S_{4} flavored CP symmetry for neutrinos

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S_4 Flavored CP Symmetry for Neutrinos

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A generalized CP symmetry for leptons is presented where CP transformations are part of an S_4 symmetry that connects different families. We study its implications for lepton mixings in a gauge model realization of the idea using type II seesaw for neutrino masses. The model predicts maximal atmospheric mixing, nonzero θ_{13} and maximal Dirac phase $\delta_D = \pm\frac{\pi}{2}$.

I. INTRODUCTION

The recent measurement of the leptonic mixing angle θ_{13} in experiments searching for the oscillations of electron anti-neutrinos emitted from reactors [1] and from the accelerator based experiments with muon neutrino beam [2] has generated considerable excitement in the field of neutrino physics. Taken together with already measured solar angle θ_{12} and atmospheric angle θ_{23}, this almost completes the CP conserving part of the lepton mixing matrix, under the assumption that there are no sterile neutrinos. This narrows the focus of the field to three remaining unknowns of neutrino flavor physics: (i) Dirac versus Majorana nature of the neutrino masses; (ii) mass hierarchy among them, namely, normal versus inverted and (iii) leptonic CP violating phases. The last item has two parts to it: Dirac phase which is analogous to the CKM phase in quark sector and Majorana phases which are exclusive to neutrino sector for Majorana neutrinos. The former can be measured in oscillation experiments whereas the latter may play a role in neutrinoless double beta decay searches. All these phases may play a role in understanding the origin of matter.

On the theoretical side, despite such a vast amount of information, the nature of BSM physics responsible for neutrino flavor properties remains largely unknown and is the subject of extensive investigation. There are two generic approaches: one based on symmetries in the lepton sector leaving the quarks aside and a second one based on grand unified theories where both quarks and leptons are considered together.

The quark-lepton unified grand unification based approach provides not only a very natural embedding of the seesaw mechanism to explain small neutrino masses but also, in a very economical class of renormalizable SO(10) models, turns to be very predictive. Indeed, the recently measured value of θ_{13} agrees with predictions made for this parameter in a minimal model of this type in 2003-2005 [3]. While this agreement is impressive, until there is some other evidence directly connecting the grand unification properties to seesaw physics (e.g., B-L violation as in [4]), one cannot test the GUT seesaw approach.

The symmetry approach, on the other hand, derives its appeal from the fact that two of the observed neutrino mixing angles θ_{23} (atmospheric) and θ_{12} (solar) are close to values that look like group theoretical numbers, and find easy explanation in terms of simple discrete family symmetry based models. For example, the observed near maximal atmospheric mixing can be easily understood if, in a basis where charged lepton masses are diagonal (to be called "flavor basis" from here on), the Majorana neutrino mass matrix satisfies the Z_2 μ-τ symmetry [5]. The simple versions of this symmetry, however, predict vanishing θ_{13}, a result which is contradicted by recent reactor [1] and accelerator experiments [2]. There is a vast literature on the corrections to μ-τ symmetry that come either from allowing general forms for the charged lepton matrix or from changing the neutrino mass matrix itself or combining simple μ-τ symmetry with simultaneous CP conjugation [6, 7]. All these cases lead to nonzero θ_{13}. Many such models are also now ruled out since they predict values of θ_{13} much smaller than the measured value. If, in addition to maximal atmospheric mixing, we consider the value of the solar angle $\tan\theta_{12} \simeq \frac{1}{\sqrt{2}}$, we obtain the so-called tri-bimaximal mixing [8] and it suggests more complicated groups such as $Z_2 \times Z_2$ [9] or S_3 [10] or A_4 [11] but some of them also imply θ_{13} zero or small after charged lepton corrections are taken into account and are not any more phenomenologically viable. Thus the measurement of θ_{13} has had a great impact on neutrino model building.

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The discovery of large θ_{13} however does not rule out the generic symmetry approach and many examples have been discussed where new symmetries do allow for a large nonzero θ_{13} [12–15]. We discuss one such approach in this paper which has the virtue of not only allowing large θ_{13} but also predicts all the leptonic CP phases. The approach is somewhat different from many papers in the sense that we use a generalized definition of CP transformation among leptons [16] embedded in a S_4 lepton family symmetry. We will call this new symmetry $\tilde{S}_4$ symmetry. We present a gauge model for leptons invariant under this symmetry which, not only accommodates a large θ_{13}, but it also predicts maximal θ_{23} and maximal Dirac CP phase, i.e., $\delta_D = \pm\frac{\pi}{2}$. The maximal θ_{23} is still consistent with latest global analysis [17, 18], although there are indications that it may be smaller [17].

This paper is organized as follows: in Sec. II, we present the $\tilde{S}_4$ model and the generalized CP transformation used in it; in Sec. III we present the various predictions of the model. In Sec. IV, we give some comments and conclude with a summary of the results. In an appendix, we discuss the representations of the $\tilde{S}_4$ symmetry that we use in the paper.

II. MODEL

Our model is based on the standard model gauge group $SU(2)_L \otimes U(1)_Y$ with the usual assignment for leptons, namely, the left-handed leptons L_i transform as $SU(2)_L$ doublets with $Y = -1$ and the right-handed charged leptons l_i $(= l_{iR})$ as singlets with $Y = -2$. The charged leptons gain masses through the Yukawa interactions with three Higgs doublets $\phi_i \sim (2, 1)$, $i = 1, 2, 3$. Neutrino masses and mixing are generated through type II seesaw mechanism [19] which requires the introduction of $Y = 2$ $SU(2)_L$ triplets. In order to implement the symmetry in our model, we introduce four SM triplets, Δ_0 and $\Delta_i \sim (3, 2)$, $i = 1, 2, 3$, whose neutral members acquire small vevs, induced by trilinear couplings of the form $\phi\phi\Delta^\dagger$. We assume only three families of leptons and no singlet sterile neutrinos.

We assume the theory to be invariant under a flavor symmetry acting in the horizontal space of the replicated fields. The chosen group is isomorphic to S_4, but will *contain* generalized CP transformations (GCP) defined below; we denote this group by $\tilde{S}_4$. Note that the group S_4 has been pointed out as the group for tri-bimaximal mixing [20], although some subgroup of it may turn out to be just accidental [9, 21]. The action of $\tilde{S}_4$ on complex fields will be nontrivial. It is constructed as a subgroup of $S_4 \otimes \langle \mathsf{CP} \rangle$ as follows. We remind the reader that S_4 has generators S and T which satisfy the properties: $S^4 = T^3 = \mathbb{1}$ and $ST^2S = T$.

Let us consider the (faithful) three-dimensional representation $\mathbf{3}$ of S_4 generated by [22]

$$\mathbf{3}: \quad S = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{1}$$

For complex fields, we can adjoin the usual CP transformation, denoted by the operator CP, to obtain $S_4 \otimes \langle \mathsf{CP} \rangle$. Note that S_4 transformations and the CP transformation commute because all representations of S_4 are real. We then extract the subgroup of $S_4 \otimes \langle \mathsf{CP} \rangle$ generated by

$$\mathbf{3}: \quad \tilde{S} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \mathsf{CP}, \quad T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{2}$$

Notice the charge conjugation part in $\tilde{S}$ is trivial for real fields. This group is isomorphic to S_4 after we factor the subgroup generated by $\mathsf{CP}^2 = -\mathbb{1}$ for fermions. Such a factor group is $\tilde{S}_4$. We keep the notation $\mathbf{3}$ for the representation generated by (2). The other representations of $\tilde{S}_4$ should be constructed in a similar manner from the representations $\mathbf{3}', \mathbf{2}, \mathbf{1}', \mathbf{1}$ of S_4. It is important to point out that $\tilde{S}$ is a nontrivial GCP transformation that does not reduce to the usual CP transformation by basis change [16].

Let us list the irreducible representations (irreps) of $\tilde{S}_4$, constructed from the irreps $\mathbf{1}, \mathbf{1}', \mathbf{2}, \mathbf{3}, \mathbf{3}'$ of S_4. They are led to peculiar representations of $\tilde{S}_4$ when complex fields are considered: the real irreps $\mathbf{1}$ and $\mathbf{1}'$ ($\mathbf{3}$ and $\mathbf{3}'$) are interwoven in one equivalent (complex) representation $\mathbf{1}$ ($\mathbf{3}$) whereas $\mathbf{2}$ splits into two inequivalent complex one-dimensional representations which we denote by $\mathbf{1}_\omega$ and $\mathbf{1}_{\omega^2}$; see Appendix A for an explanation. They are quite similar to the representations $\mathbf{1}', \mathbf{1}''$ of A_4.

We assign the representations of $\tilde{S}_4$ as follows:

$$L_i \sim \mathbf{3}, \quad l_1 \sim \mathbf{1},\ l_2 \sim \mathbf{1}_\omega,\ l_3 \sim \mathbf{1}_{\omega^2}, \quad \phi_i \sim \mathbf{3}. \tag{3}$$

The fields assigned to the triplet representation (2) transform explicitly as

$$\begin{aligned} L_i(x) &\xrightarrow{\tilde{S}} S_{ij} C L_j^*(\hat{x})\,, \quad L_i(x) \xrightarrow{T} T_{ij} L_j(x)\,; \\ \phi_i(x) &\xrightarrow{\tilde{S}} S_{ij} \phi_j^*(\hat{x})\,, \qquad \phi_i(x) \xrightarrow{T} T_{ij} \phi_j(x)\,, \end{aligned} \tag{4}$$

where $\hat{x} = (x_0, -\mathbf{x})$ for $x = (x_0, \mathbf{x})$ arises because of space inversion and C is the charge conjugation matrix. On the other hand, the right-handed lepton fields transform as

$$\begin{aligned} l_1(x) &\xrightarrow{\tilde{S}} C l_1^*(\hat{x})\,, \quad l_1(x) \xrightarrow{T} l_1(x)\,; \\ l_2(x) &\xrightarrow{\tilde{S}} C l_2^*(\hat{x})\,, \quad l_2(x) \xrightarrow{T} \omega\, l_2(x)\,; \\ l_3(x) &\xrightarrow{\tilde{S}} C l_3^*(\hat{x})\,, \quad l_3(x) \xrightarrow{T} \omega^2 l_3(x)\,. \end{aligned} \tag{5}$$

The Yukawa interactions for charged leptons invariant under these transformations is given by

$$\begin{aligned} -\mathcal{L}_Y^l = {} & y_1(\bar{L}_1\phi_1 + \bar{L}_2\phi_2 + \bar{L}_3\phi_3)l_1 + y_2(\bar{L}_1\phi_1 + \omega^2\bar{L}_2\phi_2 + \omega\bar{L}_3\phi_3)l_2 \\ & + y_3(\bar{L}_1\phi_1 + \omega\bar{L}_2\phi_2 + \omega^2\bar{L}_3\phi_3)l_3 + h.c., \end{aligned} \tag{6}$$

with the important restriction that all couplings y_i are *real* due to invariance by $\tilde{S}$.

When the neutral parts of the Higgs doublets acquire the vevs

$$\langle\phi_i\rangle = \frac{v}{\sqrt{3}}(1,1,1)\,, \tag{7}$$

the Lagrangian (6) gives rise to the charged-lepton mass matrix

$$M_l = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \mathrm{diag}(m_e, m_\mu, m_\tau)\,. \tag{8}$$

The correspondence is $(m_e, m_\mu, m_\tau) = v(y_1, y_2, y_3)$ and we identify U_ω^* in (8) by defining

$$U_\omega \equiv \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}\,. \tag{9}$$

We can see $M_l M_l^\dagger$ has circulant form [8] and it is invariant by T and any transposition of family indices composed with complex conjugation (CP transformation), i.e., an $\tilde{S}_3$ subgroup of $\tilde{S}_4$. The matrix (8) is identical to the one obtained in A_4 models. The potential for ϕ_i is in fact the same as the general A_4 invariant potential [11], implying that A_4 invariance leads automatically to $\tilde{S}_4$ invariance for the potential of three Higgs doublets. For that potential, it has been shown that (7) is a possible minimum [23].

To generate neutrino masses, we introduce four Higgs triplets transforming under $\tilde{S}_4$ as

$$\Delta_0 \sim \mathbf{1}, \quad \Delta_i \sim \mathbf{3}\,. \tag{10}$$

The $\tilde{S}_4$ invariant Lagrangian is then

$$-\mathcal{L}^\nu = \tfrac{1}{2} f_0 \overline{L_i^c}\epsilon\Delta_0 L_i + f_1\big(\overline{L_2^c}\epsilon\Delta_1 L_3 + \overline{L_3^c}\epsilon\Delta_2 L_1 + \overline{L_1^c}\epsilon\Delta_3 L_2\big) + h.c., \tag{11}$$

where f_0, f_1 are also real due to $\tilde{S}$.

Given the large vev hierarchy, we can assume the potential allows arbitrary vevs for the neutral components of Δ_0, Δ_i,

$$\langle\Delta_0^{(0)}\rangle = u_0\,, \quad \langle\Delta_i^{(0)}\rangle = u_i\,. \tag{12}$$

The Lagrangian (11) then induces the neutrino mass matrix

$$M_\nu = \begin{pmatrix} a & f & e \\ f & a & d \\ e & d & a \end{pmatrix}\,, \tag{13}$$

where $a = f_0 u_0$, $d = f_1 u_1, e = f_1 u_2, f = f_1 u_3$. Notice the tri-bimaximal limit corresponds to $e = f = 0$ [8]. For real a, d and complex $e = f^*$, the symmetry corresponding to 23-transposition and complex conjugation (corresponding to an element of $\tilde{S}_4$) would remain unbroken in the theory as symmetries of M_ν and $M_l M_l^\dagger$. This would lead to CP invariance and nonzero θ_{13}. In contrast, if $e = f$ we would obtain $\theta_{13} = 0$. In our case, CP violation and $\theta_{13} \neq 0$ are allowed because there is no relation between e and f.

If we assume the vevs (12) are real, the neutrino mass matrix, in the basis where the charged-lepton mass matrix is diagonal, is given by

$$U_\omega^\dagger M_\nu U_\omega^* = \begin{pmatrix} x & z & z^* \\ z & -2z^* & y \\ z^* & y & -2z \end{pmatrix} , \tag{14}$$

where x, y are real while z is in general complex; they are independent combinations of the four parameters a, d, e, f in (13). This matrix has the same form as in [24], invariant by $\mu\tau$ exchange composed with complex conjugation (called $\mu\tau$-reflection in [6]), with additional constraints so that it depends only on four real parameters. It has been shown that this form of the mass matrix leads to maximal θ_{23} and maximal CP violation [7], with $\theta_{13} \neq 0$.

The lepton mixing matrix $V_{\rm MNS}$ will be the matrix that diagonalizes (14). It is experimentally known that $V_{\rm MNS}$ is close to the tri-bimaximal mixing matrix

$$U_{\rm TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} . \tag{15}$$

Therefore we parametrize

$$V_{\rm MNS} = U_{\rm TB}\, {\rm diag}(1, 1, i) U_\epsilon \, , \tag{16}$$

where U_ϵ is the matrix that diagonalizes

$$M'_\nu = U^{\mathsf{T}} M_\nu U = \begin{pmatrix} a + d & b & 0 \\ b & a & c \\ 0 & c & a - d \end{pmatrix} , \tag{17}$$

for $U = U_\omega^* U_{\rm TB}\, {\rm diag}(1, 1, i)$, $b = \frac{e+f}{\sqrt{2}}$ and $c = \frac{e-f}{\sqrt{2}}$, with a, d, e, f being the original real parameters in (13). This mass matrix has the same form as in the A_4 model of [12] but our definition differs from [12] in that (16) includes an additional factor of i in ${\rm diag}(1, 1, i)$. Therefore, our case corresponds to taking c purely imaginary in [12]. However, this case was not considered there because it was focused on non-maximal θ_{23} and both real and imaginary parts were allowed to vary. In contrast, real a, d, e, f in the matrix (13) and, consequently, maximal θ_{23}, are natural consequences of our choice of symmetry.

We can assume $c > 0$ and consider the case $c < 0$ by replacing i by $-i$ in (16). This means that the sign of the Dirac phase $\delta_D = \pm\frac{\pi}{2}$ is not predicted in this model. Note that c controls $\theta_{13} \neq 0$ (and CP violation) and therefore it must be nonzero.

The limit $b, c \to 0$ leads to the tri-bimaximal form as $U_\epsilon = \mathbb{1}$. As $c \neq 0$ to guarantee $\theta_{13} \neq 0$, U_ϵ should deviate from the identity. That means M'_ν must be nearly diagonal, i.e., $|b|, |c| \ll |a|, |d|$. Having four parameters to describe 9 quantities, we have 5 predictions, some of which are independent of the values of a, b, c, d. This is a consequence of the specific form of the mass matrix (14), i.e., maximal θ_{23} and maximal CP violation [7]. In our specific model, the Majorana phases are also fixed: one is maximal and the other is zero. Only normal mass hierarchy for neutrinos is allowed. The remaining 5 physical quantities — two angles θ_{12}, θ_{13} and three neutrino masses m_1, m_2, m_3 — are correlated as they depend only on four parameters a, b, c, d as discussed in the next section.

A few comments are in order before we proceed to present the detailed numerical analysis of the model.

- It is worth noting that in our model the lightest two neutrino eigenstates are almost degenerate in mass and are about a factor of three lighter than the third eigenstate unlike most normal hierarchy models where $m_2/m_3 \sim 0.2$ or so.

- The Higgs potential for doublet fields in our model is the same as in the A_4 models discussed in [23] and it is easy to see from there that there is a range of parameters in the scalar self couplings where the vacuum alignment of the doublet fields in our model is justified.

III. PREDICTIONS

In the limit $b, c \to 0$, the neutrino masses, i.e., the absolute values of the eigenvalues of (17), are given by

$$m_1 = |a + d|\,, \quad m_2 = |a|\,, \quad m_3 = |a - d|\,. \tag{18}$$

We can choose $a > 0$. From $\Delta m^2_{12} = m_2^2 - m_1^2 > 0$, we can see $d < 0$ and normal hierarchy is the only possibility. The experimental information $\Delta m^2_{23} = m_3^2 - m_2^2 \gg \Delta m^2_{12}$ allows us to eliminate the modulus symbols in (18) as

$$m_1 = |d| - a\,, \quad m_2 = a\,, \quad m_3 = a + |d|\,. \tag{19}$$

We then arrive at the sum-rule

$$m_3 - 2m_2 - m_1 = 0\,, \tag{20}$$

which commonly arises in models with discrete flavor symmetries [25]. The difference here is that the sum-rule (20) applies to the neutrino masses themselves without additional Majorana phases or signs.

When we allow $b, c \neq 0$, the sum-rule (20) is still exactly satisfied provided that $b = \pm c$. This can be seen from the eigenvalues of

$$\delta M'_\nu \equiv M'_\nu - a\mathbb{1}_3\,, \tag{21}$$

which has characteristic equation

$$-\det(\delta M'_\nu - \lambda\mathbb{1}_3) = \lambda^3 - (d^2 + b^2 + c^2)\lambda - d(b^2 - c^2) = 0\,. \tag{22}$$

The eigenvalues of M'_ν can be obtained from the roots of (22) by adding a.

For general b and c the sum-rule (20) is only valid approximately. The violation of the sum-rule is quantified by

$$\epsilon_b \equiv -\frac{b}{d}\,, \quad \epsilon_c \equiv -\frac{c}{d}\,, \tag{23}$$

which controls the deviation of the PMNS matrix (16) from the tri-bimaximal mixing (15). The characteristic equation (22) shows that neutrino masses depend, apart from a, only on two combinations of d, c, b which can be chosen as

$$d' \equiv |d|\sqrt{1 + \epsilon_b^2 + \epsilon_c^2}\,, \quad \delta \equiv \frac{\epsilon_c^2 - \epsilon_b^2}{[1 + \epsilon_b^2 + \epsilon_c^2]^{3/2}}\,. \tag{24}$$

We can see δ quantifies the violation of the sum-rule.

We can seek approximate roots to (22) for $|\delta| \ll d'$, which leads to

$$\begin{aligned} -m_1 &= a - d'(1 - \tfrac{1}{2}\delta)\,,\\ m_2 &= a - d'\delta\,,\\ m_3 &= a + d'(1 + \tfrac{1}{2}\delta)\,. \end{aligned} \tag{25}$$

The result is valid up to terms of order δ^2 (order ϵ^4) multiplied by d'. Theses relations can be inverted to write a, d', δ in terms of the masses. In particular, the deviation of the sum-rule is given by

$$m_3 - 2m_2 - m_1 = \frac{3}{2}\delta\,(m_3 + m_1)\,. \tag{26}$$

The knowledge of Δm^2_{23} and Δm^2_{12} determines the parameters a, d' in terms of δ. In turn, δ depends on ϵ_b and ϵ_c which affects θ_{12} and θ_{13}.

To see how the mixing angles θ_{12} and θ_{13} are affected by ϵ_b, ϵ_c, we can perform an analysis similar to [12], with the difference that we have real matrices in our case. The matrix U_ϵ quantifies the deviations of the lepton mixing matrix from the tri-bimaximal form. For M'_ν, given the eigenvalues $(-m_1, m_2, m_3)$ in (25), we can calculate the eigenvectors which make up U_ϵ. The first approximation leads to

$$U_\epsilon \approx \begin{pmatrix} 1 & \epsilon_b & 0 \\ -\epsilon_b & 1 & \epsilon_c \\ 0 & -\epsilon_c & 1 \end{pmatrix}\,, \tag{27}$$

where the real parameters ϵ_b, ϵ_c were given in (23). Notice $\epsilon_c > 0$ for $c > 0$ because $d < 0$.

To first order, θ_{13} depends only on ϵ_c while θ_{12} depends on ϵ_b as

$$\begin{aligned} \sin^2\theta_{13} &\approx \tfrac{1}{3}\epsilon_c^2\,, \\ \sin^2\theta_{12} &\approx \frac{1}{3} + \frac{2\sqrt{2}}{3}\epsilon_b\,. \end{aligned} \tag{28}$$

We can then approximate

$$\delta \approx 3\,s_{13}^2 - \frac{9}{8}\Big(s_{12}^2 - \frac{1}{3}\Big)^2\,, \tag{29}$$

where $s_{13}^2 \equiv \sin^2\theta_{13}$ and $s_{12}^2 \equiv \sin^2\theta_{12}$ as usual. This is the amount of deviation for the sum rule (26). We can see data [18] is compatible with $\epsilon_b \approx 0$.

Given the experimentally known values of $\Delta m_{12}^2, \Delta m_{23}^2, \theta_{12}, \theta_{23}, \theta_{13}$, we can determine the values of the neutrino masses

$$m_1 \approx 13.3\,\mathrm{meV}\,, \quad m_2 \approx 15.9\,\mathrm{meV}\,, \quad m_3 \approx 52.1\,\mathrm{meV}\,. \tag{30}$$

We have used the best fit vaues of Ref. [18]. A more precise numerical study reveals

$$11.8\,\mathrm{meV} \le m_1 \le 13.6\,\mathrm{meV}\,, \tag{31}$$

when the 1-σ range for the observables is allowed [18]; see figures below.

Analogously, we can see the deviation for the sum-rule is small as $\frac{3}{2}\delta \sim 0.1$. In fact, our numerical study quantifies the deviation as

$$\frac{m_3 - 2m_2 - m_1}{m_3 + m_1} = 11\% \text{ to } 15\%\,, \tag{32}$$

at the 1-σ interval.

The remaining numerical study is summarized in two figures. In Fig. 1 we display the range of $\sin^2\theta_{13}$ against the lightest neutrino mass. In Fig. 2, we display the effective light neutrino contribution m_{ee} to neutrinoless double beta decay. Even though the two light neutrinos are quite degenerate in mass and have masses near 12 meV, due to Majorana phase, the effective mass is at most 3 meV. For both graphics the points are generated numerically without the analytic approximations employed in the previous analyses. We only collect the points compatible with the observables within 1-σ as shown in Ref. [18].

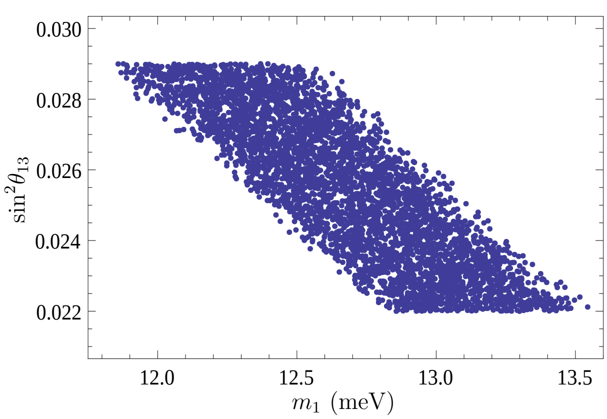

FIG. 1: Variation of θ_{13} as a function of the lightest neutrino mass. This scatter plot was generated using $\epsilon_c > 0$. There are points with $\epsilon_c < 0$ as well, corresponding to flipping the sign of the Dirac phase. However they do not introduce any perceptible change.

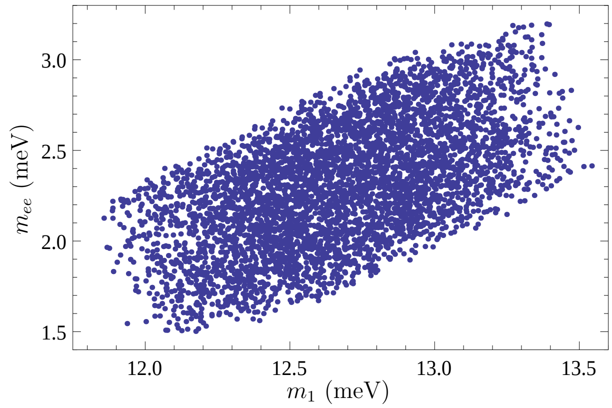

FIG. 2: The effective neutrino mass measured in neutrino-less double beta decay as a function of the lightest neutrino mass. As in Fig. 1, we have chosen $\epsilon_c > 0$ here.

IV. CONCLUSIONS

We have presented a model for leptons based on generalized CP symmetries which transform one family to another generating the non-abelian S_4 symmetry when supplemented by some permutations of families. This flavor symmetry, denoted by $\tilde{S}_4$, represents a new implementation of the S_4 symmetry where generalized CP symmetries are part of the group. This implementation shares some common features with the widely used group A_4. For example, $\tilde{S}_4$ also possesses three inequivalent one-dimensional representations that is similar to A_4 in model building. The presence of CP transformations, however, further restricts the parameters of the Lagrangian to be real. The restrictions imposed by the generalized CP transformations are such that, with the addition of another Higgs doublet, we could have easily built another variant of the model where left-handed and right-handed leptons are assigned to the same representation **3** of $\tilde{S}_4$. This could help us to embed this type of models in more symmetric theories such as left-right models. Therefore, this class of symmetries containing generalized CP transformations presents interesting features which can be further explored for flavor model-building.

Our specific model predicts maximal atmospheric mixing angle, accommodates the observed θ_{13} without any cancellation among the model parameters; it predicts normal hierarchy and maximal Dirac phase of $\pm 90°$ in the leptonic sector and should be testable in near future long baseline neutrino oscillation experiments. An important feature of the model is that the two light neutrino mass eigenstates are nearly degenerate in mass. Although the individual light eigenstates are “heavy”, i.e., near 12 meV or so, due to maximal Majorana phase their net contribution to neutrino-less double beta decay amplitude is very small. The model also predicts an approximate sum-rule relation valid for the three neutrino masses, without any Majorana phase or sign. The validity of the approximate sum-rule is around 12%.

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Appendix A: Other representations of $\tilde{S}_4$

We show here how to obtain the representations $\mathbf{1}_\omega$ and $\mathbf{1}_{\omega^2}$ of $\tilde{S}_4$ from the irreducible representation (irrep) **2** of S_4. The irreps of $\tilde{S}_4$ are constructed from the irreps of S_4 by the procedure explained in Sec. II for the representation **3**: extract the subgroup of $S_4 \otimes \langle\mathsf{CP}\rangle$ generated by $\tilde{S}$ and T in (2), instead of S, T, CP that generate $S_4 \otimes \langle\mathsf{CP}\rangle$.

Let us first explain why **1** and **1′** of S_4 generate the same representation **1** of $\tilde{S}_4$. We know that, for S_4, **1** is trivial but **1′** changes sign by S. If we follow the recipe and construct the representation of $\tilde{S}_4$ corresponding to **1** and **1′** we would obtain

$$\begin{aligned} \mathbf{1} : \tilde{S} = S \cdot \mathsf{CP} \to 1 \cdot \mathsf{CP}\,, \quad T \to 1\,, \\ \mathbf{1}' : \tilde{S} = S \cdot \mathsf{CP} \to (-1) \cdot \mathsf{CP}\,, \quad T \to 1\,. \end{aligned} \tag{A1}$$

We are using the generators (2) of **3** of $\tilde{S}_4$ as the group elements themselves, given that the representation is faithful. The CP transformation denoted by CP acts as usual. A fermion field $\psi(x)$ and a complex scalar field $\phi(x)$ transform as

$$\begin{aligned} \psi(x) &\xrightarrow{\mathsf{CP}} C\psi^*(\hat{x}) \\ \phi(x) &\xrightarrow{\mathsf{CP}} \phi^*(\hat{x}) \,. \end{aligned} \tag{A2}$$

Therefore, the representations (A1) are equivalent because if $\psi(x)$ transforms as $\mathbf{1}$ then $\psi'(x) = i\psi(x)$ transforms as $\mathbf{1'}$ (notice the field must be complex). This same reasoning leads to the equivalence of $\mathbf{3}$ and $\mathbf{3'}$ when we go from S_4 to $\tilde{S}_4$.

Let us see what happens to the representation **2** of S_4, which is equivalent to the $S_4 \to S_3$ homomorphism. To preserve the structure of $S_4 \otimes \langle \mathsf{CP} \rangle$ for which CP commutes with the elements of S_4, it is important to consider real representations of **2**. We adopt a slightly different version of Ref. [22, Eq.(640), in second reference],

$$D_2(S) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \,, \quad D_2(T) = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \,. \tag{A3}$$

Then $S_4 \otimes \langle \mathsf{CP} \rangle$ in this representation is generated by $D_2(S), D_2(T)$ and CP acting as in (A2). The subgroup $\tilde{S}_4$ would be generated by

$$D_2(\tilde{S}) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \mathsf{CP} \,, \quad D_2(T) = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \,. \tag{A4}$$

However, it is usually more convenient to work with the complex basis where T is diagonal. We change basis to

$$D'_2(\tilde{S}) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \mathsf{CP} \,, \quad D'_2(T) = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} \,, \tag{A5}$$

where

$$D'_2(T) = X^\dagger D_2(T) X \,, \tag{A6}$$

with the basis-change matrix

$$X = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} \,. \tag{A7}$$

Now, $D'_2(\tilde{S})$ in (A5) differs from

$$X^\dagger \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} X \cdot \mathsf{CP} \tag{A8}$$

because X is complex and complex basis change acts differently for CP transformations. The correct transformation is

$$D'_2(\tilde{S}) = X^\dagger \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} X^* \cdot \mathsf{CP} \,, \tag{A9}$$

which leads to (A5).

Equation (A5) defines the representation of $\tilde{S}_4$, derived from **2** of S_4. Since both transformations which generates $\tilde{S}_4$ do not mix the first and second components, they are essentially one-dimensional (complex). They correspond to the representations which we denoted by $\mathbf{1}_\omega$ and $\mathbf{1}_{\omega^2}$, corresponding to the action of (A5) to the first and second components, respectively. Explicitly, for a fermion field $\psi(x)$ (chiral or not), we have

$$\begin{aligned} \mathbf{1}_\omega : \psi(x) &\xrightarrow{\tilde{S}} C\psi^*(\hat{x}) \,, \quad \psi(x) \xrightarrow{T} \omega\, \psi(x) \,, \\ \mathbf{1}_{\omega^2} : \psi(x) &\xrightarrow{\tilde{S}} C\psi^*(\hat{x}) \,, \quad \psi(x) \xrightarrow{T} \omega^2 \psi(x) \,. \end{aligned} \tag{A10}$$

If could ignore gauge quantum numbers, the representation $\mathbf{1}_\omega$ and $\mathbf{1}_{\omega^2}$ would be equivalent because if $\psi(x) \sim \mathbf{1}_\omega$, then $C\psi^*(x) \sim \mathbf{1}_{\omega^2}$. Its real representation space is two-dimensional. In particular, $\mathbf{1}$ and $\mathbf{1}'$ would correspond to CP even and CP odd combinations of fields which have no definite transformation properties under the gauge groups. It is important to emphasize that if we were considering the whole $S_4 \otimes \langle \mathsf{CP} \rangle$, the representation $\mathbf{2}$ would remain two-dimensional (complex).

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