New simple A_{4} neutrino model for nonzero θ_{13} and large δ_{CP}

Hajime Ishimori and Ernest Ma

New Simple A_4 Neutrino Model for Nonzero θ_{13} and Large δ_{CP}

Hajime Ishimori[1] and Ernest Ma[2,3]

[1] *Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

[2] *Department of Physics and Astronomy, University of California, Riverside, California 92521, USA*

[3] *Kavli Institute for the Physics and Mathematics of the Universe, University of Tokyo, Kashiwa 277-8583, Japan*

Abstract

In a new simple application of the non-Abelian discrete symmetry A_4 to charged-lepton and neutrino mass matrices, we show that for the current experimental central value of $\sin^2 2\theta_{13} \simeq 0.1$, leptonic CP violation is necessarily large, i.e. $|\tan \delta_{CP}| > 1.3$. This result is of broad interest because CP violation in the leptonic sector is closely connected to the origin of the observed matter-antimatter asymmetry of the Universe through leptogenesis.

The origin of neutrino masses and CP violation in the leptonic sector are closely connected to the observed matter-antimatter asymmetry of our present Universe through leptogenesis, i.e. the creation of a lepton asymmetry from the decay of a heavy particle which gets converted into a baryon (matter-antimatter) asymmetry of the Universe through sphalerons during the electroweak phase transition. It is thus of theoretical importance to understand how the 3×3 neutrino mixing matrix which has a complex CP violating phase may come from a symmetry which must of course apply to both charged leptons as well as neutrinos. The mere fact that such a symmetry is possible is at first sight questionable because the three charged leptons e, μ, τ have very different masses.

In 2001, the non-Abelian discrete symmetry A_4 was introduced [1, 2, 3] to achieve this seemingly impossible result, i.e. the existence of a lepton family symmetry consistent with the three very different charged-lepton masses m_e, m_μ, m_τ. It was subsequently shown [4] to be a natural theoretical framework for neutrino tribimaximal mixing, i.e. $\sin^2 \theta_{23} = 1$, $\tan^2 \theta_{12} = 0.5$, and $\theta_{13} = 0$. This pattern was consistent with experimental data until recently, when the Daya Bay Collaboration reported [5] the first precise measurement of θ_{13}, i.e.

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst}), \tag{1}$$

followed shortly [6] by the RENO Collaboration, i.e.

$$\sin^2 2\theta_{13} = 0.113 \pm 0.013(\text{stat}) \pm 0.019(\text{syst}). \tag{2}$$

This means that tribimaximal mixing is not a good description, and more importantly, leptonic CP violation is now possible because $\theta_{13} \neq 0$, just as hadronic CP violation in the quark sector is possible because $V_{ub} \neq 0$.

In this paper, we show that A_4 is still a good symmetry for understanding this pattern, using a new simple variation of the original idea. As shown below, it predicts a correlation between θ_{13}, θ_{23}, and δ_{CP} in such a way that given the experimentally allowed ranges of

values for θ_{13} and θ_{23}, a lower bound on $|\tan\delta_{CP}|$ is obtained. In particular, for the central values of $\sin^2 2\theta_{13} = 0.1$ and $\sin^2 2\theta_{12} = 0.87$, we find $|\tan\delta_{CP}| > 1.3$ from $\sin^2 2\theta_{23} > 0.92$.

The most general 3×3 Majorana neutrino mass matrix has six complex entries, i.e. twelve parameters. Three are overall phases of the mass eigenstates which are unobservable. The nine others are three masses, three mixing angles, and three phases: one Dirac phase δ_{CP}, i.e. the analog of the one complex phase of the 3×3 quark mixing matrix, and two relative Majorana phases $\alpha_{1,2}$ for two of the three mass eigenstates. The existence of nonzero δ_{CP} or $\alpha_{1,2}$ means that CP conservation is violated. It is one of the most important issues of neutrino physics yet to be explored experimentally.

Before showing how the A_4 model is constructed, consider first the end results. In the A_4 basis, the 3×3 charged-lepton mass matrix is

$$\mathcal{M}_l = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \tag{3}$$

where $\omega = e^{2\pi i/3} = -1/2 + i\sqrt{3}/2$, and the Majorana neutrino mass matrix is

$$\mathcal{M}_\nu = \begin{pmatrix} a & f & e \\ f & a & d \\ e & d & a \end{pmatrix}. \tag{4}$$

Consider now the tribimaximal basis, i.e.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{5}$$

then

$$\mathcal{M}_\nu^{(1,2,3)} = \begin{pmatrix} a+d & b & 0 \\ b & a & c \\ 0 & c & a-d \end{pmatrix}, \tag{6}$$

where $b = (e+f)/\sqrt{2}$, $c = (e-f)/\sqrt{2}$. The advantage of using this basis is that the experimental values of the mixing angles are not too far from the tribimaximal pattern, so

3

that the unitary matrix which diagonalizes $\mathcal{M}_\nu^{(1,2,3)}$ may be approximated by

$$U_\epsilon \simeq \begin{pmatrix} 1 & \epsilon_{12} & \epsilon_{13} \\ -\epsilon_{12}^* & 1 & \epsilon_{23} \\ -\epsilon_{13}^* & -\epsilon_{23}^* & 1 \end{pmatrix}. \tag{7}$$

Suppose the parameters a, b, c, d are all real in Eq. (6), then for small b, c, we find

$$\epsilon_{12} \simeq \frac{b}{d}, \quad \epsilon_{23} \simeq \frac{c}{d}, \quad \epsilon_{13} \simeq 0. \tag{8}$$

This implies

$$\tan^2 \theta_{12} \simeq (1 - 3\sqrt{2}\epsilon_{12})/2, \quad \sin^2 2\theta_{23} \simeq 1 - (8\epsilon_{23}^2/3), \quad \sin \theta_{13} \simeq -\epsilon_{23}/\sqrt{3}. \tag{9}$$

We then have the prediction

$$\sin^2 2\theta_{23} \simeq 1 - 2\sin^2 2\theta_{13}. \tag{10}$$

Using the existing bound [7] of $\sin^2 2\theta_{23} > 0.92$, this would require $\sin^2 2\theta_{13} < 0.04$, which is of course ruled out by the recent data, i.e Eqs. (1) and (2). This result is however not negative, but rather very positive, because it says that ϵ_{23} must be complex, in which case the approximation becomes

$$\sin^2 2\theta_{23} \simeq 1 - 8[Re(U_{e3})]^2. \tag{11}$$

Now the new data can be accommodated provided that leptonic CP violation is large.

In analyzing Eq. (6), we note from Eq. (4) that whereas the parameter a may be chosen real, the others b, c, d must be kept complex. In fact, even in the tribimaximal limit ($b = c = 0$), d is in general complex, as shown already some time ago [8].

We now show how Eqs. (3) and (4) are obtained. The symmetry A_4 is that of the even permutation of four objects. It has twelve elements and is the smallest group which admits an irreducible three-dimensional representation. Its character table is given below [1]. There are 12 elements divided into 4 equivalence classes C_i, where n_i is the number of elements in each class, h_i is the order of each element, i.e. $g^h = 1$ for any group element g in this class.

class	n	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3
C_1	1	1	1	1	1	3
C_2	4	3	1	ω	ω^2	0
C_3	4	3	1	ω^2	ω	0
C_4	3	2	1	0	0	-1

Table 1: Character table of A_4.

There are 4 irreducible representations: $\underline{1}, \underline{1}', \underline{1}'', \underline{3}$, with χ_i the value of its character, i.e. the trace of its representation matrix. The basic multiplication rule of A_4 is

$$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}. \tag{12}$$

As first shown in Ref. [1], for $(\nu_i, l_i) \sim \underline{3}$, $l_i^c \sim \underline{1}, \underline{1}', \underline{1}''$, and $\Phi_i = (\phi_i^0, \phi_i^-) \sim \underline{3}$, the charged-lepton mass matrix is given by

$$\mathcal{M}_l = \begin{pmatrix} v_1 & 0 & 0 \\ 0 & v_2 & 0 \\ 0 & 0 & v_3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix}, \tag{13}$$

where $v_i = \langle \phi_i^0 \rangle$. For $v_1 = v_2 = v_3 = v/\sqrt{3}$, we then obtain Eq.(3) with $m_e = f_1 v$, $m_\mu = f_2 v$, $m_\tau = f_3 v$. The original A_4 symmetry is now broken to the residual symmetry Z_3, i.e. lepton flavor triality [9], with $e \sim 1$, $\mu \sim \omega^2$, $\tau \sim \omega$. This is a good symmetry of the Lagrangian as long as neutrino masses are zero. Exotic scalar decays are predicted and may be observable at the Large Hadron Collider (LHC) in some regions of parameter space [10, 11].

To obtain nonzero neutrino masses, we add four heavy Higgs triplets: $\Delta_0 \sim \underline{1}$, $\Delta_i \sim \underline{3}$ under A_4. Let $\langle \Delta_0^0 \rangle = u_0$, $\langle \Delta_i^0 \rangle = u_i$, then Eq. (4) is the automatic result. In previous studies, $e = f = 0$ has to be enforced to get tribimaximal mixing, which is technically an unnatural condition, requiring usually the addition of extra symmetries and auxiliary fields. Free of this burden, nonzero and arbitrary d, e, f are easily implemented. For large Higgs triplet masses, small vacuum expectation values are naturally induced [12] by the soft trilinear $\tilde{\Phi}^\dagger \Delta \Phi$ terms and leptogenesis becomes possible from the decay of these heavy Higgs

triplets. We simply assume that A_4 is broken completely by these terms to obtain different $u_{1,2,3}$. On the other hand, the tribimaximal requirement of $u_2 = u_3 = 0$ is very difficult to maintain, because it is not protected against infinite radiative corrections, which is the field theory's way of telling us that they should be nonzero and arbitrary in the first place. In retrospect, it should have been obvious that Eq. (4) is the more natural choice for the neutrino mass matrix in the A_4 basis.

The most general neutrino mass matrix in the tribimaximal basis is

$$\mathcal{M}_\nu^{(1,2,3)} = \begin{pmatrix} m_1 & m_6 & m_4 \\ m_6 & m_2 & m_5 \\ m_4 & m_5 & m_3 \end{pmatrix}. \tag{14}$$

To first order, θ_{13} and θ_{23} are sensitive to m_4 and m_5, whereas θ_{12} is sensitive to m_6. The case of $m_6 = 0$ was considered in the original proposal [4] of tribimaximal mixing using A_4, and updated recently [13]. The case of $m_5 \simeq 0$ is realized in a supersymmetric $B - L$ gauge model with T_7 symmetry discussed recently [14, 15]. The case of unbroken residual symmetries in a class of discrete symmetries has been discussed recently [16], as well as a general perturbation of the tribimaximal limit [17]. Here we consider the simplest and perhaps the most compelling case of $m_4 = 0$, which does not correspond to any unbroken residual symmetry. The fact that $m_4 = 0$ is simply the result of not having Higgs triplets which transform as $\underline{1}'$ or $\underline{1}''$ under A_4.

The neutrino mixing matrix U has 4 parameters: s_{12}, s_{23}, s_{13} and δ_{CP} [7]. We choose the convention $U_{\tau 1}, U_{\tau 2}, U_{e3}, U_{\mu 3} \to -U_{\tau 1}, -U_{\tau 2}, -U_{e3}, -U_{\mu 3}$ to conform with that of the tribimaximal mixing matrix of Eq. (5), then

$$\mathcal{M}_\nu^{(1,2,3)} = U_{TB}^T U \begin{pmatrix} e^{i\alpha_1} m_1' & 0 & 0 \\ 0 & e^{i\alpha_2} m_2' & 0 \\ 0 & 0 & m_3' \end{pmatrix} U^T U_{TB}, \tag{15}$$

where $m_{1,2,3}'$ are the physical neutrino masses, with

$$m_2' = \sqrt{m_1'^2 + \Delta m_{21}^2}, \tag{16}$$

6

$$m'_3 = \sqrt{m'^2_1 + \Delta m^2_{21}/2 + \Delta m^2_{32}} \quad \text{(normal hierarchy)}, \tag{17}$$

$$m'_3 = \sqrt{m'^2_1 + \Delta m^2_{21}/2 - \Delta m^2_{32}} \quad \text{(inverted hierarchy)}. \tag{18}$$

If U and $\alpha_{1,2}$ are known, then all $m_{1,2,3,4,5,6}$ are functions only of m'_1.

We now diagonalize $\mathcal{M}^{(1,2,3)}_\nu$ using

$$U_\epsilon \mathcal{M}^{(1,2,3)}_\nu U_\epsilon^T = \begin{pmatrix} e^{i\alpha'_1} m'_1 & 0 & 0 \\ 0 & e^{i\alpha'_2} m'_2 & 0 \\ 0 & 0 & e^{i\alpha'_3} m'_3 \end{pmatrix}, \tag{19}$$

from which we obtain $U' = U_{TB} U_\epsilon^T$. To obtain U with the usual convention, we rotate the phases of the μ and τ rows so that $U'_{\mu 3} e^{-i\alpha'_3/2}$ is real and negative, and $U'_{\tau 3} e^{-i\alpha'_3/2}$ is real and positive. These phases are absorbed by the μ and τ leptons and are unobservable. We then rotate the $\nu_{1,2}$ columns so that $U'_{e1} e^{-i\alpha'_3/2} = U_{e1} e^{i\alpha''_1/2}$ and $U'_{e2} e^{-i\alpha'_3/2} = U_{e2} e^{i\alpha''_2/2}$, where U_{e1} and U_{e2} are real and positive. The physical relative Majorana phases of $\nu_{1,2}$ are then $\alpha_{1,2} = \alpha'_{1,2} + \alpha''_{1,2}$. The three angles and the Dirac phase are extracted according to

$$\tan^2 \theta_{12} = |U'_{e2}/U'_{e1}|^2, \quad \tan^2 \theta_{23} = |U'_{\mu 3}/U'_{\tau 3}|^2, \quad \sin\theta_{13} e^{-i\delta_{CP}} = U'_{e3} e^{-i\alpha'_3/2}. \tag{20}$$

The effective Majorana neutrino mass in neutrinoless double beta decay is then given by

$$m_{ee} = |U^2_{e1} e^{i\alpha_1} m'_1 + U^2_{e2} e^{i\alpha_2} m'_2 + U^2_{e3} m'_3|. \tag{21}$$

Although b, c, d are in general complex, the structure of this model is restricted by data such that $Im(b)$ is very small, so we will assume in the following that b is real. As for $Im(d)$, it is also small and affects only m_{ee} slightly and not δ_{CP}, so we will also take d to be real. The main feature here is the complexity of c. To first approximation, we find

$$d \simeq -a, \quad U_{e3} \simeq -\frac{Re(c)}{2a} + i\frac{Im(c)}{4a}, \tag{22}$$

allowing only the normal ordering of neutrino masses.

The special case $b = 0$ is especially interesting. It may be maintained by an interchange symmetry [4, 13] such that $f = -e$. As such, it was considered in Ref. [16]. In that case,

7

Eq. (6) can be diagonalized exactly. Assuming that a, d are real and c complex, we find

$$\tan^2 \theta_{12} = \frac{1 - 3 \sin^2 \theta_{13}}{2}, \tag{23}$$

$$\tan^2 \theta_{23} = \frac{\left(1 - \frac{\sqrt{2} \sin \theta_{13} \cos \delta'_{CP}}{\sqrt{1 - 3 \sin^2 \theta_{13}}}\right)^2 + \frac{2 \sin^2 \theta_{13} \sin^2 \delta'_{CP}}{1 - 3 \sin^2 \theta_{13}}}{\left(1 + \frac{\sqrt{2} \sin \theta_{13} \cos \delta'_{CP}}{\sqrt{1 - 3 \sin^2 \theta_{13}}}\right)^2 + \frac{2 \sin^2 \theta_{13} \sin^2 \delta'_{CP}}{1 - 3 \sin^2 \theta_{13}}}, \tag{24}$$

where $\delta'_{CP} = \delta_{CP} - \alpha'_3/2$. The phase α'_3 is defined in Eq. (19) and depends on the specific values of Eq. (6). For $\sin \theta_{13} = 0.16$, corresponding to $\sin^2 2\theta_{13} = 0.1$, this predicts $\tan^2 \theta_{12} = 0.46$. If $Im(c) = 0$, then $\delta_{CP} = \alpha'_3 = 0$, so this would predict $\sin^2 2\theta_{23} = 0.80$ which is of course ruled out. Using $\sin^2 2\theta_{23} > 0.92$, we find in this case $|\tan \delta'_{CP}| > 1.2$.

For our numerical analysis, we set

$$\Delta m^2_{21} = 7.59 \times 10^{-5} \text{ eV}^2, \quad \Delta m^2_{32} = 2.45 \times 10^{-3} \text{ eV}^2, \tag{25}$$

$$\sin^2 2\theta_{12} = 0.87, \quad \sin^2 2\theta_{13} = 0.05 \text{ to } 0.15. \tag{26}$$

We then diagonalize Eq. (6) exactly and scan for solutions satisfying the above experimental inputs. We do not assume $b = 0$ or α'_3 to be necessarily small. We find that solutions exist only for the normal ordering of neutrino masses, i.e. $m'_1 < m'_2 < m'_3$, as in the tribimaximal case [8]. In Fig. 1 we show $|\tan \delta_{CP}|$ as a function of $\sin^2 2\theta_{13}$ from 0.05 to 0.15, for the central value of $\sin^2 2\theta_{12} = 0.87$ and the two fixed values of $\sin^2 2\theta_{23} = 0.92$ and 0.96. In Fig. 2 we plot the parameter b as a function of $\sin^2 2\theta_{13}$. It shows that for $\sin^2 2\theta_{12} = 0.87$, it is indeed very small. Note that for $b = 0$, we find $\sin^2 2\theta_{13} = 0.08$, i.e. $\sin^2 \theta_{13} = 0.02$. Using Eq. (23), we recover exactly $\tan^2 \theta_{12} = 0.47$, i.e. $\sin^2 2\theta_{12} = 0.87$, as expected. In Figs. 3 and 4 we plot the physical neutrino masses $m'_{1,2,3}$ together with the effective neutrino mass m_{ee} in neutrinoless double beta decay as functions of $\sin^2 2\theta_{13}$ for $\sin^2 2\theta_{23} = 0.92$ and 0.96 respectively. Note that m_{ee} is always smaller than m'_1 because $e^{i\alpha_1} = -1$ and $e^{i\alpha_2} = 1$ in Eq. (21). In Figs. 5 and 6 we plot the parameters $a, -d, Re(c), Im(c)$ as functions of of $\sin^2 2\theta_{13}$ for $\sin^2 2\theta_{23} = 0.92$ and 0.96 respectively.

In conclusion, neutrino tribimaximal mixing may be dead, but A_4 is alive and even getting healthier. In a new simple application, given the present allowed ranges of values for $\sin^2 2\theta_{13}$ and $\sin^2 2\theta_{23}$, we predict large CP violation and a normal ordering of neutrino masses. Our result is verifiable in future neutrino experiments and may offer a simple understanding of the interplay of symmetry and the values of leptonic mixing angles and phases.

References

[1] E. Ma and G. Rajasekaran, Phys. Rev. **D64**, 113012 (2001).

[2] E. Ma, Mod. Phys. Lett. **A17**, 2361 (2002).

[3] K. S. Babu, E. Ma, and J. W. F. Valle, Phys. Lett. **B552**, 207 (2003).

[4] E. Ma, Phys. Rev. **D70**, 031901 (2004).

[5] Daya Bay Collaboration: F. P. An *et al.*, Phys. Rev. Lett. **108**, 171803 (2012) [arXiv:1203.1669 [hep-ex]].

[6] RENO Collaboration: J. K. Ahn *et al.*, Phys. Rev. Lett. **108**, 191802 (2012) [arXiv:1204.0626 [hep-ex]].

[7] Particle Data Group: K. Nakamura *et al.*, J. Phys. G: Nucl. Part. Phys. **37**, 075021 (2010).

[8] E. Ma, Phys. Rev. **D72**, 037301 (2005).

[9] E. Ma, Phys. Rev. **D82**, 037301 (2010).

[10] Q.-H. Cao, S. Khalil, E. Ma, and H. Okada, Phys. Rev. Lett. **106**, 131801 (2011).

[11] Q.-H. Cao, A. Damanik, E. Ma, and D. Wegman, Phys. Rev. **D83**, 093012 (2011).

[12] E. Ma and U. Sarkar, Phys. Rev. Lett. **80**, 5716 (1998).

[13] E. Ma and D. Wegman, Phys. Rev. Lett. **107**, 061803 (2011).

[14] Q.-H. Cao, S. Khalil, E. Ma, and H. Okada, Phys. Rev. **D84**, 071302(R) (2011).

[15] H. Ishimori, S. Khalil, and E. Ma, arXiv:1204.2705 [hep-ph].

[16] D. Hernandez and A. Yu. Smirnov, arXiv:1204.0445 [hep-ph].

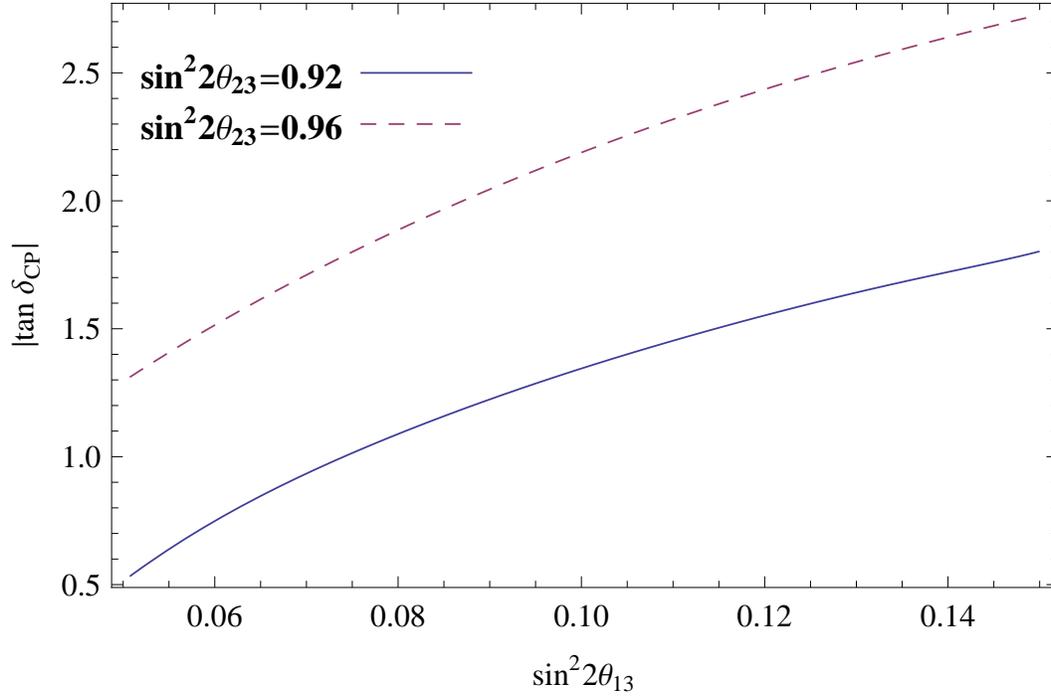[17] B. Brahmachari and A. Raychaudhuri, arXiv:1204.5619 [hep-ph].

Figure 1: $|\tan\delta_{CP}|$ versus $\sin^2 2\theta_{13}$ for $\sin^2 2\theta_{23} = 0.92$ and 0.96.
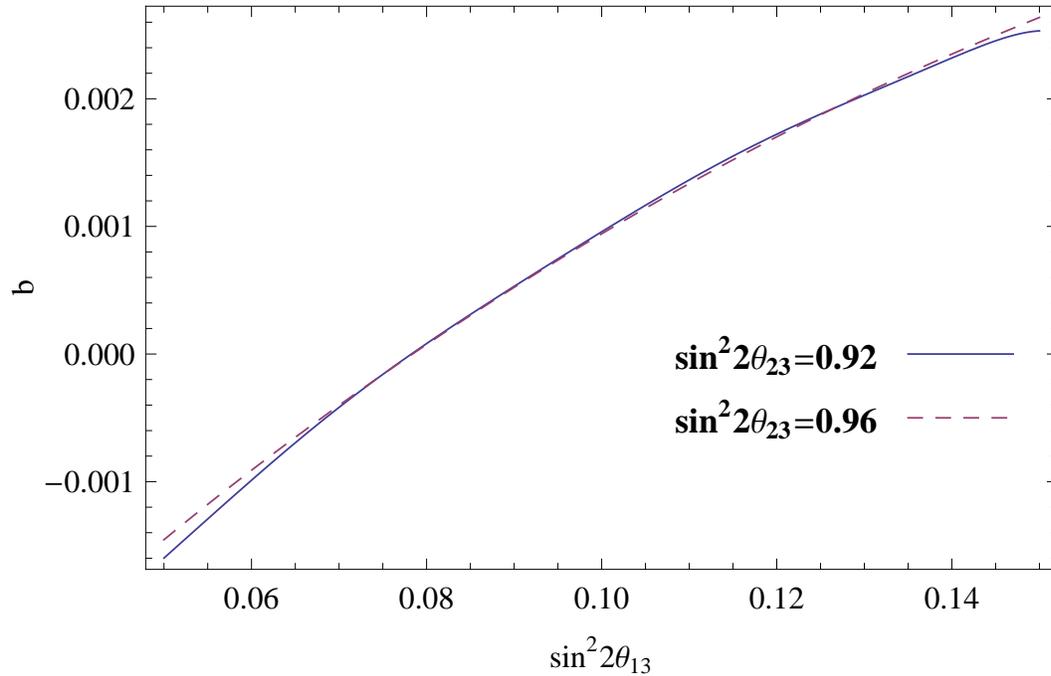

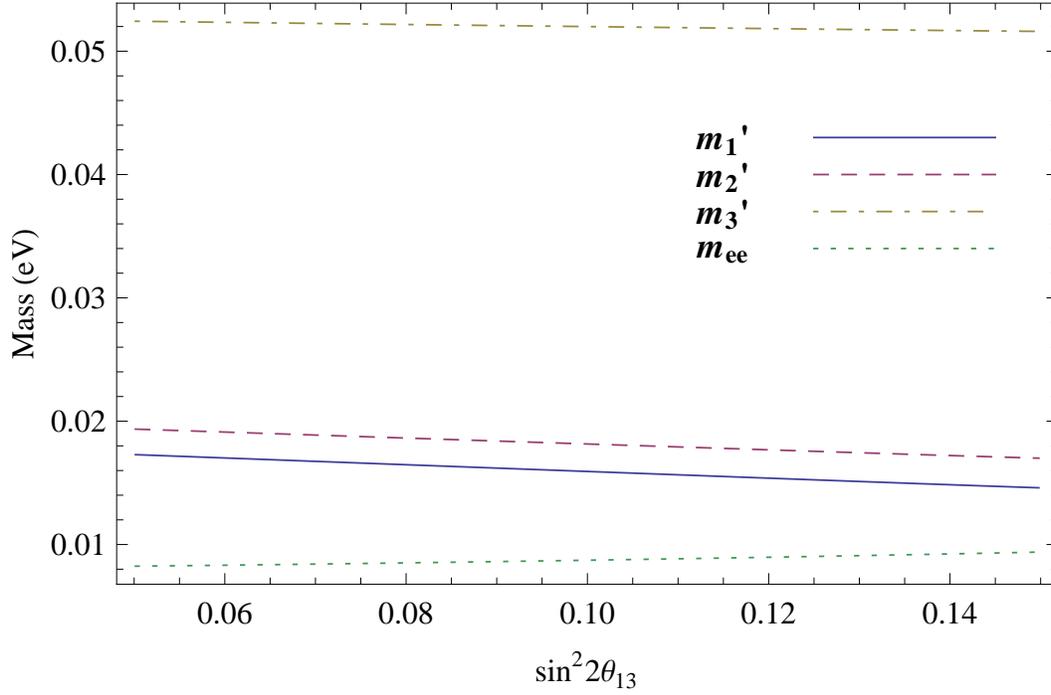Figure 2: Parameter b versus $\sin^2 2\theta_{13}$ for $\sin^2 2\theta_{23} = 0.92$ and 0.96.

Figure 3: Physical neutrino masses and the effective neutrino mass m_{ee} in neutrinoless double beta decay for $\sin^2 2\theta_{23} = 0.92$.
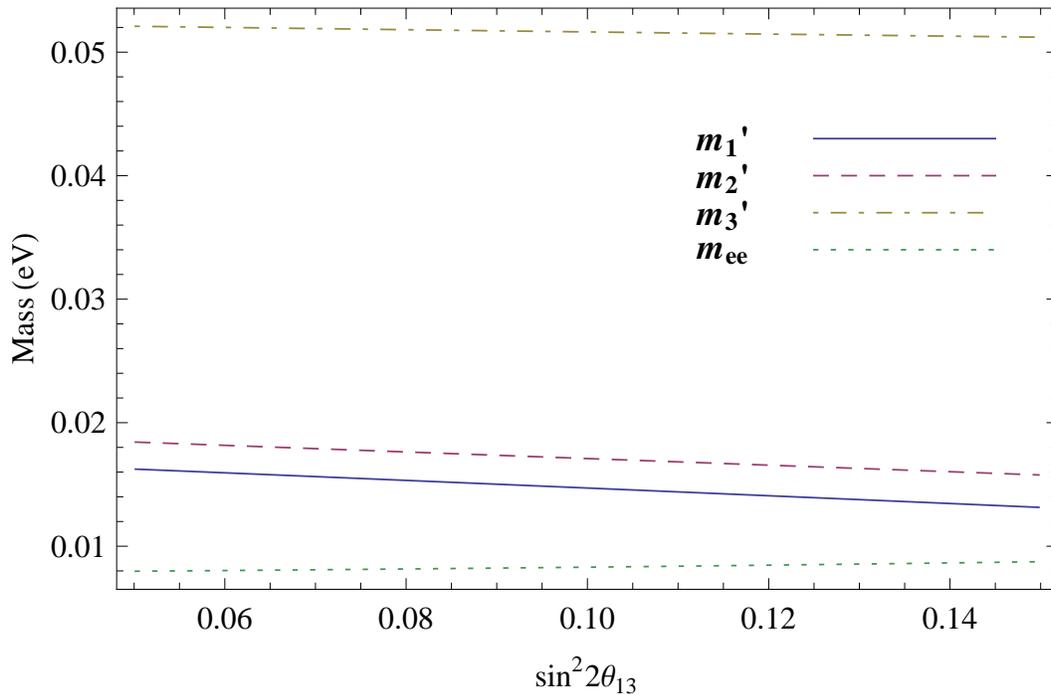

Figure 4: Physical neutrino masses and the effective neutrino mass m_{ee} in neutrinoless double beta decay for $\sin^2 2\theta_{23} = 0.96$.
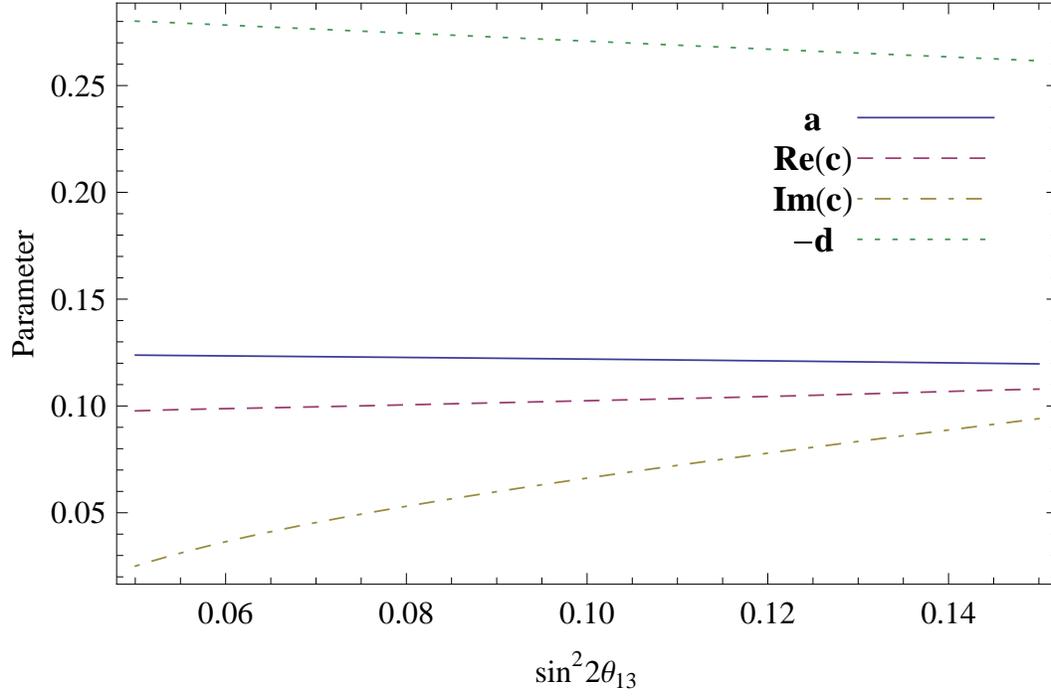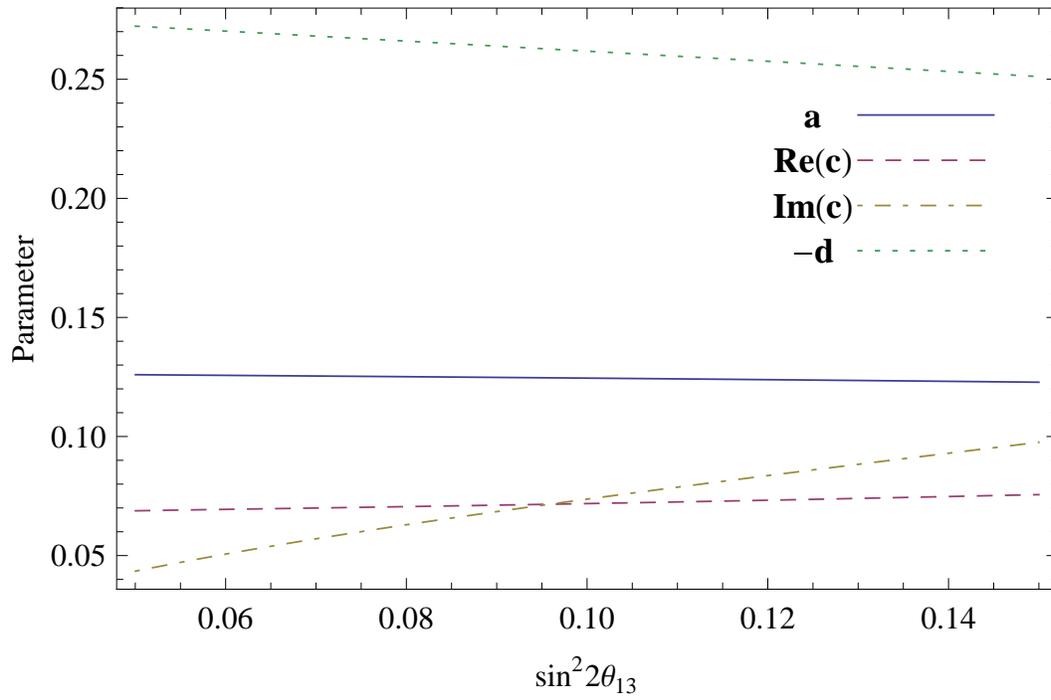
Figure 5: A_4 parameters for $\sin^2 2\theta_{23} = 0.92$.

Figure 6: A_4 parameters for $\sin^2 2\theta_{23} = 0.96$.