Non-Abelian gauged Nambu–Jona-Lasinio models on the lattice

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Non-abelian gauged NJL models on the lattice

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We use Monte Carlo simulation to probe the phase structure of a $SU(2)$ gauge theory containing N_f Dirac fermion flavors transforming in the fundamental representation of the group and interacting through an additional four fermion term. Pairs of physical flavors are implemented using the two tastes present in a reduced staggered fermion formulation of the theory. The resultant lattice theory is invariant under a set of shift symmetries which correspond to a discrete subgroup of the continuum chiral-flavor symmetry. The pseudoreal character of the representation guarantees that the theory has no sign problem. For the case of $N_f = 4$ we observe a crossover in the behavior of the chiral condensate for strong four fermi coupling associated with the generation of a dynamical mass for the fermions. At weak gauge coupling this crossover is consistent with the usual continuous phase transition seen in the pure (ungauged) NJL model. However, if the gauge coupling is strong enough to cause confinement we observe a much more rapid crossover in the chiral condensate consistent with a first order phase transition

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I. INTRODUCTION

Elucidating the nature of the electroweak symmetry breaking sector of the Standard Model (SM) is the main goal of the Large Hadron Collider currently running at CERN. It is widely believed that the simplest scenario involving a single scalar Higgs field is untenable due to the fine tuning and triviality problems which arise in scalar field theories. One natural solution to these problems can be found by assuming that the Higgs sector in the Standard Model arises as an effective field theory describing the dynamics of a composite field arising from strongly bound fermion-antifermion pairs.

One class of models that have been proposed which exhibit these features are technicolor theories (TC) [1, 2] in which a new non-abelian gauge interaction causes the condensation at low energy of fermion bound states which are presumed to carry electroweak quantum numbers. These condensates break the electroweak gauge group, giving mass to the W and Z bosons. The realization that theories of this type utilizing fermions in two index representations of the gauge group may offer an explanation of dynamical symmetry breaking which is *not* at variance with electroweak precision measurements [3] has led to numerous recent lattice studies - see the conference reviews [4–7] and references therein.

However, to obtain fermion masses in these scenarios requires additional model building, as in extended technicolor models [8–11] and models of top-condensation [12–15]. In the latter models four-fermion interactions drive the formation and condensation of a scalar top–anti-top bound state which plays the role of the Higgs at low energies.

Our motivation in this paper is to study how the inclusion of such four fermion interactions may influence the phase structure and low energy behavior of non-abelian gauge theories in general. Specifically we have examined a model with both gauge interactions and a chirally invariant four fermi interaction - a model known in the literature as the gauged NJL model [16]. The original NJL model [17] without gauge interactions is known to exhibit spontaneous breaking of chiral symmetry for large four fermi coupling. These models have been studied extensively on the lattice [18, 19]. In the vicinity of the phase transition between chirally symmetric and broken phases, the theory is thought to be renormalizable and to correspond to an elementary scalar field theory coupled to fermions [20]. As such, the continuum limit is believed to be governed by the usual IR attractive gaussian fixed point characteristic of scalar field theory[1].The abelian gauged NJL model has been studied on the lattice as well [21], in which numerical evidence for the triviality of QED was presented.

The focus of the current work is to explore the phase diagram when fermions are charged under a non-abelian gauge group. Indeed, arguments have been given in the continuum that the gauged NJL model may exhibit different critical behavior at the boundary between the symmetric and broken phases [2] corresponding to the appearance of a line of new fixed points associated with a mass anomalous dimension varying in the range $1 < \gamma_\mu < 2$ [16, 22]. The evidence for this behavior derives from calculations utilizing the ladder approximation in Landau gauge to the Schwinger-Dyson equations. A primary goal of the current study was to use lattice simulation to check the validity of these conclusions and specifically to search for qualitatively new critical behavior in the gauged model as compared to the pure NJL theory. While we will present results that indicate that the phase structure of the gauged NJL model is indeed different from pure NJL, we shall argue that our results are *not*

[1] The authors wish to thank Julius Kuti and Anna Hasenfratz for important discussions on these issues

[2] Notice that the appearance of a true phase transition in the gauged NJL models depends on the approximation that we can neglect the running of the gauge coupling

consistent with the presence of any new fixed points in the theory. Related work on the relation of the deconfinement and chiral phase transition can be found in [23].

To facilitate this study we have chosen to employ a reduced staggered fermion lattice formalism. This has the advantage of allowing us to incorporate as few as two continuum flavors of Dirac fermion and, as we will show in Section III, allows us to build in lattice four fermi terms which are invariant under a discrete subgroup of the continuum chiral symmetries [24, 26]. The presence of four fermi interactions has an additional attractive feature - it allows us to study the lattice theory with exactly zero fermion mass [27]. Thus the observation of a non zero condensate corresponds, in the infinite volume limit, to a spontaneous breaking of lattice chiral symmetry and the dynamical generation of quark masses. This discrete symmetry breaking should correspond in the continuum limit to a breaking of the usual continuous chiral-flavor symmetry. The price one pays for this simplicity is that the lattice fermion operator possesses small eigenvalues (at least for small four fermi coupling) and it has only been possible to study modest lattice volumes using a GPU accelerated code. Nevertheless the results show no strong volume dependence and should give a robust indication of the phase structure of the theory in the infinite volume limit.

In the work reported here we have concentrated on the four flavor theory corresponding to two copies of the basic Dirac doublet used in the lattice construction. The four flavor theory is expected to be chirally broken and confining at zero four fermi coupling. Understanding the effects of the four fermion term in this theory can then serve as a benchmark for future studies of theories which, for zero four fermi coupling, lie near or inside the conformal window. In the latter case the addition of a four fermion term will break conformal invariance but in principle that breaking may be made arbitrarily small by tuning the four fermi coupling. It is entirely possible that the phase diagrams of such conformal or walking theories in the presence of four fermi terms may exhibit very different features than those seen for a confining gauge theory.

In the next section we write down the continuum theory we are studying and explain how to rewrite it in a more convenient *twisted basis* in which the two usual Dirac spinors of the theory are replaced by a single matrix valued fermion field. This is the same transformation that lies at the heart of recent efforts to construct lattice theories with exact supersymmetry [28] and corresponds also to the spin-taste representation of staggered fermions [24]. We then show how to discretize this twisted two fermion theory to arrive at a reduced staggered fermion lattice theory which incorporates the Yukawa interactions needed to generate the four fermi terms [26]. We then describe the exact symmetries of the lattice action relating them to the chiral-flavor symmetries of the continuum theory. The pseudoreal character of the fundamental representation of the $SU(2)$ group allows us to avoid a potential sign problem after integration over the fermions.

We then go on to describe our numerical results on the phase diagram for the four flavor theory. We have simulated the model by sweeping in the four fermi coupling for a fixed gauge coupling. A series of these gauge couplings were examined which span the range from confined to deconfined regimes of the gauge theory in the absence of four fermion terms. We show that the chiral phase transition expected in the simple NJL model changes character in the gauged model; strictly speaking the gauge model (at least for four flavors) already breaks chiral symmetry spontaneously even for zero four fermi coupling so that no true transition is present. Nevertheless we observe a very rapid crossover behavior for strong four fermi coupling and recover evidence for would be Goldstone bosons above the crossover region. We see no evidence for the existence of new UV fixed points in the theory.

II. CONTINUUM GAUGED NJL MODEL

We will consider a model which consists of $N_f/2$ doublets of gauged massless Dirac fermions in the fundamental representation of an $SU(2)$ gauge group and incorporating an $SU(2)_L \times SU(2)_R$ chirally invariant four fermi interaction. The action for a single doublet takes the form

$$S = \int d^4x\, \overline{\psi}(i\partial\!\!\!/ - A\!\!\!/)\psi - \frac{G^2}{2N_f}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau^a\psi)^2] - \frac{1}{2g^2}Tr[F_{\mu\nu}F^{\mu\nu}], \tag{1}$$

where G is the four-fermi coupling, g the usual gauge coupling and $\tau^a, a = 1\ldots3$ are the generators of the $SU(2)$ flavour group.

This theory has been explored in the continuum using approximations to the Schwinger-Dyson equations in which sub-classes of planar loop diagrams are re-summed. This "ladder" approximation neglects the running of the four-fermion interaction, and treats the running of the gauge coupling in only a heuristic way, implementing the momentum dependence of the non-abelian gauge coupling by hand. In this approximation, the Schwinger-Dyson equation for the fermion two point function is

$$S_F^{-1}[p\!\!\!/] = (S_F^{(0)}[p\!\!\!/])^{-1} + iG^2\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\; S_F[k\!\!\!/] - 4\pi C_2(F)\int\frac{d^4k}{(2\pi)^4}\alpha[p-k]\gamma^\mu S_F[k\!\!\!/]\gamma^\nu D_{\mu\nu}[p-k], \tag{2}$$

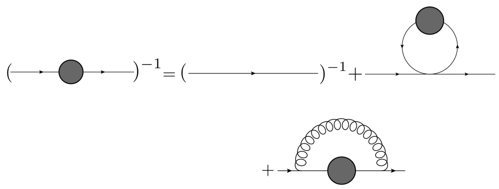

FIG. 1. Diagramatic truncated Schwinger-Dyson equation for the fermion self-energy

where the gauge boson propagator is taken to be in Landau gauge, such that this self-energy term is finite without taking into account vertex corrections. This equation is expressed diagrammatically in Figure 1. A typical ansatz for the functional form of the gauge coupling is

$$\alpha[p^2] = \begin{cases} \frac{\alpha[\Lambda^2]}{1-\alpha[\Lambda^2]\frac{b}{8\pi}\log\frac{p^2}{\Lambda^2}} & p^2 > \Lambda_c \\ \alpha(\Lambda_c^2) & p^2 \leq \Lambda_c \end{cases}, \tag{3}$$

where b is the β-function coefficient, and Λ_c is the confinement scale for the gauge theory.

Approximate analytic numerical solutions for the fermion self-energy were studied in [29]. It was argued that in asymptotically free theories which themselves confine and generate a chiral condensate, the second order NJL phase transition in the ungauged NJL case morphs with increasing gauge coupling into a cross-over phenomenon where the chiral condensate is dramatically enhanced at a critical value for the four fermi coupling. Our analysis constitutes a non-perturbative exploration of this crossover phenomenon in the two dimensional parameter space of the bare gauge and four fermi couplings.

To implement the theory in Eq. 1 on the lattice, it is convenient to reparametrize the four fermi term in the continuum action via the use of scalar auxiliary fields. Specifically the fermion interaction term is replaced by Yukawa and scalar mass terms

$$S_{\rm aux} = \int d^4x \, \frac{G}{\sqrt{N_f}} \left(\bar{\psi}\psi\phi_4 + \bar{\psi}i\gamma_5\tau^a\psi\phi_a\right) + \frac{1}{2}\left(\phi_4^2 + \phi_a^2\right). \tag{4}$$

The action is now quadratic in the fermions and that portion of the path integral can be performed analytically. This yields the usual fermion determinant as a function of the scalar field configurations which are then numerically integrated over.

It is convenient when we come to discretization to rewrite the fermionic sector of this theory in terms of a new set of matrix valued fields. To see how these arise consider first a system of four Dirac fermions ψ^i_α where $i = 1 \ldots 4$ is a flavor index and α a spinor index (initially consider a model without Yukawa interactions). If we denote the matrix implementing the usual space-time rotations by $R_{\alpha\beta}$ and the corresponding one for flavor rotations by F^{ij} then these fermions transform as

$$\psi^i_\alpha = F_{ij} R^{\alpha\beta} \; \psi^j_\beta. \tag{5}$$

Using only lower indices this can be trivially rewritten as

$$\psi_{i\alpha} = F_{ij} \; \psi_{j\beta} \; R^T_{\beta\alpha}. \tag{6}$$

Thus under the diagonal subgroup corresponding to equal rotations in flavor and space, $R = F$, one can treat the fermions as matrix valued fields, Ψ.

In this formalism there is a natural way to reduce the number of degrees of freedom from four to two; introduce the projected matrices

$$\Psi \to \frac{1}{2}\left(\Psi - \gamma_5\Psi\gamma_5\right), \qquad \overline{\Psi} \to \left(\overline{\Psi} + \gamma_5\overline{\Psi}\gamma_5\right) \tag{7}$$

More explicitly in a chiral basis this implies that the matrix fields take the block matrix form

$$\Psi = \begin{pmatrix} 0 & \psi_R \\ \psi_L & 0 \end{pmatrix}, \qquad \overline{\Psi} = \begin{pmatrix} \overline{\psi}_L & 0 \\ 0 & \overline{\psi}_R \end{pmatrix}. \tag{8}$$

Note that while Ψ is a 4×4 matrix field the fields ψ_R and ψ_L are just 2×2 matrix fields each of which can be thought of as corresponding to 2 flavors of Weyl fermion. This can be confirmed by computing the kinetic term which now reads

$$\int \mathrm{Tr}(\overline{\Psi}\gamma_\mu\partial_\mu\Psi) = \overline{\psi}_L\sigma_\mu\partial_\mu\psi_L + \overline{\psi}_R\overline{\sigma}_\mu\partial_\mu\psi_R \tag{9}$$

where $\sigma_\mu = (\sigma_i, iI)$. Furthermore, Yukawa type interactions of the form $\overline{\psi}_L\phi\psi_R + \overline{\psi}_R\phi^\dagger\psi_L$ can also be written in (projected) matrix form as

$$\mathrm{Tr}\left(\overline{\Psi}\Psi\Phi\right), \tag{10}$$

where

$$\Phi = \begin{pmatrix} 0 & \phi \\ \phi^\dagger & 0 \end{pmatrix} = \phi_\mu\gamma_\mu, \tag{11}$$

with the 2×2 matrix $\phi = \phi_4 I + i\phi_i\tau_i$. These Yukawa interactions are chirally invariant if the scalar field ϕ_μ transforms appropriately. In the end we can use these Yukawa terms to build four fermi interactions by adding a quadratic term for the scalar field of the form $\frac{1}{2}\phi_\mu^2$ and subsequently integrating out ϕ_μ.

III. DISCRETIZATION ON A LATTICE

The reason that we have recast the continuum theory in this language of matrix twisted fields is that it admits a simple transcription to the lattice where it becomes the well known reduced staggered formulation of lattice fermions.

We start with the matrix fields Ψ and $\overline{\Psi}$ introduced in the last section, for the moment considering the unprojected matrices. We then expand these matrices on a basis corresponding to products of gamma matrices and associate these products with staggered fields χ, $\overline{\chi}$.

$$\Psi(x) = \frac{1}{8}\sum_b \gamma^{x+b}\chi(x+b), \tag{12}$$

$$\overline{\Psi}(x) = \frac{1}{8}\sum_b (\gamma^{x+b})^\dagger\overline{\chi}(x+b), \tag{13}$$

where $\gamma^{x+b} = \prod_{i=1}^4 \gamma_i^{x_i+b_i}$ and the sums correspond to the vertices in an elementary hypercube associated with lattice site x as the components vary $b_i = 0, 1$ [26, 30]. It is easy to see that the projected matrix fields introduced in the continuum construction then merely correspond to restricting the staggered fields χ and $\overline{\chi}$ to odd and even lattice sites respectively via

$$\chi(x) \to \frac{1}{2}[1-\epsilon(x)]\,\chi(x)\ \ ,\ \ \overline{\chi}(x) \to \frac{1}{2}[1+\epsilon(x)]\,\overline{\chi}(x), \tag{14}$$

where, $\epsilon(x) = (-1)^{x_1+x_2+x_3+x_4}$. Furthermore since χ and $\overline{\chi}$ now live in different sites on the lattice we can drop the bar on $\overline{\chi}$ and write everything in terms of a single staggered field χ defined on all sites. This restriction of the single component fields χ and $\overline{\chi}$ reduces the number of degrees of freedom by a factor of two so the continuum limit of this lattice theory contains two Dirac fermion flavors. The free reduced staggered kinetic action can therefore be recast as

$$\begin{aligned} S_{\rm kin} &= \frac{1}{64}\sum_{x,\mu} {\rm Tr}\left[\overline{\Psi}(x)\gamma_\mu\Psi(x+\mu)\right] \qquad (15)\\ &= \frac{1}{64}\sum_{x,\mu,b,b'} \chi(x+b)\chi(x+\mu+b') \\ &\quad\times {\rm Tr}\left((\gamma^{x+b})^\dagger\gamma_\mu\gamma^{x+b'+\mu}\right) \\ &= \sum_{x,\mu}\eta_\mu(x)\chi(x)\chi(x+\mu) \qquad (16)\end{aligned}$$

Here, we have substituted the matrix expressions given in Eq. (13) into the free Dirac action having replaced the continuum derivative with a symmetric difference operator and evaluated the trace as $4\delta_{b,b'+\mu}\eta_\mu(x)$ where $\eta_\mu(x) = (-1)^{\sum_1^{\mu-1} x_\mu}$ is the usual staggered quark phase. Gauging the reduced staggered theory we obtain [24]

$$S_{kin} = -\sum_{x,\mu}\frac{1}{2}\eta_\mu(x)[\overline{\chi}^T(x)\mathcal{U}_\mu(x)\chi(x+\mu)] \tag{17}$$

where

$$\mathcal{U}_\mu(x) = \frac{1}{2}[1+\epsilon(x)]\,U_\mu(x) + \frac{1}{2}[1-\epsilon(x)]\,U^*_\mu(x). \tag{18}$$

Finally, the Yukawa interactions from equation (10) on the lattice take the form:

$$\begin{aligned} S_{\rm Yuk} &= {\rm Tr}\left(\overline{\Psi}(x)\Psi(x)\Phi(x)\right) \qquad (19)\\ &= \sum_{x,b,b'}\overline{\chi}(x+b)\chi(x+b')\phi_\mu(x) \\ &\quad\times {\rm Tr}\left((\gamma^{x+b})^\dagger\gamma_\mu\gamma^{x+b'}\right) \\ &= \sum_{x,\mu}\chi(x)\chi(x+\mu)\overline{\phi}_\mu(x)\epsilon(x)\xi_\mu(x), \qquad (20)\end{aligned}$$

where the trace evaluation now leads to $4\delta_{b,b'\pm\mu}\xi_\mu(x)$ with the phase $\xi_\mu(x) = (-1)^{\sum_{i=\mu+1}^4 x_i}$ and

$$\overline{\phi}_\mu(x) = \frac{1}{16}\sum_b \phi_\mu(x-b). \tag{21}$$

Notice that if we assign the scalar to the dual lattice this latter expression simply represents the average of the scalar field over the dual hypercube associated with a given lattice site. Combining Eqs. (17) and (20), the gauged massless action including Yukawa interactions can be written in terms of a reduced staggered field as

$$S = \sum_{x,\mu}\chi^T(x)\,\mathcal{U}_\mu(x)\,\chi(x+\mu)\,[\eta_\mu(x)+G\,\overline{\phi}_\mu(x)\,\epsilon(x)\,\xi_\mu(x)]. \tag{22}$$

Notice that no single site mass term is allowed in this model and the mass term that appears in eqn. 22 corresponds to one out of the sixteen possible mass terms defined in the hypercube that are allowed for a staggered fermion action - see the work by Goltermann and Smit [25]. The two staggered tastes become the two physical quark flavors in the continuum limit and as we will see the lattice action possesses additional discrete symmetries which form a subgroup of the continuum chiral-flavor symmetries.

IV. SYMMETRIES OF THE LATTICE THEORY

Clearly the theory is invariant under the $U(1)$ symmetry $\chi(x) \to e^{i\alpha\epsilon(x)}\chi(x)$ which is to be interpreted as the $U(1)$ symmetry corresponding to fermion number. More interestingly it is also invariant under certain shift symmetries given by

$$\chi(x) \to \xi_\rho(x)\chi(x+\rho), \tag{23}$$

$$U_\mu(x) \to U^*_\mu(x+\rho), \tag{24}$$

$$\phi_\mu(x) \to (-1)^{\delta_{\mu\rho}}\phi_\mu(x+\rho). \tag{25}$$

The transformed action is given by

$$S = \sum_{x,\mu} \xi_\rho(x)\chi^T(x+\rho)\,\mathcal{U}_\mu(x+\rho)\xi_\rho(x+\mu)\chi(x+\mu+\rho)\eta_\mu(x)\left(1+G\overline{\phi}(x+\rho)(-1)^{\delta_{\mu\rho}}(-1)^{x_\mu}\right), \tag{26}$$

where we have used the result $\xi_\mu(x)\epsilon(x) = (-1)^{x_\mu}\eta_\mu(x)$. Therefore, shifting the summation vector $x \to x-\rho$ and assuming periodic boundary conditions, the transformed action can then be rewritten

$$S = \sum_{x,\mu} \chi(x)^T\,\mathcal{U}_\mu(x)\chi(x+\mu)A(\mu,\rho)\left(1+G\overline{\phi}(x)(-1)^{x_\mu}\right) \tag{27}$$

where we have used the additional identities

$$\xi_\rho(x)\xi_\rho(x+\mu) = \xi_\rho(\mu) \tag{28}$$

$$\eta_\mu(x+\rho) = \eta_\mu(x)\eta_\mu(\rho). \tag{29}$$

and

$$A(\mu,\rho) = [\xi_\rho(\mu)\eta_\mu(\rho)] = 1 \tag{30}$$

Hence the action is invariant under the original shift symmetry. These shift symmetries correspond to a *discrete* subgroup of the continuum axial flavor transformations which act on the matrix field Ψ according to

$$\Psi \to \gamma_5\Psi\gamma_\rho \tag{31}$$

V. NUMERICAL RESULTS

We have used the RHMC algorithm to simulate the lattice theory with a standard Wilson gauge action being employed for the gauge fields. Upon integration over the basic fermion doublet we obtain a Pfaffian Pf(M(U)) depending on the gauge field [3]. The required pseudofermion weight for N_f flavors is then $\mathrm{Pf}(M)^{N_f/2}$. The pseudoreal character of $SU(2)$ allows us to show that the Pfaffian is purely real [4] and so we are guaranteed to have no sign problem if we use multiples of four flavors corresponding to a pseudofermion operator of the form $(M^\dagger M)^{-\frac{N_f}{8}}$. The results in this paper are devoted to the case $N_f = 4$. Notice that while the four flavor theory inherits an additional $SU(2)$ vector symmetry associated to having now two reduced staggered fields no new continuous chiral symmetries appear. We have utilized a variety of lattice sizes: 4^4, 6^4, 8^4 and $8^3\times 16$ and a range of gauge couplings $1.8 < \beta \equiv 4/g^2 < 10.0$. To determine where the pure gauge theory is strongly coupled and confining we have examined the average Polyakov line as β varies holding the four fermi coupling fixed at $G = 0.1$. This is shown in Figure 2. We see a strong crossover between a confining regime for small β to a deconfined regime at large β. The crossover coupling is volume dependent and takes the value of $\beta_c \sim 2.4$ for lattices of size $L = 8$. For $\beta < 1.8$ the plaquette drops below 0.5 which we take as indicative of the presence of strong lattice spacing artifacts and so we have confined our simulations to larger values of β. We have set the fermion mass to zero in all of our work so that our lattice action possesses the series of exact chiral symmetries discussed earlier.

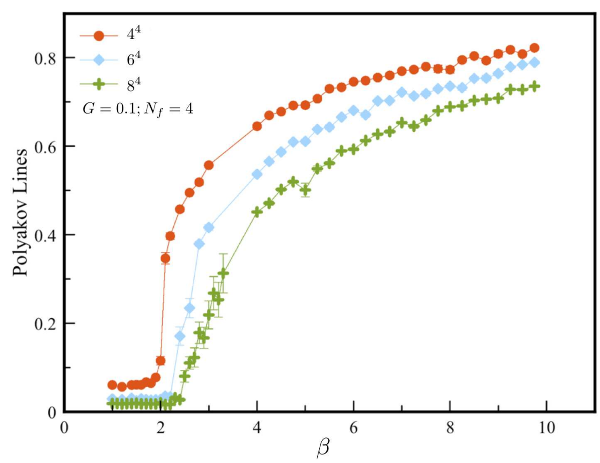

FIG. 2. Polyakov loop vs β at $G = 0.1$ with $N_f = 4$

One of the primary observables used in this study is the chiral condensate which is computed from the gauge invariant one link operator

$$\chi(x)\left(\mathcal{U}_\mu(x)\chi(x+\mu) + \mathcal{U}^\dagger_\mu(x-\mu)\chi(x-e_\mu)\right)\epsilon(x)\xi_\mu(x) \tag{32}$$

Because of the absence of spontaneous symmetry breaking in finite volume we measure the absolute value of this operator. In a chirally broken phase we expect this to approach a constant as the lattice volume is sent to infinity. Conversely if chiral symmetry is restored this observable will approach zero in the same limit. In all our runs we observe that the the only component of the auxiliary field to develop a vacuum expectation value corresponds to the Dirac mass term represented by the component $\mu = 4$. This is consistent with the usual conjecture that the chiral symmetries break to the maximal subgroup. In principle the direction of this breaking is arbitrary and on a finite lattice one might have expected the system to tunnel between four discrete vacua corresponding to giving a vacuum expectation value to each of the four components of ϕ_μ. In practice we have not observed

[3] Note that the fermion operator appearing in eqn. 22 is antisymmetric

[4] In practice we observe that the Pfaffian is in fact not only real but also always positive definite so multiples of two flavors should be possible too.

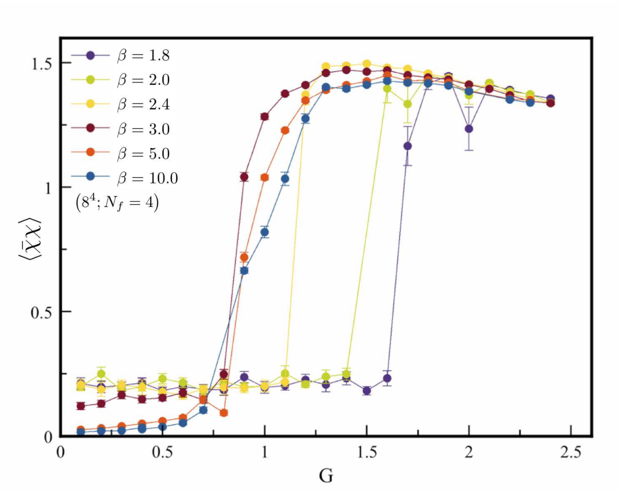

FIG. 3. $\langle\overline{\chi}\chi\rangle$ vs G for varying β for the 8^4 lattice with $N_f = 4$.

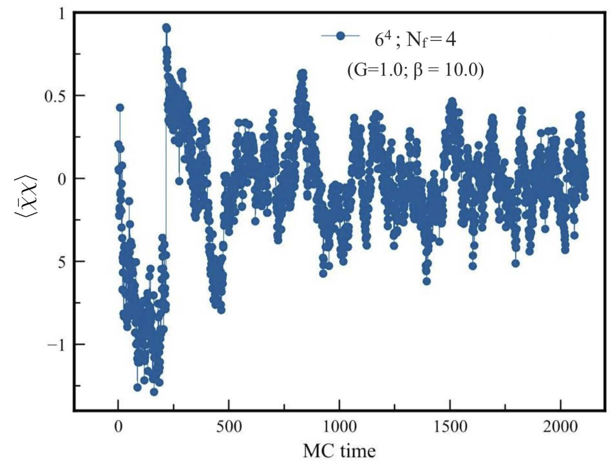

FIG. 4. $\langle\overline{\chi}\chi\rangle$ vs Monte Carlo time, t, for $\beta = 10.0$ at $G = 1.0$ for the 6^4 lattice with $N_f = 4$. Note that here we do not take the absolute value. In this case, $G = 1.0 \equiv G_{cr}$ is the point at which the transition occurs

such changes in the direction of the vev in our simulations which presumably indicates the tunneling time is very long. In Figure 3 we show a plot of the absolute value of the condensate at a variety of gauge couplings β on 8^4 lattices. Notice the rather smooth transition between symmetric and broken phases around $G \sim 0.9$ for $\beta = 10$. This is consistent with earlier work using sixteen flavors of naive fermion reported in [20] which identified a line of second order phase transitions in this region of parameter space. It also agrees with the behavior seen in previous simulations using conventional staggered quarks [18]. Furthermore for weak gauge couplings we see that the transition to large values of the condensate occurs at smaller four fermi coupling in agreement with the naive expectation that the presence of a chiral condensate due to gauge interactions facilitates further chiral symmetry breaking by the four fermi term. Notice this trend is reversed for large gauge coupling which we interpret as due to the presence of significant lattice artifacts[5]. We are currently experimenting with a smeared fermion action to see whether this is the case.

The second order nature of this transition, for large β values, can be confirmed by examining the Monte Carlo time series for the condensate close to the transition as shown in Figure 4. Large fluctuations are observed but there is no sign of metastability or a two state signal in the Monte Carlo evolution. This behavior should be contrasted with the behavior of the condensate for strong

[5] It is also possible that at strong gauge coupling the four flavor system is able to generate a conventional single site condensate by coupling the two reduced staggered fermions. This pairing must then be broken to generate the condensate favored by the four fermi term which requires a larger four fermi coupling. We thank Don Sinclair for this suggestion

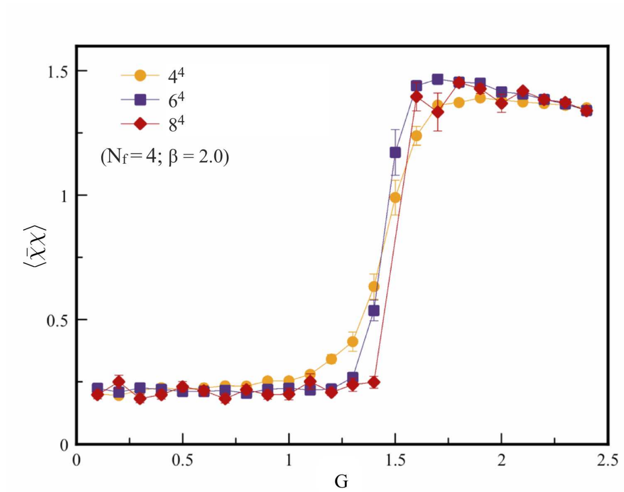

FIG. 5. $\langle\overline{\chi}\chi\rangle$ vs G at $\beta = 2.0$ for lattices 4^4, 6^4 and 8^4 with $N_f = 4$.

gauge coupling $\beta \leq 2.4$. Here a very sharp transition can be seen reminiscent of a first order phase transition. In Figure 5 we highlight this by showing a plot of the condensate versus four fermi coupling at the single gauge coupling $\beta = 2.0$ for a range of different lattice sizes. The chiral condensate is now non-zero even for small four fermi coupling and shows no strong dependence on the volume consistent with spontaneous chiral symmetry breaking in the pure gauge theory. However, it jumps abruptly to much larger values when the four fermi coupling exceeds some critical value. This crossover or

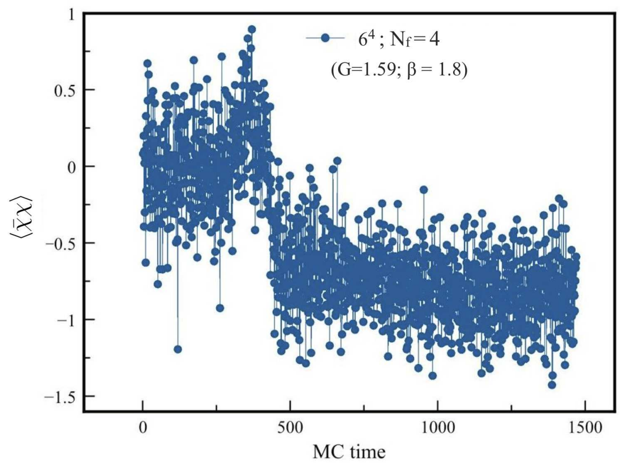

FIG. 6. $\langle\overline{\chi}\chi\rangle$ vs Monte Carlo time, t, for $\beta = 1.8$ at $G = 1.59$ for the 6^4 lattice with $N_f = 4$. Note that here we do not take the absolute value. $G = 1.59 \equiv G_{cr}$ is the point at which the transition occurs.

transition is markedly discontinuous in character - reminiscent of a first order phase transition. Indeed, while the position of the phase transition is only weakly volume dependent it appears to get sharper with increasing volume. To try to see whether the jump is indeed first order we have once again examined the Monte Carlo time series for the condensate close to the jump - the results are shown in Figure 6 for a lattice with $L = 6$ at $\beta = 1.8$. Clearly the system suffers from extremely long relaxation times close to the transition region - only finding the correct ground state after hundreds of Monte Carlo sweeps. However, we have not observed a tunneling between two competing minima as one would expect of a true first order transition and so it is hard to state with certainty that the transition is indeed first order.

What seems clear is that the second order transition seen in the pure NJL model is no longer present when the gauge coupling is strong. In the next section we will argue that this is to be expected – in the gauged model one can no longer send the fermion mass to zero by adjusting the four fermi coupling since it receives a contribution from gauge mediated chiral symmetry breaking. Indeed the measured one link chiral condensate operator is not an order parameter for such a transition since we observe it to be non-zero for all G. Notice however that we see no sign that this condensate depends on the gauge coupling β in the confining regime at small G. This is qualitatively different from the behavior of regular staggered quarks and we attribute it to the fact that the reduced formalism does not allow for a single site mass term or an exact *continuous* chiral symmetry. Thus the spontaneous breaking of the residual discrete lattice chiral symmetry by gauge interactions will not be signaled by a light Goldstone pion and the measured condensate will receive contributions only from massive states. The transition we observe is probably best thought of as a crossover phenomenon corresponding to the sudden onset of a new mechanism for dynamical mass generation due to the strong four fermi interactions.

In the continuum limit we nevertheless expect that the discrete lattice chiral symmetry will be enhanced to the full continuous symmetry $SU_L(2) \times SU_R(2)$. In this case we expect the auxiliary fields ϕ_i, $i = 1 \ldots 3$ to behave as would be Goldstone bosons. Evidence in favor of this is shown in Figure 7. which shows a plot of $\langle\phi_1(t)\phi_1(0)\rangle$ for lattices $8^3 \times 16$ at $\beta = 2.0$ in the strongly broken regime with four fermi coupling $G = 2.2$. We see indeed that the auxiliary fields have developed dynamics and propagate as light quasi Goldstone bosons.

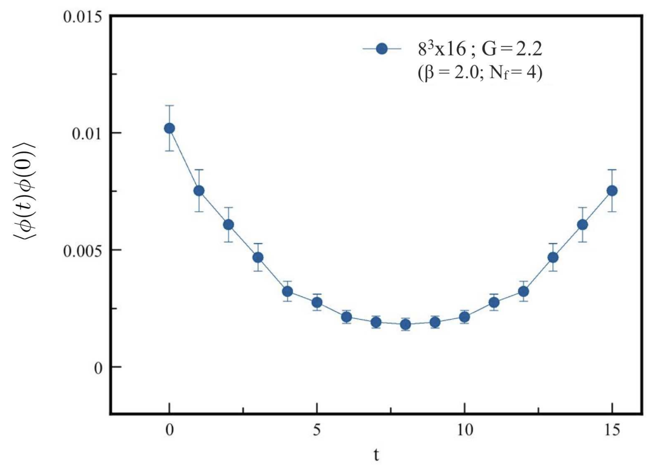

FIG. 7. Pion correlator for different G at $\beta = 2.2$ for $8^3 \times 16$ lattice with $N_f = 4$

VI. SUMMARY

In this paper we have conducted numerical simulations of the gauged NJL model for four flavors of Dirac fermion in the fundamental representation of the $SU(2)$ gauge group. We have employed a reduced staggered fermion discretization scheme which allows us to maintain an exact subgroup of the continuum chiral symmetries.

We have examined the model for a variety of values for lattices size, gauge coupling, and four fermi interaction strength. In the NJL limit $\beta \to \infty$ we find evidence for a continuous phase transition for $G \sim 1$ corresponding to the expected spontaneous breaking of chiral symmetry. However, for gauge couplings that generate a non-zero chiral condensate even for $G = 0$ this transition or crossover appears much sharper and there is no evidence of critical fluctuations in the chiral condensate.

Thus our results are consistent with the idea that the second order phase transition which exists in the pure NJL theory ($\beta = \infty$) survives at weak gauge coupling. However our results indicate that any continuous transition ends if the gauge coupling becomes strong enough to cause confinement. In this case we do however see evidence of additional dynamical mass generation for sufficiently large four fermi coupling associated with an observed rapid crossover in the chiral condensate and a possible first order phase transition. These results are consistent with the numerical solution of an augmented ladder calculation [29] reviewed in Section II.

The fact that we find the condensate non-zero and constant for strong gauge coupling and $G < G_{\rm cross}$ shows that the chiral symmetry of the theory is already broken as expected for $SU(2)$ with $N_f = 4$ flavors. This breaking of chiral symmetry due to the gauge interactions is accompanied by the generation of a non-zero fermion mass even for small four fermi coupling. Notice that this type of scenario is actually true of top quark condensate models in which the strong QCD interactions are already expected to break chiral symmetry independent of a four fermion top quark operator. The magnitude of this residual fermion mass is *not* controlled by the four fermi coupling and cannot to sent to zero by tuning the four fermi coupling - there can be no continuous phase transition in the system as we increase the four fermi coupling - rather the condensate becomes strongly enhanced for large G.

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