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Acceleration and Vacuum Temperature

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The quantum fluctuations of an 'accelerated' vacuum state, that is vacuum fluctuations in presence of a constant electromagnetic field, can be described by a temperature $T_{\rm M}$. Considering $T_{\rm M}$ for the gyromagnetic factor $g = 1$ we show that $T_{\rm M}(g = 1) = T_{\rm U}$, where $T_{\rm U}$ is the Unruh temperature experienced by an accelerated observer. We conjecture that both particle production and nonlinear field effects inherent in the Unruh accelerated observer case are described by the case $g = 1$ QED of strong fields. We present rates of particle production for $g = 0, 1, 2$ and show that the case $g = 1$ is experimentally distinguishable from $g = 0, 2$. Therefore either accelerated observers are distinguishable from accelerated vacuum or there is unexpected modification of the theoretical framework.

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Introduction: A detector in a matter- and field-free spacetime undergoing constant acceleration $a_{\rm U}$ is found to be embedded in a thermal background at the Unruh temperature ($\hbar = c = 1 = k_{\rm B}$)

$$T_{\rm U} = \frac{a_{\rm U}}{2\pi}. \tag{1}$$

The statistics of the thermal distribution are bosonic considering the vacuum of a scalar particle [1, 2] and fermionic in the vacuum of a Fermi particle [3]. Said differently, the free and unstructured vacuum fluctuations appear to an accelerated observer as having an effective temperature $T_{\rm U}$ with statistics corresponding to the fluctuation of either Fermi or Bose type.

A complementary effect was recognized by Müller et al [4] who found that the structured vacuum fluctuations induced by an exactly constant electric field $\mathcal{E}$ (or magnetic field) can be understood as a thermal background characterized by the temperature parameter

$$T_{\rm M} = \frac{e\mathcal{E}}{m\pi}\,. \tag{2}$$

$T_{\rm M}$ arises from the exact solution introduced by Heisenberg and Euler [5] and generalized by Schwinger [6] of vacuum fluctuation properties for constant electromagnetic fields in QED evaluated at lowest order in α.

Since an electric field accelerates all charged particles and in particular the electron-positron pairs whose fluctuations are considered, it is natural to introduce the global acceleration $a_v = e\mathcal{E}/m$ [7] (see p.569 ff) and consider this equivalent to an 'accelerated quantum vacuum' state. A succinct discussion is found in the work of Pauchy Hwang and Kim [8].

Comparing to the Unruh temperature,

$$T_{\rm M} = \frac{e\mathcal{E}}{m\pi} = \frac{a_v}{\pi} = 2T_{\rm U}\,. \tag{3}$$

The factor two in temperature is not the only difference between the accelerated vacuum and accelerated observer. For the case of the accelerated vacuum, Müller et al. [4] show the associated thermal distribution to be opposite expectation, being bosonic for spin-1/2 electron fluctuations and fermionic for spin-0 charged particle fluctuations.

The difference between the physical conditions giving rise to the Unruh and Müller temperatures is *whether it is the observer or the vacuum state that is accelerated.* While frame independence of physics phenomena is assured for inertial observers, there is no imperative need for the two cases we consider, accelerated observer and accelerated vacuum, to yield equivalent results. Owing to the mathematical similarity the two different acceleration cases can be solved by similar methods [9]. For this reason the difference in outcomes Eq. (3) is somewhat unexpected, and our objective here is to identify a possible origin of the discrepancy, which could point to new physics: either modification of the theoretical framework, or inequivalence of accelerated quantum observers, however small acceleration can be.

In QED, the structure of vacuum fluctuations is encoded in the effective action, from which one derives spontaneous particle creation and the associated temperature. The difference arises in connection with the spin and statistics of the particle. Therefore we study the structure of the QED vacuum fluctuations in presence of strong fields for different values of the g-factor. We show that the specific value $g = 1$ reconciles the temperatures and statistics and discuss pair production in strong fields which can help distinguish the accelerated observer from accelerated vacuum state.

Temperature of electron fluctuations: Separate conservation of charge-convective and spin currents means that for any particle the value of the gyromagnetic ratio g can be arbitrary. For point-like electrically charged leptons, quantum corrections result in $g - 2 = \alpha/\pi + ...$, and composite spin 1/2 particles have values which can significantly differ from the Dirac value $g = 2$.

The dynamics of a particle ψ with arbitrary g is generated by the equation of motion

$$\left[D^2 + m^2 - \frac{g}{2}\frac{e\sigma_{\mu\nu}F^{\mu\nu}}{2}\right]\psi = 0, \tag{4}$$

where $D = i\partial + eA$ is the covariant derivative, $F^{\mu\nu}$ the electromagnetic field strength tensor and $\sigma_{\mu\nu} = (i/2)[\gamma_\mu, \gamma_\nu]$. The Eq. (4) comprises a doubling of dynamical components since the 'squared' equation commutes with γ_5. For the specific case $g = 2$ one can cast Eq. (4) in the form of product of two Dirac equations with $\pm m$. We will explicitly show the number of physical degrees of freedom. The effect of g on the vacuum fluctuations is determined computing the effective potential

$$V_{\rm eff} = -\frac{i}{2}{\rm tr}\,\ln\left[D^2 + m^2 - \frac{g}{2}\frac{e\sigma_{\mu\nu}F^{\mu\nu}}{2}\right]. \qquad (5)$$

The Schwinger proper time method [6] can be applied to evaluate Eq. (5) and one finds

$$V_{\rm eff} = \frac{\gamma_s}{32\pi^2}\int_0^\infty e^{-im^2u}\left(\frac{au\cosh(\frac{g}{2}au)}{\sinh(au)}\frac{bu\cos(\frac{g}{2}bu)}{\sin(bu)} - 1\right)\frac{du}{u^3} \qquad (6)$$

in which γ_s counts the number of degrees of freedom. With only bosonic particle and antiparticle degrees of freedom $\gamma_s = -2$ for $g = 0$. When $g = 2$, we have spin-1/2 Dirac fermions, and counting spin degrees of freedom, $\gamma_s = +4$. The -1 inside the parentheses removes the field-independent constant. In Eq. (6), we use a the electric-like and b the magnetic-like eigenvalues of $eF^{\mu\nu}$, which are related to the field strengths by

$$a^2 - b^2 = e^2(\vec{E}^2 - \vec{B}^2) \quad \text{and} \quad (ab)^2 = e^4(\vec{E}\cdot\vec{B})^2. \quad (7)$$

The a eigenvalue is electric-like because $a \to e|\vec{E}|$ in the limit $b \to 0$, and similarly $b \to e|\vec{B}|$ in the limit $a \to 0$.

We discuss here the temperature and statistics for the case of electric-only field; a transformation similar to that detailed below is possible for the general case Eq. (6) [10]. For an electric-only field of strength $\mathcal{E} \equiv |\vec{E}|$, the $b \to 0$ limit of Eq. (6) yields

$$V_{\rm eff} = \frac{\gamma_s}{32\pi^2}\int_0^\infty e^{-im^2u}\left(\frac{e\mathcal{E}u\cosh(\frac{g}{2}e\mathcal{E}u)}{\sinh e\mathcal{E}u} - 1\right)\frac{du}{u^3}. \quad (8)$$

Transforming $V_{\rm eff}$ to a statistical format proceeds via meromorphic expansion of the integrand of Eq. (8) [4]. We introduce the identity

$$1 - \frac{z\cosh(zy)}{\sinh(z)} = -2z^2\sum_{n=1}\frac{\cos n\pi(y+1)}{(n\pi)^2} + 2z^4\sum_{n=1}\frac{\cos n\pi(y+1)}{(n\pi)^2(z^2+(n\pi)^2)} \qquad (9)$$

The first term ($\propto x^2$) is identified as the logarithmically divergent contribution and displays the renormalization of charge.

The finite (regularized and renormalized) effective potential is obtained by inserting only the second term of Eq. (9) in the integrand of Eq. (8). Scaling $u \to -inu\pi/e\mathcal{E} = -inu/mT_{\rm M}$,

$$V_{\rm eff}=\frac{\gamma_s m^2 T_{\rm M}^2}{32\pi^2}\int_0^\infty\frac{2u\,du}{u^2-1+i\epsilon}\sum_{n=1}^\infty\frac{e^{-nu\frac{m}{T_{\rm M}}}}{n^2}\cos\left(n\pi(\frac{g}{2}+1)\right) \qquad (10)$$

Note that we have rotated the integration contour onto the real axis and defined the integration contour in accordance with the assignment

$$m^2 \to m^2 - i\epsilon \equiv m_-^2, \qquad (11)$$

which defines the imaginary part discussed further below. While the real part of $V_{\rm eff}$ controls nonlinear electromagnetic field-field interactions, its imaginary part controls the rate at which the electromagnetic field decays into electron-positron pairs.

Setting $g = 2$ for a spin-1/2 (Dirac) electron, $\cos 2n\pi = 1$ for all n, and setting $g = 0$ for a spin-0 electron, $\cos n\pi = (-1)^n$ producing an alternating sum. In each case, integrating by parts twice and summing the series yields the results of Müller et al [4] which with arbitrary g we present as

$$V_{\rm eff} = \frac{\gamma_s m^2 T_{\rm M}}{64\pi^2}\int_0^\infty dE\,\ln(E^2-m_-^2)\sum_\pm\ln(1+e^{\pm i\pi\frac{g}{2}}e^{-E/T_{\rm M}}). \qquad (12)$$

The sum over $\pm$ ensures the distribution is real so that the imaginary part arises only from the branch cut in the first log factor. The exponential weights of the terms in the series in Eq. (10) generate for integer values of g an exact thermal distribution, and the statistics of the distribution are determined by the phase of the terms in the series.

For $g = 1$ (and more generally for any odd integer value of g) summing in Eq. (12) over $\pm$ simplifies to

$$V_{\rm eff}\Big|_{g=1} = \frac{\gamma_s m^2 T_{\rm U}}{32\pi^2}\int_0^\infty dE\,\ln(E^2-m_-^2)\ln(1+e^{-E/T_{\rm U}}) \quad (13)$$

exhibiting in the second log factor a thermal fermionic distribution controlled by the Unruh temperature, $T_{\rm U}$. The effective potential of a 'classical' spinning electron with $g = 1$ in a constant field thus has the format of a thermodynamic potential with temperature parameter and statistics in agreement with expectations based on the result obtained for an accelerated observer in the (unaccelerated) vacuum of a fermion field.

We thus find that when the gyromagnetic moment of the electron is that of the 'classical' spinning particle $g = 1$, the differences disappear between accelerated observer and accelerated vacuum in both temperature and statistics. This situation is summarized in Table I. It seems that reconciliation of the physics arising under Unruh and Müller experimental conditions implies that we can no longer distinguish an accelerated observer from an accelerated vacuum state. However, in our opinion one must take the computation with $g = 1$ as a method to evaluate the prior result attributed to the accelerated observer case.

Observables: We discuss two observable effects inherent in $V_{\rm eff}$: spontaneous pair production and light-by-light scattering. Experiments seeking either of these effects may one day help resolve the question whether or

	Detector acceleration a_{U} relative to vacuum		Constant Electric Field acceleration $a_v = e\mathcal{E}/m$		
	$g=0$	$g=2$	$g=0$	$g=1$	$g=2$
T	$\frac{a_{\mathrm{U}}}{2\pi}$	$\frac{a_{\mathrm{U}}}{2\pi}$	$\frac{a_v}{\pi}$	$\frac{a_v}{2\pi}$	$\frac{a_v}{\pi}$
statistics	boson	fermion	fermion	fermion	boson

TABLE I: Relation between of accelerated observer in quantum vacuum (Unruh case) to quantum vacuum accelerated by external field (Müller et al. case).

not the two cases, accelerated observer and accelerated vacuum, lead to different physics.

The analyticity of quantum field theory demands that aside of heat fluctuations the accelerated observer also sees a rate of real $e\bar{e}$-pair production. Assuming that $g = 1$ provides an accurate model of the physics seen by an accelerated observer, pair production in this case is obtained according to Heisenberg-Euler-Schwinger for $g = 1$ with the field strength written in terms of acceleration. On the other hand, a strong field applied to the vacuum is expected to produce the usual $g = 2$ pair production [11–19].

We obtain the rate (per unit volume) of spontaneous field decay by pair emission, an effect possible only in the presence of an electric field, equivalently whenever the field invariant $a > 0$, see Eq. (7). The decay rate is controlled by the imaginary part of V_{eff}, which arises from the poles in the integrand of Eq. (8) at $u = in\pi/e\mathcal{E}$ for integer n (or equivalently in Eq. (10) at $u = 1$). The integration contour is defined as in Eq. (11) by assigning a small imaginary constant to the mass before rotating onto the positive real u axis. For the electric-only field,

$$\mathtt{Im}\, V_{\mathrm{eff}} = \frac{\gamma_s m^2 T_{\mathrm{M}}^2}{32\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos(n\pi \frac{g}{2})\, e^{-nm/T_{\mathrm{M}}}. \quad (14)$$

The total probability per unit volume per unit time of decay of the field is twice this imaginary part, $d\Gamma/d^4x = 2\mathtt{Im}\, V_{\mathrm{eff}}$.

Setting $g = 1$ (accelerated observer case) changes the analytic structure of V_{eff}, giving odd-n terms in the sum zero weight. The argument of the exponential is thus doubled,

$$\frac{d\Gamma}{d^4x} = 2\mathtt{Im}\, V_{\mathrm{eff}}\big|_{g=1} = \frac{\gamma_s m^2 T_{\mathrm{U}}^2}{16\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} e^{-nm/T_{\mathrm{U}}}. \quad (15)$$

This change is especially visible in the rate per unit volume of particle emission $d\langle N\rangle/d^4x$, which is given by the first term of the series in Eq. (14) [16]. Seeing that the $n = 1$ term vanishes in Eq. (14), the $n = 2$ term becomes the first term in the series, *effectively* halving the temperature to become the Unruh temperature,

$$\frac{d\langle N\rangle}{d^4x}\bigg|_{g=1} = \frac{\gamma_s m^2 T_{\mathrm{U}}^2}{32\pi} e^{-m/T_{\mathrm{U}}}. \quad (16)$$

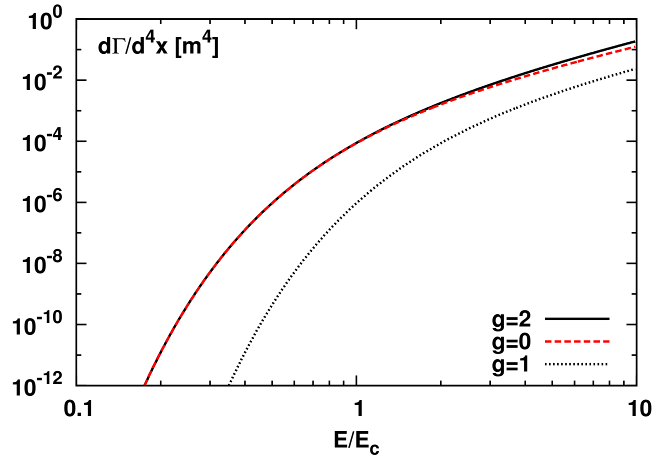

FIG. 1: The rate per unit volume of decay of the field $d\Gamma/d^4x = 2\mathtt{Im}\, V_{\mathrm{eff}}$ with $\mathtt{Im}\, V_{\mathrm{eff}}$ given by Eq. (14). The electric field magnitude is normalized to $\mathcal{E}_c = m^2/e$ the critical field strength, at which $T_{\mathrm{M}} \to m/\pi$. For $g \neq 2$ the rate of field decay is reduced with the largest reduction for $g = 1$. Above $\mathcal{E}_c$ we see suppression due to the g-factor modifying weights in the sum in Eq. (14).

This notably shows the same numerical factors as the analogous result for $g = 0, 2$ after substitution of the Unruh temperature $T_{\mathrm{U}} = T_{\mathrm{M}}/2$, as can be expected considering the analytic properties of the effective action Eq. (12) and the $g = 1$ form Eq. (13).

Figure 1 shows Eq. (14) for the values $g = 0; 1; 2$. The results for $g = 0; 2$ are very similar and yield the largest total decay probability as function of g. The reduction in the rate driven by the effective temperature parameter is largest for the particular case $g = 1$. Due to the exponential dependence, the reduction in the temperature parameter by factor 2 reduces spontaneous pair production below the critical field $\mathcal{E}_c = m^2/e$ by many orders of magnitude.

The real part of V_{eff} leads to the nonlinear field-field interaction. For $g = 1$ one finds,

$$V_{\mathrm{eff}}\Big|_{g=1} \simeq \frac{\gamma_s}{32\pi^2} \frac{e^4}{m^4} \frac{-1}{5760} \left(7(\vec{B}^2 - \vec{E}^2)^2 + 4(\vec{E}\cdot\vec{B})^2\right) \quad (17)$$

The terms higher order in the field are given in [20]. Relative to the $g = 2$ values, the coefficients of $(\vec{B}^2 - \vec{E}^2)^2$ and $(\vec{E}\cdot\vec{B})^2$ in Eq. (17) are opposite in sign and suppressed: for light-by-light scattering experiments the important $(\vec{E}\cdot\vec{B})^2$ term is 224 times smaller.

Discussion and conclusions: In a constant electric field $\mathcal{E} \equiv a > 0$, the electron fluctuations display a thermal Bose spectrum with temperature $T_{\mathrm{M}} = e\mathcal{E}/m\pi = a_v/\pi$. This result contrasts with the Fermi spectrum and the Unruh temperature $T_{\mathrm{U}} = a_{\mathrm{U}}/2\pi$ experienced by an accelerated observer. We discovered and exploited the coincidence that case $g = 1$ used in accelerated vacuum produces physics relevant to the case of an accelerated observer. It is important to recognize that we have not, and in general cannot resolve the question why we should, or should not, expect that the two cases, accelerated observer and accelerated vacuum to yield different or the same physics.

We have evaluated the effective QED potential of a $g = 1$ 'electron' in presence of a constant electric field Eq. (13) finding the form of the QED effective potential with the Unruh temperature and fermionic statistics appropriate for the physics of an observer accelerated in the electromagnetic force field. Considering the quantum fluctuations of a 'classical spinning particle' $g = 1$ thus *describes* the Unruh result within the effective Heisenberg-Euler-Schwinger action. We argued that the computation with $g = 1$ is providing the complete effective potential generating the physics of an accelerated observer.

Two effects could be used to distinguish future the accelerated observer $g = 1$ from the QED vacuum $g = 2$: $e\bar{e}$-pair production in strong electric fields and nonlinear field-field interaction. We have shown that both are greatly suppressed in the case $g = 1$ relative to the QED $g = 2$ expectation. QED strong field experiments such as light-field scattering [21, 22] will, if accelerated observer case prevails, be seeking a much weaker signal.

This proves measurability of the difference between the frames down to arbitrarily small acceleration. Being able to determine who is accelerated means that there is a universal class of inertial reference frames. Introduction of a class of inertial reference frames realizes within the quantum theory the Einstein view of the Mach Principle. The Einstein-Mach Principle is incorporated in both the Unruh-type calculation (by comparing to the vacuum of flat (Minkowski) space) and the QED effective action (by renormlizing with respect to the zero-particle state).

Experiment has not yet tested properties of the vacuum of quantum electrodynamics associated with the critical field strength $\mathcal{E}_c = m^2/e$, a value considerably beyond the limiting field of Born-Infeld theory [23] and even beyond limits set considering precision strong field tests [24]. For this reason it is necessary to ascertain that QED of strong fields, which differs from the expectations based on equivalent accelerated observer is indeed different.

Should the strong-field QED experiment observe the original $g = 2$ results, one would infer a difference in temperatures Eq. (1) and Eq. (2), and it follows that the two views of acceleration are not equivalent for any magnitude of the acceleration. Note that the limit of weak acceleration is achieved in QED by considering fields smoothly varying on compact spatial domain. On the other hand, the authors are not aware of a treatment of the Unruh detector in which the accelerated observer is smoothly connected to asymptotic inertial frames. If one insists on equivalence of the accelerated observer and accelerated vacuum, our result therefore suggests that there is additional, undiscovered physics content in the properties of the Unruh accelerated detector.

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