{5}$ matter field. This will allow unequal mixings of the down quarks and charged leptons with these fields, thus correcting the wrong mass relations. Simultaneously this set-up would provide a new set of color triplet/weak doublet fields, which allows for a precise unification of gauge couplings by choosing the color triplet somewhat lighter than the weak doublet. Note that such a choice does not run afoul with $d = 5$ proton decay rates, unlike the minimal SUSY $SU(5)$ model, since the $5 + \bar{5}$ fields do not acquire vacuum expectation values (VEVs). As in minimal SUSY $SU(5)$ we assume R -parity conservation, and we take the vector-like $5 + \bar{5}$ pair to be fermion-like. Had we chosen Higgs-like multiplets such as $45 + \overline{45}$, the wrong fermion mass relations could have been corrected [17], however in this case quantitative predictions for proton decay would be difficult to make owing to the large number of parameters that would be introduced. Another possible solution to the wrong mass problem of the minimal SUSY $SU(5)$ model

is through supersymmetric threshold corrections arising from soft SUSY breaking terms with a particular form, see for example Ref. [18, 19]. Here we shall assume that the SUSY spectrum is such that such threshold corrections remain small. Yet another possibility is to utilize large Yukawa couplings involving vector-like multiplets. This can raise the unification scale when two-loop RGE effects are included, which would allow for a better prediction for $\alpha_3(M_Z)$ [20, 21].

We now turn to the discussion of fermion masses in presence of a $5 + \bar{5}$ matter fields and show how the mixing of these fields with the MSSM fermions corrects the wrong mass relations. We then derive the baryon number violating effective $d = 5$ superpotential and study its implications for nucleon lifetime. The small number of new parameters that are introduced with the addition of a $5 + \bar{5}$ vector-like fermions allows the model to be consistent with current proton lifetime limits, but at the same time we find that at least some modes should have partial lifetime less than about 2×10^{34} yrs. In our analysis we assume that the GUT scale stays well below the Planck scale (by a factor of 20 to 50) so that quantum gravity effects can be ignored, and the approximate unification of the gauge couplings that occurs in the MSSM is not a complete accident. For supersymmetric spectrum, we assume that all super-particles have masses less than about 3 TeV, which would make them detectable at the LHC, while at the same time providing a solution to the gauge hierarchy problem.

2 Fermion masses with vector-like $5 + \bar{5}$ matter fields

Before discussing the modifications of the fermion mass relations with the inclusion of a $5 + \bar{5}$ matter fields in SUSY $SU(5)$, let us briefly summarize the situation in the minimal renormalizable SUSY $SU(5)$ model.

2.1 Fermion Masses in minimal SUSY $SU(5)$

The matter fields of the model consist of three generations in representations $10_i + \bar{5}_i$, $i = 1, 2, 3$. The Higgs sector consists of an adjoint 24_H used for breaking $SU(5)$ symmetry down to the SM symmetry, and a pair of $5_H + \bar{5}_H$ fields for electroweak symmetry breaking. The renormalizable superpotential of the adjoint field relevant for $SU(5)$ symmetry breaking is

$$W_{24} = \frac{m}{2} \text{Tr} (24_H^2) + \frac{\lambda}{3} \text{Tr} (24_H^3) . \quad (2.1)$$

The scalar potential induced by this superpotential has a ground state with a non-zero vacuum expectation value,

$$\langle 24_H \rangle = v \text{ diag } (2, 2, 2, -3, 3) \quad (2.2)$$

which spontaneously breaks $SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$. The VEV v is determined to be

$$v = \frac{m}{\lambda} . \quad (2.3)$$

The simplicity of Eq. (2.1) fixes the masses of the color octet (the $(8, 1, 0)$ fragment of 24_H which is a physical Higgs particle) M_8 and the weak triplet (the $(1, 3, 0)$ fragment of 24_H) M_3 to be

$$M_3 = M_8 = 5m . \quad (2.4)$$

The same VEV sets the super-heavy $SU(5)$ gauge boson masses to be

$$M_X = M_Y = 5\sqrt{2}g\frac{m}{\lambda} . \quad (2.5)$$

The two MSSM Higgs doublets H_u and H_d live in the pair of Higgs fundamentals $5_H + \bar{5}_H$ and have Yukawa couplings with the matter fields given by

$$W_Y = 10_i Y_{10}^{ij} 10_j 5_H + \bar{5}_i Y_5^{ij} 10_j \bar{5}_H . \quad (2.6)$$

The equality of the down-type quark masses and charged lepton masses follows from this superpotential:

$$M_D = \langle \bar{5}_H \rangle Y_5^T = M_E^T . \quad (2.7)$$

The color triplets from $5_H + \bar{5}_H$ have the same Yukawa couplings as the Higgs doublets and would mediate rapid proton decay via $d = 5$ baryon number violating operators. For this reason they must be ultra-heavy, preferably with a mass above the GUT scale. In the superpotential terms

$$W_5 = \bar{5}_H (m_H + \eta_H 24_H) 5_H \quad (2.8)$$

this can be arranged by a fine-tuning:

$$m_H = 3\eta_H \frac{m}{\lambda} . \quad (2.9)$$

The color triplet mass is thus

$$M_T = 5\eta_H \frac{m}{\lambda} \quad (2.10)$$

which shows that m_T cannot be arbitrarily large if we demand (as we do) perturbativity of the couplings:

$$\frac{M_T}{M_X} = \frac{\eta_H}{\sqrt{2}g} \lesssim \mathcal{O}(1) . \quad (2.11)$$

Due to the relation in Eq. (2.4), the requirement of gauge coupling unification would imply that the color triplet mass is actually much lower, around or even smaller than 10^{15} GeV [11].⁴ Such a light color triplet would mediate too fast a proton decay, which is a problem with the minimal model.

2.2 Mixing of chiral families with $5 + \bar{5}$ fields

To the minimal SUSY $SU(5)$ described in the previous subsection we now add a vector-like pair of matter fields⁵ denoted as $5_4 + \bar{5}_4$. With their R -parity assumed to be identical to that of the chiral families $10_i + \bar{5}_i$ (or equivalently odd matter parity), the most general renormalizable addition to the superpotential of minimal $SU(5)$ is

$$W_4 = \bar{5}_a (\mu_a + \eta_a 24_H) 5_4, \quad a = 1, \dots, 4 . \quad (2.12)$$

Notice that, without loss of generality, by an appropriate choice of the basis, the terms $\bar{5}_4 10_i \bar{5}_H$ can be rotated away. Thus, the whole Yukawa superpotential reads as

$$W_Y = 10_i Y_{10}^{ij} 10_j 5_H + \bar{5}_i Y_5^{ij} 10_j \bar{5}_H + \bar{5}_a (\mu_a + \eta_a 24_H) 5_4 . \quad (2.13)$$

One can work in a basis where the 3×3 coupling matrix Y_5^{ij} is diagonal:

$$Y_5^{ij} = y_i \delta_{ij} .$$

Plugging the VEVs $\langle 5_H \rangle = v_u$, $\langle \bar{5}_H \rangle = v_d$, $\langle 24_H \rangle = v \text{ diag } (2, 2, 2, -3, -3)$ into Eq. (2.13) and keeping color triplet states T, \bar{T} (from $5_H, \bar{5}_H$), the relevant terms involving the MSSM fields and the additional vector-like states will be

$$\begin{aligned} W_Y = & L^T M_l^{4 \times 4} E^c + D^{cT} M_d^{4 \times 4} D + u^T M_U^0 u^c + l^T Y_5 q \bar{T} + \frac{1}{v_u} u^T M_U^0 d T \\ & + d^{cT} Y_5 u^c \bar{T} + \frac{1}{v_u} e^{cT} M_U^0 u^c T , \end{aligned} \quad (2.14)$$

where

$$L^T = (l_1, l_2, l_3, l_4) , \quad E^{cT} = (e_1^c, e_2^c, e_3^c, \bar{l}_4) ,$$

⁴An exception would be to choose very special MSSM soft parameters [22]. This may however require very particular and exotic hidden and messenger sectors of SUSY breaking.

⁵The use of heavy vector-like matter to correct the bad mass relations in GUTs is long known. For an incomplete list see for example [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34].

$$D^{cT} = (d_1^c, d_2^c, d_3^c, d_4^c) , \quad D^T = (d_1, d_2, d_3, \bar{d}_4^c) , \quad (2.15)$$

$$M_l^{4 \times 4} = \begin{pmatrix} y_i \delta_{ij} v_d & M_i^l \\ 0 & |M_4^l| \end{pmatrix} , \quad M_d^{4 \times 4} = \begin{pmatrix} y_i \delta_{ij} v_d & M_i^d \\ 0 & |M_4^d| \end{pmatrix} , \quad (2.16)$$

$$M_i^l = \mu_a - 3\eta_i v , \quad M_i^d = \mu_i + 2\eta_i v , \quad M_U^0 = Y_{10} v_u . \quad (2.17)$$

Let us now focus on the light (MSSM) charged lepton and down-type quark masses arising from Eq. (2.16). These are obtained by removing the heavy vector-like state from the spectrum. The mass matrices of Eq. (2.16) can be block-diagonalized so as to bring the mass terms in the superpotential to the form

$$W_{mass} = e^T \hat{M}_E e^c + d^T \hat{M}_D d^c + u^T \hat{M}_U u^c + M_D D \bar{D} + M_C C \bar{C} . \quad (2.18)$$

The reduced mass matrices \hat{M}_E and \hat{M}_D , derived in Appendix A.1, can be made real and have forms

$$\hat{M}_E = \begin{pmatrix} d_1 c_1^e & 0 & 0 \\ -d_1 s_1^e s_2^e & d_2 c_2^e & 0 \\ -d_1 c_2^e s_1^e s_3^e & -d_2 s_2^e s_3^e & d_3 c_3^e \end{pmatrix} , \quad \hat{M}_D = \begin{pmatrix} d_1 c_1^d & -d_1 s_1^d s_2^d & -d_1 c_2^d s_1^d s_3^d \\ 0 & d_2 c_2^d & -d_2 s_2^d s_3^d \\ 0 & 0 & d_3 c_3^d \end{pmatrix} \quad (2.19)$$

with

$$d_i = |y_i v_d| , \quad c_i^{e,d} \equiv \cos \theta_i^{e,d} , \quad s_i^{e,d} \equiv \sin \theta_i^{e,d} , \quad t_i^{e,d} \equiv \tan \theta_i^{e,d} , \\ t_1^{e,d} = \frac{|M_1^{l,d}|}{|M_4^{l,d}|} , \quad t_2^{e,d} = \frac{|M_2^{l,d}|}{|M_4^{l,d}|} c_1^{e,d} , \quad t_3^{e,d} = \frac{|M_3^{l,d}|}{|M_4^{l,d}|} c_1^{e,d} c_2^{e,d} . \quad (2.20)$$

Note that since $M_i^l \neq M_i^d$, the wrong GUT scale asymptotic relation $\hat{M}_E(M_G) = \hat{M}_D^T(M_G)$, which is problematic for the minimal renormalizable $SU(5)$ model, is avoided here. In Eq. (2.16) $\hat{M}_U = M_U^0 = Y_{10} v_u$, since the up-type quarks do not mix with any of the vector-like field.

From Eq. (2.19), it follows that realizing the mass hierarchy between different families is possible only when the diagonal factors d_i are hierarchical, $d_1 \ll d_2 \ll d_3$, in which case we can write down very simple formulas for the masses:

$$m_i^{e,d} \simeq d_i \cos \theta_i^{e,d} . \quad (2.21)$$

Thus, it is possible to fit all quark and lepton masses consistently to the observed values. The mixing angles are related by the ratios:

$$\frac{m_i^d}{m_i^e} \simeq \frac{\cos \theta_i^d}{\cos \theta_i^e} . \quad (2.22)$$

The 3×3 light fermion mass matrices are diagonalized via bi-unitary transformations

$$\hat{M}_E = U_E^\dagger M_{diag}^E V_E, \quad \hat{M}_D = U_D^\dagger M_{diag}^D V_D, \quad \hat{M}_U = V_u^\dagger M_{diag}^U V_u^*, \quad (2.23)$$

by going from the flavor to the mass eigenstate basis:⁶

$$\begin{aligned} d &\rightarrow U_D^T \hat{P} d, & e &\rightarrow U_E^T e, & u &\rightarrow V_u^T P^{1/2} u, & \nu &\rightarrow U_E^T \nu \\ d^c &\rightarrow V_D^\dagger \hat{P}^* d^c, & e^c &\rightarrow V_E^\dagger e^c, & u^c &\rightarrow V_u^T \sqrt{P^*} u^c. \end{aligned} \quad (2.24)$$

The diagonal phase matrices P and \hat{P} are introduced (see Appendix A.1 for details) so that the CKM matrix can be written as

$$V_{CKM} = \sqrt{P^*} V_u^* U_D^T \hat{P} \quad (2.25)$$

in a standard parametrization with a single phase:

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & \hat{s}_{13}^* \\ -s_{12}c_{23} - c_{12}s_{23}\hat{s}_{13} & c_{12}c_{23} - s_{12}s_{23}\hat{s}_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}\hat{s}_{13} & -c_{12}s_{23} - s_{12}c_{23}\hat{s}_{13} & c_{23}c_{13} \end{pmatrix}. \quad (2.26)$$

The entries of Eq. (2.26) can be parameterized by four Wolfenstein parameters λ , A , $\bar{\rho}$ and $\bar{\eta}$ as follows:

$$\begin{aligned} s_{12} &= \lambda, & c_{12} &= \sqrt{1 - \lambda^2}, & s_{23} &= A\lambda^2, & c_{23} &= \sqrt{1 - A^2\lambda^4} \\ \hat{s}_{13} &= \frac{A\lambda^3(\bar{\rho} + i\bar{\eta})\sqrt{1 - A^2\lambda^4}}{\sqrt{1 - \lambda^2}[1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta})]}, & s_{13} &= |\hat{s}_{13}|, & c_{13} &= \sqrt{1 - s_{13}^2}. \end{aligned} \quad (2.27)$$

With the central values of these parameters taken from PDG [37]

$$\lambda = 0.2253, \quad A = 0.808, \quad \bar{\rho} = 0.132, \quad \bar{\eta} = 0.341 \quad (2.28)$$

we can calculate the CKM elements at M_Z scale. The corresponding CKM elements at the GUT scale are obtained from $V_{CKM}(M_Z)$ by dividing the 13, 23, 31 and 32 elements by a common RGE factor ($\simeq 1.055$ for $\tan\beta = 7$), while keeping the remaining elements intact.

⁶Neutrino masses are ignored for simplicity, since they are irrelevant for our studies. They can of course be included via the seesaw mechanism with right-handed singlet neutrinos fields introduced. This would have very little effects on our discussions. Another possibility would be to include bilinear R-parity violating couplings, see for example [35].

As far as the charged fermion masses are concerned, their Yukawa couplings at the GUT scale, taken to be $M_G \approx 2 \cdot 10^{16}$ GeV, for $\tan \beta = 7$, are taken to be

$$\begin{aligned} M_{diag}^U/v_u &= \text{diag} (5.49 \cdot 10^{-6}, 0.00323, 1) \lambda_t(M_G), \quad \lambda_t(M_G) \simeq 0.44 , \\ M_{diag}^D/v_d &= \text{diag} (0.000886 , 0.01646 , 1) \lambda_b(M_G), \quad \lambda_b(M_G) \simeq 0.038 , \\ M_{diag}^E/v_d &= \text{diag} (0.0002777, 0.05862, 1) \lambda_\tau(M_G), \quad \lambda_\tau(M_G) \simeq 0.047 . \end{aligned} \quad (2.29)$$

These values correspond to central values of these masses at low energy scale, see for eg., Ref. [36]. These numerical values will be used below for the study of proton decay. We emphasize that realistic fermion masses are obtained in this model, unlike the minimal renormalizable $SU(5)$ model.

3 The value of $\alpha_3(M_Z)$

Since in the model under study we have additional states D, \bar{D}, C, \bar{C} beyond those of minimal SUSY $SU(5)$, if their masses lie below the GUT scale (M_G), the unification of three gauge couplings will be modified. The masses of these extra states are given by

$$\begin{aligned} M_D &= \sqrt{|M_1^l|^2 + |M_2^l|^2 + |M_3^l|^2 + |M_4^l|^2} , \\ M_C &= \sqrt{|M_1^d|^2 + |M_2^d|^2 + |M_3^d|^2 + |M_4^d|^2} . \end{aligned} \quad (3.1)$$

Since in M_a^l, M_a^d there are $SU(5)$ symmetry breaking effects (see Eq. (2.17)), in general these two masses differ: $M_D \neq M_C$. We will exploit this fact for improving the value of $\alpha_3(M_Z)$ predicted by the demand that the three gauge couplings unify. Assuming that $M_D \simeq M_G$ and $M_C < M_G$, we will have:

$$\frac{1}{\alpha_3(M_Z)} \simeq \frac{1}{\alpha_3^0(M_Z)} - \frac{9}{14\pi} \ln \frac{M_C}{M_G} , \quad (3.2)$$

where $\alpha_3^0(M_Z)$ denotes the value of the strong coupling constant one would have obtained in minimal SUSY $SU(5)$ GUT. The second term on the right-hand side of Eq. (3.2) is due to the one-loop contribution of the extra color triplet pair from the vector-like fermions with mass $M_C < M_G$. With the choice of super-particle spectrum inspired by supergravity (see below Eq. (4.20) and Table 1 for the spectral values we use), and with all the GUT-scale states (besides C, \bar{C}) having masses $\simeq M_G$ one would obtain $\alpha_3^0(M_Z) \simeq 0.127$. To bring this somewhat large value down we take $\frac{M_C}{M_G} \simeq 0.061$. Using this in Eq. (3.2),

we obtain $\alpha_3(M_Z) \simeq 0.1184$ - the central value of the experimentally determined strong coupling constant.

Note that from Eq. (3.2) the ratio $\frac{M_G}{M_C}$ is determined. The value of M_G should be found from the meeting point of three gauge couplings. Because of the fact that the dependence of M_G on $\alpha_i(M_Z)$ is exponential, we are able to determine M_G , and therefore also M_T , only to an accuracy of about 22%. This will cause an uncertainty of about 45% in the $d = 5$ proton decay lifetime estimate. Further uncertainty is caused by the uncertainty in the ratio $r = M_8/M_X$. The natural value of r is of order one, but $r \ll 1$ cannot be excluded. Choosing $r \ll 1$ would result in larger values of the unification scale, which we shall demand to lie at least a factor 20 – 50 below the Planck scale, so that quantum gravitational corrections to the gauge coupling evolution remain small.

4 Effective baryon number violating operators and nucleon decay

In studying nucleon decay, we will need to derive the relevant $d = 5$ baryon number violating effective operators. These operators are obtained by integrating out the extra vector-like matter superfields, as well as the states T, \bar{T} from the couplings given in Eq. (2.14). Details of this procedure are given in Appendix A.2. Here we present the relevant effective superpotential couplings:

$$W_{eff} = W_{mass} + W_L^{d=5} + W_R^{d=5}, \quad (4.1)$$

where W_{mass} is given in Eq. (2.18),

$$W_L^{d=5} = \frac{\epsilon^{abc}}{M_T v_u v_d} (u_a^T \hat{M}_U d_b) (\nu^T \hat{M}_E P' d_c - e^T \hat{M}_E P' u_c), \quad (4.2)$$

and

$$W_R^{d=5} = \frac{\epsilon^{abc}}{M_T v_u v_d} (u_a^{cT} \hat{M}_U P'^* e^c) (d_b^{cT} \hat{M}_D^T u_c^c). \quad (4.3)$$

Here a, b, c are color indices. P' is a phase matrix $P' = \text{diag}(e^{i\delta_1}, e^{i\delta_2}, 1)$. M_D and M_C are the masses of the extra vector-like weak doublets (D, \bar{D}) and color triplets (C, \bar{C}) respectively. Note that all these coupling are written in the flavor basis of MSSM quarks and leptons.⁷ The couplings given in (4.1)-(4.3) will be needed for the discussion of nucleon decay. Now we turn to the estimate of $d = 5$ proton decay rates.

⁷These states differ from those of initial superpotential (2.14) due to various rotations (discussed in the Appendix). However, in Eqs. (4.1)-(4.3) we use the same notation (without primes) for simplicity.

4.1 Effective $d = 5$ operators in the mass eigenstate basis

With the basis change given in Eq. (2.24) and using Eqs. (2.23), (2.25), the baryon number violating operators of Eqs. (4.2), (4.3) will have the following form in the mass eigenstate basis:

$$W_L^{d=5} = \frac{\epsilon^{abc}}{M_T v_u v_d} (u_a^T P M_{diag}^U V_{CKM} d_b) \left(\nu^T M_{diag}^E V d_c - e^T M_{diag}^E V V_{CKM}^\dagger u_c \right) \quad (4.4)$$

$$W_R^{d=5} = \frac{\epsilon^{abc}}{M_T v_u v_d} (u_a^{cT} M_{diag}^U V_{CKM} V^\dagger e^c) \left(d_b^{cT} M_{diag}^D V_{CKM}^\dagger P^* u_c^c \right) . \quad (4.5)$$

The matrices V and P are given in Eqs. (A.10)-(A.12).

The $d = 6$ four fermion operator obtained from $W_L^{d=5}$ by wino dressing and involving the neutrino has the form

$$\mathcal{O}_{\nu L}^{d=6} = \frac{\epsilon^{abc}}{M_T} \mathcal{C}_{\delta\alpha\gamma\rho}^\nu (u_a^\delta d_b^\alpha) (d_c^\gamma \nu^\rho) , \quad (4.6)$$

where

$$\begin{aligned} \mathcal{C}_{\delta\alpha\gamma\rho}^\nu &= g_2^2 \sum_{\beta, \sigma} (c_{\beta\sigma\gamma\rho} - c_{\beta\gamma\sigma\rho})|_{\mu=M_G} (V_{CKM})_{\beta\alpha} (V_{CKM}^*)_{\delta\sigma} I(\tilde{u}^\beta, \tilde{d}^\sigma, \tilde{W}) \bar{A}_S(d^\gamma, u^\beta, d^\sigma) \\ &+ g_2^2 \sum_{\beta} (\bar{c}_{\delta\alpha\beta\rho} - \bar{c}_{\beta\alpha\delta\rho})|_{\mu=M_G} (V_{CKM})_{\beta\gamma} I(\tilde{u}^\beta, \tilde{e}^\rho, \tilde{W}) \bar{A}_S(d^\alpha, u^\delta, u^\beta) , \\ \text{with ,} \quad c_{\beta\sigma\gamma\rho} &= \frac{1}{v_u v_d} (M_{diag}^U P V_{CKM})_{\beta\sigma} (V^T M_{diag}^D)_{\gamma\rho} , \\ \bar{c}_{\delta\alpha\beta\rho} &= \frac{1}{v_u v_d} (M_{diag}^U P V_{CKM})_{\delta\alpha} (V_{CKM}^* V^T M_{diag}^E)_{\beta\rho} . \end{aligned} \quad (4.7)$$

Here I is the loop integral defined as

$$I(i, j, k) = \frac{1}{16\pi^2} \frac{m_k}{m_i^2 - m_j^2} \left(\frac{m_i^2}{m_i^2 - m_k^2} \ln \frac{m_i^2}{m_k^2} - \frac{m_j^2}{m_j^2 - m_k^2} \ln \frac{m_j^2}{m_k^2} \right) , \quad (4.8)$$

while \bar{A}_S accounts for short distance renormalization factor of the corresponding $LLLL$ $d=5$ operator. Here we present some of these RG factors, which will be needed later on for numerical calculations:

$$\begin{aligned} \bar{A}_S(d^\gamma, u^\beta, d^\sigma)_{\gamma, \beta, \sigma \neq 3} &= \bar{A}_S(d^\alpha, u^\delta, u^\beta)_{\alpha, \delta, \beta \neq 3} \simeq 6.88 , \\ \bar{A}_S(d^\gamma, u^\beta, b)_{\gamma, \beta \neq 3} &= \bar{A}_S(d^\gamma, t, d^\sigma)_{\gamma, \sigma \neq 3} = \bar{A}_S(d^\alpha, u^\delta, t)_{\alpha, \delta \neq 3} \simeq 6.54 , \\ \bar{A}_S(d^\gamma, t, b)_{\gamma \neq 3} &\simeq 6.2 . \end{aligned} \quad (4.9)$$

These expressions are valid for low to moderate values of $\tan \beta$.

The $d = 6$ four fermion operator obtained from $W_R^{d=5}$ by higgsino dressing and involving the neutrino has the form

$$\mathcal{O}_{\nu R}^{d=6} = \frac{\epsilon^{abc}}{M_T} \mathcal{R}_{\delta\alpha\gamma\rho}^\nu \left(\overline{u}_a^{c\delta} \overline{d}_b^{c\alpha} \right) (d_c^\gamma \nu^\rho) , \quad (4.10)$$

where

$$\mathcal{R}_{\delta\alpha\gamma\rho}^\nu = \frac{1}{v_u v_d} \sum_\sigma (\overline{\omega}_{\delta\rho\alpha\sigma}^* - \overline{\omega}_{\sigma\rho\alpha\delta}^*) \Big|_{\mu=M_G} (M_{diag}^U V_{CKM})_{\sigma\gamma} (M_{diag}^E)_\rho I(\tilde{e}^{c\delta}, \tilde{u}^{c\sigma}, \tilde{H}^\pm) \bar{A}_{S,R}(u^{c\delta}, u^{c\sigma}) ,$$

$$\text{with } \overline{\omega}_{\delta\rho\alpha\sigma} = \frac{1}{v_u v_d} (M_{diag}^U V_{CKM} V^\dagger)_{\delta\rho} \left(M_{diag}^D V_{CKM}^\dagger P^* \right)_{\alpha\sigma} . \quad (4.11)$$

$\bar{A}_{S,R}$ accounts for short distance renormalization factor of the corresponding $RRRR$ $d=5$ operator. Here we give values of those, which will be needed for further calculations:

$$\bar{A}_{S,R}(u^c, u^{c\sigma})_{\sigma \neq 3} \simeq 4.44 , \quad \bar{A}_{S,R}(u^c, t^c) \simeq 4.0 . \quad (4.12)$$

4.2 Nucleon decay

The operators responsible for $p \rightarrow \bar{\nu}_\rho K^+$ decay are

$$\frac{\epsilon^{abc}}{M_T} \left[\mathcal{C}_{112\rho}^\nu (u_a d_b) (s_c \nu_\rho) + \mathcal{C}_{121\rho}^\nu (u_a s_b) (d_c \nu_\rho) + \mathcal{R}_{112\rho}^\nu (\overline{u}_a^c \overline{d}_b^c) (s_c \nu_\rho) + \mathcal{R}_{121\rho}^\nu (\overline{u}_a^c \overline{s}_b^c) (d_c \nu_\rho) \right] . \quad (4.13)$$

From these expressions we can calculate the partial widths for nucleon decay:

$$\Gamma(p \rightarrow \bar{\nu}_\rho K^+) = \frac{(m_p^2 - m_K^2)^2}{32\pi m_p^3 f_\pi^2} \left| \frac{R_L}{M_T} \left\{ (\beta_H \mathcal{C}_{121\rho}^\nu + \alpha_H \mathcal{R}_{121\rho}^\nu) \frac{2m_p}{3m_B} D + \right. \right. \\ \left. \left. (\beta_H \mathcal{C}_{112\rho}^\nu + \alpha_H \mathcal{R}_{112\rho}^\nu) \left(1 + \frac{m_p}{3m_B} (D + 3F) \right) \right\} \right|^2 . \quad (4.14)$$

Here α_H, β_H are hadronic matrix elements and at $\mu = 2$ GeV scale are [38] $|\alpha_H| \simeq |\beta_H| \simeq 0.012 \text{ GeV}^3$, while the values of other parameters are $m_p = 0.94 \text{ GeV}$, $m_K = 0.494 \text{ GeV}$, $f_\pi = 0.131 \text{ GeV}$, $m_B = 1.15 \text{ GeV}$, $D = 0.8$, $F = 0.47$. The factor $R_L \simeq 1.25$ is a long distance renormalization factor.

Note that, different from the minimal SUSY $SU(5)$ model, in Eqs. (4.4) and (4.5) the unitary matrix V appears. This matrix, by proper selection of its mixing angles, allows us to suppress proton decay so as to bring the partial lifetime within experimental limits.

Before demonstrating this with numerical results, in order to get a better feeling, we present an analytic study to leading order in certain small parameters. To leading order, let us ignore (i.e., set to zero) the $2-3$ and the $1-3$ mixing angles in the CKM matrix and in the \hat{V} matrix. Let us also take the limit $m_u, m_d, m_e \rightarrow 0$. In this limit, we get

$$\mathcal{C}_{1211}^\nu = \mathcal{C}_{1213}^\nu = \mathcal{C}_{1121}^\nu = \mathcal{C}_{1123}^\nu = 0 . \quad (4.15)$$

Similar results hold for the corresponding \mathcal{R}^ν amplitudes. Therefore

$$\Gamma(p \rightarrow \bar{\nu}_e K^+) = \Gamma(p \rightarrow \bar{\nu}_\tau K^+) = 0 . \quad (4.16)$$

Only $\Gamma(p \rightarrow \bar{\nu}_\mu K^+)$ will be non-zero due to the non-zero elements \mathcal{C}_{1212}^ν and \mathcal{C}_{1122}^ν ⁸ which are given by

$$\mathcal{C}_{1212}^\nu = \mathcal{C}_{1122}^\nu \simeq g_2^2 \left(I(\tilde{u}, \tilde{d}) + I(\tilde{u}, \tilde{e}) \right) \bar{A}_S^\alpha e^{i\omega_2} \lambda_s \lambda_\mu \sin \theta_c \left(\sin \theta_c e^{i(\phi_2 + \delta_2)} + \hat{V}_{21} e^{i\phi_1} \right) . \quad (4.17)$$

Note that in the limit $\hat{V}_{21} \rightarrow 0$ the expressions of Eq. (4.17) will coincide with those of minimal SUSY $SU(5)$. Now, we can select the matrix element \hat{V}_{21} in such a way that these coefficients vanish (or are suppressed): $\sin \theta_c e^{i(\phi_2 + \delta_2)} + \hat{V}_{21} e^{i\phi_1} = 0$, or

$$|\hat{V}_{21}| = \sin \theta_c , \quad \text{Arg}(\hat{V}_{21}) = \pi + \phi_2 + \delta_2 - \phi_1 . \quad (4.18)$$

With this conditions satisfied we get $\Gamma(p \rightarrow \bar{\nu}_\mu K^+) \simeq 0$ and the decay $p \rightarrow \bar{\nu} K^+$ will be eliminated. Note that the conditions in Eq. (4.18) are easily satisfied. This is true for the second relation because all phases entering there are free. As far as the condition $|\hat{V}_{21}| = \sin \theta_c$ is concerned, from (A.12), with $t_1^e s_2^e \lesssim 5 t_1^d s_2^d$ we have $|\hat{V}_{21}| \approx \frac{m_d}{m_s} t_1^d s_2^d$. With the selection $t_1^d s_2^d \approx 4$ we get $|\hat{V}_{21}| \approx 0.2 \approx \sin \theta_c$.

With the inclusion of $1-3$ and $2-3$ mixings, and $m_{u,d,e} \neq 0$, the expressions get more lengthy, making analytical treatment harder. Thus, in the following we proceed with a numerical study, demonstrating the possibility of proton lifetime suppression.

4.3 Exact numerical results

Following Eq. (2.22) we choose

$$\theta_1^l = \arccos \left(\frac{m_e}{m_d} \cos \theta_1^d \right) , \quad \theta_2^d = \arccos \left(\frac{m_s}{m_\mu} \cos \theta_2^l \right) , \quad \theta_3^d = \arccos \left(\frac{m_b}{m_\tau} \cos \theta_3^l \right) . \quad (4.19)$$

⁸The elements \mathcal{R}_{1212}^ν , \mathcal{R}_{1122}^ν are suppressed strongly and can be ignored.

h	A	H^0	H^\pm	$\tilde{\chi}_1^\pm$	$\tilde{\chi}_2^\pm$	$\tilde{\chi}_1^0$	$\tilde{\chi}_2^0$	$\tilde{\chi}_3^0$	$\tilde{\chi}_4^0$	\tilde{g}
125	1000	1000	1003	145	497	132	-158	259	497	1450
	\tilde{t}_1	\tilde{t}_2	\tilde{u}_1, \tilde{c}_1	\tilde{u}_2, \tilde{c}_2	\tilde{b}_1	\tilde{b}_2	\tilde{d}_1, \tilde{s}_1	\tilde{d}_2, \tilde{s}_2		
	554	2197	3144	3241	2186	3096	3145	3118		
	$\tilde{\tau}_1$	$\tilde{\tau}_2$	$\tilde{e}_1, \tilde{\mu}_1$	$\tilde{e}_2, \tilde{\mu}_2$	$\tilde{\nu}_\tau$	$\tilde{\nu}_e, \tilde{\nu}_\mu$				
	2849	3062	3073	2871	3061	3072				

Table 1: Particle masses (in GeV) obtained by the input given in Eq. (4.20) in MSSM.

Then there are only three independent angles. We treat θ_1^d , θ_2^l and θ_3^l as free parameters and select them in such a way as to suppress $d = 5$ proton decay rates adequately. We also have the free phases $\delta_{1,2}, \omega_{1,2}, \phi_{1,2}$, which we vary so as to suppress proton decay rate.

For soft SUSY breaking parameters we adopt supergravity-inspired spectrum. However, we deviate from mSUGRA and allow for non-universality in the Higgs boson mass. This is implemented by taking the pseudoscalar Higgs mass M_A and μ as independent parameters. At the GUT scale we take as input, inspired by the “natural SUSY” spectrum of Ref. [39],

$$\begin{aligned}
M_0 = 3 \text{ TeV}, \quad M_{1/2} = 568.3 \text{ GeV}, \quad A_0 = -5 \text{ TeV}, \\
\tan \beta = 7, \quad \mu = 150 \text{ GeV}, \quad M_A = 1 \text{ TeV},
\end{aligned} \tag{4.20}$$

where M_0 ($M_{1/2}$) is the usual universal soft mass for chiral matter superfields (gauginos) at the GUT scale, A_0 the common trilinear term, while the Higgs sector is not universal ($M_{H_{u,d}}^2 \neq M_0^2$). The value of $\tan \beta$ given is at the weak scale, corresponding to $\tan \beta = 6.75$ at the GUT scale. The parameters are chosen so that the SUSY spectrum is lighter than approximately 3 TeV, which can be discovered at LHC. For numerical calculations we used the code SuSpect [40], through which we make sure that the lightest (SM like) Higgs mass is $\simeq 125$ GeV. The spectrum (at weak scale) we get for the input of Eq. (4.20) is given in Table 1. These values will be used in the calculation of proton lifetime.

One choice of the three free angles and phases giving adequate suppression of proton decay rate is:

$$\begin{aligned}
\theta_1^d = 1.3433, \quad \theta_2^l = 1.016, \quad \theta_3^l = 0.10275, \\
\phi_1 = \delta_1 = 0, \quad \phi_2 = 3.3065, \quad \delta_2 = 1.883, \\
\omega_1 = 2.515, \quad \omega_2 = 1.748.
\end{aligned} \tag{4.21}$$

With these input values we obtain for the decay rate $p \rightarrow \bar{\nu} K^+$

$$\Gamma_{d=5}^{-1}(p \rightarrow \bar{\nu} K^+) = \frac{1}{\sum_{i=1}^3 \Gamma_{d=5}(p \rightarrow \bar{\nu}_i K^+)} \simeq$$

$$4 \cdot 10^{33} \text{ yrs} \times \left(\frac{0.012 \text{ GeV}^3}{\beta_H} \right)^2 \left(\frac{1.25}{R_L} \right)^2 \left(\frac{M_T}{4.8 \cdot 10^{16} \text{ GeV}} \right)^2. \quad (4.22)$$

In Table 2 we summarize the partial lifetimes for this and other decay modes. Not all decay modes (induced by the $d = 5$ operators) are listed, those with lifetimes exceeding $\sim 5 \cdot 10^{36}$ years are not shown. Note that with further tuning of parameters, we may suppress even more the $p \rightarrow \bar{\nu} K^+$ decay. However, we can not decrease much further the value of M_T because that would decrease the lifetime $\Gamma_{d=5}^{-1}(p \rightarrow \mu^+ K^0)$ whose value is already near at the experimental limit [41] (see Table 2).

Note that with the value $M_T = 4.8 \cdot 10^{16} \text{ GeV}$ (used in Eq. (4.22)), the mass of the $SU(5)$ gauge bosons (X, Y) should be greater than about $2 \times 10^{16} \text{ GeV}$ in order to be consistent with perturbativity [26]. Such a value for M_X would mean that there is some chance for the observation of the gauge boson mediated nucleon decay such as $p \rightarrow e^+ \pi^0$, but this will be challenging.

One can try to increase the color triplet mass to further suppress the rates for the $d = 5$ modes. Due to the perturbativity constraint (see Eq. (2.11)) one needs first to increase the heavy gauge boson mass. For $m_3 = m_8$ this equals

$$M_X = M_X^0 / r^{1/3} \quad (4.23)$$

where $M_X^0 \approx 2 \cdot 10^{16} \text{ GeV}$. By choosing $r \approx 1/10$ or so M_X and thus M_T can be increased by a factor of 2. The color triplet mass can now be raised to $M_T \approx 10^{17} \text{ GeV}$, which would imply the scaling of all lifetimes for all modes in Table 2 upward by a factor of 4. Further increase of the triplet mass could jeopardize the expansion in inverse powers of the Planck scale, so we will not consider it. We see that, with the assumption that SUSY particles masses lie below about 3 TeV, which is testable at the LHC, proton lifetime cannot exceed about 2×10^{34} years. This is within reach of ongoing and proposed experiments.

We have not included gluino dressing of the effective $d = 5$ operators in order to obtain four fermion operators for proton decay. When universality is assumed, as we do, for the masses of the superpartners of the chiral fermions, the gluino dressing diagrams are highly suppressed [42] compared to the Wino dressing diagrams. This is primarily due to the antisymmetric nature of the $QQQL$ operator in flavor. With the SUSY particle masses taken to be less than about 3 TeV, universality in the soft scalar masses is almost a necessity in order to suppress flavor changing neutral currents (FCNC) arising from the exchange of SUSY particles. If the third family squark and slepton masses are taken to be different from those of the (degenerate) first two families, FCNC processes may not be excessive. In this case, the gluino dressing contributions to nucleon decay may become

Table 2: Inverse widths for nucleon decay. Calculations are carried out for the SUSY parameters (spectrum) given in Eq. (4.20), Table 1. The model parameters are given in Eqs. (4.21), (4.19), along with $M_T = 4.8 \times 10^{16}$ GeV. Other parameters used can be found right after Eq. (4.14).

$\Gamma_{d=5}^{-1}(p \rightarrow \bar{\nu} K^+)$	$4 \cdot 10^{33}$ yrs.
$\Gamma_{d=5}^{-1}(n \rightarrow \bar{\nu} K^0)$	$2 \cdot 10^{33}$ yrs.
$\Gamma_{d=5}^{-1}(p \rightarrow \mu^+ K^0)$	$1.0 \cdot 10^{34}$ yrs.
$\Gamma_{d=5}^{-1}(p \rightarrow \mu^+ \pi^0)$	$1.8 \cdot 10^{34}$ yrs.
$\Gamma_{d=5}^{-1}(p \rightarrow \bar{\nu} \pi^+)$	$7.3 \cdot 10^{33}$ yrs.
$\Gamma_{d=5}^{-1}(n \rightarrow \bar{\nu} \pi^0)$	$1.5 \cdot 10^{34}$ yrs.

important, but typically the amplitude is not much more than that arising from the Wino dressing, see for eg. discussions in Ref. [43]. Thus, variation of SUSY spectrum would not significantly alter the upper limit on nucleon lifetime derived above, as long as the sparticle masses lie below 3 TeV or so.

5 Conclusions

In this paper we have shown that the main problems of the minimal renormalizable model based on SUSY $SU(5)$ can be cured by adding a vector-like pair of $5 + \bar{5}$ matter fields. This allows for the mixing of chiral families with the vector-like fields, which we show corrects the wrong mass relations of minimal $SU(5)$. The mass splitting between the color triplets and the weak doublets of this vector-like fields improves the unification of the three gauge couplings. The color triplets from the $5_H + \bar{5}_H$ fields, which mediate $d = 5$ proton decay can have GUT scale masses, thus avoiding the rapid proton decay problem of the minimal model. The small number of couplings of this model enables us to make quantitative predictions for partial lifetimes for proton decay. We find that, in the favorable case that the LHC is sensitive to the discovery of the whole SUSY spectrum (corresponding to all the super-partner masses and Higgs boson masses $\lesssim 3$ TeV), at least some of the modes should have partial lifetimes shorter than about 2×10^{34} yrs, which is within reach of proposed experiments.

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A Deriving W_{eff}

In this Appendix, we give details of obtaining the effective superpotential, both for the light fermion mass matrices, and for the $d = 5$ baryon number violating superpotential couplings. The effective superpotential is obtained by decoupling the extra heavy vector like states. First we integrate out the extra matter states. This is performed by block-diagonalization of the first two coupling matrices in Eq. (2.14).

A.1 Derivation of W_{mass}

With the transformation

$$L = P_l V_l^\dagger L' , \quad E^c = P_{e^c} E^{c'} , \quad D^c = P_{d^c} V_d^\dagger D^{c'} , \quad D = P_d D' , \quad (\text{A.1})$$

the matrices $M_l^{4 \times 4}$ and $M_d^{4 \times 4}$ get transformed to [30]

$$M_l^{4 \times 4} \rightarrow V_l P_l M_l^{4 \times 4} P_{e^c} = \begin{pmatrix} \hat{M}_E & 0 \\ \mathcal{O}(v_d) & M_D \end{pmatrix} , \quad (\text{A.2})$$

$$M_d^{4 \times 4} \rightarrow V_d P_{d^c} M_d^{4 \times 4} P_e = \begin{pmatrix} \hat{M}_D^T & 0 \\ \mathcal{O}(v_d) & M_C \end{pmatrix} . \quad (\text{A.3})$$

The matrices in Eq. (A.1) are given by

$$P_l = e^{i\omega_l} \text{Diag} \left(e^{-i\phi_{M_1^l}} , e^{-i\phi_{M_2^l}} , e^{-i\phi_{M_3^l}} , 1 \right)$$

$$P_{e^c} = e^{-i\omega_l} \text{Diag} \left(e^{i(\phi_{M_1^l} - \phi_{y_1 v_d})} , e^{i(\phi_{M_2^l} - \phi_{y_2 v_d})} , e^{i(\phi_{M_3^l} - \phi_{y_3 v_d})} , 1 \right)$$

$$\begin{aligned}
P_{d^c} &= e^{i\omega_{d^c}} \text{Diag} \left(e^{-i\phi_{M_1^d}}, e^{-i\phi_{M_2^d}}, e^{-i\phi_{M_3^d}}, 1 \right) \\
P_q &= e^{-i\omega_{d^c}} \text{Diag} \left(e^{i(\phi_{M_1^d} - \phi_{y_1 v_d})}, e^{i(\phi_{M_2^d} - \phi_{y_2 v_d})}, e^{i(\phi_{M_3^d} - \phi_{y_3 v_d})}, 1 \right)
\end{aligned} \tag{A.4}$$

$$V_{l,d} = \begin{pmatrix} c_1^{e,d} & 0 & 0 & -s_1^{e,d} \\ -s_1^{e,d} s_2^{e,d} & c_2^{e,d} & 0 & -c_1^{e,d} s_2^{e,d} \\ -c_2^{e,d} s_1^{e,d} s_3^{e,d} & -s_2^{e,d} s_3^{e,d} & c_3^{e,d} & -c_1^{e,d} c_2^{e,d} s_3^{e,d} \\ c_2^{e,d} c_3^{e,d} s_1^{e,d} & c_3^{e,d} s_2^{e,d} & s_3^{e,d} & c_1^{e,d} c_2^{e,d} c_3^{e,d} \end{pmatrix}, \tag{A.5}$$

where definitions for the entries of Eq. (A.5) see Eq. (2.20). We use the notation ϕ_X to denote the phase of a complex parameter X . Thus $\phi_{y_1 v_d}$ is the argument of $y_1 v_d$, etc. With all these, one can easily check that the matrices \hat{M}_E , \hat{M}_D and masses M_D, M_C are given by Eqs. (2.19) and (3.1) respectively. The entries $\mathcal{O}(v_d)$ in Eqs. (A.2), (A.3) can be safely ignored. Thus, the diagonal block-entries in these matrices, together with \hat{M}_U , coincide with the terms of Eq. (2.18).

A.2 Deriving effective $d = 5$ operators

Now we turn to the derivation of the effective $d = 5$ baryon number violating superpotential couplings. With the transformations of Eq. (A.1) and with

$$q = P'_q q', \quad u^c = P'_q u^{c'}, \tag{A.6}$$

where

$$P'_q = e^{-i\omega_{d^c}} \text{Diag} \left(e^{i(\phi_{M_1^d} - \phi_{y_1 v_d})}, e^{i(\phi_{M_2^d} - \phi_{y_2 v_d})}, e^{i(\phi_{M_3^d} - \phi_{y_3 v_d})} \right), \tag{A.7}$$

one can derive the couplings of the light states with the color triplets T, \bar{T} :

$$\frac{1}{v_d} l^T \hat{M}_E P'_q q \bar{T} + \frac{1}{v_u} u^T \hat{M}_U d T + \frac{1}{v_d} d^{cT} \hat{M}_D^T u^c \bar{T} + \frac{1}{v_u} e^{cT} P'^* \hat{M}_U u^c T, \tag{A.8}$$

where we have omitted primes for the quark and lepton states. The matrix P' , without loss of generality, can be parameterized as:

$$P' = \text{Diag} (e^{i\delta_1}, e^{i\delta_2}, 1). \tag{A.9}$$

Further, integrating out the states T, \bar{T} with mass M_T , from Eq. (A.8) we derive the effective $d = 5$ operators given in Eqs. (4.2), (4.3). These are written in a flavor basis.

Finally, we present the matrices which appear in the $d = 5$ couplings written in the mass eigenstate basis, using the transformations given in Eq. (2.24). These are the phase matrix P

$$P = \text{Diag} (e^{i\omega_1}, e^{i\omega_2}, 1) , \quad (\text{A.10})$$

and the matrix

$$V = \hat{V} \hat{P} , \quad \text{with} \quad \hat{V} = V_E P' U_D^T , \quad \hat{P} = \text{Diag} (e^{i\phi_1}, e^{i\phi_2}, 1) . \quad (\text{A.11})$$

The elements of the matrix \hat{V} are:

$$\begin{aligned} \hat{V}_{11} &\simeq e^{i\delta_1} , & \hat{V}_{12} &\simeq -\frac{m_d}{m_s} t_1^d s_2^d e^{i\delta_1} + \frac{m_e}{m_\mu} t_1^e s_2^e e^{i\delta_2} , \\ \hat{V}_{13} &\simeq -\frac{m_d}{m_b} t_1^d c_2^d s_3^d e^{i\delta_1} - \frac{m_s}{m_b} \frac{m_e}{m_\mu} t_1^e s_2^e t_2^d s_3^d e^{i\delta_2} + \frac{m_e}{m_\tau} t_1^e \frac{s_3^e}{c_2^e} , \\ \hat{V}_{21} &\simeq \frac{m_d}{m_s} t_1^d s_2^d e^{i\delta_2} - \frac{m_e}{m_\mu} t_1^e s_2^e e^{i\delta_1} , & \hat{V}_{22} &\simeq e^{i\delta_2} , & \hat{V}_{23} &\simeq -\frac{m_s}{m_b} t_2^d s_3^d e^{i\delta_2} + \frac{m_\mu}{m_\tau} t_2^e s_3^e , \\ \hat{V}_{31} &\simeq \frac{m_d}{m_b} t_1^d \frac{s_3^d}{c_2^d} - \frac{m_d}{m_s} \frac{m_\mu}{m_\tau} t_2^e s_3^e t_1^d s_2^d e^{i\delta_2} - \frac{m_e}{m_\tau} t_1^e s_3^e c_2^e e^{i\delta_1} , \\ \hat{V}_{32} &\simeq \frac{m_s}{m_b} t_2^d s_3^d - \frac{m_\mu}{m_\tau} t_2^e s_3^e e^{i\delta_2} , & \hat{V}_{33} &\simeq 1 . \end{aligned} \quad (\text{A.12})$$

A.3 An alternative derivation of W_{mass}

Here we provide an alternative, perhaps more intuitive, derivation of the effective mass matrices for the down-type quarks and charged leptons that follow from Eq. (2.16). We write down these matrices in a unified $SU(5)$ notation,

$$\mathcal{L} = (\bar{5}_i \quad \bar{5}_4) \begin{pmatrix} m_{ij}^0 & M_i \\ 0 & M_4 \end{pmatrix} \begin{pmatrix} 10_j \\ 5_4 \end{pmatrix} \quad (\text{A.13})$$

where

$$m_{ij}^0 = y_i \delta_{ij} \langle \bar{5}_H \rangle \quad (\text{A.14})$$

$$M_a = \mu_a + \eta_a \langle 24_H \rangle , \quad a = 1 \dots 4 \quad (\text{A.15})$$

Here $\langle 24_H \rangle = 2v$ for the color triplet quark fields, while $\langle 24_H \rangle = -3v$ for the $SU(2)_L$ doublet lepton fields from the $\bar{5}_a + 5_4$. Now we make a unitary rotation parametrized by

$$(\bar{5}_i \quad \bar{5}_4) \rightarrow (\bar{5}_i \quad \bar{5}_4) U \quad (\text{A.16})$$

with

$$U = \begin{pmatrix} \Lambda & -\Lambda x \\ x^\dagger \bar{\Lambda} & \bar{\Lambda} \end{pmatrix} \quad (\text{A.17})$$

$$x^T = (M_1, M_2, M_3)/M_4 \quad (\text{A.18})$$

$$\Lambda = (1 + x x^\dagger)^{-1/2} \quad , \quad \bar{\Lambda} = (1 + x^\dagger x)^{-1/2} = (1 + |x|^2)^{-1/2} \quad (\text{A.19})$$

Note that the unitary matrix U is different for the quarks and leptons, since the M_i factors that enter into U are different. Similarly, the x_i factors are not the same in these two sectors. We shall not explicitly show here the dependence of U or x_i on the fermion flavor, but it is to be understood.

With the rotation of Eq. (A.16) , Eq. (A.13) becomes

$$\mathcal{L} \rightarrow (\bar{5}_i \quad \bar{5}_4) \begin{pmatrix} (\Lambda m^0)_{ij} & 0 \\ (x^\dagger \bar{\Lambda} m^0)_i & x^\dagger \bar{\Lambda} M + \bar{\Lambda} M_4 \end{pmatrix} \begin{pmatrix} 10_j \\ 5_4 \end{pmatrix} \quad (\text{A.20})$$

The heavy pair is now $\bar{5}_4 - 5_4$, and the light mass matrices for down quarks and charged leptons become

$$M^D = \Lambda^d m^0 \quad M^E = m^0 \Lambda^{eT} \quad (\text{A.21})$$

with

$$x_{Di} = \frac{\mu_i + 2\eta_i v}{\mu_4 + 2\eta_4 v} \quad , \quad x_{Ei} = \frac{\mu_i - 3\eta_i v}{\mu_4 - 3\eta_4 v} \quad (\text{A.22})$$

where we have explicitly shown the separate matrices for down type quarks and charged leptons, using the GUT scale VEV v given in Eq. (2.2).

The matrix Λ from (A.19) (for each sector separately) can be written explicitly as

$$\begin{aligned} \Lambda &= 1 - \frac{x x^\dagger}{\sqrt{1 + |x|^2} (\sqrt{1 + |x|^2} + 1)} \\ &= \begin{pmatrix} 1 - c x_1 x_1^* & -c x_1 x_2^* & -c x_1 x_3^* \\ -c x_2 x_1^* & 1 - c x_2 x_2^* & -c x_2 x_3^* \\ -c x_3 x_1^* & -c x_3 x_2^* & 1 - c x_3 x_3^* \end{pmatrix} \end{aligned} \quad (\text{A.23})$$

with

$$c = \frac{1}{\sqrt{1 + |x|^2} (\sqrt{1 + |x|^2} + 1)} \quad (\text{A.24})$$

The down quark and charged lepton mass matrices of Eq. (A.21) can be diagonalized

readily. Their eigenvalues are given by:

$$\begin{aligned}
m_1^2 + m_2^2 + m_3^2 &= \frac{|d_1|^2(1 + |x_2|^2 + |x_3|^2) + |d_2|^2(1 + |x_3|^2 + |x_1|^2) + |d_3|^2(1 + |x_1|^2 + |x_2|^2)}{1 + |x|^2} \\
m_1^2 m_2^2 + m_1^2 m_3^2 + m_2^2 m_3^2 &= \frac{|d_1|^2 |d_2|^2 (1 + |x_3|^2) + |d_2|^2 |d_3|^2 (1 + |x_1|^2) + |d_3|^2 |d_1|^2 (1 + |x_2|^2)}{1 + |x|^2} \\
m_1^2 m_2^2 m_3^2 &= \frac{|d_1|^2 |d_2|^2 |d_3|^2}{1 + |x|^2},
\end{aligned} \tag{A.25}$$

where d_i 's are common for M_D and M_E , while the x_i 's are different. From Eq. (A.25), it follows that realizing the mass hierarchy is possible only when $|d_i|$ are hierarchical, $|d_1| \ll |d_2| \ll |d_3|$, in which case we can write down very simple formulas for the three masses:

$$m_i = |d_i| \cos \theta_i. \tag{A.26}$$

Here we define three mixing angles as:

$$\tan \theta_1 = |x_1|, \quad \tan \theta_2 = \frac{|x_2|}{\sqrt{1 + |x_1|^2}}, \quad \tan \theta_3 = \frac{|x_3|}{\sqrt{1 + |x_1|^2 + |x_2|^2}} \tag{A.27}$$

with $0 \leq \theta_i \leq \pi/2$. These are the same definitions used in Eq. (2.20).

Noting that the mass matrix elements of Eq. (A.13) can be all made real by redefinitions of fields, we also obtain the unitary matrices that diagonalize M^D and M^E :

$$U^T M^D V = M_{diag}^D \tag{A.28}$$

$$V^T M^E U = M_{diag}^E \tag{A.29}$$

We interchanged the notation $U \leftrightarrow V$ passing from D to E , because it is M_E^T that has the same form as M_D . Again, the matrices U, V are different for down type quarks and charged leptons, we use the same symbol however. The unitary matrices U and V are given as (with $|d_1| \ll |d_2| \ll |d_3|$)

$$U \simeq \begin{pmatrix} 1 & -\frac{m_1}{m_2} t_1 s_2 & -\frac{m_1}{m_3} t_1 c_2 s_3 \\ \frac{m_1}{m_2} t_1 s_2 & 1 & -\frac{m_2}{m_3} t_2 s_3 \\ \frac{m_1}{m_3} \frac{t_1 s_3}{c_2} & \frac{m_2}{m_3} t_2 s_3 & 1 \end{pmatrix}, \tag{A.30}$$

$$V \simeq \frac{1}{1 + c_1 c_2 c_3} \begin{pmatrix} c_1 + c_2 c_3 & -s_1 s_2 & -s_1 c_2 s_3 \\ s_1 s_2 c_3 & c_2 + c_3 c_1 & -s_2 s_3 \\ s_1 s_3 & c_1 s_2 s_3 & c_3 + c_1 c_2 \end{pmatrix}. \tag{A.31}$$

Here $c_i = \cos \theta_i$, $s_i = \sin \theta_i$, $t_i = \tan \theta_i$. Terms of order (m_2^2/m_3^2) and (m_1^2/m_2^2) are ignored in the derivation of these matrices.

It is possible to fit all quark and lepton masses consistently to the observed values. The mixing angles are related by the ratios given in Eq. (2.22).

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