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LHC Phenomenology and Cosmology of String-Inspired Intersecting D-Brane Models

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Abstract

We discuss the phenomenology and cosmology of a Standard-like Model inspired by string theory, in which the gauge fields are localized on D-branes wrapping certain compact cycles on an underlying geometry, whose intersection can give rise to chiral fermions. The energy scale associated with string physics is assumed to be near the Planck mass. To develop our program in the simplest way, we work within the construct of a minimal model with gauge-extended sector $U(3)_B \times Sp(1)_L \times U(1)_{I_R} \times U(1)_L$. The resulting $U(1)$ content gauges the baryon number B , the lepton number L , and a third additional abelian charge I_R which acts as the third isospin component of an $SU(2)_R$. All mixing angles and gauge couplings are fixed by rotation of the $U(1)$ gauge fields to a basis diagonal in hypercharge Y and in an anomaly free linear combination of I_R and $B - L$. The anomalous Z' gauge boson obtains a string scale Stückelberg mass via a 4D version of the Green-Schwarz mechanism. To keep the realization of the Higgs mechanism minimal, we add an extra $SU(2)$ singlet complex scalar, which acquires a VEV and gives a TeV-scale mass to the non-anomalous gauge boson Z'' . The model is fully predictive and can be confronted with dijet and dilepton data from LHC8 and, eventually, LHC14. We show that $M_{Z''} \approx 3 - 4$ TeV saturates current limits from the CMS and ATLAS collaborations. We also show that for $M_{Z''} \lesssim 5$ TeV, LHC14 will reach discovery sensitivity $\gtrsim 5\sigma$. After that, we demonstrate in all generality that Z'' milli-weak interactions could play an important role in observational cosmology. Finally, we examine some phenomenological aspects of the supersymmetric extension of the D-brane construct.

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I. INTRODUCTION

With the turn on of the Large Hadron Collider (LHC) at CERN, a new era of discovery has just begun [1–4]. The $SU(3)_C \times SU(2)_L \times U(1)_Y$ Standard Model (SM) of electroweak and strong interactions was once again severely tested with a dataset corresponding to an integrated luminosity of $\sim 5 \text{ fb}^{-1}$ of pp collisions collected at $\sqrt{s} = 8 \text{ TeV}$. The LHC8 data have shown no evidence for new physics beyond the SM.

However, there is another side to the story. The concordance model of cosmology –the flat expanding universe containing 5% baryons, 20% dark matter, and 75% dark energy– continues to be put on a firmer footing through observations of the Supernova Search Team [5–7], the Supernova Cosmology Project [8–10], the Wilkinson Microwave Anisotropy Probe (WMAP) [11, 12], the Sloan Digital Sky Survey (SDSS) [13–16], and the Hubble Space Telescope [17]. While not yet rock solid experimentally, from these observations it is evident that in order to describe the physics of the early universe, and thereupon particle interactions at sub-fermi distances, new theoretical concepts are necessary, which go beyond the SM.¹

Arguably, another major driving force behind the consideration of physics beyond the SM is the huge disparity between the strength of gravity and of the SM forces. This hierarchy problem suggests that new physics could be at play at the TeV-scale. To be more specific, the non-zero vacuum expectation value of the scalar Higgs doublet sets the scale of electroweak interactions. However, due to the quadratic sensitivity of the Higgs mass to quantum corrections from an arbitrarily high mass scale, with no new physics between the energy scale of electroweak unification, $M_{\text{EW}} \sim 1 \text{ TeV}$, and the vicinity of the Planck mass, $M_{\text{Pl}} \sim 10^{19} \text{ GeV}$, the Higgs mass must be fine-tuned to an accuracy of $\mathcal{O}(10^{32})$. Therefore, it is of interest to identify univocal footprints that can plausibly arise in theories with the capacity to describe physics over this colossal range of scales. Among various attempts in this direction, string theory is perhaps the most successful candidate and also the most ambitious approach since besides the SM gauge interactions it includes also the gravitational force at the quantum level [19, 20].

In recent years there has been achieved substantial progress in connecting string theory

¹ It appears likewise from experimental evidence of neutrino flavor oscillations by the mixing of different mass eigenstates that the SM has to be extended [18].

with particle physics and cosmology. Important advances were fueled by the realization of the vital role played by D-branes [21, 22] in connecting string theory to phenomenology. This has permitted the formulation of string theories with string scale setting in at TeV scales, and together with large extra dimensions [23].

There are two peerless phenomenological consequences for TeV scale D-brane string compactifications: the emergence of Regge recurrences at parton collision energies $\sqrt{\hat{s}} \sim$ string scale $\equiv M_s$; and the presence of one or more additional $U(1)$ gauge symmetries, beyond the $U(1)_Y$ of the SM. The latter follows from the property that the gauge group for open strings terminating on a stack of N identical D-branes is $U(N)$ rather than $SU(N)$ for $N > 2$. (For $N = 2$ the gauge group can be $Sp(1) \cong SU(2)$ rather than $U(2)$.) In a series of recent publications we have exploited both these properties to explore and anticipate new-physics signals that could potentially be revealed at LHC. Regge recurrences most distinctly manifest in the $\gamma + \text{jet}$ [24, 25] and dijet [26–30] spectra resulting from their decay.² The extra $U(1)$ gauge symmetries beyond hypercharge have (in general) triangle anomalies, but are cancelled by the Green-Schwarz mechanism [33]. In addition there can be also massive $U(1)$ gauge bosons, which are associated to 4D non-anomalous Abelian gauge symmetries, but however originate from anomalous $U(1)$'s in six dimensions. In both cases, these $U(1)$ gauge bosons get Stückelberg masses. Since in these D-brane models M_s is assumed to be $\mathcal{O}(\text{TeV})$, the presence of these generic $U(1)$'s may be amenable to experimental tests at the LHC [34–36].

In this work we take a related but different approach studying new physics effects of D-brane models with the conventional assumption $\text{TeV} \ll M_s \lesssim M_{\text{Pl}}$. The gauge symmetry also arises from a product of $U(N)$ groups, guaranteeing extra $U(1)$ gauge bosons in the spectrum. The weak hypercharge is identified with a linear combination of anomalous $U(1)$'s which itself is anomaly free. As indicated in the preceding paragraph, the extra anomalous $U(1)$ gauge bosons generically obtain a string scale Stückelberg mass. The $U(1)$ symmetries

² The amplitudes of lowest massive Regge excitations that include $2 \rightarrow 2$ scattering processes involving 4 gauge bosons, or 2 gauge bosons and 2 quarks, are *universal* [26]. Therefore, the s -channel pole terms of the average square amplitudes contributing to $\gamma + \text{jet}$ and dijet topologies can be obtained independent of the details of the compactification scheme. For phenomenological purposes, the poles need to be softened to a Breit-Wigner form by obtaining and utilizing the correct total widths of the resonances [31]. The recent search for such narrow resonances in data collected during the LHC8 run, excludes a string scale below 4.69 TeV [32].

survive as global selection rules in the effective low energy theory. Such anomalous gauge bosons are now very heavy and out of the LHC reach.

However, in some D-brane models there exists non-anomalous and also massless $U(1)$ gauge symmetries in addition to hypercharge. Namely, under certain topological conditions the associated gauge bosons can remain massless and obtain a low mass scale via the ordinary Higgs mechanism. Some phenomenological aspects of these kind of $U(1)$ gauge bosons were recently discussed in [37]. In this paper we first revisit the prospects of detecting such TeV-scale gauge bosons in particular at the LHC, and then we show in all generality that their milli-weak interactions could play an important role in observational cosmology.

Before proceeding with an outline of the paper, we sketch some issues surrounding the choice of a non-supersymmetric formulation. To avoid the fine tuning inherent in the hierarchy problem, the overwhelmingly favored approach is the introduction of supersymmetry (SUSY). However, for the present study, this presents a difficult technical problem: the full complexity of the scale of SUSY breaking has been pushed by experiment into the TeV region, which coincides with the energy scale involved in searching for the extra $U(1)$ gauge bosons. In the absence of an experimental signal for the onset of SUSY breaking, we will extract from string theory the choice of the $U(1)$ gauge assignments, as well as the quiver structure of the fermionic couplings. In principle, SM-like non-SUSY vacua exist in the string landscape [38–40]. Throughout most of this work we will operate within that vacuum structure. However, before concluding we will also discuss in some detail the phenomenology of supersymmetric vacua and the technical problems associated with a phenomenologically viable breaking of an additional $U(1)$ symmetry in a SUSY background.

The layout of the paper is as follows. In Sec. II we outline the basic setting of intersecting D-brane models and discuss general aspects of the effective low energy theory inherited from properties of the overarching string theory. After that, we particularize the discussion to the $U(3)_B \times Sp(1)_L \times U(1)_L \times U(1)_{I_R}$ intersecting D-brane configuration that realizes the SM by open strings [41]. In Sec. III we study the associated phenomenological aspects of non-anomalous $U(1)$ gauge bosons related to experimental searches for new physics at the LHC. In Sec. IV we explore cosmological predictions of intersecting D-brane models in light of recent data, which seem to favor the existence of roughly one additional neutrino species (in addition to the 3 contained in the SM), challenging the earliest observationally verified landmarks: big bang nucleosynthesis (BBN) and the cosmic microwave background (CMB).

The gist of Sec. IV extends the previous study of TeV-scale string compactifications [42] to D-brane models where some of the $U(1)$ masses are at a high string scale. In Sec. V we examine the consequences of possible supersymmetric extensions. Our conclusions are collected in Sec. VI.

II. STANDARD MODEL FROM INTERSECTING D-BRANES

D-brane string compactifications provide a collection of building block rules that can be used to build up the SM or something very close to it [43–57]. In this section, we will briefly review the basics of constructing such D-brane models. More comprehensive treatments can be found in [58–61].

A. Construction Rules and Generalities of D-brane Models

The details of the D-brane construct depend a lot on whether we use oriented string or unoriented string models. The basic unit of gauge invariance for oriented string models is a $U(1)$ field, so that a stack of N identical D-branes eventually generates a $U(N)$ theory with the associated $U(N)$ gauge group. In the presence of many D-brane types, the gauge group becomes a product form $\prod U(N_P)$, where N_P reflects the number of D-branes in each stack. As an illustration, consider Type IIA string theory compactified on a six dimensional manifold \mathcal{M} . A specific configuration will be given by K stacks of intersecting D6-branes filling 4-dimensional Minkowski spacetime M_4 and wrapping internal homology 3-cycles of \mathcal{M} . Each stack consists of N_P coincident D6 branes whose world-volume is $M_4 \times \Pi_P$, where Π_P is the corresponding homology class of each 3-cycle, with $P = 1, \dots, K$. The closed string degrees of freedom reside in the entire ten dimensional space, which in addition to the gravitational fields, contain the geometric scalar moduli fields of the internal space. The open string degrees of freedom give rise to the gauge theory on the D6-brane world-volumes, with gauge group $\prod U(N_P)$. In addition, there are open string modes which split into states with both ends on the same stack of branes as well as those connecting different stacks of branes. The latter are particularly interesting: there is a chiral fermion living at each four-dimensional intersection of two branes P and Q , transforming in the bifundamental representation of $U(N_P) \times U(N_Q)$ [62]. The intersection number of these two

branes, $I_{PQ} \equiv [\Pi_P] \cdot [\Pi_Q]$, is a topologically invariant integer whose modulus gives us the multiplicity of such massless fermionic content and its sign depends on the chirality of such fermions. A particularly simple subfamily of the configurations described above consist of taking \mathcal{M} as a factorizable six-torus: $T^6 = T^2 \times T^2 \times T^2$. We can then further simplify the configurations assuming that the 3-cycles can be factorized as three 1-cycles, each of them wrapping on a different T^2 . In this case the homology 3-cycle Π_P can be expressed as

$$[\Pi_P] = [(n_P^1, m_P^1), (n_P^2, m_P^2), (n_P^3, m_P^3)], \quad (1)$$

where (n_P^i, m_P^i) , are the wrapping numbers of each $D6_P$ -brane, on the i^{th} torus, with n_P^i and (m_P^i) being the number of times the brane is wrapping around the i^{th} torus. The intersection number takes a simple form

$$I_{PQ} = \prod_{i=1}^3 (n_P^i m_Q^i - m_P^i n_Q^i). \quad (2)$$

In orientifold brane configurations, which are necessary for tadpole cancellation [63, 64], and thus consistency of the theory, open strings become in general non-oriented. For unoriented strings the above rules still apply, but we are allowed many more choices because the branes come in two different types. There are the branes whose images under the orientifold are different from themselves and their image branes, and also branes who are their own images under the orientifold procedure. Stacks of the first type combine with their mirrors and give rise to $U(N)$ gauge groups, while stacks of the second type give rise to only $SO(N)$ or $Sp(N)$ gauge groups.

Generally speaking, intersecting D-brane models involve at least three kinds of generic mass scales. First, of course, there is the fundamental string scale,

$$M_s = \frac{1}{\sqrt{\alpha'}}, \quad (3)$$

where α' is the slope parameter of the well known Regge trajectories of vibrating strings

$$j = j_0 + \alpha' M^2, \quad (4)$$

with j and $M = \sqrt{n}M_s$ the spin and mass of the resonant state, respectively ($n = 1, \dots$). Second, compactification from ten to four dimensions on an internal six-dimensional space of volume V_6 defines a mass scale:

$$M_6 = \frac{1}{V_6^{1/6}}. \quad (5)$$

Third, wrapping a stack P of $D(p+3)$ -branes around the internal p -cycle defines an internal world-volume $V_p^{(P)} = (2\pi)^p v_p^{(P)}$ of this D-branes stack and an associated (Kaluza-Klein) mass:

$$M_p^{(P)} = \frac{1}{\left(v_p^{(P)}\right)^{1/p}}. \quad (6)$$

These three types of fundamental dimensional parameters of D-brane models are linked to four-dimensional physical observables. First, the Planck mass given by

$$M_{\text{Pl}}^2 = 8 e^{-2\phi_{10}} M_s^8 \frac{V_6}{(2\pi)^6} \quad (7)$$

determines the strength of gravitational interactions. Here, the dilaton field ϕ_{10} is related to the string coupling constant through $g_s = e^{\phi_{10}}$. Thus, for a string scale $M_s \approx \mathcal{O}(1 \text{ TeV})$, the volume of the internal space M_6 needs to be as large as $V_6 M_s^6 = \mathcal{O}(10^{32})$. Second, the four-dimensional gauge couplings of the strong and weak interactions are given in terms of the respective volumes V_p^P , where P runs over the corresponding gauge group factors, as

$$g_P^{-2} = (2\pi)^{-1} M_s^p e^{-\phi_{10}} v_p^{(P)}. \quad (8)$$

Again for a string scale $M_s \approx \mathcal{O}(1 \text{ TeV})$ and using the known values of the strong ($g_3^2/4\pi \approx 0.1$) and the weak ($g_2^2/4\pi \approx g_3^2/12\pi$) gauge coupling constants at the string scale ($g_2^2/4\pi = \alpha_{\text{EM}}/\sin^2\theta_W$, $\sin^2\theta_W \approx 0.23$, $\alpha_{\text{EM}} \approx 1/128$) we can compute the volumes of the internal cycles, assuming weak string coupling. To be specific, we choose $g_s = 0.2$, and then we obtain

$$M_s^p v_p^{(3)} \approx 1, \quad M_s^p v_p^{(2)} \approx 3. \quad (9)$$

For $\text{TeV} \ll M_s \lesssim M_{\text{Pl}}$, V_6 and $V_p^{(P)}$'s are $\mathcal{O}(1)$ in string units. In general, there are different volumes $V_p^{(P)}$'s for different stacks, and therefore the abelian gauge couplings associated to $U(1)$ symmetries of different D-brane stacks are not equal.

This approach to string model building leads to a variety of low energy theories including the SM as well as its supersymmetric extensions. Throughout most of this paper we consider theories which are non-supersymmetric all the way up to the UV cutoff of the effective theory; of course the deep UV theory of quantum gravity may well be supersymmetric. Even though SUSY introduces special advantages over completely non-SUSY theories, our approach is distinguished by its simplicity to describe very appealing phenomenological possibilities that

best display the dynamics involving the extra $U(1)$ symmetries. The study of some aspects of the supersymmetric version of these models will be postponed until Sec. V.

The minimal embedding of the SM particle spectrum requires at least three brane stacks [65] leading to three distinct models of the type $U(3) \times U(2) \times U(1)$ that were classified in [65, 66]. Only one of them (model C of [66]) has baryon number as symmetry that guarantees proton stability (in perturbation theory), and can be used in the framework of TeV-scale strings. Moreover, since the charge associated to the $U(1)$ of $U(2)$ does not participate in the hypercharge combination, $U(2)$ can be replaced by the symplectic $Sp(1)$ representation of Weinberg-Salam $SU(2)_L$, leading to a model with one extra $U(1)$ (the baryon number) besides hypercharge [67].

The SM embedding in four D-brane stacks leads to many more models that have been classified in [68, 69]. In order to make a phenomenologically interesting choice, herein we focus on models where $U(2)$ can be reduced to $Sp(1)$. Besides the fact that this reduces the number of extra $U(1)$'s, one avoids the presence of a problematic Peccei-Quinn symmetry, associated in general with the $U(1)$ of $U(2)$ under which Higgs doublets are charged [65]. To develop our program in the simplest way, we will work within the construct of a minimal model, $U(3)_B \times Sp(1)_L \times U(1)_L \times U(1)_{I_R}$, which has the attractive property of elevating the two major global symmetries of the SM (baryon number B and lepton number L) to local gauge symmetries [41]. We turn now to discuss the compelling properties of this model.

B. Standard Model⁺⁺

In this paper we are interested in the minimal 4-stack gauge-extended sector $U(3)_B \times Sp(1)_L \times U(1)_L \times U(1)_{I_R}$ [41]. A schematic representation of the D-brane structure is shown in Fig. 1 and the brane content is given in Table I. Note that for the $Sp(1)$ stack P , the mirror brane P^* lies on top of P . So even though $N_P = 1$, it can be thought of as a stack of two D6 branes, which give an $Sp(1) \cong SU(2)$ group under the orientifold projection. Concretely, in the bosonic sector the open strings terminating on the stack of “color” branes contain, in addition to the $SU(3)$ octet of gluons

$$G_{\mu\nu}^a = (\partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_3 f^{abc} G_\mu^b G_\nu^c), \quad i f^{abc} T^a = [T^b, T^c], \quad T^a \in SU(3),$$

TABLE I: D-brane content of $U(3)_B \times Sp(1)_L \times U(1)_L \times U(1)_{I_R}$; the mirror branes O^*, P^*, Q^*, R^* are not shown.

Label	Stack	Number of Branes	Gauge Group
$1 \equiv R$	Right	$N_R = 1$	$U(1)_{I_R}$
$2 \equiv P$	Left	$N_P = 1$	$Sp(1)_L \cong SU(2)_L$
$3 \equiv Q$	Baryonic	$N_Q = 3$	$U(3)_B = SU(3)_C \times U(1)_B$
$4 \equiv O$	Leptonic	$N_O = 1$	$U(1)_L$

an extra $U(1)$ boson C_μ . On the $Sp(1)$ stack the open strings correspond to the weak gauge bosons

$$W_{\mu\nu}^a = (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_2 \epsilon^{abc} W_\mu^b W_\nu^c), \quad i\epsilon^{abc} \tau^a = [\tau^b, \tau^c], \quad \tau^a \equiv \sigma^a/2 \in SU(2).$$

The $U(1)_{I_R}$ D-brane is a terminus for the B_μ gauge boson, and there is a third additional $U(1)$ field X_μ terminating on the $U(1)_L$ brane. The resulting $U(1)$ content gauges B [with $U(1)_B \subset U(3)_B$], L , and a third additional abelian charge I_R which acts as the third isospin component of an $SU(2)_R$. The usual electroweak hypercharge is a linear combination of these three $U(1)$ charges:

$$Q_Y = c_1 Q_{I_R} + c_3 Q_B + c_4 Q_L, \quad (10)$$

with $c_1 = 1/2$, $c_3 = 1/6$, $c_4 = -1/2$, $B = Q_B/3$ and $L = Q_L$. Alternatively, inverting the above relations, one finds:

$$Q_B = 3B \quad ; \quad Q_L = L \quad ; \quad Q_{I_R} = 2Q_Y - (B - L). \quad (11)$$

The chiral particle spectrum from these intersecting branes consists of six sets (labeled by an index $i = 1, \dots, 6$) of Weyl fermion-antifermion pairs, whose quantum numbers are given in Table II. Note that the combination $B - L$ is anomaly free, while both B and L are anomalous.

As mentioned already, the Q_B (gauged baryon number) is anomalous. This anomaly is canceled by the 4D version [70–74] of the Green-Schwarz mechanism [33]. Non anomalous $U(1)$'s can acquire masses due to effective six-dimensional anomalies associated for instance

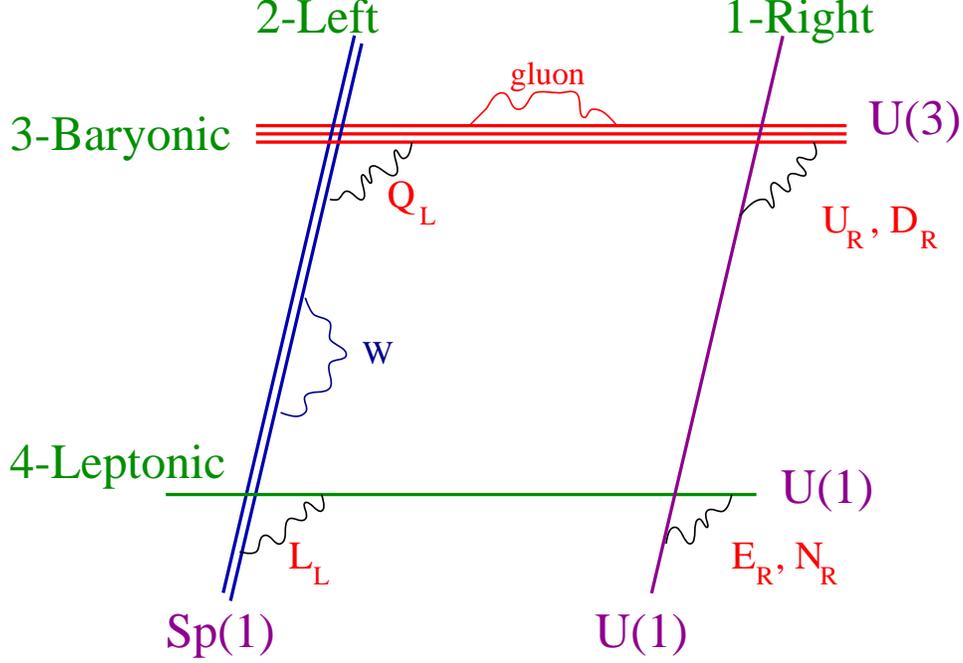


FIG. 1: Pictorial representation of the $U(3)_B \times Sp(1)_L \times U(1)_L \times U(1)_{I_R}$ D-brane model.

TABLE II: Chiral fermion spectrum of the $U(3)_B \times Sp(1)_L \times U(1)_L \times U(1)_{I_R}$ D-brane model.

Label	Fields	Sector	I_{PQ}	Representation	Q_B	Q_L	Q_{I_R}	Q_Y
1	U_R	(Q, R^*)	3	$(3, 1)$	1	0	1	$\frac{2}{3}$
2	D_R	(Q, R)	3	$(3, 1)$	1	0	-1	$-\frac{1}{3}$
3	L_L	(O, P)	3	$(1, 2)$	0	1	0	$-\frac{1}{2}$
4	E_R	(O, R)	3	$(1, 1)$	0	1	-1	-1
5	Q_L	(Q, P)	3	$(3, 2)$	1	0	0	$\frac{1}{6}$
6	N_R	(O, R^*)	3	$(1, 1)$	0	1	1	0

to sectors preserving $\mathcal{N} = 2$ supersymmetry [75, 76].³ These two-dimensional ‘bulk’ masses

³ In fact, also the hypercharge gauge boson of $U(1)_Y$ can acquire a mass through this mechanism. In order to keep it massless, certain topological constraints on the compact space have to be met.

become therefore larger than the localized masses associated to four-dimensional anomalies, in the large volume limit of the two extra dimensions. Specifically for $D(p+3)$ -branes with p -longitudinal compact dimensions the masses of the anomalous and, respectively, the non-anomalous $U(1)$ gauge bosons have the following generic scale behavior:

$$\begin{aligned} \text{anomalous } U(1)_a : \quad M_{Z'} &= g'_a M_s, \\ \text{non - anomalous } U(1)_a : \quad M_{Z''} &= g'_a M_s^3 V_2. \end{aligned} \tag{12}$$

Here g'_a is the gauge coupling constant associated to the group $U(1)_a$, given by $g'_a \propto g_s / \sqrt{V_p^{(a)}}$ where g_s is the string coupling and $V_p^{(a)}$ is the internal D-brane world-volume along the p compact extra dimensions, up to an order one proportionality constant. Moreover, V_2 is the internal two-dimensional volume associated to the effective six-dimensional anomalies giving mass to the non-anomalous $U(1)_a$.⁴ E.g. for the case of D5-branes, whose common intersection locus is just 4-dimensional Minkowski-space, $V_p^{(a)} = V_2$ denotes the volume of the longitudinal, two-dimensional space along the two internal D5-brane directions. Since internal volumes are bigger than one in string units to have effective field theory description, the masses of non-anomalous $U(1)$ -gauge bosons are generically larger than the masses of the anomalous gauge bosons.⁵

The non-anomalous $U(1)_a$ can also remain massless all the way down to the TeV-scale energy region and grow a mass through a Higgs mechanism. The absence of a Stückelberg mass term for the associated gauge bosons means that this $U(1)$ gauge symmetry is anomaly free also in six dimensions. In this case a certain topological condition has to hold, which cannot be read off from the local D-brane quiver, but can only be answered knowing the 6D compact orientifold. Specifically, just like for the SM gauge symmetry $U(1)_Y$, the absence of the Stückelberg mass term for $U(1)_a = c_3^a U(1)_B + c_4^a U(1)_L + c_1^a U(1)_{I_R}$ can be phrased by the following condition on the homology cycles Π and their orientifold images Π' of the

⁴ It should be noted that in spite of the proportionality of the $U(1)_a$ masses to the string scale, these are not string excitations but zero modes. The proportionality to the string scale appears because the mass is generated from anomalies, via an analog of the Green-Schwarz anomaly cancellations: either 4 dimensional anomalies, in which case the Green-Schwarz term is equivalent to a Stückelberg mechanism, or from effective 6 dimensional anomalies, in which case the mass term is extended in two more (internal) dimensions.

⁵ In [77] a different (possibly T-dual) scenario with $D7$ -branes was investigated. In this case the masses of the anomalous and non-anomalous $U(1)$'s appear to exhibit a dependence on the entire six-dimensional volume, such that the non-anomalous masses become lighter than the anomalous ones.

three $U(1)$ gauge groups:

$$3c_3^a(\Pi_3 - \Pi'_3) + c_4^a(\Pi_L - \Pi'_L) + c_1^a(\Pi_{I_R} - \Pi'_{I_R}) = 0. \quad (13)$$

In what follows we entertain this possibility, having two massless gauge bosons $U(1)_Y$ (associated to the SM hypercharge) and $U(1)_{Y''}$ (associated to a linear combination of anomaly-free I_R and $B - L$) and one heavy gauge boson $U(1)_{Y'}$ (associated to an anomalous combination of the three $U(1)$'s). The classical gauge invariant Lagrangian, obeying the $U(3)_B \times Sp(1)_L \times U(1)_L \times U(1)_{I_R}$ gauge symmetry, can be decomposed as:

$$\mathcal{L}_{\text{SM}^{++}} = \mathcal{L}_{\text{YM}} + \sum_{\text{generations}} (\mathcal{L}_f + \mathcal{L}_Y) + \mathcal{L}_s + \mathcal{L}_X, \quad (14)$$

where the terms on the right hand side identify the gauge (or Yang-Mills) part, the fermion part, the Yukawa part, the scalar part, and extra terms from the underlying string theory, respectively.

Electroweak symmetry breaking is achieved through the standard Higgs doublet H . The spontaneous symmetry breaking of the extra non-anomalous $U(1)$ is attained through an $SU(2)$ singlet scalar field H'' , which acquires a vacuum expectation value (VEV) at the TeV scale. The $U(1)$ quantum numbers of the Higgs sector are given in Table III.

The Yang-Mills Lagrangian reads:

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4} \left(G_{\mu\nu}^a G_a^{\mu\nu} + W_{\mu\nu}^a W_a^{\mu\nu} + F_{\mu\nu}^{(1)} F_{(1)}^{\mu\nu} + F_{\mu\nu}^{(3)} F_{(3)}^{\mu\nu} + F_{\mu\nu}^{(4)} F_{(4)}^{\mu\nu} \right), \quad (15)$$

with the non-Abelian field strengths the same as in the SM, and the Abelian $F_{\mu\nu}^{(1)} = \partial_\mu B_\nu - \partial_\nu B_\mu$, $F_{\mu\nu}^{(3)} = \partial_\mu C_\nu - \partial_\nu C_\mu$, and $F_{\mu\nu}^{(4)} = \partial_\mu X_\nu - \partial_\nu X_\mu$.

The fermion Lagrangian is given by

$$\begin{aligned} \mathcal{L}_f = & i\overline{Q}_L \gamma_\mu \mathcal{D}^\mu Q_L + i\overline{U}_R \gamma_\mu \mathcal{D}^\mu U_R + i\overline{D}_R \gamma_\mu \mathcal{D}^\mu D_R + i\overline{L}_L \gamma_\mu \mathcal{D}^\mu L_L + i\overline{E}_R \gamma_\mu \mathcal{D}^\mu E_R \\ & + i\overline{N}_R \gamma_\mu \mathcal{D}^\mu N_R, \end{aligned} \quad (16)$$

where

$$\mathcal{D}_\mu = \partial_\mu - ig_3 T^a G_\mu^a - ig'_3 Q_B C_\mu - ig_2 \tau^a W_\mu^a - ig'_1 Q_{I_R} B_\mu - ig'_4 Q_L X_\mu \quad (17)$$

are the covariant derivatives with the gauge fields specified in the D-brane basis.

The fields C_μ, X_μ, B_μ are related to Y_μ, Y'_μ and Y''_μ by the rotation matrix,

$$\mathbb{R} = \begin{pmatrix} C_\theta C_\psi & -C_\phi S_\psi + S_\phi S_\theta C_\psi & S_\phi S_\psi + C_\phi S_\theta C_\psi \\ C_\theta S_\psi & C_\phi C_\psi + S_\phi S_\theta S_\psi & -S_\phi C_\psi + C_\phi S_\theta S_\psi \\ -S_\theta & S_\phi C_\theta & C_\phi C_\theta \end{pmatrix}, \quad (18)$$

TABLE III: Higgs spectrum of the $U(3)_B \times Sp(1)_L \times U(1)_L \times U(1)_{I_R}$ D-brane model.

Fields	Sector	I_{PQ}	Representation	Q_B	Q_L	Q_{I_R}	Q_Y
H	(P, R)	1	$(1, 2)$	0	0	1	$\frac{1}{2}$
H''	(O, R)	1	$(1, 1)$	0	-1	-1	0

with Euler angles θ , ψ , and ϕ [78]. Hence, the covariant derivative for the $U(1)$ fields in Eq. (17) can be rewritten in terms of Y_μ , Y'_μ , and Y''_μ as follows

$$\begin{aligned}
 \mathcal{D}_\mu = & \partial_\mu - iY_\mu (-S_\theta g'_1 Q_{I_R} + C_\theta S_\psi g'_4 Q_L + C_\theta C_\psi g'_3 Q_B) \\
 & - iY'_\mu [C_\theta S_\phi g'_1 Q_{I_R} + (C_\phi C_\psi + S_\theta S_\phi S_\psi) g'_4 Q_L + (C_\psi S_\theta S_\phi - C_\phi S_\psi) g'_3 Q_B] \\
 & - iY''_\mu [C_\theta C_\phi g'_1 Q_{I_R} + (-C_\psi S_\phi + C_\phi S_\theta S_\psi) g'_4 Q_L + (C_\phi C_\psi S_\theta + S_\phi S_\psi) g'_3 Q_B] .
 \end{aligned} \tag{19}$$

Now, by demanding that Y_μ has the hypercharge Q_Y given in Eq. (10) we fix the first column of the rotation matrix \mathbb{R}

$$\begin{pmatrix} C_\mu \\ X_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} Y_\mu c_3 g_Y / g'_3 & \dots \\ Y_\mu c_4 g_Y / g'_4 & \dots \\ Y_\mu c_1 g_Y / g'_1 & \dots \end{pmatrix}, \tag{20}$$

and we determine the value of the two associated Euler angles

$$\theta = -\arcsin[c_1 g_Y / g'_1] \tag{21}$$

and

$$\psi = \arcsin[c_4 g_Y / (g'_4 C_\theta)]. \tag{22}$$

The couplings g'_1 and g'_4 are related through the orthogonality condition, $P(g_Y, g'_1, g'_3, g'_4) = 0$, yielding

$$\left(\frac{c_4}{g'_4}\right)^2 = \frac{1}{g_Y^2} - \left(\frac{c_3}{g'_3}\right)^2 - \left(\frac{c_1}{g'_1}\right)^2, \tag{23}$$

with g'_3 fixed by the relation $g_3(M_s) = \sqrt{6} g'_3(M_s)$ [36]. Next, by demanding that Y'' couples to a linear combination of anomaly-free I_R and $B - L$ we determine the third Euler angle

$$\tan \phi = -S_\theta \frac{3 g'_3 C_\psi + g'_4 S_\psi}{3 g'_3 S_\psi - g'_4 C_\psi}. \tag{24}$$

In the (Y, Y', Y'') basis, Y and Y'' are coupled to anomaly-free currents while the anomaly of the current associated to Y' is cancelled by the generalized Green-Schwarz mechanism. As a result, Y' acquires a mass of order of the string mass M_s , c.f. Eq. (12). Higgs VEVs will generate additional mass terms for Y' , introducing also some small mixing with other gauge gauge bosons, of order $(\text{TeV}/M_s)^2$. From now on, we neglect such small effects and take $Y' \simeq Z'$.

The Yukawa interactions are given by

$$\mathcal{L}_Y = -Y_d (\overline{Q}_L H) D_R - Y_u (\overline{Q}_L i\sigma^2 H^*) U_R - Y_e (\overline{L}_L H) E_R - Y_N (\overline{L}_L i\sigma^2 H^*) N_R + \text{h.c.}, \quad (25)$$

where the Yukawa couplings Y_i are matrices in flavor space. Note that unlike in the supersymmetric case, a single Higgs vacuum expectation value will generate masses for up and down quarks.⁶

Note that with the charge assignments of Tables II and III there are no dimension 4 operators involving H'' that contribute to the Yukawa Lagrangian. This is very important since H'' carries the quantum numbers of right-handed neutrino and its VEV breaks lepton number. However, this breaking can affect only higher-dimensional operators which are suppressed by the high string scale, and thus there is no phenomenological problem with experimental constraints for M_s higher than $\sim 10^{14}$ GeV.

The scalar Lagrangian is

$$\mathcal{L}_s = (\mathcal{D}^\mu H)^\dagger \mathcal{D}_\mu H + (\mathcal{D}^\mu H'')^\dagger \mathcal{D}_\mu H'' - V(H, H''), \quad (26)$$

with the potential

$$V(H, H'') = \mu^2 |H|^2 + \mu'^2 |H''|^2 + \lambda_1 |H|^4 + \lambda_2 |H''|^4 + \lambda_3 |H|^2 |H''|^2. \quad (27)$$

The Higgs VEVs obtained after minimizing this potential will be denoted as

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{and} \quad \langle H'' \rangle = v''. \quad (28)$$

The kinetic terms of the Higgs fields in (26) give masses to the various gauge bosons.

⁶ $i\sigma_2 H^*$ transforms in the fundamental representation of $SU(2)$.

At this point, we identify the photon A_μ and weak force mediators $W_\mu^+, W_\mu^-, \bar{Z}_\mu$ performing the usual Weinberg rotation

$$\begin{pmatrix} A_\mu \\ \bar{Z}_\mu \\ W_\mu^+ \\ W_\mu^- \end{pmatrix} = \begin{pmatrix} C_{\theta_W} & S_{\theta_W} & 0 & 0 \\ -S_{\theta_W} & C_{\theta_W} & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & i/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -i/\sqrt{2} \end{pmatrix} \begin{pmatrix} Y_\mu \\ W_\mu^3 \\ W_\mu^1 \\ W_\mu^2 \end{pmatrix}; \quad (29)$$

this gives

$$\begin{aligned} \mathcal{D}_\mu &= \partial_\mu - \frac{i}{2} g_2 \sigma^- W_\mu^+ - \frac{i}{2} g_2 \sigma^+ W_\mu^- - i g_2 \cos \theta_W (\sigma^3/2 - Q_Y \tan^2 \theta_W) \bar{Z}_\mu - i g_2 \sin \theta_W \\ &\times (\sigma^3/2 + Q_Y) A_\mu - i g_{Y'} Q_{Y'} Z'_\mu - i g_{Y''} Q_{Y''} Y''_\mu, \end{aligned} \quad (30)$$

with $\sigma^\pm = (\sigma^1 \pm i\sigma^2)/2$, $g_{Y'}/g_2 = \tan \theta_W$. From (19) and (30) we define

$$\begin{aligned} Q_Y H &= H/2, \\ g_{Y'} Q_{Y'} H &= (g'_1 C_\theta S_\phi) H, \\ g_{Y''} Q_{Y''} H &= (g'_1 C_\theta C_\phi) H, \\ Q_Y H'' &= 0, \\ g_{Y'} Q_{Y'} H'' &= -[g'_1 C_\theta S_\phi + g'_4 (C_\phi C_\psi + S_\theta S_\phi S_\psi)] H'', \\ g_{Y''} Q_{Y''} H'' &= -(g'_1 C_\theta C_\phi + g'_4 [C_\phi S_\theta S_\psi - C_\psi S_\phi]) H''. \end{aligned} \quad (31)$$

The Higgs kinetic terms of Eq.(26) together with the Green-Schwarz mass term, $\frac{1}{2} M'^2 Z'_\mu Z'^\mu$, lead to

$$\mathcal{B} = [\mathcal{D}_\mu^\dagger (0 \ v)] \left[\mathcal{D}^\mu \begin{pmatrix} 0 \\ v \end{pmatrix} \right] + (\mathcal{D}_\mu v'')^\dagger (\mathcal{D}^\mu v'') + \frac{1}{2} M'^2 Z'_\mu Z'^\mu. \quad (32)$$

Expanded this gives

$$\begin{aligned} \mathcal{B} &= \frac{1}{4} (g_2 v)^2 W_\mu^+ W^{-\mu} + \frac{1}{4} (g_2 v)^2 C_{\theta_W}^{-2} \bar{Z}_\mu \bar{Z}^\mu + g'_1 C_\theta (S_\phi Z'_\mu + C_\phi Y''_\mu) g_2 v^2 C_{\theta_W}^{-1} \bar{Z}^\mu \\ &+ v''^2 \{ g'_1 C_\theta (S_\phi Z'_\mu + C_\phi Y''_\mu) + g'_4 [(C_\phi C_\psi + S_\theta S_\phi S_\psi) Z'_\mu + S_\psi S_\theta C_\phi Y''_\mu] \}^2 \\ &+ (g'_1 v C_\theta)^2 (S_\phi Z'_\mu + C_\phi Y''_\mu) (S_\phi Z'^\mu + C_\phi Y''^\mu) + \frac{1}{2} M'^2 Z'_\mu Z'^\mu \\ &\simeq \frac{1}{4} (g_2 v)^2 W_\mu^+ W^{-\mu} + \frac{1}{4} (g_2 v)^2 C_{\theta_W}^{-2} \bar{Z}_\mu \bar{Z}^\mu + g'_1 C_\theta C_\phi Y''_\mu g_2 v^2 C_{\theta_W}^{-1} \bar{Z}^\mu \\ &+ v''^2 (g'_1 C_\theta C_\phi Y''_\mu + g'_4 S_\psi S_\theta C_\phi Y''_\mu)^2 + (g'_1 v C_\theta C_\phi)^2 Y''_\mu Y''^\mu + \dots \end{aligned} \quad (33)$$

where the omitted terms pertain only to the Z' couplings at the string scale. Recall that we have taken $M' \sim M_s$ and therefore Z' decouples from the low energy physics. By inspection

of (33) we immediately recognize the W^\pm masses and the usual tree level formula for the mass of the Z particle in the electroweak theory, $\overline{M}_Z^2 = (g_2^2 v^2 + g_Y^2 v^2)/2$, before mixing.

Now, we use the relation $g'_1 S_\theta = g'_4 C_\theta S_\psi$ to conveniently rewrite (33) as

$$\begin{aligned}
\mathcal{B} &\simeq \frac{1}{4} (g_2 v)^2 W_\mu^+ W^{-\mu} + \frac{1}{4} (g_2 v)^2 C_{\theta_W}^{-2} \overline{Z}_\mu \overline{Z}^\mu + g'_1 v C_\theta C_\phi g_2 v C_{\theta_W}^{-1} Y_\mu'' \overline{Z}^\mu + \left(\frac{v'' g'_1 C_\phi}{C_\theta} \right)^2 \\
&\times \left(1 + \left(\frac{v}{v''} C_\theta^2 \right)^2 \right) Y_\mu'' Y''^\mu + \dots \\
&\simeq \frac{1}{4} (g_2 v)^2 W_\mu^+ W^{-\mu} + \left(\frac{v'' g'_1 C_\phi}{C_\theta} \right)^2 \left[1 + \left(\frac{v}{v''} C_\theta^2 \right)^2 \right] \left\{ Y_\mu'' + \frac{g'_1 C_\theta^3 C_\phi g_2 v^2 C_{\theta_W}^{-1} \overline{Z}_\mu}{2 (v'' g'_1 C_\phi)^2 \left[1 + \left(\frac{v}{v''} C_\theta^2 \right)^2 \right]} \right\}^2 \\
&+ \left\{ \frac{1}{4} (g_2 v)^2 C_{\theta_W}^{-2} - \frac{g'_1 C_\theta^3 C_\phi g_2 v^2 C_{\theta_W}^{-1}}{2 (v'' g'_1 C_\phi)^2 \left[1 + \left(\frac{v}{v''} C_\theta^2 \right)^2 \right]} \right\} \overline{Z}_\mu \overline{Z}^\mu + \dots \quad (34)
\end{aligned}$$

Finally, if we make the expansion around $v/v'' \ll 1$, the $\overline{Z}_\mu Y''^\mu$ mass matrix is render diagonal and we obtain the desired expression for the mass terms

$$\mathcal{B} = \left(\frac{g_2 v}{2} \right)^2 W_\mu^+ W^{-\mu} + \left(\frac{g_2 v}{2 C_{\theta_W}} \right)^2 Z_\mu Z^\mu + \left(\frac{g'_1 C_\phi v''}{C_\theta} \right)^2 Z_\mu'' Z''^\mu + \mathcal{O} \left(\left(\frac{v}{v''} \right)^2 \right), \quad (35)$$

where $Z'' \simeq Y'' +$ small corrections.

In principle, in addition to the orthogonal field mixing induced by identifying anomalous and non-anomalous $U(1)$ sectors, there may be kinetic mixing between these sectors. However, in models where there is only one $U(1)$ per stack of D-branes, the relevant kinetic mixing is between $U(1)$'s on different stacks, and hence involves loops with fermions at brane intersection. Such loop terms are typically down by $g_i'^2/16\pi^2 \sim 0.01$ [79].⁷ By inspection of Table II the charges Q_B , Q_L , and Q_{I_R} are mutually orthogonal in the fermion space, *i.e.* $\sum_f Q_{i,f} Q_{j,f} = 0$ for $i \neq j$. This will maintain the orthogonality relation $P = 0$ to one loop without inducing kinetic mixing [36]. The charges assigned to H'' (see Table III) will violate the orthogonality condition. However, the H'' only contributes at the 0.9% level to the running of g'_1 from the string scale to the TeV scale, and about 0.3% to the running of g'_4 . These are of the same order as the two loop contributions from the fermion sector, so we may consistently ignore the nonorthogonality introduced by H'' in the context of one loop considerations.

⁷ The major effect of the kinetic mixing is in communicating SUSY breaking from a hidden $U(1)$ sector to the visible sector, generally in modification of soft scalar masses. For a comprehensive review of experimental limits on the mixing, see [80].

III. LHC PHENOMENOLOGY

In this section we discuss the discovery potential of the Z'' resonance at the LHC. Before proceeding, we summarize the lessons learned thus far. The initially free parameters of the model consist of three couplings g'_1, g'_3, g'_4 . These are augmented by three Euler angles to allow for a field rotation to coupling diagonal in hypercharge. This diagonalization fixes two of the angles and the orthogonal nature of the rotation introduces one constraint on the couplings $P(g_Y, g'_1, g'_3, g'_4) = 0$. The baryon number coupling g'_3 is fixed to be $\sqrt{1/6}$ of the non-abelian $SU(3)$ coupling at the scale of $U(N)$ unification, and is therefore determined at all energies through RG running. In what follows, we take $M_s = 10^{14}$ GeV as a reference point for running down the g'_3 coupling to the TeV region that is *ignoring mass threshold effects of stringy states*. This yields $g'_3(M_s) = 0.231$. We have checked that the running of the g'_3 coupling does not change significantly within the LHC range, for different values of the string scale. This leaves one free angle and two couplings with one constraint. Equation (24) fixes the third Euler angle. To comply with these assignments and ensure perturbativity of g'_4 between the TeV scale and the string scale we find from (23) that $g'_1 > 0.4845$. We also take $g'_1 \lesssim 1$ in order to ensure perturbativity at the string scale.

We first consider the case with $g'_1(M_s) \simeq 1$. This leads to $\psi(M_s) = -1.245$, $\theta(M_s) = -0.217$, $\phi(M_s) = -0.0006$, and $g'_4(M_s) = 0.232$. Substituting our fiducial values in (19) we find the non-anomalous $U(1)$ vector bosons couple to currents

$$\begin{aligned} J_Y &= 2.1 \times 10^{-1} Q_{IR} + 2.1 \times 10^{-1} (B - L) \\ J_{Y''} &= 9.8 \times 10^{-1} Q_{IR} - 4.7 \times 10^{-2} (B - L), \end{aligned} \quad (36)$$

at the string scale. Next, we run the couplings down to the TeV region. A very important point is that the couplings that are running are those of the $U(1)$ fields; hence the β functions receive contributions from fermions and scalars, but not from gauge bosons. The one loop correction to the various couplings are

$$\frac{1}{\alpha_Y(Q)} = \frac{1}{\alpha_Y(M_s)} - \frac{b_Y}{2\pi} \ln(Q/M_s), \quad (37)$$

$$\frac{1}{\alpha_i(Q)} = \frac{1}{\alpha_i(M_s)} - \frac{b_i}{2\pi} \ln(Q/M_s), \quad (38)$$

where

$$b_i = \frac{2}{3} \sum_f Q_{i,f}^2 + \frac{1}{3} \sum_s Q_{i,s}^2, \quad (39)$$

with f and s indicating contribution from fermion and scalar loops, respectively. Setting $Q = 4$ TeV, from (38) we obtain: $g'_1 = 0.406$, $g'_3 = 0.196$, $g'_4 = 0.218$, $\theta = -0.466$, $\psi = -1.215$, and $\phi = -0.0003$. This leads to

$$\begin{aligned} J_Y &= 1.8 \times 10^{-1} Q_{I_R} + 1.8 \times 10^{-1} (B - L) \\ J_{Z''} &= 3.6 \times 10^{-1} Q_{I_R} - 9.2 \times 10^{-2} (B - L), \end{aligned} \quad (40)$$

where we have assumed that H'' has developed its VEV. Since $\text{Tr} [Q_{I_R} B] = \text{Tr} [Q_{I_R} L] = 0$, the Z'' decay width is given by

$$\begin{aligned} \Gamma_{Z''} &= \Gamma_{Z'' \rightarrow Q_{I_R}} + \Gamma_{Z'' \rightarrow B-L} \\ &\propto (1.4 \times 10^{-1})^2 \text{Tr}[Q_{I_R}^2] + (9.2 \times 10^{-2})^2 \text{Tr}[(B - L)^2] \\ &= 1.0 \times 10^0 + 4.5 \times 10^{-2}. \end{aligned} \quad (41)$$

Thus, the corresponding branching fractions are $\text{BR } Z'' \rightarrow Q_{I_R} = 0.959$ and $\text{BR } Z'' \rightarrow B - L = 0.041$. Though not relevant for LHC phenomenology, a straightforward calculation shows that Z' is very nearly diagonal in B , with $\text{BR } Z' \rightarrow B = 0.946$ and $\text{BR } Z' \rightarrow L = 0.054$. Of course, since the quiver construction has each particle straddling two adjacent branes, there can be considerable variation in decay channels particle by particle. This is evident in Table IV.⁸ The dominance of B for the Z' decay channel and I_R for the Z'' decay channel is valid after averaging over decay channels.⁹

Now, duplicating the procedure for $g'_1(M_s) = 0.4845$ we obtain

$$\begin{aligned} \text{BR } Z' \rightarrow B : \text{BR } Z' \rightarrow L : \text{BR } Z'' \rightarrow Q_{1R} : \text{BR } Z'' \rightarrow B - L \\ 0.066 : 0.934 : 0.039 : 0.961 . \end{aligned} \quad (42)$$

The chiral couplings of Z' and Z'' gauge bosons which are mostly L and $B - L$, respectively are given in Table V. The variation of the branching fractions within the allowed range of $g'_1(M_s)$ is shown in Fig. 2.

⁸ The physical couplings of the Z'' to fermions fields given in Table IV are consistent with the bounds presented in [81] from a variety of experimental constraints.

⁹ An analogue is in the SM. The Z couples to a current $J_Z \propto T_3 - \tan^2 \theta_W \frac{Y}{2}$, where $Q = T_3 - \frac{Y}{2}$. In this case, $\sum (\frac{Y}{2})^2 = \frac{17}{6}$ and $\text{Tr}[T_3^2] = 2$; we have $\text{BR } Z \rightarrow T_3 : \text{BR } Z \rightarrow \frac{Y}{2} = 2 : \frac{17}{6} \tan^4 \theta_W = 2 : 0.25 = 8 : 1$. However, this certainly does not hold particle by particle; e.g., for the neutrino electron doublet: $\Gamma_{Z \rightarrow \nu} \propto (1 + \tan^2 \theta_W)^2 \sim 1.7$, whereas $\Gamma_{Z \rightarrow e} \propto (1 - \tan^2 \theta_W)^2 \sim 0.5$.

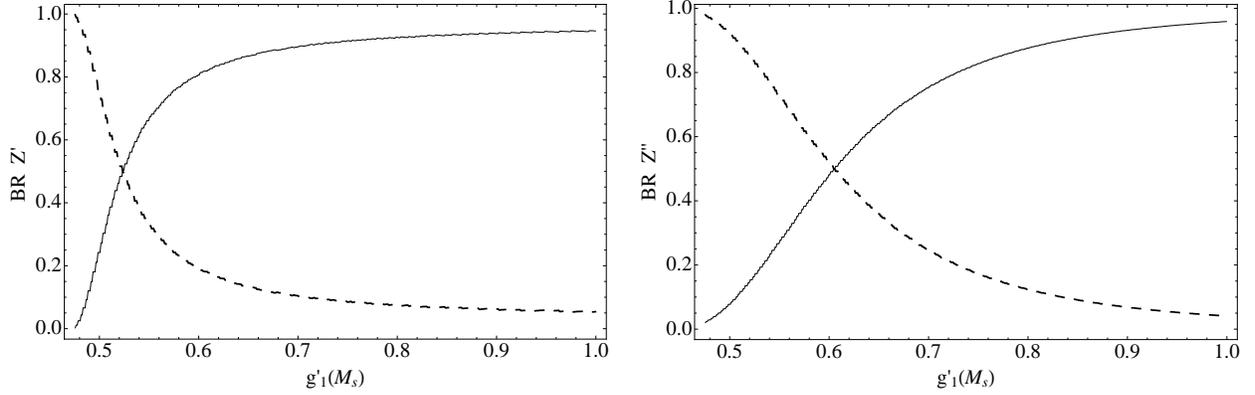


FIG. 2: Branching fractions of Z' (left) and Z'' (right) as a function of $g'_1(M_s)$. The solid lines denote the branching into B (left) and I_R (right). The dashed lines denote the branching into L (left) and B-L (right).

The LHC discovery potential for Z'' gauge boson as a mass peak above a small background in the reactions $pp \rightarrow Z'' \rightarrow jj$ and $pp \rightarrow Z'' \rightarrow \ell^+\ell^-$, with $\ell = e, \mu$, is well known. The required luminosity to discover a Z'' basically depends only on its cross section, and therefore on its mass and couplings. Experimental effects due to mass resolution are known to result in an only minor reduction of the sensitivity.

Using a data set of pp collisions at $\sqrt{s} = 8$ TeV, with an integrated luminosity of 4.0 fb^{-1} , the CMS Collaboration has searched for narrow resonances in the dijet invariant mass spectrum [32]. Each event in the search is required to have its two highest- p_T jets with (pseudorapidity) $|\eta_j| < 2.5$. The acceptance \mathcal{A} of selection requirements is reported to be ≈ 0.6 . The spectra are consistent with SM expectations and thus upper limits on the cross section times branching fraction for Z'' into two jets have been set. Similar upper limits have been obtained by the ATLAS Collaboration using 5.8 fb^{-1} of data collected at $\sqrt{s} = 8$ TeV [82]. These results, which are display in Fig. 3, extend previous exclusion limits from LHC7 [83–87].

The ATLAS Collaboration has searched for narrow resonances in the invariant mass spectrum of dimuon and dielectron final states in event samples at $\sqrt{s} = 7$ TeV corresponding to an integrated luminosity of 4.9 fb^{-1} and 5.0 fb^{-1} , respectively [88]. The spectra are consistent with SM expectations and thus upper limits on the cross section times branching fraction for Z'' into lepton pairs have been set. More recently, the CMS Collaboration updated the LHC7 results using 4.1 fb^{-1} of data collected at $\sqrt{s} = 8$ TeV [89]. The combined

upper limits from LHC7 and LHC8 are shown in Fig. 4. Previous dilepton searches by the LHC experiments have been reported in [90, 91].

In order to compare with these results we now turn to compute these cross sections in our model. The relevant part of (16), $f\bar{f}Z''$ coupling, is of the form

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\sqrt{g_Y^2 + g_2^2} \sum_f \left(\epsilon_{f_L^i} \bar{f}_L^i \gamma^\mu f_L^i + \epsilon_{f_R^i} \bar{f}_R^i \gamma^\mu f_R^i \right) Z''_\mu \\ &= \sum_f \left((g_{Y''} Q_{Y''})_{f_L^i} \bar{f}_L^i \gamma^\mu f_L^i + (g_{Y''} Q_{Y''})_{f_R^i} \bar{f}_R^i \gamma^\mu f_R^i \right) Z''_\mu,\end{aligned}\quad (43)$$

where $f_{L(R)}^i$ are fermion fields and $\epsilon_{f_{L,R}^i} = v_q \pm a_q$, with v_q and a_q , the vector and axial couplings respectively. To compare our predictions with LHC experimental searches in dilepton and dijets it is sufficient to consider the production cross section in the narrow Z'' width approximation,

$$\hat{\sigma}(q\bar{q} \rightarrow Z'') = K \frac{2\pi}{3} \frac{G_F M_Z^2}{\sqrt{2}} [v_q^2(\phi, g'_1) + a_q^2(\phi, g'_1)] \delta(\hat{s} - M_{Z''}^2), \quad (44)$$

where G_F is the Fermi coupling constant and the K -factor represents the enhancement from higher order QCD processes estimated to be $K \simeq 1.3$ [92]. After folding $\hat{\sigma}$ with the CTEQ6 parton distribution functions [93], we determine (at the parton level) the resonant production cross section. In Figs. 3 and 4 we compare the predicted $\sigma(pp \rightarrow Z'') \times \text{BR}(Z'' \rightarrow jj)$ and $\sigma(pp \rightarrow Z'') \times \text{BR}(Z'' \rightarrow \ell\ell)$ production rates with 95% CL upper limits recently reported by the CMS and ATLAS collaborations. Selection cuts will probably reduce event rates by factors of 20%. Keeping this in mind, we conclude that if Z'' is mostly I_R , then the predicted production rates for $M_{Z''} \approx 4$ TeV at $\sqrt{s} = 8$ TeV saturate the current dijet limits. On the other hand, if Z'' is mostly $B - L$ the lower limit on the gauge boson mass, $M_{Z''} \gtrsim 3$ TeV, is determined primarily from dilepton searches.

For the discovery potential in the high mass region the dijet channel is statistically a better discriminator than lepton pairs. Therefore, we investigate (at the parton level) the LHC14 sensitivity for a Z'' resonance (which is mostly I_R) in data binned according to the dijet invariant mass M , after setting cuts on the different jet rapidities, $|y_1|, |y_2| \leq 1$ and transverse momenta $p_T^{1,2} > 50$ GeV. With the definitions $Y \equiv \frac{1}{2}(y_1 + y_2)$ and $y \equiv \frac{1}{2}(y_1 - y_2)$,

TABLE IV: Chiral couplings of Y , Z' , and Z'' gauge bosons. All fields in a given set have a common $g_{Y'}Q_{Y'}$, $g_{Y''}Q_{Y''}$ couplings. We have taken Z' to be mostly B and Z'' to be mostly I_R .

Fields	$g_Y Q_Y$	$g_{Y'} Q_{Y'}$	$g_{Y''} Q_{Y''}$
U_R	0.2434	0.1836	0.3321
D_R	-0.1214	0.1838	-0.3933
L_L	-0.1826	0.0759	0.0918
E_R	-0.3650	0.0760	-0.2709
Q_L	0.0610	0.1837	-0.0306
N_R	0.0000	0.0758	0.4545
H	0.1824	0.0000	0.3627
H''	0.0000	-0.0758	-0.4545

TABLE V: Chiral couplings of Y , Z' , and Z'' gauge bosons. All fields in a given set have a common $g_{Y'}Q_{Y'}$, $g_{Y''}Q_{Y''}$ couplings. We have taken Z' to be mostly L and Z'' to be mostly $B - L$.

Fields	$g_Y Q_Y$	$g_{Y'} Q_{Y'}$	$g_{Y''} Q_{Y''}$
U_i	0.2435	0.1101	-0.0763
D_i	-0.1217	0.1101	-0.2242
L_i	-0.1825	0.7165	0.4509
E_i	-0.3651	0.7165	0.3769
Q_i	0.0609	0.1101	-0.1503
N_i	0.0000	0.7165	0.5248
H	0.1826	-0.0000	0.0739
H''	-0.0000	-0.7165	-0.5248

the cross section per interval of M for $pp \rightarrow$ dijet is given by

$$\begin{aligned}
\frac{d\sigma}{dM} = M\tau \sum_{ijkl} & \left[\int_{-Y_{\max}}^0 dY f_i(x_a, M) f_j(x_b, M) \int_{-(y_{\max}+Y)}^{y_{\max}+Y} dy \frac{d\sigma}{d\hat{t}} \Big|_{ij \rightarrow kl} \frac{1}{\cosh^2 y} \right. \\
& \left. + \int_0^{Y_{\max}} dY f_i(x_a, M) f_j(x_b, M) \int_{-(y_{\max}-Y)}^{y_{\max}-Y} dy \frac{d\sigma}{d\hat{t}} \Big|_{ij \rightarrow kl} \frac{1}{\cosh^2 y} \right], \quad (45)
\end{aligned}$$

where $f(x, M)$'s are parton distribution functions (we use CTEQ6 [93]), $\tau = M^2/s$, $x_a =$

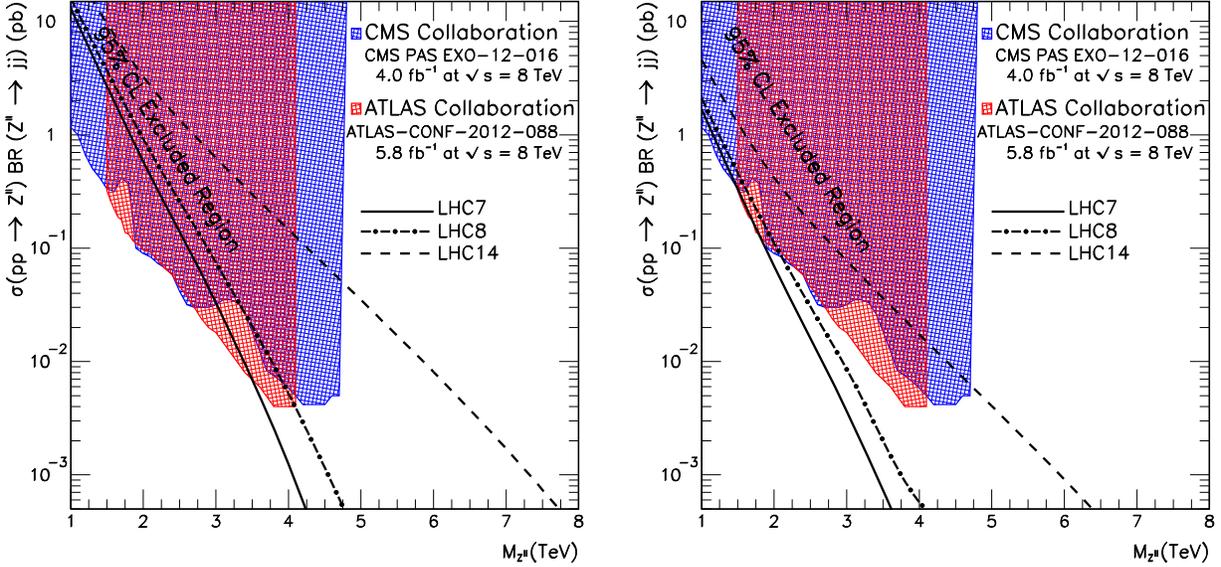


FIG. 3: Comparison of the (pre-cut) total cross section for the production of $pp \rightarrow Z'' \rightarrow jj$ with the 95% CL upper limits on the production of a gauge boson decaying into two jets as reported by the CMS and ATLAS collaborations (corrected by acceptance). For isotropic decays (independently of the resonance), the acceptance for the CMS detector has been reported to be $\mathcal{A} \approx 0.6$. The ATLAS acceptance ranges from 11% to 54% varying from 1 TeV to 4.25 TeV, and is never lower than 48% for masses above 2 TeV. The case in which Z'' is mostly diagonal in I_R is shown in the left panel and the case in which it is mostly $B-L$ in the right panel.

$\sqrt{\tau}e^Y$, $x_b = \sqrt{\tau}e^{-Y}$, and

$$|\mathcal{M}(ij \rightarrow kl)|^2 = 16\pi\hat{s}^2 \left. \frac{d\sigma}{d\hat{t}} \right|_{ij \rightarrow kl}; \quad (46)$$

we specify partonic subprocesses with caret notation $(\hat{s}, \hat{t}, \hat{u})$. The Y integration range in Eq. (45), $Y_{\max} = \min\{\ln(1/\sqrt{\tau}), y_{\max}\}$, comes from requiring $x_a, x_b < 1$ together with the rapidity cuts $y_{\min} < |y_1|, |y_2| < y_{\max}$. The kinematics of the scattering also provides the relation $M = 2p_T \cosh y$, which when combined with $p_T = M/2 \sin \theta^* = M/2\sqrt{1 - \cos^2 \theta^*}$, yields $\cosh y = (1 - \cos^2 \theta^*)^{-1/2}$, where θ^* is the center-of-mass scattering angle. Finally, the Mandelstam invariants occurring in the cross section are given by $\hat{s} = M^2$, $\hat{t} = -\frac{1}{2}M^2 e^{-y}/\cosh y$, and $\hat{u} = -\frac{1}{2}M^2 e^{+y}/\cosh y$.

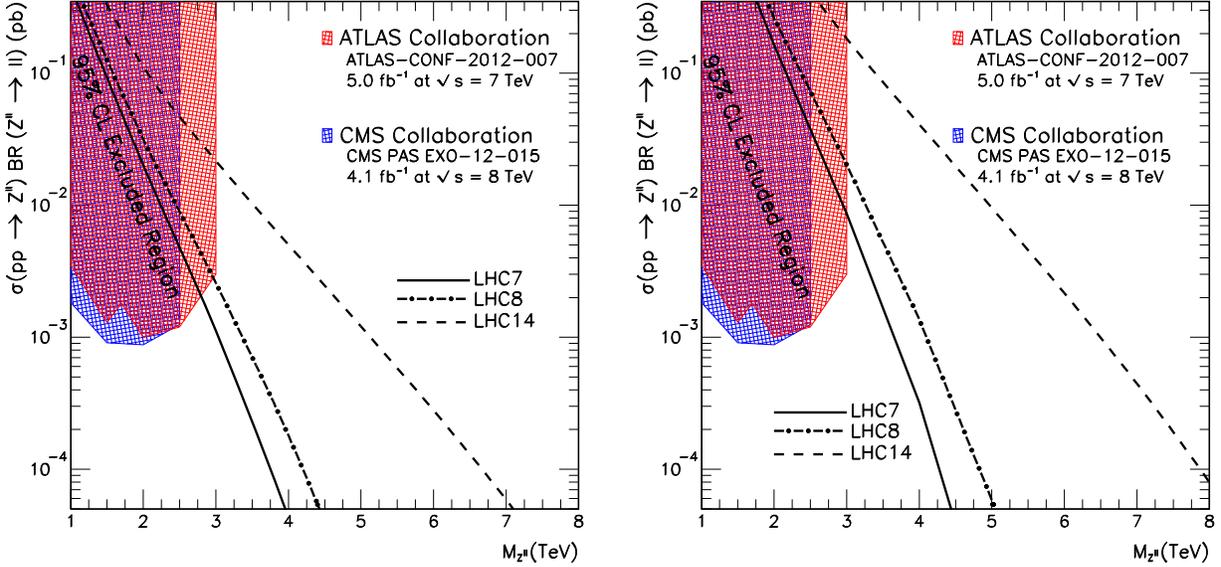


FIG. 4: Comparison of the (pre-cut) total cross section for the production of $pp \rightarrow Z'' \rightarrow \ell\ell$ with the 95% CL upper limits on the production of a gauge boson decaying into two leptons, as reported by the ATLAS and CMS collaborations. The case in which Z'' is mostly diagonal in I_R is shown in the left panel and the case in which it is mostly $B - L$ in the right panel.

The average square amplitude (for incoming quark q and outgoing quark q') is given by,

$$\begin{aligned}
 |\mathcal{M}(q\bar{q} \xrightarrow{Z''} q'\bar{q}')|^2 &= \frac{1}{4} [g_{Y''}^2 Q_{Y''}^2(q_L) + g_{Y''}^2 Q_{Y''}^2(q_R)] [g_{Y''}^2 Q_{Y''}^2(q_L') + g_{Y''}^2 Q_{Y''}^2(q_R')] \\
 &\times \left[\frac{2(u^2 + t^2)}{(s - M_{Z''}^2)^2 + (\Gamma_{Z''} M_{Z''})^2} \right], \quad (47)
 \end{aligned}$$

where $g_{Y''} Q_{Y''}(q_L)$ and $g_{Y''} Q_{Y''}(q_R)$ are the couplings of Z'' to quarks. (Note that we have not summed over the flavors, but we did average and sum the colors).

The decay width of $Z'' \rightarrow f\bar{f}$ is given by

$$\Gamma(Z'' \rightarrow f\bar{f}) = \frac{G_F M_Z^2}{6\pi\sqrt{2}} N_c M_{Z''} \sqrt{1 - 4x} [v_f^2(1 + 2x) + a_f^2(1 - 4x)] \left(1 + \frac{\alpha_s}{\pi}\right), \quad (48)$$

where $\alpha_s = \alpha_s(M_{Z''})$ is the strong coupling constant at the scale $M_{Z''}$, $x = m_f^2/M_{Z''}^2$, v_f and a_f are the vector and axial couplings, and $N_c = 3$ or 1 if f is a quark or a lepton, respectively [94]. For our fiducial values of g'_1 and ϕ we obtain $v_u^2 + a_u^2 = 0.396$ and $v_d^2 + a_d^2 = 0.554$.

We calculate a signal-to-noise ratio, with the signal rate (\mathcal{S}) estimated in the invariant mass window $[M_{Z''} - 2\Gamma, M_{Z''} + 2\Gamma]$. To accommodate the minimal acceptance cuts on dijets from the CMS and ATLAS proposals [95], an additional kinematic cut, $|y_{\max}| < 1.0$, has

been included in the calculation. The noise (\mathcal{N}) is defined as the square root of the number of QCD background events (\mathcal{B}) in the same dijet mass interval for the same integrated luminosity. In Table. VI we show the behavior of the signal-to-noise ratio as a function of the mass of Z'' at LHC14 for different integrated luminosities. We conclude that the LHC provides a generous discovery potential for Z'' which is mostly I_R . The discovery potential of a Z'' which is mostly $B - L$ is controlled by the sensitivity of LHC14 to dilepton final states. For 300 fb^{-1} , the projected sensitivity is $M_{Z''} \lesssim 5 \text{ TeV}$ [96].

TABLE VI: Signal-to-noise ratio at LHC14 for different integrated luminosities.

$M_{Z''}$ (TeV)	10 fb^{-1}			100 fb^{-1}			1000 fb^{-1}		
	\mathcal{S}	\mathcal{B}	\mathcal{S}/\mathcal{N}	\mathcal{S}	\mathcal{B}	\mathcal{S}/\mathcal{N}	\mathcal{S}	\mathcal{B}	\mathcal{S}/\mathcal{N}
4	39	579	1.62	391	5789	5.14	3910	57895	16.25
5	7	176	0.50	67	1759	1.60	670	17590	5.05
6	1	66	0.14	11	664	0.44	113	6646	1.39

If the Z'' is observed at the LHC, we will obviously measure its mass, its total width and cross section. In addition, the off- and on resonance peak forward-backward charge asymmetries A_{FB}^ℓ would provide additional information about Z'' couplings and interference effects with the Z boson and the photon [97–99].¹⁰ Besides, the Z'' rapidity distribution is sensitive to the $u\bar{u}Z''$ and $b\bar{b}Z''$ gauge couplings. Since the W^\pm and Z boson rapidity distributions will be measured in great detail at the LHC, rapidity spectra for the mass region of interest can be calculated separately for $u\bar{u}$, $d\bar{d}$, and sea quark antiquark annihilation. A combined fit to the relative parton distribution functions and the Z'' rapidity distribution would allow us to obtain the fraction of Z'' bosons produced from $u\bar{u}$ and $d\bar{d}$ initial states [100].

¹⁰ The leptonic forward-backward charge asymmetry A_{FB}^ℓ is defined from the lepton angular distribution with respect to the quark direction in the centre-of-mass frame as $d\sigma/d\cos\theta^* \propto \frac{3}{8}(1+\cos^2\theta^*) + A_{\text{FB}}^\ell \cos\theta^*$. The lepton angle θ^* in the dilepton center-of-mass frame can be calculated using the measured four momenta of the dilepton system; A_{FB}^ℓ can then be determined with an unbinned maximum likelihood fit to the $\cos\theta^*$ distribution.

IV. NEUTRINO COSMOLOGY REDUX

In this section we reexamine some critical cosmological issues surrounding the presence of the six additional neutrino degrees of freedom correlated to the presence of Z'' in our dynamical D-brane model. These considerations, when viewed in the context of most recent cosmological observations are found to constrain the mass of the Z'' to an interesting narrow band, which will be directly probed by LHC14. To provide a starting point, we first summarize the “best-fit” cosmological parameters to recent data.

A. Beyond Λ CDM

Our universe seems, according to the present-day evidence, to be spatially flat and to possess a non-vanishing cosmological constant (Λ) plus cold dark matter (CDM), corresponding respectively to roughly 70% and 25% of the total density, with the remaining 5% in baryons. The standard Λ CDM cosmology provides a rather good fit of existing data from BBN (~ 20 minutes), the CMB (~ 380 kyr), and the galaxy formation epochs of the universe ($\gtrsim 1$ Gyr). However, there are also tantalizing hints for the presence of an extra relativistic component, dubbed dark radiation.

Taking these hints at face value, the most straightforward variation of standard Λ CDM is “extra” energy contributed by new relativistic particles “ X .” When the X ’s don’t share in the energy released by e^+e^- annihilation, it is convenient to account for the extra contribution to the SM energy density, by normalizing it to that of an “equivalent” neutrino species [101]

$$\rho_X \equiv \Delta N_\nu \rho_\nu = \frac{7}{8} \Delta N_\nu \rho_\gamma, \quad (49)$$

where ρ_ν is the energy density in neutrinos and ρ_γ is the energy density in photons (which by today have redshifted to become the CMB photons at a temperature of about 2.7 K). For each additional “neutrino-like” particle (*i.e.* any two-component fermion), if $T_X = T_{\nu_L}$, then $\Delta N_\nu = 1$; if X is a scalar (and $T_X = T_{\nu_L}$), then $\Delta N_\nu = 4/7$. However, it may well be that the X ’s have decoupled even earlier in the evolution of the universe and have failed to profit from the heating when various other particle-antiparticle pairs annihilated (or unstable particles decayed). In this case, the contribution to ΔN_ν from each such particle will be < 1 and $< 4/7$, respectively. The contribution of the 2.984 ± 0.009 neutrino species

(measured from the width for invisible $\nu\bar{\nu}$ decays of the Z boson [102]) to $N_\nu^{\text{eff}} = N_\nu + \Delta N_\nu$ is $N_\nu = 3.046$; the small deviation from 3 is due to partial heating of neutrinos in the early universe by e^+e^- annihilation, see *e.g.* [103].

The competition between gravitational potential and pressure gradients is responsible for the peaks and troughs in the CMB temperature angular power spectrum, see *e.g.* [104]. The redshift z_{eq} of matter-radiation equality,

$$1 + z_{\text{eq}} = \frac{\Omega_m h^2}{\Omega_R h^2} = \frac{\Omega_m h^2}{\Omega_\gamma h^2} \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_\nu^{\text{eff}} \right]^{-1}, \quad (50)$$

affects the time (redshift) duration over which this competition occurs. Here, $\Omega_m h^2$ is the total matter density (comprised, for nearly massless neutrinos, of baryons and CDM), h ($H_0 \equiv 100h$ km/s/Mpc) is the normalized Hubble constant, and $\Omega_\gamma h^2 = 2.469 \times 10^{-5}$ is the present-day photon energy density. The primary effect of extra relativistic degrees of freedom on the CMB results essentially from changing the redshift of matter-radiation equality. If the radiation content is increased, matter-radiation equality is delayed, and occurs closer (in time and/or redshift) to the epoch of recombination. This implies the universe is younger at recombination with a correspondingly smaller sound horizon s_* . Since the location of the n^{th} peak in the angular power spectrum scales roughly as $n\pi D_*/s_*$ (where D_* is the comoving angular diameter distance to recombination¹¹), if $\Delta N_\nu > 0$ the peaks shift to smaller angular scales and with greater separation [104]. Therefore, the equality redshift is one of the fundamental observables that one can extract from WMAP data, mainly from the height of the third acoustic peak relative to the first peak.

The variation in N_ν^{eff} reads [12]

$$\frac{\delta N_\nu^{\text{eff}}}{N_\nu^{\text{eff}}} \simeq 2.45 \frac{\delta(\Omega_m h^2)}{\Omega_m h^2} - 2.45 \frac{\delta z_{\text{eq}}}{1 + z_{\text{eq}}}. \quad (51)$$

The latest distance measurements from the Baryon Acoustic Oscillations (BAO) in the distribution of galaxies [16] and precise measurements of the Hubble constant H_0 [17] provide an independent determination in the fractional error in $\Omega_m h^2$ and allow a precise determination of N_ν^{eff} . The parameter constraints from the combination of WMAP 7-year data,

¹¹ The angles on the sky are related to actual physical distance via the angular diameter distance d , defined as the ratio of the physical length (transverse to the line of sight) and the angle it covers $d \equiv \lambda_{\text{phys}}/\vartheta$. Likewise, $D \equiv \lambda^c/\vartheta$, where $\lambda^c = (1+z)\lambda_{\text{phys}}$ is the corresponding comoving length and z the redshift; $D = (1+z)d$.

BAO, and H_0 lead to $N_\nu^{\text{eff}} = 4.34_{-0.88}^{+0.86}$ [12]. Similarly, a combination of BAO and H_0 with data from the Atacama Cosmology Telescope (ACT) yields $N_\nu^{\text{eff}} = 4.6 \pm 0.8$ [105], whereas data collected with the South Pole Telescope (SPT) combined with BAO and H_0 arrive at $N_\nu^{\text{eff}} = 3.86 \pm 0.42$ [106]. Although none of these measurements individually deviates from the standard value by more than about two standard deviations, they collectively rule out $N_\nu = 3.046$ at the approximately 99% CL, and instead prefer roughly one extra effective neutrinos species [107].¹²

The expansion rate of the universe at early times increases with the number of relativistic particle species in thermal equilibrium, and this in turn sets timescales for BBN [109–111]. One can then use the BBN yields of light nuclei to constrain the number of light species quantitatively. The nucleosynthesis chain begins with the formation of deuterium in the process $p(n, \gamma)\text{D}$. However, photo-dissociation by the high number density of photons delays production of deuterium (and other complex nuclei) until well after T drops below the binding energy of deuterium, $\Delta_{\text{D}} = 2.23$ MeV. The number of photons per baryon above the deuterium photo-dissociation threshold, $\eta^{-1}e^{-\Delta_{\text{D}}/T}$, falls below unity at $T \simeq 0.1$ MeV, where $\eta \equiv n_B/n_\gamma \sim 5 \times 10^{-10}$ is the baryon to photon number density. Nuclei can then begin to form without being immediately photo-dissociated again. Only 2-body reactions such as $\text{D}(p, \gamma)^3\text{He}$, $^3\text{He}(\text{D}, p)^4\text{He}$, are important because the density is rather low at this time. Nearly all the surviving neutrons when nucleosynthesis begins end up bound in the most stable light element ^4He . Heavier nuclei do not form in any significant quantity both because of the absence of stable nuclei with mass number 5 or 8 (which impedes nucleosynthesis via $n^4\text{He}$, $p^4\text{He}$, or $^4\text{He}^4\text{He}$ reactions) and the large Coulomb barriers for reactions such as $\text{T}(^4\text{He}, \gamma)^7\text{Li}$ and $^3\text{He}(^4\text{He}, \gamma)^7\text{Be}$. Hence the primordial mass fraction of ^4He , conventionally referred to as Y_{p} , can be estimated by the simple counting argument

$$Y_{\text{p}} = \frac{2(n/p)}{1 + n/p}. \quad (52)$$

For $T \gtrsim 1$ MeV, weak interactions were in thermal equilibrium, thus fixing the ratio of the neutron and proton number densities to be $n/p = e^{-Q/T}$, where $Q = 1.293$ MeV is the neutron-proton mass difference. As the temperature dropped, the neutron-proton inter-conversion rate, $\Gamma_{n \rightleftharpoons p} \sim G_F^2 T^5$, fell faster than the Hubble expansion rate, $H \approx$

¹² A more recent study seems to indicate $3.0 < N_\nu^{\text{eff}} < 4.1$ [108].

$\sqrt{N(T)} T^2/M_{\text{Pl}}$ (see *e.g.* [112]). Since $N(T)$ counts the number of relativistic particle species determining the energy density in radiation, the neutron fraction n/p is directly sensitive to ΔN_ν . For standard ΛCDM , the freeze-out temperature of ν_L is

$$T_{\text{FO}} \sim \left[\frac{\sqrt{N(T_{\text{FO}})}}{M_{\text{Pl}} G_F^2} \right]^{1/3} \simeq 1 \text{ MeV}, \quad (53)$$

yielding $n/p \simeq 1/6$ and $Y_p \simeq 0.25$.

The evidence for extra radiation from Y_p data is, however, somewhat ambiguous. The observationally-inferred primordial fractions of baryonic mass in ${}^4\text{He}$ ($Y_p = 0.2472 \pm 0.0012$ [113], $Y_p = 0.2477 \pm 0.0029$ [114], and $Y_p = 0.250 \pm 0.004$ [115]) have been constantly favoring $N_\nu^{\text{eff}} \lesssim 3$ [116]. Unexpectedly, two recent independent studies determined $Y_p = 0.2565 \pm 0.001(\text{stat}) \pm 0.005(\text{syst})$ [117] and $Y_p = 0.2561 \pm 0.0108$ [118]. For $\tau_n = 885.4 \pm 0.9 \text{ s}$ and $\tau_n = 878.5 \pm 0.8 \text{ s}$, the updated effective number of light neutrino species is reported as $N_\nu^{\text{eff}} = 3.68_{-0.70}^{+0.80} (2\sigma)$ and $N_\nu^{\text{eff}} = 3.80_{-0.70}^{+0.80} (2\sigma)$, respectively [117].¹³

As mentioned above, the primordial deuterium abundance depends not just on N_ν^{eff} but also on the cosmological baryon density, $\Omega_b h^2$. Prior to the precise inference of $\Omega_b h^2$ from CMB measurements, the strongest constraint on N_ν^{eff} came from using the deuterium-to-hydrogen number ratio D/H to restrict $\Omega_b h^2$ and then exploiting the N_ν^{eff} dependence of the primordial helium mass fraction Y_p . However, D/H has its own dependence on N_ν^{eff} [123, 124]; a strong external constraint on $\Omega_b h^2$ allows BBN constraints on N_ν^{eff} that are independent of Y_p . Since precise measurements of Y_p are difficult, the constraint on N_ν^{eff} from D/H is found to be competitive with that from Y_p . A recent analysis, which combines the CMB results with BBN theory and the observed D/H, suggests $N_\nu^{\text{eff}} = 3.90 \pm 0.44$ [125].¹⁴

In summary, though uncertainties remain large, the most recent cosmological observations show a consistent preference for additional relativistic degrees of freedom (r.d.o.f.) during BBN and the CMB epochs. We take these hints as motivation for the subsequent analysis, which consists of the following tasks: (1) to explain the dark radiation using the

¹³ For several years the Particle Data Group recommended $\tau_n = 885.7 \pm 0.8 \text{ s}$ [119]. More recently, conflicting lifetimes $\tau_n = 878.5 \pm 0.7 \pm 0.3 \text{ s}$ [120] and $\tau_n = 880.7 \pm 1.3 \pm 1.2 \text{ s}$ [121] have been reported. The Particle Data Group now recommends a world average that includes the conflicting values, $\tau_n = 881.5 \pm 1.5 \text{ s}$ [122], with errors that have been inflated to reflect the discrepancy.

¹⁴ The BBN calculations in this analysis includes updates of nuclear rates in light of recent experimental and theoretical information, with the most significant change occurring for the $d(p, \gamma){}^3\text{He}$ cross section.

non-supersymmetric $U(3)_B \times Sp(1)_L \times U(1)_L \times U(1)_{I_R}$ D-brane model, in which the additional r.d.o.f. are the three flavors of light right-handed neutrinos which interact with the SM fermions via the exchange of heavy vector fields Z' and Z'' ; (2) to suppress the six additional fermionic r.d.o.f. to levels in compliance with BBN and CMB. This is accomplished by imposing the decoupling of ν_R 's from the plasma *early enough* so that they undergo incomplete reheating during the quark-hadron transition; and *late enough* so as to leave an excess neutrino density suggested by the data. These requirements strongly constrain the masses of the heavy vector fields. Together with the couplings given in Table IV, the model is fully predictive, and can be confronted with dijet and dilepton data from LHC8 and, eventually, LHC14.

B. Cosmology of Intersecting Branes

The ensuing discussion will be framed in the context of a Z'' which is mostly I_R , and we will comment on the case in which Z'' is mostly $B - L$ after presenting our results.

We begin by first establishing, in a model independent manner, the range of decoupling temperatures implied by the BBN and CMB analyses. For the subsequent study, the physics of interest will be taking place at energies in the region of the quark-hadron transition, so that we will restrict ourselves to the following fermionic fields, and their contribution to r.d.o.f.: $[3u_R] + [3d_R] + [3s_R] + [3\nu_L + e_L + \mu_L] + [e_R + \mu_R] + [3u_L + 3d_L + 3s_L] + [3\nu_R]$. This amounts to 28 Weyl fields, translating to 56 fermionic r.d.o.f.

Next, in line with our stated plan, we use the data estimate to calculate the range of decoupling temperature. The effective number of neutrino species contributing to r.d.o.f. can be written as $N_\nu^{\text{eff}} = 3[1 + (T_{\nu_R}/T_{\nu_L})^4]$; therefore, taking into account the isentropic heating of the rest of the plasma between ν_R decoupling temperature T_{dec} and the end of the reheating phase,

$$\Delta N_\nu = 3 \left(\frac{N(T_{\text{end}})}{N(T_{\text{dec}})} \right)^{4/3}, \quad (54)$$

where T_{end} is the temperature at the end of the reheating phase, and $N(T) = r(T)(N_B + \frac{7}{8}N_F)$ is the effective number of r.d.o.f. at temperature T , with $N_B = 2$ for each real vector field and $N_F = 2$ for each spin- $\frac{1}{2}$ Weyl field. The coefficient $r(T)$ is unity for the lepton and photon contributions, and is the ratio $s(T)/s_{\text{SB}}$ for the quark-gluon plasma. Here $s(T)$ (s_{SB}) is the actual (ideal Stefan-Boltzmann) entropy. Hence $N(T_{\text{dec}}) = 47.5 r(T_{\text{dec}}) + 14.25$. We

take $N(T_{\text{end}}) = 10.75$ reflecting $(e_L^- + e_R^+ + e_R^- + e_L^+ \nu_{eL} + \bar{\nu}_{eR} + \nu_{\mu L} + \bar{\nu}_{\mu R} + \nu_{\tau L} + \bar{\nu}_{\tau R} + \gamma_L + \gamma_R)$. We consistently omit ν_R in considering the thermodynamics part of the discussion, but will include it when dealing with expansion. As stated in the introduction

$$\Delta N_\nu = \begin{cases} 0.68_{-0.35}^{+0.40} & (1\sigma) \quad \text{BBN} + Y_p \\ 0.90_{-0.44}^{+0.44} & (1\sigma) \quad \text{CMB} + \text{BBN} + \text{H/D} \end{cases} \quad (55)$$

so the excess r.d.o.f. will lie within 1σ of the central value of each set of observations if $0.46 < \Delta N_\nu < 1.08$. From Eqs. (54) and (55), the allowable range for N is $23 < N(T_{\text{dec}}) < 44$. This is achieved for $0.18 < r(T_{\text{dec}}) < 0.63$. By comparing to Fig. 8 in Ref. [126], this can be translated into a temperature range

$$175 \text{ MeV} < T_{\text{dec}} < 250 \text{ MeV}, \quad (56)$$

with the lower temperature coinciding with the region of most rapid rise of the entropy. Thus, the data implies that the ν_R decoupling takes place during the quark-hadron transition.

We now turn to use our model in conjunction with the decoupling condition to constrain its parameters. To this end we calculate the interaction rate $\Gamma(T)$ for a right-handed neutrino and determine T_{dec} from the plasma via the prescription

$$\Gamma(T_{\text{dec}}) = H(T_{\text{dec}}). \quad (57)$$

Let f_L^i be a single species of Weyl fermion, representing the two r.d.o.f. $\{f_L^i, \bar{f}_R^i\}$, where the superscript indicates bins $i = 3, 5$. Similarly $f_R^i \in \{f_R^i, \bar{f}_L^i\}$, for $i = 1, 2, 4, 6$. Notice that the subscripts L, R denote the actual helicities of the massless particles in question, not the chirality of the fields. With this said, we may write the amplitude for f_L^i scattering

$$\mathcal{M}(\nu_R(p_1) f_L^i(p_2) \rightarrow \nu_R(p_3) f_L^i(p_4)) = \frac{G_i}{\sqrt{2}} [\bar{u}(p_3) \gamma^\mu (1 + \gamma_5) u(p_1)] [\bar{u}(p_4) \gamma_\mu (1 - \gamma_5) u(p_2)]. \quad (58)$$

The other 3 amplitudes are obtained by the crossing substitutions in the second square bracket; for scattering from

$$\begin{aligned} \bar{f}_R^i &\rightarrow \bar{v}(p_2) \gamma_\mu (1 - \gamma_5) v(p_4) \\ f_R^i &\rightarrow \bar{u}(p_4) \gamma_\mu (1 + \gamma_5) u(p_2) \\ \bar{f}_L^i &\rightarrow \bar{v}(p_2) \gamma_\mu (1 + \gamma_5) v(p_4). \end{aligned} \quad (59)$$

The cross sections for the four scattering processes (no average over helicities) are

$$\sigma(\nu_R f_L^i \rightarrow \nu_R f_L^i) = \frac{1}{3} \sigma(\nu_R \bar{f}_R^i \rightarrow \nu_R \bar{f}_R^i) = \frac{2 G_i^2 s}{3 \pi} \quad (\text{for bins } i = 3, 5) \quad (60)$$

and

$$\sigma(\nu_R \bar{f}_L^i \rightarrow \nu_R \bar{f}_L^i) = \frac{1}{3} \sigma(\nu_R f_R^i \rightarrow \nu_R f_R^i) = \frac{2 G_i^2 s}{3 \pi} \quad (\text{for bins } i = 1, 2, 4, 6). \quad (61)$$

In addition to these scattering processes, the ν_R interacts with the plasma through the annihilation processes: $\nu_R \bar{\nu}_L \rightarrow f_L^i \bar{f}_R^i$, for bins $i = 3, 5$, and $\nu_R \bar{\nu}_L \rightarrow f_R^i \bar{f}_L^i$, for bins $i = 1, 2, 4, 6$. These all yield cross sections $2G_i^2 s / (3\pi)$ due to forward and backward suppression. Assuming all chemical potentials to be zero, the plasma will have an equal number density $n(T) = 0.0913T^3$, for each fermion r.d.o.f. Thus,

$$\Gamma^{\text{scat}}(T) = n(T) \left\langle \sum_{i=1}^6 \sigma_i(s) v_M \mathcal{N}_i \right\rangle, \quad (62)$$

where $v_M = 1 - \cos \theta_{12}$ is the Moller velocity, $s = 2k_1 k_2 (1 - \cos \theta_{12})$ is the square of the center-of-mass energy, and \mathcal{N}_i is the multiplicity of Weyl fields in each bin (e.g., for $i = 3$, $\mathcal{N}_3 = 3 + 2 = 5$). The scattering cross section is given by

$$\sigma_i^{\text{scat}} = \sigma(\nu_R f_L^i \rightarrow \nu_R f_L^i) + \sigma(\nu_R \bar{f}_R^i \rightarrow \nu_R \bar{f}_R^i) = \frac{4 G_i^2 s}{3 \pi} \quad \text{for each } i = 1, \dots, 6; \quad (63)$$

similarly,

$$\sigma_i^{\text{ann}}(s) = \sigma(\nu_R \bar{\nu}_L \rightarrow f_L^i \bar{f}_R^i + f_R^i \bar{f}_L^i) = \frac{1}{3} \frac{2 G_i^2 s}{\pi} \quad \text{for each } i = 1, \dots, 6. \quad (64)$$

Since $s = 2k_1 k_2 (1 - \cos \theta_{12})$ and $v_M = 1 - \cos \theta_{12}$, we perform an approximate angular average $\langle (1 - \cos \theta_{12})^2 \rangle = 4/3$, followed by a thermal averaging $\langle 2k_1 k_2 \rangle = 2(3.15^2 T^2)$ to give

$$\Gamma^{\text{scat}}(T) = \left(\frac{4}{3}\right)^2 \frac{2}{\pi} 2 (3.15T)^2 (0.0919T^3) \underbrace{\left(\sum_{i=1}^6 G_i^2 \mathcal{N}_i\right)}_{G_{\text{eff}}^2} \simeq 2.05 G_{\text{eff}}^2 T^5. \quad (65)$$

From (63), (64), and (65),

$$\Gamma^{\text{ann}}(T) = \frac{1}{4} \Gamma^{\text{scat}}(T) \simeq 0.50 G_{\text{eff}}^2 T^5. \quad (66)$$

Each of the G_i is given by the sum of the contributions from Z' and Z'' exchange,

$$4 \frac{G_i}{\sqrt{2}} = \frac{g'_6 g'_i}{M_{Z'}^2} + \frac{g''_6 g''_i}{M_{Z''}^2}. \quad (67)$$

The Hubble expansion parameter during this time is

$$H(T) = 1.66 \langle N(T) \rangle^{1/2} T^2 / M_{\text{Pl}}, \quad (68)$$

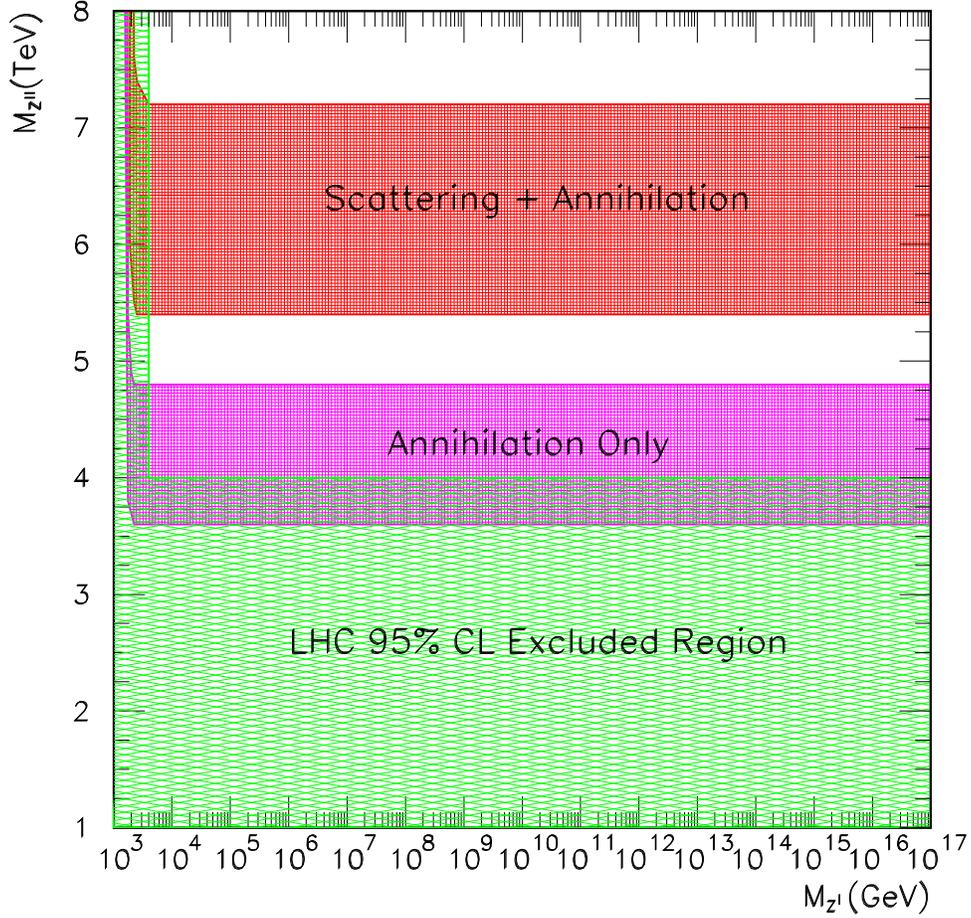


FIG. 5: The shaded areas show the region allowed from decoupling requirements to accommodate CMB and BBN data. The hatched region indicates the masses excluded by the LHC8 dijet searches. The lower and upper shaded areas pertain to chemical and thermal equilibrium, respectively. These two estimates should serve to bracket the size of the actual effect.

where M_{Pl} is the Planck mass. Since the quark-gluon energy density in the plasma has a similar T dependence to that of the entropy (see Fig. 7 in [126]), we take $N(T) = 47.5 r(T) + 19.5$, so that $H(T) = 10.3 T^2/M_{\text{Pl}}$. (The first factor provides an average for $r(T)$ over the temperature region, and we have now included the six ν_R r.d.o.f.) Since $\Gamma \propto T^5$ and $H \sim T^2$, it is clear that if at some temperature T_{dec} , $H(T_{\text{dec}}) = \Gamma_i(T_{\text{dec}})$, the ratio Γ/H will fall rapidly on further cooling. Thus from (57) and (68) the equation determining T_{dec} depends on: (1) whether we need to preserve the absence of a chemical potential, or (2) whether we need simply to maintain physical equilibrium. The decoupling condition in these two cases is: (1)

$\Gamma^{\text{ann}}(T_{\text{dec}}) = H(T_{\text{dec}})$ and (2) $\Gamma^{\text{scat}}(T_{\text{dec}}) + \Gamma^{\text{ann}}(T_{\text{dec}}) = H(T_{\text{dec}})$; or numerically: (1)

$$0.50 G_{\text{eff}}^2 T_{\text{dec}}^5 = 10.3 T_{\text{dec}}^2/M_{\text{pl}} \Rightarrow T_{\text{dec}}^3 = 20.6 (G_{\text{eff}}^2 M_{\text{Pl}})^{-1}, \quad (69)$$

and (2)

$$2.50 G_{\text{eff}}^2 T_{\text{dec}}^5 = 10.3 T_{\text{dec}}^2/M_{\text{Pl}} \Rightarrow T_{\text{dec}}^3 = 4.1 (G_{\text{eff}}^2 M_{\text{Pl}})^{-1}. \quad (70)$$

T_{dec} as determined from these equations must lie in the band (56).

Since all freedom of determining coupling constant and mixing angles has been exercised, there remains only constraints on the possible values of $M_{Z'}$ and $M_{Z''}$. For high mass string scales the contribution from $M_{Z'}$ to G_{eff} is negligible. We find that for certain ranges of $M_{Z''}$ the decoupling of the ν_R 's occurs during the course of the quark-hadron transition, just so that they are only partially reheated compared to the ν_L 's — the desired outcome. Since our aim is to match the data, which has lower and upper bounds on the neutrino “excess”, we obtain corresponding upper and lower bounds on the Z'' gauge field mass. Roughly speaking, if decoupling requires a freezout of the annihilation channel (loss of chemical equilibrium), then $3.6 \text{ TeV} < M_{Z''} < 4.8 \text{ TeV}$. This range will be probed at LHC14. If thermal equilibrium via scattering is sufficient, then $5.4 \text{ TeV} < M_{Z''} < 7.4 \text{ TeV}$.

Depending on the details of the string type model and M_s some of the couplings may go up and some may go down, but the net result for G_{eff} involving the product of all these couplings is virtually unchanged. Moreover, we have verified that if M_s is pushed downwards to the TeV-scale region both $M_{Z'}$ and $M_{Z''}$ contribute to G_{eff} and are within the LHC reach. A summary of LHC7 constraints and $M_{Z'} - M_{Z''}$ mass regions consistent with CMB + BBN + Y_p + H/D data (within 1σ) is encapsulated in Fig. 5.

We comment briefly on the case in which Z'' is mostly $B - L$. By comparing Tables IV and V it becomes evident that the Z'' coupling to neutrinos is stronger when the extra gauge boson is almost diagonal in $B - L$. As a consequence, the allowed range of masses from decoupling requirements to accommodate CMB and BBN data is shifted to higher values: $4.5 \text{ TeV} < M_{Z''} < 6.1 \text{ TeV}$ if decoupling requires a freezout of the annihilation channel, and $6.3 \text{ TeV} < M_{Z''} < 8.2 \text{ TeV}$ if thermal equilibrium via scattering is sufficient.

The first cosmology results from the Planck satellite anticipated in early 2013 would allow determination of N_{ν}^{eff} with a standard deviation of about 0.3, whereas the future Large Synoptic Survey Telescope (LSST) could determine N_{ν}^{eff} with a standard deviation of

about 0.1 [127]. These observations when combined with future LHC results can directly test the viability of our model.

V. SUPERSYMMETRIC EXTENSION

When the string scale is at high energies, supersymmetry is in principle welcome for the hierarchy problem. Gauge bosons of the brane stacks belong then to $\mathcal{N} = 1$ vector multiplets together with the corresponding gauginos, while at brane intersections chiral fermions belong to chiral multiplets denoted by their left-handed fermionic components Q, L, U^c, D^c, E^c, N^c , where the superscript c stands for the charged conjugate in the familiar notation. Moreover, in the (P, R) intersection, one should have the two usual Higgs doublets chiral multiplets H_1, H_2 with the quantum numbers of H^* and H , respectively. Finally, the extra Higgs singlet H'' becomes naturally the superpartner of the right-neutrino superfield N^c . The Yukawa interactions (25) are now replaced by the superpotential:

$$W_Y = Y_u Q H_2 U^c + Y_d Q H_1 D^c + Y_e L H_1 E^c + Y_N L H_2 N^c. \quad (71)$$

On electroweak symmetry breaking, H_2 develops a VEV, as a result of which N^c couples with ν_L to form a Dirac neutrino. Since superpotentials such as MN^cN^c or SN^cN^c are precluded by the $U(1)_L$ and $U(1)_{I_R}$ gauge invariances, there seems no equivalent of the seesaw mechanism to generate the Weinberg term [128] which gives rise to Majorana neutrinos.¹⁵ Here M is a Majorana mass matrix in flavor space and S is a gauge singlet. In addition, the existence of the VEV $\langle N^c \rangle$ breaks the $U(1)_L$ lepton gauge symmetry which allows the Z'' to grow a mass. It also generates the R -parity breaking term LH_2 , whose coefficient is subject to a variety of phenomenological constraints [131].

A superfield H'' with $I_R = L = +1$ opposite to N^c presents difficulties. A VEV for this version of H'' serves equally well for the purpose of mass growth for Z'' . However, its presence introduces a non-zero anomaly in $B-L$ and I_R . The anomaly free status of I_R and $B-L$ can be regained by introducing a fourth flavor N^c . With this extension, the dimension 5 operator $(N^c H'')^2$ is permitted. This gives rise to a Majorana mass contribution $\propto v''^2/M_s$ and to a pseudo-Dirac neutrino mass matrix [134, 135]. Present limits on pseudo-Dirac splittings

¹⁵ However it is possible that D-brane instantons can generate Majorana masses for these perturbatively forbidden operators [129, 130].

arise from the solar and atmospheric neutrino measurements. Splitting of less than about 10^{-12}eV^2 (for ν_1 and ν_2) have no effect on the solar neutrino flux, while a pseudo-Dirac splitting of ν_3 could be as large as 10^{-4}eV^2 before affecting the atmospheric neutrinos [136]. An even stronger bound emerges if we require the extra relativistic degrees of freedom not to exceed 1 as indicated by recent cosmological observations. To see this, we note that the effective thermalization of the right handed neutrinos can occur through mixing. This will occur if the oscillation length is less than horizon size during the CMB era. For a typical neutrino mass of $\sim 0.1\text{eV}$, this requires that the Majorana mass is less than $\mathcal{O}(10^{-25}\text{eV})$. At present we have no understanding of the origin of such a hierarchy (*i.e.* 10^{-13} beyond the ordinary suppression of the Yukawa), and as a consequence we discard the assignment $I_R = L = +1$ on phenomenological grounds.

Like other broad frameworks for model-building, supersymmetric D-brane models do not lead uniquely to a single theory.¹⁶ However, the conjectured models are rather rigidly constrained, and lead to LHC predictions that are qualitatively different from the conventional minimal supersymmetric SM extensions [133].

We turn now to discuss some specifics of the SUSY extension to our analysis. The first and obvious change is the modification of the β functions for the running of the couplings. However, these changes will be minor: the phenomenological requirements at the TeV scale will effectively fix the $U(1)$ couplings at that scale. Since unification is not a requirement of D-brane models, the coupling constants at the string scale will differ somewhat due to the change in the β functions, but string scale couplings do not alter our phenomenological predictions. The only caveat is to ensure that, as a result of the enhanced β functions, none of the couplings which comply with TeV data acquire non-perturbative components at the string scale. We have verified that the variation of the $g'_1(M_s)$ parameter space is hardly noticeable. This gives scarcely any change in the production cross section and/or branching fractions, even in the extreme cases shown in Figs. 3 and 4, in which Z'' is mostly diagonal in $B - L$ or mostly diagonal in I_R . Furthermore, the milli-weak interactions required to explain the extra relativistic degrees of freedom during BBN and CMB epochs are largely independent of these changes.

¹⁶ Some phenomenological aspects of the $U(3)_B \times Sp(1)_L \times U(1)_{I_R} \times U(1)_L$ SUSY extension have been discussed in [132].

Much more serious considerations come to light in transcribing the low energy effective theory into a broken SUSY background. The technical problem arises most prominently in finding a broken SUSY framework that will accommodate the hierarchy between the mass of the Z and the mass of Z'' . *Breaking of the extra $U(1)$ via the Higgs mechanism modeled on the radiative breaking of $SU(2) \times U(1)$ driven by a large top Yukawa coupling is not an option in the present model.* The introduction of an added D-term, a Fayet-Iliopoulos term, and an extended set of soft breaking masses, requires a sizable enlargement of the parameter space of the model. In order to incorporate this parameter space in a phenomenological study it is imperative to have additional experimental constraints on the SUSY spectrum.

The approach we have taken here can be regarded as an effective theory with a new and novel phenomenology, as well as interesting theoretical characteristics (*e.g.*, conservation of B to prevent proton decay and violation of L without Majorana masses). Of course such an effective theory requires a high level of fine tuning, which could be resolved in a more complete broken SUSY framework. However, we do not expect the phenomenology to differ in any substantial degree with the one presented in this paper.

VI. CONCLUSIONS

The main purpose of this paper has been to cast D-brane ideology in as bottoms-up, phenomenologically driven a way as possible. The energy scale associated with string physics is assumed to be near the Planck mass. To develop our program in the simplest way, we considered a minimal model with gauge-extended sector $U(3)_B \times Sp(1)_L \times U(1)_{I_R} \times U(1)_L$. The resulting $U(1)$ content gauges the baryon number B , the lepton number L , and a third additional abelian charge I_R which acts as the third isospin component of an $SU(2)_R$. Rotation of the $U(1)$ gauge fields to a basis exactly diagonal in hypercharge Y and very nearly diagonal in (anomalous) B and (non-anomalous) I_R fixes all mixing angles and gauge couplings. The anomalous Z' gauge boson obtains a string scale Stückelberg mass via a 4D version of the Green-Schwarz mechanism, $\text{TeV} \ll M_{Z'} \lesssim M_s \lesssim M_{\text{Pl}}$. To keep the realization of the Higgs mechanism minimal, we add an extra $SU(2)$ singlet complex scalar, which acquires a VEV and gives a TeV-scale mass to the non-anomalous gauge boson Z'' . It is noteworthy that there are no dimension 4 operators involving H'' that contribute to the Yukawa Lagrangian in our D-brane construct. This is very important since H'' carries the

quantum numbers of right-handed neutrino and its VEV breaks lepton number. However, this breaking can affect only higher-dimensional operators which are suppressed by the high string scale, and thus there is no phenomenological problem with experimental constraints for M_s higher than $\sim 10^{14}$ GeV. Since all freedom of determining coupling constant and mixing angles has been exercised, there remains only constraints on the possible value of $M_{Z''}$. We have shown that $M_{Z''} \approx 3 - 4$ TeV saturates current limits from the CMS and ATLAS collaborations. We have also shown that for $M_{Z''} \lesssim 5$ TeV, LHC14 will reach discovery sensitivity $\gtrsim 5\sigma$.

Armed with our D-brane construct, we developed a dynamic explanation of recent hints that the relativistic component of the energy during the CMB and BBN epochs is equivalent to about 1 extra Weyl neutrino. Requiring that the $B - L$ current be anomaly free implies existence of 3 right-handed Weyl neutrinos. The task then reverts to explain why there are not 3 additional r.d.o.f. We showed that for certain ranges of $M_{Z''}$ the decoupling of the ν_R 's occurs during the course of the quark-hadron crossover transition, just so that they are only partially reheated compared to the ν_L 's — the desired outcome. Roughly speaking, if decoupling requires a freezout of the annihilation channel (loss of chemical equilibrium), then for a Z'' which is mostly I_R , $3.6 \text{ TeV} < M_{Z''} < 4.8 \text{ TeV}$, whereas for a Z'' which is mostly $B - L$, $4.5 \text{ TeV} < M_{Z''} < 6.1 \text{ TeV}$. This range will be probed at LHC14. If thermal equilibrium via scattering is sufficient, for a Z'' which is mostly I_R , $5.4 \text{ TeV} < M_{Z''} < 7.4 \text{ TeV}$, and for a Z'' which is mostly $B - L$, $6.3 \text{ TeV} < M_{Z''} < 8.2 \text{ TeV}$. To carry out this program, we needed to make use of some high statistics lattice simulations of a QCD plasma in the hot phase, especially the behavior of the entropy during the confinement-deconfinement changeover. Interestingly, the behavior of the trace anomaly (shown in Fig. 15 of [126]), which is very sensitive to the nature of the crossover region, shows a sharp peak at 200 MeV and our range for T_{dec} straddles this region.

Throughout this paper we remained agnostic with respect to SUSY breaking and the details of the low energy effective potential. However, we do subject the choice of quantum numbers for H'' to the stringent holonomic constraints of the superpotential at the string scale. This forbids the simultaneous presence of scalar fields and their complex conjugate. As an illustration, if the quantum numbers of H'' are those of N_R^c , then higher dimensional operators such as $\overline{N}_R N_R^c H''^2$, which can potentially generate a Majorana mass, are absent. Because of holonomy this absence cannot be circumvented by including $\overline{N}_R N_R^c H''^{*2}$.

In summary, we have studied the $U(1)$ phenomenology of D-brane models endowed with a high mass string scale. We have incorporated some elements of SUSY, discussing evolution of the gauge couplings to the string scale and enforcing the holonomic constraints on the superpotential. We have shown that LHC8 data set upper limits on the mass of the Z'' gauge boson: $M_{Z''} \lesssim 3 - 4$ TeV. We have also shown that Z'' milli-weak interactions, which are within reach of LHC14, could play an important role in observational cosmology. It is important to stress that the Z'' production cross section and its branching fractions are *universal* and have been evaluated in a parameter-free manner. Therefore, the $U(1)$ phenomenology presented in this paper is completely independent of the details of the compactification scheme, such as the configuration of branes, the geometry of the extra dimensions, and whether the low energy theory is supersymmetric or not.

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