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Quark-hadron phase transition in a chameleon Brans-Dicke model of brane gravity

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Abstract

In this work, the quark hadron phase transition in a chameleon Brans-Dicke model of brane world cosmology within an effective model of QCD is investigated. Whereas in CBD model of brane world cosmology the Friedmann equation and conservation of density energy are modified, resulting in an increased expansion at early Universe. These have important effect on quark hadron phase transition. We investigate the evolution of the physical quantities relevant to quantitative description of the early times, namely, the energy density, ρ, temperature, T, and the scale factor, a, before, during and after the phase transition. We do this for smooth crossover formalism which lattice QCD data is used for obtaining the matter equation of state and first order phase transition formalism. Our analyzes show that, the quark hadron phase transition has occurred at approximately nanosecond after the big bang and the general behavior of temperature is similar in both of two approaches.

1 Introduction

As the universe expanded and cooled it passed through a series of symmetry-breaking phase transitions which can generate topological defects. Here we study the quark-gluon (QG) to hadron phase transition. This early universe phase transition has been studied in detail for over three decades [1, 2, 3, 4, 5, 6]. It could be a first, second, or higher order phase transition. In addition, the possibility of no phase transition was considered in Ref. [7]. The order of the phase transition depends strongly on the mass and flavor of the quarks.

For an early study of a first-order quark-hadron phase transition in the expanding universe see Ref. [8]. As the deconfined quark-gluon plasma cools below the critical temperature $T_c, = 150$ MeV, it becomes energetically favorable to form color-confined hadrons (mainly pions and a few of neutrons and protons, due to the conserved net baryon number). However, this new phase does not form immediately. As is characteristic of a first-order phase transition, some supercooling is needed to overcome the energy expense of forming the surface of the bubble and the new hadron phase. When a hadron bubble is nucleated, latent heat is released, and a spherical shock wave expands into the surrounding supercooled

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quark-gluon plasma. This reheats the plasma to the critical temperature, preventing further nucleation in regions passed through by one or more shock fronts. Generally, bubble growth is described by deflagrations, with a shock front preceding the actual transition front. The nucleation stops when the universe has reheated to T_c. After that, the hadron bubbles grow at the expense of the quark phase and eventually percolate or coalesce. The transition ends when all quark-gluon plasma has been converted to hadrons, neglecting possible quark nugget production. The physics of the quark-hadron phase transition, as well as the cosmological implications of this process, has been extensively discussed in the framework of general relativistic cosmology in Refs. [9, 10, 11, 12, 13, 14, 15].

As an alternative to general relativity, the scalar-tensor theory was conceived originally by Jordan, who embedded a four-dimensional curved manifold in five-dimensional flat space-time [16]. Scalar-tensor models include a scalar field, ϕ, with non-minimal coupling to the geometry in the gravitational action, as introduced by Brans and Dicke (BD) [17]. Brans-Dicke models have proved to be useful as a setting for discussing some of the outstanding puzzles in cosmology [18, 19]. The mechanism that creates a non-minimal scalar field coupling to the geometry could also lead to a coupling between the scalar and matter fields. Two such examples are a generalization of quintessence [20] and the chameleon mechanism [21, 22]. The scalar field in the generalized quintessence scenario has a very small mass and couples to matter with gravitational strength. The authors of Ref. [21] study a chameleon mechanism where the scalar field couples directly to matter with order unity strength. In this mechanism the mass of the scalar field depends on the local mass density. The chameleon proposal provides a way to generate an effective mass for a light scalar field via field self-interaction and the interaction between matter and scalar fields. When the chameleon coupling is used in the Brans-Dicke model, this is called the chameleon-Brans-Dicke model [23]. Solar system observational constraints on the chameleon-Brans-Dicke model have been studied in Ref. [24].

Over the past decade the possibility that our four dimensional universe is a brane embedded in a higher dimensional space-time has attracted considerable interest [25]. This scenario has been investigated for the case in which the bulk is five dimensional and it has been shown that it can result in a theory of gravity which mimics purely four-dimensional gravity, both with respect to the classical gravitational potential and with respect to gravitational radiation [26].

Of interest in the present study are brane-world models in the context of chameleon-Brans-Dicke (CBD) gravity. Interestingly, it will be show that in such a scenario, and in the presence of a CBD field in the bulk, due to non-minimal coupling between the scalar field and matter, the energy conservation equation on the brane for matter fields is modified. It is of interest to study the quark-gluon to hadrons phase transition in the context of the CBD brane world theory of gravity. The quark-hadron phase transition in the context of conventional brane-world gravity and in Brans-Dicke brane-world gravity have been studied in Refs. [27, 28, 29].

Recently, based on the particle physics motivation, there has been interested in the possibility of energy exchange between the brane and bulk. Observational constraints on cosmological models in the brane-world scenario in which the bulk is not empty, and that allow for the exchange of mass-energy between the bulk and the brane have been studied [30]. The evolution of matter fields on the brane is modified due to new terms in the energy momentum tensor that describe this exchange. This model can account for the observed suppression of the cosmic microwave background (CMB) power spectrum at low multipoles, and in this model the observed recent cosmic acceleration is attributable to the flow of matter from the bulk to the brane. The cosmological evolution of a brane with chameleon scalar field and general matter content in the bulk was considered in Ref. [31]. Also the reheating the universe in brane-world model of cosmology with bulk-brane energy transfer has been studied in Refs. [32, 33].

In fact existence an energy dissipation from the bulk scalar field into the matter field on the brane, shows an interaction between matter and scalar field. **But when a chameleon scalar field interacts with perfect fluid, this interaction produce a fifth force on the matter which may violate the equivalence principle (EP) and creates a non-geodesic motion. This kind of interactions have attracted much attention [21, 23, 24, 31, 34, 35, 36]. As was mentioned earlier, the mass of chameleon scalar field is a function of local density and in the high density regions[6] the fifth force effects are confined to an undetectable small distances. Therefore the violation of EP is not observed in the laboratory [21, 35]. Moreover, In [36] has been shown that the motion of perfect fluid in this model is the same as the motion of perfect fluid in Einsteinian theory. In fact it is shown that for $L_m = -p$ as a Lagrangian density of perfect fluid, the motion of perfect fluid is geodesic. Therefore we consider this model due to the following reasons.**

- **The brane world theory is an outstanding motivation in cosmology.**
- **This model can create a bulk-brane energy transfer which has been studied in early time a lot.**
- **The motion of perfect fluid in this model is geodesic.**
- **Study of quark-hadron phase transition in this model shows some interesting results.**

This paper is organized as follows. In Sec. 2, we introduce the model and derive the equations of motion. We review the first-order phase transition and consider it in our model in Sec. 3. In Sec. 4 we investigate our model for a typical example. We study the smooth crossover approach in Sec. 5 and Sec. 6 summarizes our results.

2 General framework

We consider the five-dimensional chameleon-Brans-Dicke model with action

$$S = -\frac{1}{2\kappa^2_{(5)}} \int d^5x\sqrt{-g}\Big[\phi R - \frac{\omega}{\phi}\partial_A\phi\partial^A\phi - V(\phi)\Big] + \int d^5x\sqrt{-g}f(\phi)L_m. \qquad (1)$$

Here g is the determinant of the five-dimensional metric g_{AB}, R is the Ricci scalar constructed from g_{AB}, ϕ is the CBD scalar field, ω is a dimensionless coupling constant which determines the coupling between gravity and ϕ, L_m is the Lagrangian density for the matter fields, and $V(\phi)$ is the scalar field potential energy density. Latin indices label five-dimensional components $(A, B = 1, \ . \ . \ . \ , 5)$ and for convenience we shall set $\kappa^2_{(5)} = 8\pi G_{(5)} = 1$, where $G_{(5)}$ is the five-dimensional Newtonian gravitational constant. The last term on the right hand side of Eq. (1), $f(\phi)L_m$, indicates non-minimal coupling between the scalar field and matter, where $f(\phi)$ is an analytical function of ϕ.

[6] Where observation and experiments are performed such as Earth.

One can obtain the gravitational and scalar field equations of motion by varying the action (1) with respect to g_{AB} and ϕ. The gravitational field equation is

$$
{}^5G_{AB} \equiv {}^5R_{AB} - \frac{1}{2}g_{AB}R = \frac{1}{\phi}\left(T^{\phi}_{AB} + f(\phi)T_{AB}\right), \tag{2}
$$

where ${}^5G_{AB}$ is the five-dimensional Einstein tensor, ${}^5R_{AB}$ is the five-dimensional Ricci tensor. The five-dimensional energy-momentum tensor of the matter, T_{AB}, is given by

$$
T_{AB} = \frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{AB}}, \tag{3}
$$

and the scalar field energy-momentum tensor, T^{ϕ}_{AB}, is

$$
T^{\phi}_{AB} = \frac{\omega}{\phi}\Big[\nabla_A\phi\nabla_B\phi - \frac{1}{2}g_{AB}\nabla_C\phi\nabla^C\phi\Big] + \Big[\nabla_A\nabla_B\phi - g_{AB}\nabla_A\nabla^A\phi\Big] - \frac{1}{2}g_{AB}V(\phi). \tag{4}
$$

The scalar field equation of motion is

$$
(3\omega+4)\nabla_A\nabla^A\phi = \Big[Tf(\phi) + 3\tilde{f}(\phi)\phi L_m\Big] + \frac{3}{2}\phi\tilde{V}(\phi) - \frac{5}{2}V(\phi), \tag{5}
$$

where T is the trace of T_{AB} and $\tilde{X} := \frac{dX}{d\phi}$. Setting $f(\phi) = 1$ and $V(\phi) = 0$, the above equations reduce to those of Ref. [29].

Note that to solve Eq. (5) we need an explicit form of the perfect fluid Lagrangian density. In [37] have been shown that, for perfect fluid that does not couple explicitly to the other components of the system, there are different Lagrangian densities which are perfectly equivalent. In fact, they have shown that, by using Eq. (3), the two Lagrangian densities $L_{m_1} = -p$ and $L_{m_2} = \rho$ give the same stress-energy tensor, moreover the equation of motions for all components of the system for these two different Lagrangian densities are similar. But according to [36], when perfect fluid couple explicitly to the scalar field, these two perfect Lagrangian densities are not equivalent and for $L_m = -p$ the motion of perfect fluid is geodesic. Therefore in this work we choose $L_m = -p$.

We consider a five-dimensional flat metric of the form

$$
ds^2 = -n^2(t,y)dt^2 + a^2(t,y)\gamma_{ij}dx^i dx^j + b^2(t,y)dy^2, \tag{6}
$$

where $i,j = 1, 2, 3$. We also assume an orbifold symmetry along the fifth direction $y = -y$.

One can define the energy-momentum tensor as

$$
T^A{}_B = T^A{}_B\,|_{bu} + T^A{}_B\,|_{br}, \tag{7}
$$

where the subscripts br and bu refer to the corresponding energy-momentum tensors in the brane and bulk respectively. We assume the brane has tension λ and is filled with perfect fluid matter and the bulk has no ordinary matter. I. e., the matter energy-momentum tensors are taken to be

$$
\begin{aligned}
T^A{}_B\,|_{br} &= \frac{\delta(y)}{b}\mathrm{diag}(-\rho,p,p,p,0), && (8)\\
T^A{}_B\,|_{bu} &= \mathrm{diag}(0,0,0,0,0), && (9)
\end{aligned}
$$

where $\delta(y)$ is the Dirac delta function. It is assumed that the brane is held at $y = 0$, and

$$\rho = \rho_m + \lambda, \tag{10}$$

$$p = p_m - \lambda, \tag{11}$$

where the subscript m denotes matter. There are several suggestions for the appropriate numerical value of the brane tension λ. From the success of big bang nucleosynthesis $\lambda \geq 1$ MeV^4 [38]. A much stronger bound for λ comes from the null results of submillimeter tests of Newton's inverse-square law of gravity, giving $\lambda \geq 10^8$GeV^4 [39]. An astrophysical limit on λ, independent of Newton's law of gravity, and cosmological limits, have been studied in Ref. [38], leading to $\lambda > 5 \times 10^8$ MeV^4.

We assume that the five dimensional metric (6) is continuous, but the first derivative with respect to y is discontinuous, so that the second derivative with respect to y includes a Dirac delta function. Making use of Eq. (6), one can obtain the non-vanishing components of the Einstein tensor. The (0,0) component of the Einstein equation is

$$^5G_{00} = 3\left[\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}+\frac{\dot{b}}{b}\right) - \frac{n^2}{b^2}\left\{\frac{a''}{a}+\frac{a'}{a}\left(\frac{a'}{a}-\frac{b'}{b}\right)\right\}\right] = \frac{1}{\phi}\left[T^{\phi}_{00} + f(\phi)T_{00}\right], \tag{12}$$

where

$$T^{\phi}_{00} = -\dot{\phi}\left(3\frac{\dot{a}}{a}+\frac{\dot{b}}{b}-\frac{\omega}{2}\frac{\dot{\phi}}{\phi}\right) + \left(\frac{n}{b}\right)^2\left[\phi''+\phi'\left(3\frac{a'}{a}-\frac{b'}{b}+\frac{\omega}{2}\frac{\phi'}{\phi}\right)\right] + \frac{n^2}{2}V(\phi). \tag{13}$$

The (i,j) component of the Einstein equation is

$$\begin{aligned} ^5G_{ij} = {} & \frac{a^2}{b^2}\gamma_{ij}\left[\frac{a'}{a}\left(\frac{a'}{a}+2\frac{n'}{n}\right) - \frac{b'}{b}\left(\frac{n'}{n}+2\frac{a'}{a}\right) + 2\frac{a''}{a}+\frac{n''}{n}\right] \\ & + \frac{a^2}{n^2}\gamma_{ij}\left[\frac{\dot{a}}{a}\left(-\frac{\dot{a}}{a}+2\frac{\dot{n}}{n}\right) - 2\frac{\ddot{a}}{a} + \frac{\dot{b}}{b}\left(-2\frac{\dot{a}}{a}+\frac{\dot{n}}{n}\right) - \frac{\ddot{b}}{b}\right] = \frac{1}{\phi}\left[T^{\phi}_{ij}+f(\phi)T_{ij}\right], \end{aligned} \tag{14}$$

where

$$\begin{aligned} T^{\phi}_{ij} = {} & \left[\left(\frac{a}{n}\right)^2\left\{\ddot{\phi}+\dot{\phi}\left(\frac{\omega}{2}\frac{\dot{\phi}}{\phi}+2\frac{\dot{a}}{a}-\frac{\dot{n}}{n}+\frac{\dot{b}}{b}\right)\right\}\right. \\ & \left. -\left(\frac{a}{b}\right)^2\left\{\phi''+\phi'\left(\frac{\omega}{2}\frac{\phi'}{\phi}+2\frac{a'}{a}-\frac{n'}{n}+\frac{b'}{b}\right)\right\} + \frac{1}{2}a^2V(\phi)\right]\delta_{ij}. \end{aligned} \tag{15}$$

The (0,5) and (5,5) components of the Einstein equation are

$$^5G_{05} = 3\left(\frac{n'}{n}\frac{\dot{a}}{a}+\frac{a'}{a}\frac{\dot{b}}{b}-\frac{\dot{a}'}{a}\right) = \frac{1}{\phi}T^{\phi}_{05}, \tag{16}$$

$$^5G_{55} = 3\frac{b^2}{n^2}\left[-\frac{a'}{a}\left(\frac{a'}{a}+\frac{n'}{n}\right) + \frac{b^2}{n^2}\left\{\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a}-\frac{\dot{n}}{n}\right)\right\}\right] = \frac{1}{\phi}\left[T^{\phi}_{55}+f(\phi)T_{55}\right], \tag{17}$$

where

$$T^{\phi}_{05} = \dot{\phi}' - \dot{\phi}\left(\frac{n'}{n}-\omega\frac{\phi'}{\phi}\right) - \frac{\dot{b}}{b}\phi', \tag{18}$$

$$T^{\phi}_{55} = \ddot{\phi}+\dot{\phi}\left(3\frac{\dot{a}}{a}-\frac{\dot{n}}{n}+\frac{\omega}{2}\frac{\dot{\phi}}{\phi}\right) - \left(\frac{n}{b}\right)^2\phi'\left(3\frac{a'}{a}+\frac{n'}{n}-\frac{\omega}{2}\frac{\phi'}{\phi}\right). \tag{19}$$

The equation of motion of the CBD scalar field is

$$\begin{aligned}\ddot{\phi}+\dot{\phi}\left(3\frac{\dot{a}}{a}+\frac{\dot{b}}{b}-\frac{\dot{n}}{n}\right) \quad &- \quad \left(\frac{n}{b}\right)^2\left[\phi''+\phi'\left(\frac{n'}{n}+3\frac{a'}{a}-\frac{b'}{b}\right)\right]= \\ &- \quad \frac{n^2}{(3\omega+4)}\left[\left\{Tf(\phi)-3p\tilde{f}(\phi)\phi\right\}+\frac{3}{2}\phi\tilde{V}(\phi)-\frac{5}{2}V(\phi)\right].\end{aligned} \tag{20}$$

In these equations, $\dot{X}:=\frac{dX}{dt}$ and $X':=\frac{dX}{dy}$.

Since the second derivative of the metric includes a Dirac delta function, according to Ref. [40], one can define

$$W''=\hat{W}''+[W']\delta(y) \tag{21}$$

where $\hat{W}''$ is the non-distributional part of the double derivative of $W(t,y)$, and $[W']$ is the jump in the first derivative across $y=0$, which is defined by

$$[W']=W'(0^+)-W'(0^-).$$

The junction relations can be obtain by matching the coefficient of the Dirac delta function on both sides of the Einstein equation. From the $(0,0)$ and (i,j) components of the field equation we have, respectively

$$\frac{[a_0']}{a_0b_0} = \frac{-1}{(3\omega+4)\phi_0}\left\{\Big[p+(\omega+1)\rho\Big]f(\phi_0)-p\phi_0\tilde{f}(\phi_0)\right\}, \tag{22}$$

$$\frac{[n_0']}{n_0b_0} = \frac{1}{(3\omega+4)\phi_0}\left\{\Big[3(\omega+1)p+(2\omega+3)\rho\Big]f(\phi_0)+p\phi_0\tilde{f}(\phi_0)\right\}, \tag{23}$$

$$\frac{[\phi_0']}{\phi_0b_0} = \frac{1}{(3\omega+4)\phi_0}\left\{(3p-\rho)f(\phi_0)-3p\phi_0\tilde{f}(\phi_0)\right\}. \tag{24}$$

For $f(\phi)=1$ these equations reduce to the junction relations of Refs. [29, 41].

Using the $(0,0)$ component of the Einstein field equation for a brane which is located at $y=0$ and the equations which represent the jump conditions, (22), (23) and (24), one can derive the Friedmann equation

$$\begin{aligned}H^2 \quad + \quad &\Upsilon\Big(H-\frac{\omega}{6}\Upsilon\Big) \\ = \quad &\frac{1}{24(3\omega+4)^2\phi_0^2}\Bigg[\Big\{\omega(3p-\rho)^2+6(2+3\omega+\omega^2)\rho^2-6\omega\rho p-12p^2\Big\}f^2(\phi_0) \\ &\qquad +3(3\omega-4)p^2\phi_0^2\tilde{f}^2(\phi_0)+4(3\omega+4)^2\phi_0V(\phi_0) \\ &\qquad\qquad +6\big[(4-3\omega)p+2\omega\rho)\big]p\phi_0f(\phi_0)\tilde{f}(\phi_0)\Bigg],\end{aligned} \tag{25}$$

where $H=\dot{a}/a$ is the Hubble parameter, $\Upsilon=\dot{\phi}/\phi$, and the subscript 0 indicates the quantity is on the brane.

From the $(0,5)$ component of the field equation, using Eqs. (22), (23) and (24), we obtain the energy conservation equation on the brane

$$\dot{\rho}+3H(\rho+p)=-(p+\rho)\frac{\tilde{f}(\phi_0)}{f(\phi_0)}\dot{\phi}_0. \tag{26}$$

As expected, due to the interaction between the matter and scalar field, the energy conservation relation is modified. Note that in this Section we have used $n_0 = 1$ and $b_0 = 1$, without loss of generality. Using Eqs. (21) and (27) we can obtain the equation of motion for ϕ on the brane

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 = -\frac{1}{8(3\omega+4)^2\phi_0}\Bigg[4(3\omega+4)\phi_0\Big\{3\phi_0 V'(\phi_0) - 5V(\phi_0)\Big\} \tag{27}$$
$$-(3p-\rho)^2\omega f^2(\phi_0) + 12p^2\phi_0^2\tilde{f}^2(\phi_0) - (4-3\omega)p(3p-\rho)\phi_0 f(\phi_0)\tilde{f}(\phi_0)\Bigg],$$

where we have assumed $\hat{\phi}'' = 0$.

For simplicity, we assume

$$\phi_0 = N a_0^n, \tag{28}$$

where N and n are constants. For small n this ansatz has been shown to lead to consistent results [42]. With this choice the energy conservation equation becomes

$$\dot{\rho} + (3 + n\phi_0\bar{F})H(\rho + p) = 0, \tag{29}$$

where $\bar{F} = \tilde{f}/f$.

3 Quark-hadron phase transition

In this Section we study a first-order quark-hadron phase transition in the early universe within the CBD brane-world scenario. For a review of a first-order quark-hadron phase transition see Ref. [27] and references therein.

The energy density and pressure of matter in the quark-gluon phase at temperature T are [27]

$$\rho_q = 3a_q T^4 + U(T), \qquad p_q = a_q T^4 - U(T). \tag{30}$$

Here the subscript q denotes quark-gluon matter and $a_q = 61.75(\pi^2/90)$. The potential energy density, $U(T)$, is [40]

$$U(T) = B + \gamma_T T^2 - \alpha_T T^4, \tag{31}$$

where B is the bag pressure constant, $\alpha_T = 7\pi^2/20$, and $\gamma_T = m_s^2/4$, where m_s, the mass of the strange quark, is in the range $m_s \in (60 - 200)$ MeV. This form of U is for a model in which the quark fields interact with a chiral field formed from the π meson field and a scalar field [12]. Results obtained in low energy hadron spectroscopy, heavy ion collisions, and from phenomenological fits of light hadron properties, give $B^{1/4}$ between 100 and 200 MeV.

In the hadron phase one takes the cosmological fluid to be an ideal gas of massless pions and nucleons described by the Maxwell-Boltzmann distribution function, with energy density ρ_h and pressure p_h. Hence the equation of state in the hadron phase is

$$p_h = \frac{1}{3}\rho_h = a_\pi T^4, \tag{32}$$

where $a_\pi = 17.25(\pi^2/90)$.

The critical temperature T_c is defined by the condition $p_q(T_c) = p_h(T_c)$ [7], and, for $m_s = B^{1/4} = 200$ MeV, is given by

$$T_c = \left[\frac{\gamma_T + \sqrt{\gamma_T^2 + 4B(a_q + \alpha_T - a_\pi)}}{2(a_q + \alpha_T - a_\pi)}\right]^{\frac{1}{2}} \approx 125\text{MeV}. \quad (33)$$

Since the phase transition is first order, all physical quantities, such as the energy density, pressure, and entropy, exhibit discontinuities across the critical curve.

3.1 Evolution of temperature in quark-gluon phase (QGP) for general $U(T)$

In this Subsection we study the quark-hadron phase transition in the chameleon Brans-Dicke brane-world scenario for the potential energy density of Eq. (31). The quantities we want to trace through the quark-hadron phase transition are the temperature T and the scale factor a. To accomplish this we use the equations obtained in Sec. 2. In the quark-gluon phase with $T > T_c$, from Eqs. (30), (31) and (29), we have

$$H = -\frac{2(3a_q - \alpha_T)T^2 + \gamma_T}{2(3 + n\phi_0\bar{F})a_q T^3}\dot{T}. \quad (34)$$

This equation can be used to determined the scale factor as a function of T. Using Eqs. (8), (11) and (28), the Friedmann equation, Eq. (25), becomes

$$\begin{aligned} H^2 &= \frac{1}{4(3\omega+4)^2\theta^2\phi_0^2}\Bigg[6\Big\{(4-3\omega)(p_q-\lambda) + 2\omega(\rho_q+\lambda)\Big\}(p_q-\lambda)\phi_0 f(\phi_0)\tilde{f}(\phi_0) \\ &\quad + \Big\{\omega H_1^2 + 6(2+3\omega+\omega^2)(\rho_q+\lambda)^2 - 6\omega(\rho_q+\lambda)(p_q-\lambda) - 12(p_q-\lambda)^2\Big\}f^2(\phi_0) \\ &\quad + 3\Big(3\omega-4\Big)(p_q-\lambda)^2\phi_0^2\tilde{f}^2(\phi_0) + 4(3\omega+4)^2\phi_0 V(\phi_0)\Bigg], \end{aligned} \quad (35)$$

where

$$\theta^2 = (6 + 6n - \omega n^2),$$
$$H_1 = 3p_q - \rho_q - 4\lambda.$$

Combining these equations, one obtains the expression governing the evolution of temperature in the quark phase

$$\begin{aligned} \dot{T} &= -\frac{a_q(3+n\phi_0 F)T^3}{(3\omega+4)\theta\phi_0\Big[2(3a_q-\alpha_T)T^2 + \gamma_T\Big]} \quad (36) \\ &\quad \times\Bigg[\Big\{\omega T_1^2 + 6(2+3\omega+\omega^2)T_3^2 - 6\omega T_3 T_2 - 12T_2^2\Big\}f^2(\phi_0) + 4(3\omega+4)^2\phi_0 V(\phi_0) \\ &\quad + 6\Big\{(4-3\omega)T_2 + 2\omega T_3\Big\}T_2\phi_0 f(\phi_0)\tilde{f}(\phi_0) + 3\Big(3\omega-4\Big)T_2^2\phi_0^2\tilde{f}^2(\phi_0)\Bigg]^{1/2}, \end{aligned}$$

where

$$T_1 = 4\alpha_T T^4 - 4\gamma_T T^2 - 4B - 4\lambda,$$
$$T_2 = (a_q + \alpha_T)T^4 - \gamma_T T^2 - B - \lambda,$$
$$T_3 = (3a_q - \alpha_T)T^4 + \gamma_T T^2 + B + \lambda.$$

3.2 Evolution of temperature in QGP for $U(T) = B$

When dealing with quark confinement, one popular model is that of an elastic bag which allows the quarks to move around freely, and the potential energy density is constant. In this case the equation of state for quark matter is $p_q = (\rho_q - 4B)/3$. In this Subsection we assume an equation of state of quark matter given by this bag model. For this case, the expression (34) becomes

$$H = -\frac{3\dot{T}}{(3 + n\phi_0 \bar{F})T}. \tag{37}$$

From this relation we can obtain an expression for the scale factor as a function of T. Also, Eq. (36) reduces to

$$\begin{aligned}
\dot{T} \quad = \quad & -\frac{(3 + n\bar{F}(\phi_0))T}{3(3\omega + 4)\theta\phi_0} \\
& \times \Bigg[\Big\{ \omega T_{10}^2 + 6(2 + 3\omega + \omega^2)T_{30}^2 - 6\omega T_{30}T_{20} - 12T_{20}^2 \Big\} f^2(\phi_0) + 4(3\omega + 4)^2 \phi_0 V(\phi_0) \\
& + 6\Big\{ 2\omega T_{30} + (4 - 3\omega)T_{20} \Big\} T_{10}\phi_0 f(\phi_0)\tilde{f}(\phi_0) + 3\Big(3\omega - 4\Big) T_{20}^2 \phi_0^2 \tilde{f}^2(\phi_0) \Bigg]^{1/2},
\end{aligned} \tag{38}$$

where

$$T_{10} = -4(B + \lambda),$$
$$T_{20} = a_q T^4 - B - \lambda,$$
$$T_{30} = 3a_q T^4 + B + \lambda.$$

In Eq. (38) the scalar field ϕ_0 is a function of the scale factor a whose dependence on the temperature T is determined from Eq. (37).

3.3 Evolution of hadron volume fraction

During the quark-hadron phase transition $\rho_q(t)$ decreases from $\rho_q(T_c) = \rho_Q$ to $\rho_h(T_c) = \rho_H$, but the temperature and pressure stay constant. At the phase transition temperature $T_c = 125$ MeV we have $\rho_q \approx 5 \times 10^9 \mathrm{MeV}^4$, $\rho_h \approx 1.38 \times 10^9$ MeV^4, and $p_c \approx 4.6 \times 10^8 \mathrm{MeV}^4$ is constant during the phase transition. Following Refs. [27, 29, 7], one can replace $\rho(t)$ by $h(t)$, the volume fraction of matter in the hadron phase, by defining

$$\rho_q(t) = \Big[1 + mh(\tau)\Big]\rho_Q, \tag{39}$$

where $m = \rho_H/\rho_Q - 1 =$ constant. At the beginning of the phase transition $\rho(\tau_c) = \rho_Q$ and $h(\tau_c) = 0$, where τ_c is the time at the beginning of the phase transition, while at the end of the transition $\rho(\tau_h) = \rho_H$ and $h(\tau_h) = 1$, where τ_h is the time at the end of the phase transition.

For $\tau > \tau_h$ the universe is in the hadronic phase. Then, from Eqs. (29) and (39), we arrive at

$$H = -\frac{r\dot{h}}{(3 + n\phi_0 \bar{F})(1 + rh)} \tag{40}$$

where

$$r = \frac{\rho_H - \rho_Q}{p_c + \rho_Q} = \text{constant}, \tag{41}$$

and

$$A_0 = \frac{3p_c + \rho_Q - 2\lambda}{p_c + \rho_Q}. \tag{42}$$

Using Eqs. (25), (39) and (40), the evolution of the hadron fraction during the phase transition is governed by

$$\begin{aligned} \dot{h} &= -\frac{(3+n\phi_0\bar{F})(1+rh)}{2r(3\omega+4)\theta\phi_0}\Bigg[3\Big(3\omega-4\Big)(p_c-\lambda)^2\phi_0^2\tilde{f}^2(\phi_0) \\ &\quad +\Big\{\omega A_1^2 + 6(2+3\omega+\omega^2)A_2^2 - 6\omega A_2(p_c-\lambda) - 12(p_c-\lambda)^2\Big\}f^2(\phi_0) \\ &\quad +6\Big\{(4-3\omega)(p_c-\lambda)+2\omega A_2\Big\}(p_c-\lambda)\phi_0 f(\phi_0)\tilde{f}(\phi_0) + 4(3\omega+4)\phi_0 V(\phi)\Bigg]^{1/2}, \end{aligned} \tag{43}$$

where

$$A_1 = 3p_c - \rho_Q - (\rho_H - \rho_Q)h - 4\lambda,$$
$$A_2 = \rho_Q + (\rho_H - \rho_Q)h + \lambda.$$

3.4 Evolution of temperature in the hadronic era

In the hadronic phase, the equation of state is given by Eq. (32), and from Eq. (29) one can obtain

$$H = -\frac{3\dot{T}}{(3+n\phi_0\bar{F})T}, \tag{44}$$

while from the Friedmann equation, (25), and Eqs. (32) and (44) we arrive at

$$\begin{aligned} \dot{T} &= -\frac{(3+n\phi_0\bar{F})T}{6(3\omega+4)\theta\phi_0}\Bigg[3\Big(3\omega-4\Big)(a_\pi T^4-\lambda)\phi_0^2\tilde{f}^2(\phi_0) \\ &+ 6\Big\{(4+6\omega+3\omega^2)a_\pi^2 T^8 \omega(1+\omega)\lambda a_\pi T^4 + \omega(6+\omega)\lambda^2\Big\}f^2(\phi_0) \\ &\quad +4(3\omega+4)^2\phi_0 V(\phi_0) + 6\Big\{(3\omega+4)a_\pi T^4 + \lambda(5\omega-4)\Big\}(a_\pi T^4-\lambda)\phi_0 f(\phi_0)\tilde{f}(\phi_0)\Bigg]^{1/2}. \end{aligned} \tag{45}$$

4 An example

In this Section we study a model with definite, simple, functional forms for $f(\phi)$ and $V(\phi)$. Two scalar field potential energy densities, exponential and inverse power law, are commonly used in discussion of the chameleon mechanism. Here we consider the inverse power law potential energy density [20]

$$V(\Phi) = M^5\left(\frac{M^2}{\phi}\right)^{\xi}, \tag{46}$$

where $\xi > 0$, M is a constant mass scale and the scalar field, ϕ, has mass^2 dimension. The authors of Ref. [21] consider the solar system constraints for a model with this potential and find that for small values of $\xi \in (0,2)$ the magnitude of M is $\sim 10^{-3}$ eV. Therefore, the potential may be written as

$$V(\phi) \sim \frac{10^{-3(2\xi+5)}}{\phi^{\xi}}\ \text{eV}^5. \tag{47}$$

Since the characteristic energy density scales of other quantities such as ρ, p, λ, and constants of the model, are of order an MeV, the scalar field potential energy density term is very small compared to other terms in the Lagrangian density and we can ignore it. Also to make the equations tractable, we consider a simple functional form for $f(\phi)$, $f(\phi) = \phi$.

4.1 Evolution of temperature in the QGP for general $U(T)$

To determine the relevant quantities we use equations derived in Sec. 2. Matching $f(\phi) = \phi$ and $\tilde{f}(\phi) = 1$, Eq. (29) becomes

$$\dot{\rho_q} + H(3+n)(\rho_q + p_q) = 0, \tag{48}$$

and the Hubble parameter is given by

$$H = -\frac{2(3a_q - \alpha_T)T^2 + \gamma_T}{2(3+n)a_q T^3}\dot{T}, \tag{49}$$

Integrating Eq. (49) gives the scale factor

$$a(T) = CT^{-K}e^{B_0/2T^2}, \tag{50}$$

where C is a constant of integration and the other constants are

$$K = -\frac{3a_q - a_T}{(3+n)a_q}, \tag{51}$$

$$B_0 = \frac{\gamma_T}{2(3+n)a_q}, \tag{52}$$

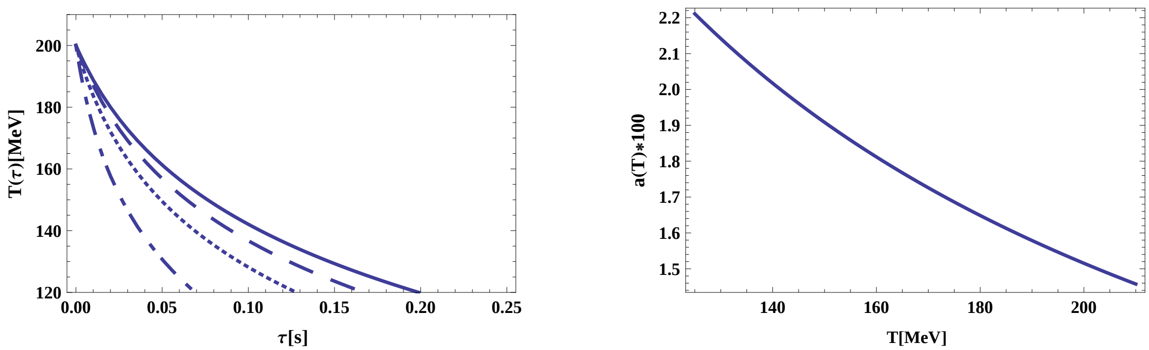
Also, the Friedmann equation is

$$H^2 = \frac{1}{4(3\omega+4)^2\theta^2}\left(12 + 19\omega + 6\omega^2\right)\rho^2. \tag{53}$$

Combining Eqs. (49), (50) and (53), the equation for $\dot{T}$ in the quark gluon phase (QGP) is

$$\dot{T} = -\frac{(3+n)a_q}{(3\omega+4)}\sqrt{\frac{12+19\omega+6\omega^2}{6+6n-\omega n^2}}\left\{\frac{T^3\left[(3a_q - a_T)T^4 + 2\gamma_T T^2 + B + \lambda\right]}{2(3a_q - \alpha_T)T^2 + \gamma_T}\right\}. \tag{54}$$

We numerically integrate this equation and the results are shown in Fig. 1. **Figure 1a shows the decreasing rate of temperature as a function of cosmic time, τ, in quark-gluon phase (QGP), for different values of ω, with $n = 0.015$, $N_0 = 2\times 10^5$, and $\lambda = 10\times 10^8$ MeV^4. This plot shows that by increasing ω the decreasing rate of temperature will be faster and this decreasing is occurred at about $(0.05-0.25)$ nanosecond after the big bang when $T = T_c \approx 125$ MeV. Figure 1b shows the scale factor as a function of temperature, T, in QGP and it clearly indicates an expanding Universe at that time.**



(a) (b)

Figure 1: (a) Temperature as a function of cosmic time in the QGP for $\omega = 10^4$ (solid line), 1.5×10^4 (dashed), 2×10^4 (dot), and 2.5×10^4 (dot-dashed). (b) Scale factor as a function of temperature in the QGP for general $U(T)$. We have set $N = 2 \times 10^5$, $\lambda = 10^9$ Mev^4, $n = 0.015$, and $B^{1/4} = 200$ Mev.

4.2 Evolution of temperature in the QGP for $U(T) = B$

By matching $f = \phi$ in Eq. (37), we have

$$H = -\frac{3}{(3+n)}\frac{\dot{T}}{T}. \tag{55}$$

Integrating of this equation gives the scalar field as a function of temperature,

$$a(T) = cT^{-3/(3+n)}. \tag{56}$$

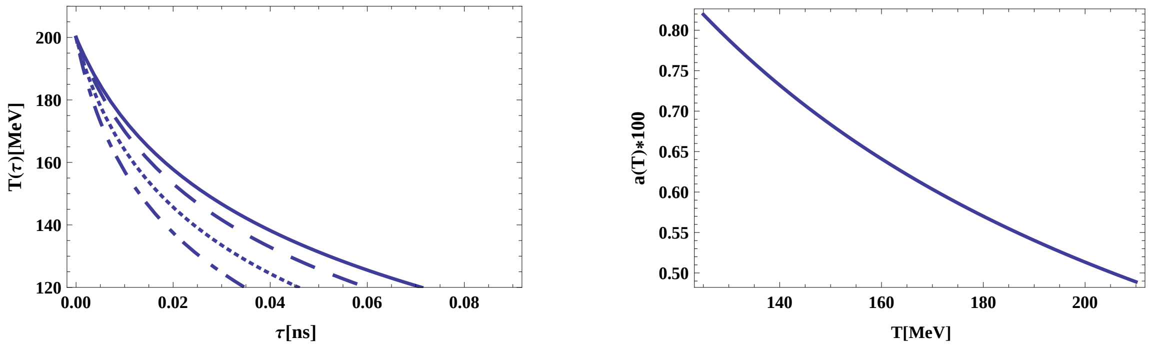
Eq. (35) is became

$$H^2 = \left(\frac{12 + 19\omega + 6\omega^2}{6 + 6n - \omega n^2}\right)\frac{(\rho_q + \lambda)^2}{4(3\omega + 4)^2} \tag{57}$$

Using Eqs. (30), (31),(55), and (57) one obtains an expression for $\dot{T}$

$$\dot{T} = -\frac{(3+n)}{6(3\omega+4)}\sqrt{\frac{12 + 19\omega + 6\omega^2}{6 + 6n - \omega n^2}}\Big[T\big(3a_q T^4 + B + \lambda\big)\Big] \tag{58}$$

We numerically solved this equation and plot the result in Fig. 2. **Figure 2a shows the decreasing of temperature as a function of cosmic time, τ, in quark-gluon phase (QGP), for different values of ω, with $n = 0.015$, $N_0 = 2 \times 10^5$, $\lambda = 10 \times 10^8$ MeV^4 , and $U(T) = B$. This plot shows that by increasing ω the decreasing of temperature will be faster and this decreasing is occurred at about $(0.03 - 0.08)$ nanosecond after the big bang when $T = T_c \approx 125$ MeV. Figure 2a indicates in the $U(T) = B$ case the QGP is occurred earlier than general case, $U(T) = B + \gamma_T T^2 - \alpha_T T^4$. Figure 2b shows the scale factor as a function of temperature, T, in QGP in the $U(T) = B$ case and it clearly indicates an expanding Universe at that time.**



(a) (b)

Figure 2: (a) Temperature as a function of cosmic time in the QGP for $\omega = 8 \times 10^3$ (solid line), 1.4×10^4 (dashed), 2×10^4 (dotted), and 2.6×10^4 (dotted-dashed) and for $U(T) = B$. (b) Scale factor as a function of temperature in QGP. We have set $N = 2 \times 10^5$, $\lambda = 10^9$ Mev^4, $n = 0.015$, and $U(T) = B^{1/4} = 200$ Mev.

4.3 Evolution of hadron volume fraction

As mentioned above the pressure during the phase transition is constant, $p_c \approx 4.6\times10^8\text{MeV}^4$ and the density of quark matter and hadron matter at the transition are $\rho_q \approx 5\times10^9\text{MeV}^4$, $\rho_h \approx 1.38 \times 10^9$ MeV^4, respectively. Therefore, by matching $f(\phi) = 1$ in Eq. (40) we have

$$H = -\frac{r\dot{h}}{(3+n)\big(1+rh\big)} \tag{59}$$

where

$$r \;=\; \frac{\rho_H - \rho_Q}{p_c + \rho_Q} = -5.6 \times 10^{-6}, \tag{60}$$

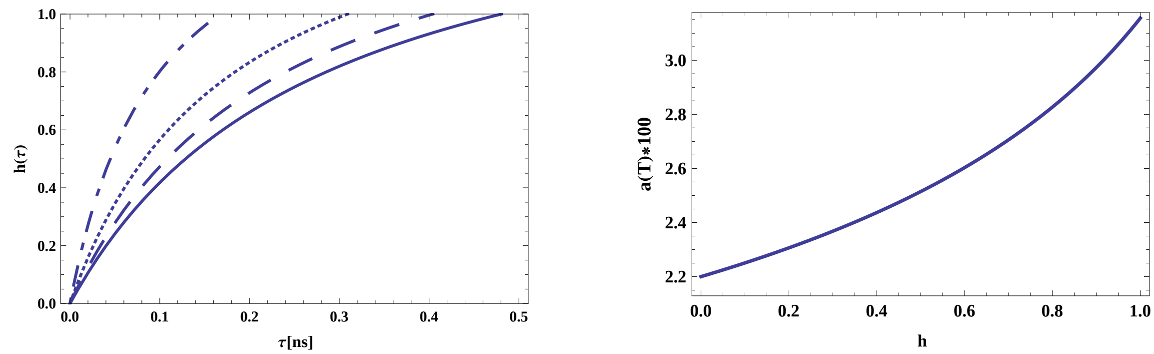
Integrating Eq. (59) gives the scale factor on the brane as a function of the hadronic volume fraction, $h(\tau)$,

$$a(\tau) = a(\tau_c)\Big[1 + rh(\tau)\Big]^{-1/(3+n)}, \tag{61}$$

here we have assumed $h(\tau_c) = 0$. So, using Eq. (53), the time evolution equation of the matter fraction in the hadronic phase is

$$\dot{h} \;=\; -\frac{(3+n)}{2(3\omega+4)}\sqrt{\frac{12+19\omega+6\omega^2}{6+6n-\omega n^2}}\;\frac{(1+rh)}{r}\Big[(1+mh)\rho_Q + \lambda\Big] \tag{62}$$

Numerically evaluated $h(\tau)$'s are presented in Fig. 3 for various values of ω. **Figure 3a shows the hadron volume fraction during the QHPT for $\omega = 10^4$ (solid line), 1.5×10^4 (dashed), 2×10^4 (dotted), 2.5×10^4 (dotted-dashed), as a function of cosmic time. This figure indicates that by increasing the dimensionless parameter of BD model, ω, the rate of quark-hadron phase transition will be faster and QH phase transition takes about $(0.1-0.5)$ nanosecond in a constant temperature. Figure 3b shows the scale factor of the universe during the QHPT as a function of the hadron volume fraction. It is well known that when the QHPT accurse the density of quark gluon plasma decreases but the hadron volume fraction and the scale factor of universe increase. Moreover Fig. 3b states that during the QH phase transition the Universe is expanding.**



(a) (b)

Figure 3: (a) Hadron volume fraction as a function of cosmic time for $\omega = 10^4$ (solid), 1.5×10^4 (dashed), 2×10^4 (dotted), and 2.5×10^4 (dotted-dashed). (b) Scale factor as a function of hadron volume fraction. We have set $N = 2 \times 10^4$, $\lambda = 10^9$ Mev^4, $n = 0.015$, and $B^{1/4} = 200$ Mev.

4.4 Evolution of temperature in the hadronic area

Using the equation of state, Eq. (32), and the energy conservation relation, Eq. (29), we have

$$H = -\frac{3\dot{T}}{(3+n)T}. \tag{63}$$

Integrating this equation gives

$$a(T) = cT^{-3/(3+n)}, \tag{64}$$

and from the Friedmann equation one arrives at

$$\dot{T} \quad = \quad -\frac{(3+n)}{6(3\omega+4)}\sqrt{\frac{12+19\omega+6\omega^2}{6+6n-\omega n^2}}\left[T(3a_\pi T^4+\lambda)\right] \tag{65}$$

We numerically solved Eq. (65) and plot the results in Fig. 4. **Figure 4a shows the temperature as a function of cosmic time in the hadron phase for** $\omega = 10^4$ (solid line)**,** 1.5×10^4 (dashed)**,** 2×10^4 (dotted)**,** 2.5×10^4 (dotted-dashed)**. This figure indicates that by increasing the dimensionless parameter of BD model,** ω**, the rate of decreasing of temperature will be faster and the hadron phase is occurred about** $(1.2-2.5)ns$ **after the big bang. This result is in a good agreement with the expanding Universe in Fig. 1, general case of** $U(T)$**. Figure 3b shows the scale factor of the universe in hadron phase as a function of cosmic time and indicates an expanding Universe in this phase.**

The effective temperature is plotted in Fig. 5 for various (ω, n) for which $\omega n = c$. Figures 5a and 5b show $T(\tau)$ for $(\omega, n) = (10^5, 10^{-3})$, and $(3 \times 10^4, 10^{-2}/3)$ with $\omega n = 100$, and $(\omega, n) = (10^6, 10^{-3})$, and $(3 \times 10^5, 10^{-2}/3)$ with $\omega n = 1000$ respectively. It is seen that these curves are slightly different. We found that for $n \leq 0.015$, $c \leqslant 120$, and $100 \leqslant \omega \leqslant 10^6$, all curves are very similar function of cosmic time while for $c > 120$ the temperature curves are differ slightly.

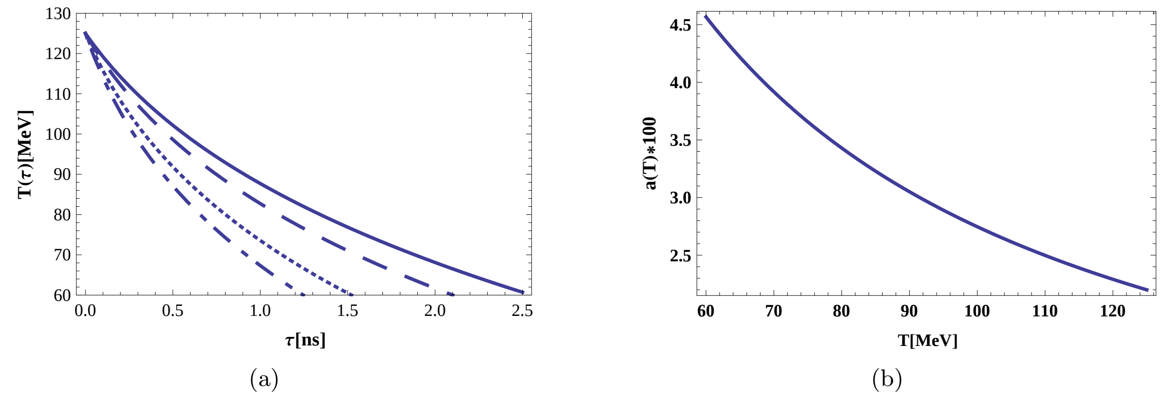


Figure 4: (a) Temperature as a function of cosmic time in the hadronic phase for $\omega = 10^4$ (solid line), 1.55×10^4 (dashed), 2.1×10^4 (dotted), and 2.55×10^3(dotted-dashed). (b) Scale factor as a function of temperature in the hadronic phase. We have set $N = 2 \times 10^5$, $\lambda = 10^9$ Mev^4, $n = 0.015$, and $B^{1/4} = 200$ Mev.

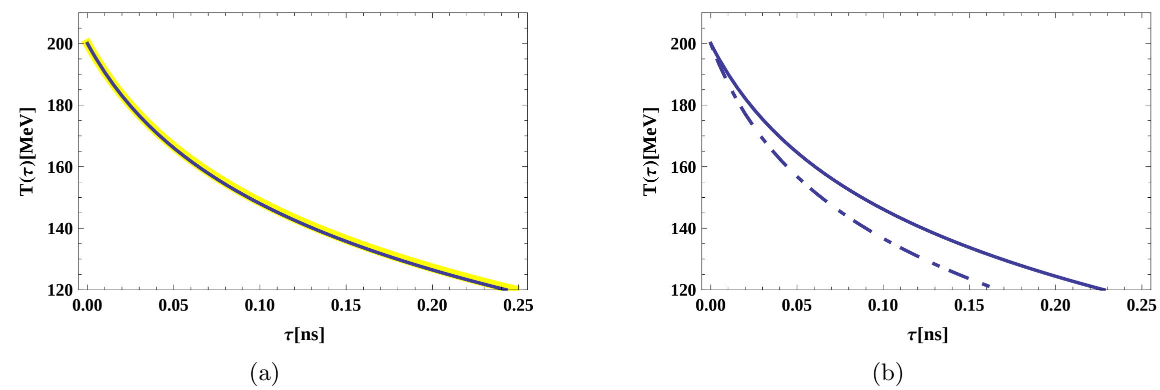


Figure 5: (a) Temperature as a function of cosmic time in the QGP for $(\omega, n) = (10^5, 10^{-3})$, and $(3 \times 10^4, 10^{-2}/3)$ for which $\omega \times n = 100$. (b) Temperature in the QGP for $(\omega, n) = (10^6, 10^{-3})$solid line), and $3 \times 10^5, 0.01/3$(dashed line) for which $\omega \times n = 1000$. We have set $N = 2 \times 10^5$, $\lambda = 10^9$ Mev^4, and $B^{1/4} = 200$ Mev.

5 QCD phase transition

Depending on the values of the quark masses, the phase transition in QCD, characterized by the singular behavior of the partition function, could be a first or second order phase transition, or it could be only a crossover with rapid changes in some observables. In this Section we examine physical quantities related to the quark-hadron phase transition, based on the assumption of a smooth crossover approach, in the CBD model of the brane world scenario.

As mentioned earlier, to study the quark-hadron phase transition we need the equation of state of matter in both the quark and the hadron phase regimes. Different approaches have been used to obtain the equation of state. Recently, detailed computations of the equation of state have been performed using the fermion formulation on lattices with temporal extent $N_t = 4, 6$ [45, 46], $N_t = 8$ [47] and $N_t = 6, 8, 10$ [48]. In the high temperature region, where $T > 250$ MeV, the trace anomaly can be precisely calculated. So one can use the lattice data for the trace anomaly in the high temperature to construct a realistic equation of state. On the other hand in the low temperature region, where $T \lesssim 180$ MeV, the trace anomaly is affected by large discretization effects, but the hadronic resonance gas (HRG) model can be used to determine a realistic low temperature equation of state [49].

5.1 High temperature region

As mentioned above, lattice data for the trace anomaly can be used to determined the equation of state at high temperature, $T > 250$ MeV [49]. In this regime the gluons and quarks are effectively massless so behave like radiation, and one can fit the data to a simple equation of state

$$\rho(T) \approx \alpha_r T^4, \tag{66}$$
$$p(T) \approx \sigma_r T^4. \tag{67}$$

Here $\alpha_r = 14.9702 \pm 009997$ and $\sigma_r = 4.99115 \pm 004474$ are found using a least squares fit [46]. Substituting Eqs. (66) and (67) into Eq. (29), we obtain

$$H = -\frac{4\alpha_r \dot{T}}{(3+n)(\alpha_r+\sigma_r)T}. \tag{68}$$

Integrating Eq. (68) we have

$$a(T) = a_c T^{-4\alpha_r/(3+n)(\alpha_r+\sigma_r)}, \tag{69}$$

where a_c is the constant of integration and, by using Eqs. (35), (66), (67), and (68), $\dot{T}$ is

$$\dot{T} = -\frac{(3+n)(\alpha_r+\sigma_r)}{8\alpha_r(3\omega+4)}\sqrt{\frac{12+19\omega+6\omega^2}{6+6n-\omega n^2}}\,T(\alpha_r T^4+\lambda). \tag{70}$$

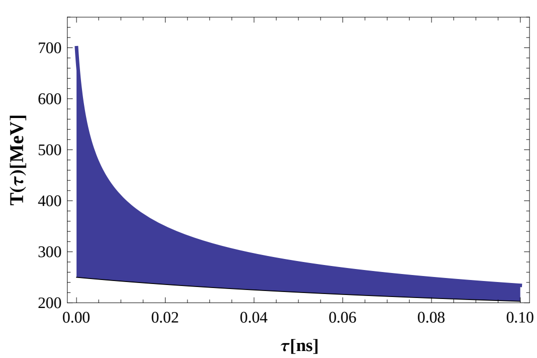


Figure 6: T as a function of cosmic time, τ, for $250 < T < 750$ MeV. We have set $N = 2\times10^5$ and $\lambda = 10^9$.

We numerically integrate Eq. (70) and plot the results in Fig. 6. This figure shows the effective temperature in the QGP in the CBD model of brane gravity for $T > 250$ MeV, obtained for the smooth crossover approach. We see that b the Universe become cooler and the temperature drops to 250 MeV at about $0.02 - 0.1ns$ after the big bang.

5.2 Low temperature region

As mentioned above, the hadronic resonance gas (HRG) model can be used to build a realistic equation of state at low temperatures, $T \lesssim 180$ MeV [49]. In the HRG scenario QCD is treated as a non-interacting gas of fermions and bosons [50]. In fact, the fermions and bosons in this model are mesons and baryons. The basic idea of the HRG model is to implicitly account for the strong interaction in the confinement phase by looking only at hadronic resonances, since these are the relevant low temperature degrees of freedom. In this regime, it is believed that the HRG model provides a reasonable description of thermodynamic quantities.

The HRG result can also be parameterized for the trace anomaly as [49]

$$\frac{I(T)}{T^4} = \frac{\rho - 3p}{T^4} = a_1 T + a_2 T^3 + a_3 T^4 + a_4 T^{10}, \tag{71}$$

where $I(T) = \rho(T) - 3p(T)$ is the trace anomaly, $a_1 = 4.654\ \mathrm{GeV}^{-1}$, $a_2 = -879\ \mathrm{GeV}^{-3}$, $a_3 = 8081\ \mathrm{GeV}^{-4}$, $a_4 = -7039000\ \mathrm{GeV}^{-10}$. In lattice QCD, through the computation of the trace anomaly $I(T)$, one can estimate the pressure, energy density, and entropy density, with the help of the thermodynamics identities. The pressure difference at two temperatures T and $T_{\rm low}$ is an integral of the trace anomaly

$$\frac{p(T)}{T^4} - \frac{p(T_{\rm low})}{T_{\rm low}^4} = \int_{T_{\rm low}}^{T} \frac{dT'}{T'^5} I(T'). \tag{72}$$

By choosing a sufficiently small lower integration limit, $p(T_{\rm low})$ can be neglected due to the exponential suppression. The energy density $\rho(T) = I(T) + 3p(T)$ can be computed. This procedure is known as the integral method [51].

Using Eqs. (71) and (72) we obtain

$$\rho(T) = 3a_0 T^4 + 4a_1 T^5 + 2a_2 T^7 + \frac{7a_3}{4} T^8 + \frac{13a_4}{10} T^{14}, \tag{73}$$

$$p(T) = a_0 T^4 + a_1 T^5 + \frac{a_2}{3} T^7 + \frac{a_3}{4} T^8 + \frac{a_4}{10} T^{14}, \tag{74}$$

where $a_0 = -0.112$. In this step, we consider the era before phase transition at low temperature during which the quark become confinement which can treated as non-interaction gases of fermions and bosons [50]. From the conservation relation, during this epoch we have

$$H = -\frac{\left[12a_0 T^3 + 20a_1 T^4 + B_0(T)\right]\dot{T}}{(3+n)\left[4a_0 T^4 + 5a_1 T^5 + B_1(T)\right]}, \tag{75}$$

where

$$B_0(T) = 14a_2 T^6 + 14a_3 T^7 + \frac{92}{5} a_4 T^{13}, \tag{76}$$

$$B_1(T) = \frac{7}{3} a_2 T^7 + 2a_3 T^8 + \frac{7}{5} a_4 T^{14}, \tag{77}$$

To obtain the scale factor as a function of temperature we must integrate Eq. (75). This can be expended the time derivative of temperature

$$\begin{aligned} \dot{T} &= -\frac{(3+n)}{2(3\omega+4)} \sqrt{\frac{12 + 19\omega + 6\omega^2}{6 + 6n - \omega n^2}} \\ &\times \left\{ \frac{4a_0 T^4 + 5a_1 T^5 + B_1(T)}{12a_0 T^3 + 20a_1 T^4 + B_0(T)} \left(3a_0 T^4 + 4a_1 T^5 + 2a_2 T^7 + \frac{7a_3}{4} T^8 + \frac{13a_4}{10} T^{14} + \lambda \right) \right\} \end{aligned} \tag{78}$$

We numerically integrate Eq. (78) and plot the result in Fig. 7 for the interval $50 < T < 180$ MeV. Figure 7 shows temperature as a function of the cosmic time, τ, in the low energy region for CBD model of brane gravity. This figure shows that, in the low temperature regime of QCD phase transition (crossover transition) where HRD is used, the QGP of the Universe is about 1-10 nanoseconds after the bib bang. One can clearly seen that the QGP in the low region of the smooth crossover approach is occurred later than first order phase transition formalism.

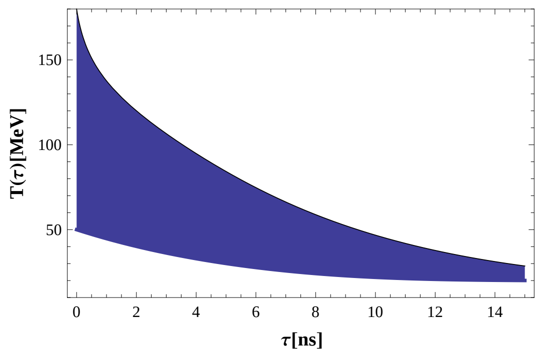


Figure 7: T as a function of cosmic time, τ, in the interval $50 < T < 180$ MeV in the CBD model of brane gravity. We have set $\omega = 10^4$, $n = 0.002$, and $\lambda = 10^9\mathrm{MeV}^4$.

6 Conclusion

In this paper, we have studied the quark-hadron phase transition in a chameleon Brans-Dicke brane-world scenario. We investigated the evolution of the physical quantities relevant to the physical description of the early times such as; energy density, temperature and scale factor, before, during, and after the phase transition. We found that for different values of $n < 0.015$ and $100 \leqslant \omega \leqslant 10^6$ phase transition occur and with increasing time the effective temperature of the quark-gluon plasma and the hadronic fluid will be decrease. We plot the effective temperature and scale factor of the Universe on different stage of phase transition for various values of ω and n. All plots show that the effective temperature and the FLRW scale factor decrease and increase respectively by possing time. **Especially the increasing behavior of the scale factor during the phase transition in first order formalism indicates that in this stage the Universe is expanding although temperature and pressure of the Universe is constant. Our analysis in the first order phase transition formalism shows that the QGP has taken place at about $0.05 - 0.2ns$ after the big bang and phase transition has taken about $0.1 - 0.5ns$ and after that we have the hadronic phase at about $1.2 - 2.5ns$ after the big bang.**

We compared our results with the results presented in [27, 28, 29]. In [27] the authors studied quark hadron phase transition in a Randall Sundrum brane model and have shown that for different values of the barne tension, λ, phase transition occurs about at 10^{-6}s after the big bang. Also in [28, 29], the authors investigated the quark-hadron phase transition in a brane-world scenario where the localization of matter on the brane is achieved through the action of a confining potential and have shown that for different values of parameters in their model, phase transition takes place. They found that for various value of ω the phase transition has taken place about microsecond after the big bang, but our investigation shows that the quark-hadron phase transition has occurred at about nanosecond after the big bang. This is a difference between the results of our study and the studies of other researchers. **This means that due to the interaction between scalar field and matter has made a brane-bulk energy transfer and conservation equation of energy density has been modified. Actually this phenomena change the functionality of the effective temperature with respect to cosmic time and therefore the rate of expansion of the Universe is increased at the early times.**

At last, we studied the smooth crossover approach for quark-hadron phase transition in high and low region of temperature in Sec. 5. We have used the equation of state which are obtained from lattice QCD data. The results of our calculations show that the general behavior of temperature in both of approaches (smooth crossover and first order phase transition) is similar, although the differences in the energy should be taken into account. **In fact, by considering in detailed, one can see that the dropping of temperature in the QGP phase of the Universe in the first order phase transition approach is slower than the high temperature region of the smooth crossover formalism which lattice QCD is used to investigate the equation of state and faster than the low temperature regime of QCD phase transition (crossover transition) where HRD is used for obtaining the matter equation of state.**

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References

[1] K. Olive, Nucl. Phys. B **190**, 483 (1981)

[2] E. Suhonen, Phys. Lett. B **119**, 81 (1982).

[3] M. Crawford and D. Schramm, Nature **298**, 538 (1982).

[4] E. Kolb and M. Turner, Phys. Lett. B **115**, 99 (1982).

[5] D. Schramm and K. Olive, Nucl. Phys. A **418**, 289c (1984).

[6] M. I. Gorenstein, V. K. Petrov and G. M. Zinovjev, Phys. Lett. B **106**, 327 (1981); M. I. Gorenstein, *et al.*, Theor. Math. Phys. **52**, 843 (1982).

[7] M. I. Gorenstein, W. Greiner and Yang Shin Nan, J. Phys. G: Nucl. Part. Phys. **24**, 725 (1998); M. I. Gorenstein, M. Gazdzicki and W. Greiner, Phys. Rev. C **72**, 024909 (1998); I. Zakout, C. Greiner and J. Schaffner-Bielich, Nucl. Phys. A **781**, 150 (2007); I. Zakout and C. Greiner, Phys. Rev. C **78**, 034916 (2008); A. Bessa, E. S. Fraga and B. W. Mintz, Phys. Rev. D **79**, 034012 (2009).

[8] K. Kajantie and H. Kurki- Suonio, Phys.Rev. D **34**, 1719 (1986).

[9] J. Ignatius, K. Kajantie, H. Kurki-Suonio and M. Laine, Phys. Rev. D **49**, 3854 (1994), Phys. Rev. D **50**, 3738 (1994); H. Kurki- Suonio and M. Laine, Phys. Rev. D **51**, 5431 (1995), Phys. Rev. D **54**, 7163 (1996).

[10] M. B. Christiansen, J. Madsen, Phys. Rev. D **53**, 5446 (1996).

[11] L. Rezzolla, J. C. Millerand and O. Pantano, Phys. Rev. D **52**, 3202 (1995); L. Rezzolla and J. C. Miller, Phys. Rev. D **53**, 5411 (1996); L. Rezzolla, Phys. Rev. D **54**, 1345 (1996), Phys. Rev. D **54**, 6072 (1996).

[12] A. C. Davis and M. Lilley, Phys. Rev. D **64**, 043502 (2000).

[13] N. Borghini, W. N. Cottingham and R. Vinh Mau, J. Phys. G **26**, 771 (2000).

[14] H. I. Kim, B. H. Lee and C. H. Lee, Phys. Rev. D **64**, 067301 (2001).

[15] J. Ignatius and D. J. Schwarz, Phys. Rev. Lett. **86**, 2216 (2001).

[16] P. Jordan and Schwerkraft und Weltall (Friedrich Vieveg Und Sohn, Braunschweig, 1955).

[17] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1962).

[18] C. Mathiazhagan and V. B. Johri, Class. Quantum Grav. **1**, L29 (1984); D. La and P. J. Steinhardt, Phys. Rev. Lett.**62**, 376 (1989).

[19] O. Bertolami and P. J. Martins, Phys. Rev. D. **61**, 064007 (2000); N. Banerjee and D. Pavon, Class. Quantum Grav. **18**, 593 (2001); A. A. Sen and S. Sen, Mod. Phys. Lett. A. **16**, 1303 (2001); S. Sen and A. A. Sen, Phys. Rev. D. **63**, 124006 (2001); S. Sen and T. R. Seshadri, Int. J. Mod. Phys. D. **12**, 445 (2003); V. K. Onemli and R. P. Woodard, Class. Quantum Grav. **19**, 4607 (2002).

[20] P. J. E. Peebles and B. Ratra, Astrophys. J. Lett. **325**, L17 (1998); B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1998).

[21] D. F. Mota and J. D. Barrow, Phys. Lett. B **581**, 141 (2004); J. Khoury and A. Weltman, Phys. Rev. Lett. **93**, 171104 (2004), Phys. Rev. D **69**, 044026 (2004).

[22] Ph. Brax, C. van de Bruck, A. C. Davis, J. Khoury and A. Weltman, [astro-ph/0410103].

[23] T. Clifton and J. D. Barrow, [gr-qc/0603116]; S. Das and N. Banerjee [gr-qc/0803.3936]. (2010).

[24] Kh. Saaidi, A. Mohammadi and H. Sheikhahmadi, Phys. Rev. **D 83**, 104019 (2011).

[25] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **125**, 136 (1983).

[26] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999), Phys. Rev . Lett **83**, 4690 (1999).

[27] G. De Risi, T. Harko, F. S. N. Lobo and C. S. J. Pun, Nucl. Phys. B **805**, 190 (2008).

[28] M. Heydari-Fard and H. R. Sepangi, Class. Quantum Grav. **26**, 235021 (2009).

[29] K. Atazadeh, A. M. Ghezelbash and H. R. Sepangi, Class. Quantum. Grav. **28**, 085013 (2011).

[30] K. Umezu, K. Ichiki, T. Kajino, G. J. Mathews, R. Nakamura and M. Yahiro, Phys. Rev. D **73**, 063527 (2006).

[31] Kh. Saaidi, A. Mohammadi, Phys. Rev. D **85**, 023526 (2012); C. Bogdanos and K. Tamvakis, Phys. Lett. B **646**, 39 (2007).

[32] Y. Himemoto and T. Tanaka, Phys. Rev. D **67**, 084014 (2003).

[33] J. H. Brodie and D. A. Easson, JCAP **0312**, 004 (2003); T. Harko, W. F. Choi, K. C. Wong and K. S. Cheng, JCAP **0806**, 002 (2008).

[34] H. Farajollahi and A. Salehi, JCAP **1011**, 006 (2010); H. Farajollahi, A. Salehi, F. Tayebi and A. Ravanpak, JCAP **05**, 017 (2011); H. Farajollahi, A. Ravanpak and G. Farpour Fadakar, arXiv:1106.2659; H. Farajollahi and A. Salehi, JCAP **07**, 036 (2011).

[35] Kh. Saaidi and A. Aghamohammadi, Astrophys.Space Sci. **333**, 327 (2011).

[36] Kh. Saaidi; arXiv:1205.3542.

[37] J. D. Brown and J. W. York, Phys. Rev. D **47**, 1420 (1993); J. D. Brown, Class. Quant. Grav. **10**, 1579 (1993).

[38] C. Germani and R. Maartens, Phys. Rev. D **54**, 124010 (2001).

[39] R. Maartens, D. Wands, B. A. Bassett and I. P. C. Heard, Phys. Rev. D **62**, 041301 (2000).

[40] N. Borghini, W. N. Cottingham and R. Vinh Mau, J. Phys. G **26**, 771 (2000).

[41] P. Binetruy, C. Deffayet and D. Langlois, Nucl. Phys. B **565**, 269 (2000); P. Binetruy, C. Deffayet, U. Ellwanger and D. Langlois, Phys. Lett. B **477**, 285 (2000).

[42] N. Banerjee and D. Pavon, Phys. Lett. B **647**, 447 (2007).

[43] N. Banerjee and D. Pavon, Phys. Rev. D **63**, 043504 (2001), Class. Quantum Grav. **18**, 593 (2001).

[44] B. Bertotti, L. Iess and P. Tortora, Nature **425**, 374 (2003); C. M. Will, Theory and Experiment in Gravitational Physics (Cambridge University Press, Cambridge 1993); V. Acquaviva and L. Verde, JCAP **12**, 001 (2007).

[45] C. Bernard *et al.*, Phys. Rev. D **75**, 094505 (2007); F. Wu, X. Chen, Phys. Rev. D **82**, 083003 (2010).

[46] M. Cheng *et al.*, Phys. Rev. D **77**, 014511 (2008), Phys. Rev. D **81**, 054405 (2010).

[47] A. Bazavov *et al.*, Phys. Rev. D **80**, 014504 (2009).

[48] S. Borsanyi *et al.*, JHEP **1011**, 077 (2010).

[49] P. Huovinen and P. Petreczky, Nucl. Phys. A **837**, 26 (2010).

[50] F. Karsch, K. Redlich and A. Tawfik, Eur. Phys. J. C **29**, 549 (2003), Phys. Lett. B **571**, 67 (2003).

[51] G. Boyd, *et al.*, Nucl. Phys. B **469**, 419 (1996).