What can the observation of nonzero curvature tell us?

Alan H. Guth and Yasunori Nomura

What can the observation of nonzero curvature tell us?

Alan H. Guth[a] and Yasunori Nomura[b]

[a] *Center for Theoretical Physics, Laboratory for Nuclear Science, and Department of Physics,*
Massachusetts Institute of Technology, Cambridge, MA 02139, USA
[b] *Berkeley Center for Theoretical Physics, Department of Physics,*
and Theoretical Physics Group, Lawrence Berkeley National Laboratory,
University of California, Berkeley, CA 94720, USA

Abstract

The eternally inflating multiverse provides a consistent framework to understand coincidences and fine-tuning in the universe. As such, it provides the possibility of finding another coincidence: if the amount of slow-roll inflation in our past was only slightly more than the anthropic threshold, then spatial curvature might be measurable. We study this issue in detail, particularly focusing on the question: "If future observations reveal nonzero curvature, what can we conclude?" We find that whether an observable signal arises or not depends crucially on three issues: the cosmic history just before the observable inflation, the measure adopted to define probabilities in the eternally inflating spacetime, and the sign and strength of the correlation between the tunneling and slow-roll parts of the potential. We find that if future measurements find positive curvature at the level $\Omega_k \lesssim -10^{-4}$, then the framework of the eternally inflating multiverse, as currently understood, is excluded with high significance. If the measurements instead reveal negative curvature at the level $\Omega_k \gtrsim 10^{-4}$, then we can conclude that (1) diffusive (new or chaotic type) eternal inflation did not occur in our immediate past; (2) our pocket universe was born by a bubble nucleation; (3) the probability measure does not reward volume increase; and (4) the origin of the observed slow-roll inflation is an accidental feature of the potential, presumably selected by anthropic conditions, and not due to a theoretical mechanism ensuring the flatness of the potential. Discovery of $\Omega_k \gtrsim 10^{-4}$ would also give us nontrivial information about the correlation between the tunneling and slow-roll parts of the potential; for example, a strong correlation favoring large N would be ruled out in certain measures. We also address the question of whether the current constraint on Ω_k is consistent with multiverse expectations; we find the answer to be yes, except that current observations, for many choices of measure, rule out the possibility of strong correlations in the potential which favor small values of N. In the course of this work we were led to consider vacuum decay branching ratios, and found that it is more likely than one might guess that the decays are dominated by a single channel. Planned future measurements of spatial curvature provide a valuable opportunity to explore the structure of the multiverse as well as the cosmic history just before the observable inflation.

1 Introduction

Evidence for cosmic inflation in the early history of our universe is mounting. In addition to the original motivation of explaining flatness and homogeneity of the observable universe [1], we now have precision data from the cosmic microwave background (CMB) [2] that is in beautiful agreement with the predictions of the simplest inflationary models [3]. The details of this cosmic inflation, however, remain very uncertain. We do not know its energy scale, its duration, or the circumstances that led to its onset.

In the last decade, we have been learning that many of the structures of our own universe may be understood as a result of environmental, or anthropic, selection in the multiverse [4]. The most successful outcome of this picture was the prediction of a nonzero cosmological constant, made already in the 1980's [5] and confirmed in 1998 by the discovery of an accelerating expansion of the universe [6]. The picture of the multiverse is motivated theoretically by eternal inflation [7, 8, 9] and the landscape [10] of string theory, which together provide a consistent framework for explaining the nonzero cosmological constant and other examples of fine-tuning in the universe. The onset of cosmic inflation itself can perhaps be understood in the same way: since excessive curvature suppresses structure formation [11], it is possible that we are living in the aftermath of an era of inflation because otherwise intelligent observers would not have evolved.

An interesting consequence of this picture is that the observable era of inflation—i.e., the last $N \approx (40 - 60)$ e-folds of inflation, which are probed by the density perturbations in the CMB and in the matter distribution—may have been "just so." That is, the number of e-folds of the slow-roll inflation may have been very close to the minimal number needed to ensure the flatness required for the evolution of life. Such a coincidence would seem unlikely in a more conventional picture, in which the flatness of the inflaton potential might be ensured, for example, by some approximate symmetry. But in the context of the multiverse, such a coincidence is very plausible. This leads to a number of potentially observable signatures, especially in structures at large scales, including nonzero curvature of the universe [12, 13]. Studies along these lines have been performed, e.g., in Refs. [14].

Whether an observable signal actually arises or not, however, depends on at least three issues: 1) What was the cosmic history just before the observable era of inflation; 2) What probability "measure" is adopted to define probabilities in the eternally inflating spacetime, where anything that can happen will happen an infinite number of times; and 3) In tunneling transitions from one vacuum to another, how strong are the correlations between the tunneling rate and the properties of any slow-roll inflation that might follow the tunneling? In this paper, we explore these issues, focusing on the question: "If future observations reveal nonzero curvature, what can we conclude?" We take a bottom-up approach—we consider a variety of possibilities for the pre-inflationary history and the multiverse measure, which we think are reasonably exhaustive, and we consider both strong

1

and weak tunneling rate correlations. For the pre-inflationary history, we consider four different classes of models, characterized by the behavior of the inflaton field prior to the observable era of inflation. For the multiverse measure, we consider various geometric cutoff measures [15] as well as the recently proposed quantum measure [16], in which the probability is given by the quantum-mechanical Born rule applied to the multiverse state. We will see that the observation of curvature beyond the level of $\sim 10^{-4}$ can either exclude the multiverse framework itself (if it is positive) or exclude certain pre-inflationary histories *and* classes of probability measures (if it is negative), as well as constrain the nature and degree of correlation between the tunneling rate for a transition and the ensuing slow-roll inflation.

In the next section, we carefully define the framework of our analysis. We begin by classifying possible pre-inflationary histories, and then we discuss probability measures. Section 3 provides the actual analysis. The meaning of the probability distribution for curvature in the context of bubble universes is also elucidated there. We analyze all the possible scenarios for the pre-inflationary histories as well as the probability measures. Our result for the probability distribution for curvature (in the negative case) will be presented in Section 4. We finally conclude in Section 5, summarizing what we can learn from a future observation of nonzero curvature of the universe. One appendix discusses the effect of volume increase in the quantum measure, and a second appendix discusses the possibility that vacuum decays might be dominated by a single channel.

While completing this paper we received Ref. [17], by Kleban and Schillo, which also discusses the issue of spatial curvature and the cosmic history before the observable inflation. Our conclusions about it are consistent with theirs. In fact our treatment of scenario (iv) in Section 2.1 is based on private communication with Kleban [18].

2 Framework

The observable era of early-universe inflation—i.e., the last $N \approx (40-60)$ e-folds of inflation—was the period during which currently observable scales went outside the Hubble horizon.[1] Cosmic history before this era, however, can leave its imprint on the present-day curvature contribution, $\Omega_k \equiv 1 - \Omega_0$. The expected amount of curvature depends strongly on the cosmic history just before the observable inflation, the measure used to define probabilities in the eternally inflating spacetime, and the nature and degree of correlation between vacuum transition tunneling rates and the ensuing slow-roll inflation. In this section, we consider a variety of assumptions on the first two issues, establishing a framework for the analyses in later sections. The issue of the correlation between tunneling and slow-roll will be discussed in Section 3.2, where it becomes relevant.

---

[1]We use the phrase "Hubble horizon" to denote the distance scale H^{-1}, where H is the Hubble parameter, although the actual causal horizon is vastly larger.

2.1 History (just) before the observable inflation

Since the observable inflation occurred with energy densities much smaller than the Planckian density, the cosmic history just before it must be describable using (semi-)classical gravity. Here we consider four scenarios, which we think cover most of the realistic possibilities:

(i) *Eternal New Inflation* — By tracing history back in time, the inflaton field φ reaches a local maximum of the potential, with an energy density significantly smaller than the Planck scale, i.e. $V_0 \ll M_{\mathrm{Pl}}^4 \equiv (8\pi G_N)^{-2}$. Denoting this point as $\varphi = 0$ (which may be a saddle point in multi-dimensional field space), the "initial conditions" are given by $\varphi \approx \dot\varphi \approx 0$. The question of how these conditions arose need not concern us here, as the results for Ω_k are insensitive to how these conditions were prepared. The dynamics beginning with these initial conditions is described by eternal inflation at $\varphi \approx 0$ [8], followed by slow-roll inflation occurring near the potential minimum that corresponds to our vacuum.

(ii) *Eternal Chaotic Inflation* — By tracing history back in time, the inflaton field climbs up a hill in the potential energy diagram to the point where the quantum fluctuation in the field $\Delta\varphi_{\mathrm{qu}} \approx H/2\pi$ (averaged over a Hubble volume during a Hubble time interval) becomes so important that the global structure of spacetime is determined by $\Delta\varphi_{\mathrm{qu}}$, rather than by classical evolution of the field. Here, $H = (V/3M_{\mathrm{Pl}}^2)^{1/2}$ is the Hubble parameter. The transition point for quantum-fluctuation dominance is at a super-Planckian field value $\varphi_* \gg M_{\mathrm{Pl}}$; for example, for $V = \frac{1}{2}m^2\varphi^2$ it is at $\varphi_* \approx M_{\mathrm{Pl}}^{3/2}/m^{1/2}$, and for $V = \frac{1}{4!}\lambda\varphi^4$ it is at $\varphi_* \approx M_{\mathrm{Pl}}/\lambda^{1/6}$, where $m \ll M_{\mathrm{Pl}}$ (or $\lambda \ll 1$) to reproduce the observed magnitude of density perturbations. The cosmic history before our big-bang universe is then described by eternal chaotic inflation [9] followed by slow-roll chaotic inflation.

(iii) *Eternal Old Inflation* — By tracing history back in time, we hit a quantum tunneling event before entering into an eternally inflating epoch. Our pocket universe then arose directly from a bubble nucleation process [19], presumably occurring in an eternally inflating region in which the inflaton field was in some local minimum of the potential [7]. The bubble nucleation is followed by a brief curvature-dominated epoch, followed by non-eternal slow-roll inflation [20]. Slow-roll inflation begins when the vacuum energy starts to dominate over the curvature and kinetic energies in determining the evolution of the bubble universe.

(iv) *A Prior Episode of Inflation* — By tracing history back in time, the dynamics of another scalar field becomes important. This is the case, for example, in double-inflation [21] or in hybrid inflation [22] if the waterfall field is fully responsible for the observable inflation. An interesting feature of this scenario is that the density fluctuation spectrum shows a sharp spike at the scale corresponding to the connection of the two inflationary periods [23]. The resulting perturbation in Ω_k can be either positive or negative, which is determined only stochastically.

For each of the four cases above, we can estimate the probability distribution for Ω_k under various assumptions about the probability measure, the a priori probability distribution for parameters in the inflaton potential, and (when relevant) the initial conditions after the tunneling event.

2.2   Measures in eternal inflation

In an eternally inflating multiverse, anything that can happen will happen infinitely many times. This implies, among others, the following two statements. First, to define the relative likelihood of different types of events, we need to regularize the infinities. Second, any prediction in the multiverse will necessarily be statistical. Here we consider the first of these statements, leaving the second to the next subsection.

Regularizing infinities in the multiverse has been an extensive area of research [15]. There have been many proposals for "measures" that provide required regularizations, and thus prescriptions for making predictions. Traditionally, these measures have been defined using "global" or "local" geometric cutoffs (although this division is not always meaningful, since the same measure can often be formulated using either a global or local description [24, 25]). Global-cutoff measures propose that relative probabilities can be determined by the ratio of the number of events that occur prior to a specified "equal-time" hypersurface, usually in the limit as the hypersurface is chosen at an arbitrarily late time. Depending on the choice of hypersurfaces, different measures can be obtained. Local-cutoff measures, on the other hand, count events inside a finite neighborhood of a single timelike geodesic, and probabilities are computed after certain averaging procedures. Different measures correspond to different choices for the neighborhood.

More recently, a framework for the eternally inflating multiverse has been proposed which does not rely on a geometric cutoff [16]. In this framework, the entire multiverse is a single quantum state as described *from a single reference frame*. It is in general a superposition of many quantum states corresponding to well-defined semi-classical geometries, each of which is defined *only in and on* the apparent horizon. (This restriction on spacetime, dictated by the principles of quantum mechanics, provides the required regularization.) The well-defined probabilities are then given by the simple Born rule extended to the whole spacetime. This framework allows us to use the same probability formula for questions regarding global properties of the universe and outcomes of particular experiments, thus providing complete unification of the eternally inflating multiverse and the many-worlds interpretation of quantum mechanics.

Is there a general classification scheme that accommodates all these measures and is relevant for our present purpose of discussing the curvature contribution to the universe? A useful classification is obtained by considering how the measure does or does not reward the exponential increase in volume that characterizes inflationary models. Here we discuss the following three classes, where examples of each appear in the literature:

(I) *Measures rewarding any volume increase* — These measures reward any volume increase in the evolution of the multiverse. The simplest example is the so-called proper-time cutoff measure [26], which defines probabilities in the global picture using hypersurfaces of equal proper time, obtained through the congruence of geodesics orthogonal to some arbitrary initial hypersurface. This class of measures, however, suffers from various difficulties. The most serious one is probably the youngness paradox [27]: because of the rapid expansion of spatial volume in the eternally inflating region, the population of pocket universes is extremely youth-dominated. The probability of observing a universe that is old like ours (with $T_{\mathrm{CMB}} \simeq 2.7$ K) becomes vanishingly small. Since this essentially excludes observationally the class of measures described here, we will not consider it further.

(II) *Measures rewarding volume increase only in the slow-roll regime* — In these measures the volume increase during the eternally inflating regime is not rewarded, so the youngness paradox does not arise. To model the behavior of these measures, suppose that the probability density for the *onset* of an episode of inflation of N e-folds is given by some function $f(N)$. That is, $f(N)\,dN$ is the probability that the number of inflationary e-folds that will follow a randomly selected onset of inflation will lie between N and $N + dN$. $f(N)$ would in principle be determined by the probability distributions for inflaton potential parameters and for the inflaton field in the multiverse. While we do not know enough to compute $f(N)$, we will argue later that we can estimate its behavior under a variety of assumptions. Once inflation begins, the volume of the inflated region is multiplied by e^{3N}, so the probability density $P(N)$ of finding oneself in a region that has undergone N e-folds of slow-roll inflation can be written as

$$P(N) \sim f(N)e^{3N}\,, \tag{1}$$

where e^{3N} is the dominant factor. While the class of measures considered here has issues that need to be addressed [28, 29], it is not clear if these measures are excluded [30, 31]. We therefore keep these measures in our consideration. An important example of this class of measures is given by the so-called pocket-based measure [32].

(III) *Measures not rewarding volume increase* — These measures do not reward volume increase due to any form of inflation. Naively, this may sound rather counter-intuitive: how can a larger spatial volume avoid giving more observers, leading to a larger weight? This picture, however, can arise naturally in several different ways. For example, we could count events along a geodesic randomly chosen on an initial spacelike hypersurface, we could measure spacetime according to its comoving volume, or we could use a global time cutoff based on the total amount of expansion (i.e., scale-factor time). The probability distribution for finding oneself in a region that has undergone N e-folds of slow-roll inflation is then simply

$$P(N) \sim f(N)\,. \tag{2}$$

The fact that volume increase is not rewarded in the final probability distribution makes it rather easy to avoid the problems encountered by measures of type (I) and (II). Two examples of geometric cutoff measures in this class are the causal patch measure [33, 25] and the scale-factor cutoff measure [34]. The recently proposed quantum-mechanical measure [16] also falls in this class, as discussed in Appendix A.

Equations (1) and (2) can be summarized by writing

$$P(N) \sim f(N)M_m(N)\,, \tag{3}$$

where the dependence on the measure m is described by the factor $M_m(N)$. In this paper we are assuming that the measure is adequately described by specifying that it belongs to class (II) or class (III) above, so

$$M_m(N) \approx \begin{cases} e^{3N} & \text{if } m \in \text{(II)} \\ 1 & \text{if } m \in \text{(III)}\,. \end{cases} \tag{4}$$

It is, in principle, possible to consider hybrids of these classes. For example, in the stationary measure of Ref. [35] features of both (I) and (II) coexist. We will also comment on these hybrid possibilities when we discuss the probability distribution of Ω_k later.

2.3 Probability distributions for current and future measurements

In order to discuss implications of a future measurement of curvature by our civilization, we can study the multiverse probability distribution for Ω_k as a conditional probability density, given the set of observed values of the physical parameters $\{Q_1, Q_2, \ldots\}$ that have already been measured. These parameters $\{Q_i\}$ include cosmological parameters such as the primordial density fluctuation amplitude $\delta\rho/\rho$, the scalar spectral index n_s, and the vacuum energy density ρ_Λ, as well as particle physics parameters such as the electron mass m_e, the proton mass m_p, the fine structure constant α, etc. The conditional probability density $f_{\text{cond}}(\Omega_k|\{Q_i = Q_{i,\text{obs}}\})$ is proportional to the full probability density function $f(\Omega_k, \{Q_i\})$ evaluated at the measured values of the parameters:

$$f_{\text{cond}}(\Omega_k|\{Q_i = Q_{i,\text{obs}}\}) \propto f(\Omega_k, \{Q_{i,\text{obs}}\})\,, \tag{5}$$

where the constant of proportionality depends on $\{Q_{i,\text{obs}}\}$, but not on Ω_k. Here f_{cond} and f refer to probability densities for the onset of inflation. This conditional probability approach does *not* address the question of whether these values $Q_{i,\text{obs}}$ are in fact typical in the multiverse, i.e. whether the multiverse hypothesis is fully consistent with the current observations. To study this question, we would need to estimate the relevant anthropic constraints on these parameters and see if the observed values are indeed consistent with possible underlying multiverse distributions. The two approaches are complementary, addressing different questions. For Ω_k, we will take both approaches—we will

study the implications of a multiverse distribution on future measurements assuming our current knowledge, and we will also ask if the predicted distribution is consistent with our current knowledge.

The present-day curvature contribution Ω_k is related to the number of e-folds of *deterministic (non-diffusive) slow-roll inflation* N. We can thus study the probability distribution for Ω_k by analyzing that for N. The relation between the two depends on the details of how the deterministic slow-roll era begins, but to a good approximation we can write

$$\Omega_k \propto \frac{1}{e^{2N}} \,, \tag{6}$$

provided that Ω_k is in the relevant parameter region, where Ω_k is smaller than 1 but larger than the contribution induced by density fluctuations, $\Omega_k \gtrsim \delta\rho/\rho \approx 10^{-5}$. The probability of observing N in the interval between N and $N + dN$ in future measurements, given our current knowledge, can then be written as

$$P_{\text{obs}}(N)\, dN \propto f(N, \{Q_{i,\text{obs}}\})\, M_m(N)\, C(N)\, n(N)\, dN \,. \tag{7}$$

Here $f(N, \{Q_i\})$ is the multiversal joint probability density for N and the set of Q_i, analogous to $f(\Omega_k, \{Q_i\})$. ($f(N, \{Q_i\})$ and $f(\Omega_k, \{Q_i\})$ are of course different functions with different arguments, but we use the same symbol f because they have the same verbal description, as the joint probability density for a randomly chosen onset of inflation in the multiverse to be characterized by the arguments of f.) $f(N, \{Q_i\})$ is in principle determined by the statistical properties of the inflaton field and its potential in the multiverse. $C(N)$ encodes our current knowledge about N, and $n(N)$ is the anthropic weighting factor. If any quantity Q_i is subject to a non-negligible observational error, then we need to integrate that parameter over the observationally allowed range.

For $C(N)$, we know from cosmological observations that N must be larger than a certain value $N_{\text{obs,min}}$, corresponding to the maximum curvature allowed observationally, $\Omega_{k,\text{max}} \simeq 0.01$ [2]. Thus, we can take $C(N) \approx \theta(N - N_{\text{obs,min}})$. The value of $N_{\text{obs,min}}$ depends on the history of our pocket universe, especially on the reheating temperature T_R, but is generically around $40 - 60$. Any extra e-folds of inflation suppress Ω_k further as described by Eq. (6), so

$$\Omega_k \approx \frac{\Omega_{k,\text{max}}}{e^{2(N-N_{\text{obs,min}})}} + O(10^{-5}) \,. \tag{8}$$

The anthropic factor $n(N)$ can be chosen to be the expected number of observers per unit volume, summed over all time within the life of the pocket universe. For fixed values of the parameters Q_i, we expect that $n(N)$ approaches some constant n_∞ at large N, since the evolution of life will not be affected by very small spatial curvature. As smaller values of N are considered, at some point $|\Omega_k|$ will suddenly become large, growing by a factor of about $e^2 \approx 7.4$ each time N decreases by 1. The probability for observers to evolve presumably decreases quickly as $|\Omega_k|$ becomes large, so we will

also approximate this function by a step function: $n(N) \approx n_\infty \, \theta(N - N_{\text{anthropic}})$. Since obviously $N_{\text{obs,min}} > N_{\text{anthropic}}$, we find

$$P_{\text{obs}}(N) \propto f(N, \{Q_{i,\text{obs}}\}) \, M_m(N) \, \theta(N - N_{\text{obs,min}}) \,. \tag{9}$$

To discuss the consistency of our current measurements of Ω_k with the predictions of the multiverse hypothesis, we need to consider the predicted probability distribution $P_{\text{obs},\mathcal{Q}_k}(N)$, defined as the conditional probability density given all of our current knowledge except for our measurements of Ω_k. When expressed in terms of N instead of Ω_k, this probability distribution is obtained from Eq. (7) by omitting the factor $C(N)$. Given our approximation for $n(N)$, we find

$$P_{\text{obs},\mathcal{Q}_k}(N) \propto f(N, \{Q_{i,\text{obs}}\}) \, M_m(N) \, \theta(N - N_{\text{anthropic}}) \,. \tag{10}$$

Using this probability distribution, we can check whether the probability of obtaining $N > N_{\text{obs,min}}$ is indeed reasonable or not.

3   Statistical Distributions for the Number of e-folds

To use Eq. (9) to estimate the probability distribution for future measurements of N, we need to know $f(N, \{Q_{i,\text{obs}}\})$, the underlying multiversal joint probability density for the onset of N e-folds of inflation with the measured values $Q_{i,\text{obs}}$ of physical parameters. This quantity depends crucially on the history of our pocket universe just before the observable inflation. In this section we discuss $f(N, \{Q_{i,\text{obs}}\})$ for each of the four scenarios, (i) – (iv), described in Section 2.1.

3.1 $f(N, \{Q_{i,\text{obs}}\})$ for scenarios (i) or (ii): new or chaotic eternal inflation

Suppose that the past history of our pocket universe was either scenario (i) or (ii); i.e., suppose that the observable era of deterministic slow-roll inflation was smoothly connected to a prior era of new or chaotic eternal inflation. In this case we find that $f(N, \{Q_{i,\text{obs}}\})$ is strongly peaked at very large N, so that the residual curvature contribution in the present universe is completely negligible.

The qualitative reason for this result is very simple. At the transition point between eternal and non-eternal inflation, $\varphi \equiv \varphi_*$, the amplitude for the scalar perturbations exiting the Hubble horizon is of order unity: $Q\big(k(\varphi_*)\big) \approx 1$. On the other hand, when the current horizon scale exits the Hubble horizon at $\varphi \equiv \varphi_0$, the perturbation amplitude is very small: $Q\big(k(\varphi_0)\big) \approx 10^{-5}$. Since the perturbation amplitude changes rather slowly with k, this large change in Q implies that there must have been a large number ΔN of e-folds of slow-roll inflation between the end of eternal inflation and the time when the current horizon scale exited the Hubble horizon. We will first show this in two simple examples, and then present a general argument.

We first consider an example of scenario (i), using the following inflaton potential:

$$V = V_0 - \frac{1}{2}\mu^2\varphi^2 + \delta V(\varphi) \qquad (V_0, \mu^2 > 0), \tag{11}$$

where $\mu^2 \lesssim H_I^2 \equiv V_0/3M_{\rm Pl}^2$ to have a flat potential at small φ. We also assume, for simplicity, that before the current horizon scale exits the Hubble horizon we can take $V \approx V_0$ and $V' \approx -\mu^2\varphi$. $\delta V(\varphi)$ is assumed to be negligible during this period, although later it controls the ending of inflation.

With the initial conditions $\varphi \approx \dot{\varphi} \approx 0$, the potential of Eq. (11) leads to eternal inflation for $0 < |\varphi| < \varphi_*$, where φ_* is determined by the condition that $\Delta\varphi_{\rm qu}$ is comparable to the classical motion of φ during a Hubble time, or

$$\Delta\varphi_{\rm qu} \approx \frac{H}{2\pi} \approx |\dot{\varphi}_{\rm classical}| \, H^{-1} \approx \frac{|V'|}{3H^2}. \tag{12}$$

For the potential of Eq. (11), this gives

$$\varphi_* \approx \frac{3}{2\pi} \frac{H_I^3}{\mu^2}. \tag{13}$$

For $|\varphi| > \varphi_*$, the evolution of φ is described by the classical equation of motion, which in this approximation gives $\varphi(t) \propto \exp\left\{\frac{\mu^2}{3H_I}t\right\}$. The scalar perturbation amplitude for single-field slow-roll inflation is given by [36]

$$Q(k) \equiv \frac{2}{5}\Delta_{\mathcal{R}}(k) \approx \frac{1}{\sqrt{75}\,\pi\,M_{\rm Pl}^3} \frac{V^{3/2}}{|V'|}, \tag{14}$$

where $V(\varphi)$ is evaluated at the value of φ when the scale k exits the Hubble horizon. For the present case, one finds $Q(k) \approx (3/5\pi)(H_I^3/\mu^2\varphi)$. Observationally, the perturbation amplitude at the current horizon scale $Q_0 \equiv Q(k = H_0) \simeq 2 \times 10^{-5}$ [2], so

$$\varphi_0 \approx \frac{3}{5\pi} \frac{H_I^3}{\mu^2 Q_0} \simeq 9.5 \times 10^3 \frac{H_I^3}{\mu^2}. \tag{15}$$

Note that our approximation $V \approx V_0$ requires that $\frac{1}{2}\mu^2\varphi_0^2 \ll V_0$, which leads to the parameter restriction $\mu^2 \gg H_I^4/(Q_0^2 M_{\rm Pl}^2)$. This is consistent with the upper bound on μ^2 provided that $H_I \ll Q_0 M_{\rm Pl} \approx 5 \times 10^{13}$ GeV. The scalar spectral index n_s (defined by $Q^2 \propto k^{n_s-1}$) is given by [37]

$$1 - n_s = 6\epsilon - 2\eta, \tag{16}$$

where the slow-roll parameters ϵ and η are defined by

$$\epsilon \equiv \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2, \tag{17}$$

$$\eta \equiv M_{\rm Pl}^2 \frac{V''}{V}. \tag{18}$$

9

For the current system, one finds $1 - n_s = 2\mu^2/3H_I^2$. Since observation gives $(1 - n_s)_{\rm obs} \simeq 0.04 \pm 0.01$ [2], we have

$$\frac{\mu^2}{H_I^2} \simeq 0.06 \pm 0.01. \tag{19}$$

Thus, in this model the number of e-folds of slow-roll inflation before the exit of the current horizon scale is given by

$$\Delta N \equiv N(\varphi_*) - N(\varphi_0) = \frac{3H_I^2}{\mu^2} \ln \frac{\varphi_0}{\varphi_*} \approx 500. \tag{20}$$

Note that ΔN is a fixed, and large, number. This implies that, in the present scenario, we would not have any possibility of observing a residual curvature contribution in the current universe.

A similar analysis can also be performed for scenario (ii), for which we choose the sample potential

$$V = \frac{1}{2} m^2 \varphi^2 \qquad (m^2 > 0). \tag{21}$$

The field values corresponding to Eqs. (13) and (15) are now

$$\varphi_* \approx \sqrt{4\pi\sqrt{6}\, \frac{M_{\rm Pl}^3}{m}} \tag{22}$$

and

$$\varphi_0 \approx \sqrt{10\pi\sqrt{6}\, Q_0 \frac{M_{\rm Pl}^3}{m}}. \tag{23}$$

The parameters are then determined uniquely by the value of n_s, since for this potential $\epsilon = \eta = 2M_{\rm Pl}^2/\varphi^2$ and therefore $1 - n_{s,0} = 8M_{\rm Pl}^2/\varphi_0^2$. Using the observed values of $n_{s,0}$ and Q_0, one has $\varphi_0 \approx 14\, M_{\rm Pl} \simeq 3.4 \times 10^{19}$ GeV, $m \approx 7.7 \times 10^{-6} M_{\rm Pl} \simeq 1.9 \times 10^{13}$ GeV, and $\varphi_* \approx 2.0 \times 10^3 M_{\rm Pl} \simeq 4.9 \times 10^{21}$ GeV. As in the previous case, the large difference between φ_* and φ_0 implies that there must have been a large number ΔN of e-folds of inflation between the end of eternal inflation and the Hubble horizon exit of the current horizon scale. To find ΔN we note that, in slow-roll approximation, this potential energy function gives

$$\frac{d\varphi^2}{dN} = -\frac{1}{H}\frac{d\varphi^2}{dt} \approx \frac{2\varphi}{H}\frac{V'}{3H} \approx 4M_{\rm Pl}^2, \tag{24}$$

so

$$\Delta N \approx \frac{1}{4M_{\rm Pl}^2}\left(\varphi_*^2 - \varphi_0^2\right) \approx 1.0 \times 10^6. \tag{25}$$

For this case, the number of e-folds of inflation is even much larger than the previous case, so again there is no possibility that curvature could be observed.

Having seen that ΔN is very large for two special cases, we can now give a general argument that ΔN is always very large for scenarios (i) and (ii). By comparing Eqs. (12) and (14), one sees that the condition for the onset of eternal inflation, $\varphi \equiv \varphi_*$, is equivalent to $Q(k(\varphi_*)) \approx 2/5$. If

$Q(k)$ varies slowly, then there must be many e-folds of inflation between the point where $Q \approx 2/5$ and the point where $Q = Q_0 \simeq 2 \times 10^{-5}$. And $Q(k)$ does vary slowly, since $Q^2(k) \propto k^{n_s-1}$, and observationally $n_s \simeq 0.96$, which is near to the scale-invariant value of $n_s = 1$. To quantitatively relate a change in Q to the number of e-folds over which it occurs, we recall that $k \propto e^{-N}$, where N is the number of e-folds of inflation that have not yet occurred when the wave number k exits the Hubble horizon. Thus $Q^2 \propto e^{(1-n_s)N}$, so

$$dN \approx \frac{2}{1 - n_s} \, d \ln Q \,. \tag{26}$$

Thus for $\varphi \approx \varphi_0$ we have

$$dN \approx 50 \, d \ln Q \,. \tag{27}$$

This implies that even a fractional change in Q of $O(1)$ around $\varphi = \varphi_0$ leads to a large number of e-folds. Since $\ln Q$ changes by about 10 as φ varies from φ_* to φ_0, the resulting ΔN is very large.

The argument described above shows that in scenario (i) or (ii), ΔN must be very large, i.e. the probability density $f(N, \{Q_{i,\mathrm{obs}}\})$ is peaked at values of N much larger than $N(\varphi_0)$. Note that this conclusion does *not* depend on the measure adopted. Therefore, if our past history is either scenario (i) or (ii), the probability of observing curvature in future measurements is completely negligible. To turn the argument around, if future measurements find a curvature contribution (beyond the 10^{-5} level), then we would learn that diffusive (new or chaotic type) eternal inflation did not occur in our "immediate" past.

3.2 $f(N, \{Q_{i,\mathrm{obs}}\})$ for scenario (iii): quantum tunneling after eternal old inflation

We now start discussion of scenario (iii): eternal old inflation. While the previous cases could be understood solely in terms of the dynamics of density perturbations, for this case we will need to consider the description of probabilities in the multiverse. Consider a diagram showing the local neighborhood of our own vacuum in the landscape, as depicted schematically in Fig. 1. The diagram shows a single scalar field, but it symbolically represents a field moving in a space with many dimensions. We are interested in the situation where our pocket universe was born by a quantum tunneling event [19], in which the scalar field φ tunneled out from a local minimum, which we call our parent vacuum. The pocket universe then experienced a period of slow-roll inflation which ended with the scalar field rolling into the local minimum of our vacuum, which in this context we call a child vacuum.

We note that the transition from one vacuum to another does not always occur through a quantum tunneling event; if the potential barrier separating the two is very broad, then the field φ climbs up the barrier [38], rather than tunnels through it. If the transition from our parent to child

Figure 1: A local neighborhood of our own vacuum in the landscape.

vacua occurred in this way, however, the field φ started rolling into our vacuum from the top of the very broad barrier. Therefore, in this case the situation is reduced to the one already discussed in the previous subsection [39].

3.2.1 The meaning of the statistical distribution for N

In the setup considered here, what exactly do we mean by $f(N, \{Q_{i,\text{obs}}\})$, which we recall was described as the multiversal joint probability density for N and Q_i? In fact, if we focus our attention on a particular region of the landscape containing only one pair of parent-child vacua, as in Fig. 1, then the number of e-folds N of slow-roll inflation is just a fixed number, determined by the shape of the potential. Since the point where fields appear after the tunneling is determined uniquely (at least in the semi-classical limit), there is no "statistical distribution" for N.[2] Nonetheless, we of course do not know the value of N, so we will describe it in terms of an estimated probability distribution, which includes uncertainties arising from at least two sources.

First, it is possible that the landscape includes many parent-child pairs that could be our pocket universe and its parent. We would in fact expect that the landscape contains a large number of vacua in which the low-energy physical laws, including the values of the parameters, are consistent with what we know about our own universe. Any one of these vacua would be a candidate for our local vacuum, and we would have no way of knowing in which one we live. There would be perhaps an even larger set of vacua which tunnel to one of the local vacuum candidates, and we would have no way of knowing which of these was the parent of our pocket universe. Since any one of these parent-child transitions could have been the transition that produced our pocket universe, the value of N can acquire a statistical distribution.

Even if there are many parent-child pairs that could be ours, however, it will not lead to an actual spread in values of N unless more than one of them occurs with significant probability. Whether or not that is the case depends on branching ratios in the landscape, which is a topic about which little is known. We discuss these branching ratios in Appendix B. We do not reach a definite conclusion, but we find that it is not implausible that the decay rates in the landscape are so diverse that the decay of any given vacuum, especially a long-lived one, is overwhelmingly dominated by a single channel.

The fastest decays are most likely the least diverse, so one plausible scenario is that a significant

---

[2]If the potential contains a (quasi-)flat direction around this point, quantum fluctuation can give a distribution for N. We ignore this effect below since it is not generic.

Figure 2: A schematic picture for a landscape leading to probability distributions for inflaton potentials and initial values.

Figure 3: A schematic picture for a landscape in which various vacuum decay chains have enormously different probabilities.

fraction of the multiverse evolves through one or more short-lived Planck-scale vacua, which decay into a large number of "second generation" vacua with nonnegligible branching ratios. Then, even if the subsequent decays are each dominated by a single channel, the large number of second generation vacua could lead to many vacua which are compatible with ours, all occurring with comparable probabilities. This situation is illustrated in Fig. 2. It leads to a probability distribution in N because pocket universes entirely consistent with what we know about ours are produced by many different parent to child transitions, each with its own value of N.

On the other hand, we can also imagine that estimates of the spread of decay rates in the landscape, like the ones in our preliminary discussion in Appendix B, will show that absolutely every decay in the landscape is almost certainly dominated by a single channel. In that case, of all the vacua that are compatible with ours, we would expect one to completely dominate the probability. Furthermore, the appearance of this vacuum would be completely dominated by the decay of a single type of parent vacuum. In this situation N would have a unique value, so the previous discussion of a probability distribution does not apply. This brings us to the second source of uncertainty, which is ignorance. Even if we conclude that single paths dominate the evolution of the multiverse, we will still not be capable of identifying the vacuum and parent that dominate the probability. We would therefore *parameterize our ignorance* about the most likely path in the form of a probability distribution for N. This situation is illustrated in Fig. 3. Note that this is a different concept from the probability distribution of the physical realization of different values of N in the multiverse—in fact, it is closer to the concept of probability used in conventional arguments for naturalness in a single vacuum theory. If we have no precise knowledge about the vacuum population mechanism, we are limited to making plausible assumptions about the probability distribution for N.

In either of the cases discussed above, the implications for future measurements of Ω_k are encoded in the probability distribution $f(N, \{Q_{i,\text{obs}}\})$, as in Eq. (9). This distribution corresponds to the multiversal joint probability distribution for the onset of N e-folds of slow-roll inflation, and the measured parameters $Q_i = Q_{i,\text{obs}}$, introduced in section 2.2. We will estimate it in the next two subsections. Our estimate does not depend much on the origin of this probability, whether it represents physical realizations in the multiverse or the parameterization of our ignorance. We therefore conclude that even if nonzero curvature is someday measured, this measurement will not tell us whether different values of N are actually realized with nonnegligible probabilities in the

multiverse.

3.2.2 Probability distribution for the inflaton potential and the starting point of slow-roll inflation

Our goal is to evaluate $f(N, \{Q_{i,\text{obs}}\})$ for scenario (iii), where slow-roll inflation follows a quantum tunneling event. We have in mind a potential of the form of Fig. 1, the form of which leads immediately to an important issue. The tunneling rate for a given transition depends on the properties of the potential function in the region of the barrier, while the number N of e-folds of slow-roll inflation depends on the properties of the potential in the slow-roll part of the potential energy curve. We do not know to what extent these two parts of the potential are correlated, but it is conceivable that the statistics of the slow-roll part of the potential could be strongly affected by the fact that some shapes are more likely to occur with a barrier that gives faster tunneling. If the correlation is strong, then it is potentially a large effect, since the tunneling rate depends exponentially on the parameters.

Since we do not know how to calculate the correlations, we consider two extreme possibilities. If these regions are only weakly correlated, then the tunneling rate will have no significant effect on $f(N, \{Q_{i,\text{obs}}\})$. If, however, the correlation is strong, then it could have a large effect, the nature of which we will discuss in the following section.

For now we write $f(N, \{Q_{i,\text{obs}}\})$ as the product of two factors,

$$f(N, \{Q_{i,\text{obs}}\}) = f_0(N, \{Q_{i,\text{obs}}\}) \, B(N, \{Q_{i,\text{obs}}\}) \,, \tag{28}$$

where $f_0(N, \{Q_{i,\text{obs}}\})$ is the answer that we would expect in the absence of correlations, and $B(N, \{Q_{i,\text{obs}}\})$ is the correction factor caused by the bias toward slow-roll potentials that correspond to faster decay rates. $f_0(N, \{Q_{i,\text{obs}}\})$ can be called the vacuum statistics probability distribution, and it can be defined more precisely by imagining that we first make a list of all the parent vacua P_α that occur in the multiverse. To weight each vacuum according to its relevance to the evolution of the multiverse, we imagine assigning each vacuum P_α a weight W_α, which we take to be proportional to the relative number of nucleation events in which bubbles of P_α are produced (as determined according to the measure of choice). The precise choice of this weighting will not affect our estimates, since we will assume that the decay properties of P_α are not correlated with the properties of its production, but we will see in the next section that this specification for W_α is particularly useful. We further imagine that we can determine all the possible transitions by which each parent vacuum P_α can decay to each child vacuum C_j. These transitions will presumably have a huge range of decay rates, but $f_0(N, \{Q_i\})$ is defined as the joint probability density for N and $\{Q_i\}$ computed with all these transitions counted equally, weighted only by W_α:

$$f_0(N, \{Q_i\}) \propto \sum_{\alpha,j} W_\alpha \, \delta\big(N - N(\alpha, j)\big) \prod_i \delta\big(Q_i - Q_i(\alpha, j)\big) \,, \tag{29}$$

where α and j are summed over all parent-child pairs, and $N(\alpha, j)$ and $Q_i(\alpha, j)$ are the values of the number N of slow-roll e-folds and the value of measurable quantity Q_i associated with this parent-child combination. The constant of proportionality is determined by requiring $f_0(N, \{Q_i\})$ to be normalized, and $f(N, \{Q_{i,\text{obs}}\})$ is obtained by setting each Q_i to its observed value $Q_{i,\text{obs}}$.

In this section we will estimate $f_0(N, \{Q_{i,\text{obs}}\})$, leaving the discussion of $B(N, \{Q_{i,\text{obs}}\})$ until the next section.

Following the approach of Freivogel, Kleban, Martínez, and Susskind [13] (hereafter called FKMS), we develop a toy model for the slow-roll part of the potential energy curve and for the value of the inflaton field at the start of the slow-roll period.[3] While FKMS used the observed value of the density perturbation amplitude Q_0 as a condition, we will use both it and the observed value of the scalar spectral index n_s. We seek only a crude approximation—which is the best we can do—so we make the simplest possible assumptions. We assume therefore that the inflaton potential, during the era of slow-roll inflation, is approximated by

$$V = V_0 + A\varphi + \frac{1}{2}\mu^2\varphi^2 \,. \tag{30}$$

We further assume that slow-roll inflation starts at $\varphi = \Delta \ (> 0)$ and ends at $\varphi = 0$ (so we take $\partial V/\partial\varphi > 0$ for $0 \leq \varphi \leq \Delta$, which implies $A > 0$). In this section we will pursue the hypothesis that the parameters V_0, A, μ^2, and Δ "scan" in the landscape, in the sense that they can be assumed to vary in the multiverse according to some smooth probability distribution function $h_0(V_0, A, \mu^2, \Delta)$. Here $h_0(V_0, A, \mu^2, \Delta)$ is defined, like $f_0(N, \{Q_{i,\text{obs}}\})$, as a vacuum statistics probability density. That is, it is defined by weighting transitions by the weight W_α of the parent vacuum, but not by the decay rate, so that correlations with the tunneling part of the potential play no role. We then study the resulting probability distribution for N, the number of e-folds of slow-roll inflation. When we consider a specific example, we will choose an h_0 that is flat.

Keeping in mind that we seek only a crude approximation, we assume that the parameters of the potential satisfy

$$V_0 \gg A\Delta \,, \qquad V_0 \gg |\mu^2|\Delta^2 \,, \qquad \text{and} \qquad A \gg |\mu^2|\Delta \,. \tag{31}$$

The total number of e-folds is then given by

$$N = \int_0^\Delta \frac{V}{M_{\text{Pl}}^2 V'} d\varphi \approx \frac{V_0\Delta}{AM_{\text{Pl}}^2} \,. \tag{32}$$

Under these approximations the density perturbation amplitude Q_0 and the spectral index n_s are constant through the slow-roll period, given by

$$Q_{0,\text{obs}} = \frac{1}{\sqrt{75}\,\pi\,M_{\text{Pl}}^3} \frac{V^{3/2}}{|V'|} \approx \frac{V_0^{3/2}}{\sqrt{75}\,\pi A M_{\text{Pl}}^3} \tag{33}$$

[3]FKMS did not discuss the possibility of correlations between the tunneling and slow-roll parts of the potential, so their $P(N)$ corresponds to our $f_0(N, \{Q_{i,\text{obs}}\})$.

and
$$1 - n_{s,\text{obs}} = (6\epsilon - 2\eta) \approx M_{\text{Pl}}^2 \left(\frac{3A^2}{V_0^2} - \frac{2\mu^2}{V_0} \right).$$ (34)

The joint probability density f_0 for N, $Q_{0,\text{obs}}$, and $n_{s,\text{obs}}$ is then given by

$$
\begin{aligned}
f_0(N, Q_{0,\text{obs}}, n_{s,\text{obs}}) &= \iiiint dV_0 \, dA \, d\mu^2 \, d\Delta \, \delta\left(N - \frac{V_0 \Delta}{A M_{\text{Pl}}^2} \right) \delta\left(Q_{0,\text{obs}} - \frac{V_0^{3/2}}{\sqrt{75}\,\pi\, A M_{\text{Pl}}^3} \right) \\
&\quad \times \delta\left((1 - n_{s,\text{obs}}) - M_{\text{Pl}}^2 \left[\frac{3A^2}{V_0^2} - \frac{2\mu^2}{V_0} \right] \right) h_0(V_0, A, \mu^2, \Delta) \quad (35) \\
&= \frac{(75\pi^2)^3 M_{\text{Pl}}^2 Q_{0,\text{obs}}^5}{N^8} \int d\Delta \, \Delta^7 \, h_0\left(\frac{75\pi^2 Q_{0,\text{obs}}^2 \Delta^2 M_{\text{Pl}}^2}{N^2}, \frac{75\pi^2 Q_{0,\text{obs}}^2 \Delta^3}{N^3}, \right. \\
&\quad \left. \frac{75\pi^2 Q_{0,\text{obs}}^2 \Delta^2}{2N^2} \left[\frac{3\Delta^2}{N^2 M_{\text{Pl}}^2} - (1 - n_{s,\text{obs}}) \right], \Delta \right).
\end{aligned}
$$ (36)

As a simple example, we assume that the distribution $h_0(V_0, A, \mu^2, \Delta)$ is constant in the range $0 < V_0 < V_{0,\text{max}}$, $0 < A < A_{\text{max}}$, $\mu_{\text{min}}^2 < \mu^2 < \mu_{\text{max}}^2$, and $0 < \Delta < \Delta_{\text{max}}$, where $\mu_{\text{min}}^2 < 0$. The approximations described by Eq. (31) are not really valid throughout this range, but in the spirit of our crude approximation we will ignore this problem. Then the integral in Eq. (36) depends on N only through the limit of integration: that is, if N is sufficiently small, then one of the first three arguments of h_0 can reach its upper limit before Δ reaches Δ_{max}. In this case the upper limit of integration becomes proportional to N, resulting in a factor of N^8, canceling the prefactor. Thus

$$f_0(N, Q_{0,\text{obs}}, n_{s,\text{obs}}) \propto \begin{cases} \dfrac{1}{N^8} & \text{if } N > N_{\text{min}}, \\ \text{const.} & \text{if } N < N_{\text{min}}, \end{cases}$$ (37)

where

$$
\begin{aligned}
N_{\text{min}} &= \max \left\{ \frac{\sqrt{75}\,\pi M_{\text{Pl}} Q_{0,\text{obs}} \Delta_{\text{max}}}{\sqrt{V_{0,\text{max}}}}, \left(\frac{75\pi^2 Q_{0,\text{obs}}^2}{A_{\text{max}}} \right)^{1/3} \Delta_{\text{max}}, \right. \\
&\quad \left. \left(\frac{6\sqrt{75}\,\pi Q_{0,\text{obs}} \Delta_{\text{max}}^2}{M_{\text{Pl}} \left[\sqrt{\left[\sqrt{75}\,\pi M_{\text{Pl}} Q_{0,\text{obs}} (1 - n_s) \right]^2 + 24\mu_{\text{max}}^2} + \sqrt{75}\,\pi M_{\text{Pl}} Q_{0,\text{obs}} (1 - n_s) \right]} \right)^{1/2} \right\}.
\end{aligned}
$$ (38)

The arguments of the max function in the above expression are the values of N for which each of the first three arguments of h_0, in Eq. (36), will reach its maximum value before Δ reaches Δ_{max}.[4]

---

[4]To be complete, there is one further complication that could occur, but which we assume does not occur. For small Δ, the μ^2 argument (i.e, the 3rd argument) of h in Eq. (36) can be negative, so the integration can be limited by μ_{min}^2, the smallest allowed value of μ^2. We will assume, however, that μ_{min}^2 is chosen to be sufficiently negative to prevent this from happening. The minimum possible value for this argument is $-75\pi^2 M_{\text{Pl}}^2 (1 - n_{s,\text{obs}})^2 Q_{0,\text{obs}}^2 / 24$, which is small because $Q_{0,\text{obs}} \simeq 2 \times 10^{-5}$, so one can easily choose μ_{min}^2 to avoid this complication.

Equation (38) is very complicated, but fortunately all we really need to know is that $N_{\min}$ is generically small. For sample values we can take all the integration limits to be at the Planck scale: i.e., $V_{0,\max} = M_{\mathrm{Pl}}^4$, $A_{\max} = M_{\mathrm{Pl}}^3$, $\mu_{\max}^2 = -\mu_{\min}^2 = M_{\mathrm{Pl}}^2$, and $\Delta_{\max} = M_{\mathrm{Pl}}$. Then with the measured values of $Q_{0,\mathrm{obs}}$ and $n_{s,\mathrm{obs}}$, the three arguments in Eq. (38) become 0.00054, 0.0067, and 0.026, respectively. Thus, for all interesting values of N, this example gives $f_0(N, Q_{0,\mathrm{obs}}, n_{s,\mathrm{obs}}) \propto 1/N^8$.

There are many variants of this analysis, however, so we do not claim that there is any particular significance to the power 8. If we had not conditioned on $n_{s,\mathrm{obs}}$, whether or not we included the μ^2 term in the potential, we would have found $f_0(N, Q_{0,\mathrm{obs}}) \propto 1/N^6$. (In this case $N_{\min}$ would be larger than before, based on the first two arguments of h_0, but it would still be less than 1 for the Planck-scale sample values.) We might also consider omitting the μ^2 term from the potential, but conditioning on $n_{s,\mathrm{obs}}$ nonetheless. In that case the power counting gives a probability density that is flat, but one also finds that the arguments of h_0 become crucial. The value of Δ will be forced outside the allowed range unless $N < N_{\max}$, where

$$N_{\max} = \frac{\sqrt{3}\Delta_{\max}}{M_{\mathrm{Pl}}\sqrt{1 - n_{s,\mathrm{obs}}}} \,. \tag{39}$$

For the Planck-scale sample values this gives $N_{\max} = 8.7$, although it can be moved up to the interesting range if we allow $\Delta_{\max}$ to be a few times larger than M_{Pl}.

FKMS used a different parameterization of the potential,

$$V(\varphi) = V_0(1 - x\varphi/\Delta) \,. \tag{40}$$

Assuming a flat probability distribution for V_0, x, and Δ, and by conditioning on $Q_{0,\mathrm{obs}}$ but not $n_{s,\mathrm{obs}}$, they found that $f_0(N, Q_{0,\mathrm{obs}}) \propto 1/N^4$. They did not specify a range of validity for this result, but we find that it is valid for $N > N_{\min}$, where

$$N_{\min} = \max\left(\frac{\sqrt{75}\,\pi\,M_{\mathrm{Pl}}Q_{0,\mathrm{obs}}\Delta_{\max}}{\sqrt{V_{0,\max}}} \,, \frac{\Delta_{\max}^2}{M_{\mathrm{Pl}}^2\,x_{\max}}\right) \,. \tag{41}$$

For Planck-scale sample values, with $x_{\max} = 1$ as used by FKMS, this gives $N_{\min} = 1$, coming from the second argument of the max function. While these estimates give $N_{\min} \ll 40$, for the FKMS parameterization it is not unreasonable to consider values of $\Delta_{\max}$ and $x_{\max}$ for which $N_{\min}$ might be larger than 60. In that case $f_0(N, Q_{0,\mathrm{obs}})$ would fall as $1/N^{3/2}$ in the range of interest.

If one conditions on both $Q_{0,\mathrm{obs}}$ and $n_{s,\mathrm{obs}}$, using Eq. (40) and a flat probability density for V_0, x, and Δ, one finds $f_0(N, Q_{0,\mathrm{obs}}, n_{s,\mathrm{obs}}) \propto N$ for $N < N_{\max}$, but $f_0(N, Q_{0,\mathrm{obs}}, n_{s,\mathrm{obs}}) = 0$ for $N > N_{\max}$, where

$$N_{\max} = \min\left(\frac{3x_{\max}}{1 - n_{s,\mathrm{obs}}} \,, \frac{\sqrt{3}\,\Delta_{\max}}{M_{\mathrm{Pl}}\sqrt{1 - n_{s,\mathrm{obs}}}}\right) \,. \tag{42}$$

As with Eq. (39), for Planck-scale sample values this gives $N_{\mathrm{max}} = 8.7$, from the second argument of the min function. Again it can be increased if we allow Δ_{max} to be larger than M_{Pl}.

One can also consider adding a $\frac{1}{2}\mu^2\varphi^2$ term to the potential of Eq. (40), assigning a flat probability density to μ^2 along with the other parameters. If one does not condition on $n_{s,\mathrm{obs}}$, then with our approximations the addition of the μ^2 term has no effect on $f_0(N, Q_{0,\mathrm{obs}})$. If one does condition on $n_{s,\mathrm{obs}}$, then $f_0(N, Q_{0,\mathrm{obs}}, n_{s,\mathrm{obs}}) \propto 1/N^6$, provided that $N > N_{\mathrm{min}}$, where N_{min} is the max of both arguments in Eq. (41) and the last argument of Eq. (38).

The details of these results are of course not to be trusted, since they are based on ad hoc assumptions about the probability distribution for potential functions in the multiverse. Nonetheless, we believe that we can reasonably infer that the function $f_0(N, \{Q_{i,\mathrm{obs}}\})$, as defined by Eq. (29), can be taken as

$$f_0(N, \{Q_{i,\mathrm{obs}}\}) \propto \frac{1}{N^p} \tag{43}$$

for some (small) power $p > 0$. Here, the positivity of p represents the improbability of finding an inflaton potential that supports many e-folds of evolution with a value of Q_0 as small as 2×10^{-5}. This result is mostly in agreement with FKMS, who find $f(N, Q_{0,\mathrm{obs}}) \propto 1/N^4$, except that we allow for the possibility that there *might* be a significant correction factor $B(N, \{Q_{i,\mathrm{obs}}\})$, as in Eq. (28), caused by correlations with the tunneling rate.

Equation (43) is the generic behavior, but there is a plausible exception. Suppose there is a mechanism which ensures the flatness of the inflaton potential in the vicinity of our (child) vacua: for example, a (softly broken) shift symmetry acting on the inflaton field φ. In terms of the model potential of Eq. (30), such a mechanism would assure that A is very small. By combining Eqs. (32) and (33), the number of e-folds of inflation can be written as

$$N(A, \Delta) = \left(\frac{75\pi^2 Q_{0,\mathrm{obs}}^2}{A}\right)^{1/3} \Delta \,, \tag{44}$$

which shows how large values of N result from small values of A. In most situations the probability of finding large values of N is suppressed by the need to find unusually small values of A, but a mechanism such as a shift symmetry can avoid that problem. If the mechanism makes it probable to find values of A so small that $N(A, \Delta) \gtrsim 60$ for $\Delta < \Delta_{\mathrm{max}}$, then we would expect the suppression of large N would be removed. The results we obtained in Eqs. (37) and (38) verify these expectations, if we describe the mechanism as one that enforces a very small value of A_{max}. By comparing Eq. (44) with the second argument of Eq. (38), we see that if A_{max} is small enough to allow 60 e-folds of inflation, then $N_{\mathrm{min}} \geq 60$, and then Eq. (37) implies that we are on the flat part of the probability density curve. Thus, a mechanism to assure the flatness of the potential can lead to

$$f_0(N, \{Q_{i,\mathrm{obs}}\}) \sim \mathrm{const.} \tag{45}$$

for the relevant range of N, so the preference to shorter inflation in Eq. (43) does not arise. In fact, the consideration here can be used to discriminate if the observable inflation arose "accidentally," which leads to Eq. (43), or due to some mechanism: if nonzero curvature is measured, this would be strong evidence against a mechanism that ensures a flat potential.

Finally, although it is not needed for the main arguments presented in this paper, it is interesting to use the probability distributions that have been modeled in this section to ask what is the absolute probability of finding instances of inflation like the one that apparently began our pocket universe. Specifically, we can use the models discussed in this section to calculate the probability $\bar{P}_1$ that a given instance of inflation will satisfy $N > \bar{N}$, $Q_0 < \bar{Q}_0$, and $|1 - n_s| < \Delta \bar{n}_s$. Here we set the bias correction factor $B(N, \{Q_{i,\text{obs}}\}) = 1$; in the following section we will see that B can decrease $\bar{P}_1$, but for most choices of measure it cannot increase it. For the model used in Eq. (36), we can assume that $\bar{N} > N_{\min}$, and then the integration extends to $\Delta = \Delta_{\max}$, giving a factor $\Delta_{\max}^8/8$. Using a normalized flat probability density for $h_0(V_0, A, \mu^2, \Delta)$, the probability described above is given by

$$
\begin{aligned}
\bar{P}_1 &= \int_{\bar{N}}^{\infty} dN \int_0^{\bar{Q}_0} dQ_0 \int_{1-\Delta\bar{n}_s}^{1+\Delta\bar{n}_s} dn_s \, f_0(N, Q_0, n_s) \\
&= \frac{(75\pi^2)^3 M_{\text{Pl}}^2 \Delta_{\max}^8}{8 V_{0,\max} A_{\max} \Delta_{\max} (\mu_{\max}^2 - \mu_{\min}^2)} \int_{\bar{N}}^{\infty} dN \int_0^{\bar{Q}_0} dQ_0 \int_{1-\Delta\bar{n}_s}^{1+\Delta\bar{n}_s} dn_s \, \frac{Q_0^5}{N^8} \\
&= \frac{(75\pi^2)^3 M_{\text{Pl}}^2 \Delta_{\max}^7 \bar{Q}_0^6 \Delta\bar{n}_s}{168 V_{0,\max} A_{\max} (\mu_{\max}^2 - \mu_{\min}^2) \bar{N}^7} \, .
\end{aligned}
\tag{46}
$$

If we take $\bar{N} = 60$, $\bar{Q}_0 = Q_{0,\text{obs}} = 2 \times 10^{-5}$, $\Delta\bar{n}_s = 0.04$, and Planck-scale parameters for the probability distribution, we find $\bar{P}_1 = 1.1 \times 10^{-36}$. If instead we ask for the probability that $N > \bar{N}$ and $Q_0 < \bar{Q}_0$, without specifying n_s, then we find

$$
\bar{P}_2 = \frac{(75\pi^2)^2 M_{\text{Pl}}^2 \Delta_{\max}^5 \bar{Q}_0^4}{60 V_{0,\max} A_{\max} \bar{N}^5} \, ,
\tag{47}
$$

which is valid whether or not the $\frac{1}{2}\mu^2\varphi^2$ term is included in the potential. For the parameters specified above, this gives $\bar{P}_2 = 1.9 \times 10^{-24}$.

For comparison, the same questions can be answered using the FKMS parameterization, and the associated flat probability distribution in V_0, x, Δ, and possibly μ^2. If we include the $\frac{1}{2}\mu^2\varphi^2$ term and ask for the probability $\bar{P}_1'$ that a given instance will satisfy $N > \bar{N}$, $Q_0 < \bar{Q}_0$, and $|1-n_s| < \Delta\bar{n}_s$, we find

$$
\bar{P}_1' = \frac{(75\pi^2)^2 \Delta_{\max}^6 \bar{Q}_0^4 \Delta\bar{n}_s}{70 V_{0,\max} x_{\max} (\mu_{\max}^2 - \mu_{\min}^2) \bar{N}^5} \, .
\tag{48}
$$

For the numbers used above, this evaluates to $\bar{P}_1' = 3.2 \times 10^{-26}$. If instead we exclude the $\frac{1}{2}\mu^2\varphi^2$ term and ask for the probability $\bar{P}_2'$ that $N > \bar{N}$ and $Q_0 < \bar{Q}_0$, without specifying n_s, then we find

$$
\bar{P}_2' = \frac{75\pi^2 \bar{Q}_0^2 \Delta_{\max}^4}{15 V_{0,\max} x_{\max} \bar{N}^3} \, ,
\tag{49}
$$

which, for the numbers used here, is equal to $\bar{P}'_2 = 9.1 \times 10^{-14}$.

The detailed answers here depend very much on the ad hoc assumptions, and are not to be trusted, but the thrust of the answers is clear. First, in this picture the probability of seeing an episode of inflation that is suitable to begin our pocket universe is very small. The key point is that 60 e-folds is large compared to one e-fold, and $Q_{0,\text{obs}} \simeq 2 \times 10^{-5}$ is small compared to one. But we have assumed probability distributions that in no way favor large numbers of e-folds or small $Q_{0,\text{obs}}$, so the required values are found only in a small corner of the probability space. This feature could be changed dramatically if the underlying theory incorporated some mechanism to favor the right kind of potential, as we discussed at Eq. (45). Nonetheless, it is certainly not clear that any such probability enhancement is needed for the picture to be viable, because with 10^{500} or more vacua estimated to exist in the landscape, probabilities like 10^{-36} are very large. We would expect the landscape to contain a colossal number of possibilities for inflation to occur in exactly the right way to produce our pocket universe. One then argues that there are selection effects that explain why we would expect to find ourselves living in such a pocket universe. FKMS argue that at least 59.5 e-folds of inflation are necessary to explain the evolution of structure even at only the level of dwarf galaxies, and that with this condition the probability of having at least 62 e-folds, which is enough to explain the observed homogeneity and flatness, is high: about 88%. We will examine this question in Section 4, finding similar results.

3.2.3 The role of nucleation rates in the statistical distribution of N

In Eq. (28) we expressed $f(N, \{Q_{i,\text{obs}}\})$—the joint probability density for the number N of e-folds of slow-roll inflation and the measured parameters $\{Q_i\}$ for a bubble universe consistent with our observations and arising from a randomly selected quantum tunneling event—as the product of two factors, $f_0(N, \{Q_{i,\text{obs}}\})$ and $B(N, \{Q_{i,\text{obs}}\})$. $f_0(N, \{Q_{i,\text{obs}}\})$ is the vacuum statistics probability density, given by Eq. (29), defined so that correlations between N and the tunneling rate are irrelevant. $B(N, \{Q_{i,\text{obs}}\})$ is the factor that corrects for any bias caused by the correlations with the tunneling rate, and it is the purpose of this section to estimate this factor.

As described at the beginning of the previous section, we are not aware of any way of estimating the strength of the correlations between the tunneling region of the potential and the slow-roll region, so we are allowing for the two extreme possibilities. If these correlations are very weak, then $B = 1$. The rest of this section will be concerned with estimating B when the correlations are strong.

A transition can be described by specifying the parent P_α and the child C_j. For each such transition there is a nucleation rate $\lambda_{j\alpha}$, the number of tunneling events per physical spacetime volume. To understand the bias factor B, we need to understand the relation between $\lambda_{j\alpha}$ and the probability $p_{j\alpha}$ that a randomly chosen quantum tunneling event is of the type $P_\alpha \to C_j$.

Whether $p_{j\alpha}$ indeed depends on the nucleation rate $\lambda_{j\alpha}$ is a measure-dependent question. Here we argue that for the measures in classes (II) and (III), which are the ones that we consider most plausible, the probability $p_{j\alpha}$ is unchanged by any overall change in the decay rates from P_α, but is proportional to the branching ratio of the decay of P_α to C_j. We first explain this result, and then discuss its consequences.

We begin with the measure of Ref. [32], an example of measures in class (II). This measure adopts the method of comoving horizon cutoff, where the probabilities are defined by the ratios of the number of bubbles whose comoving sizes are greater than some small number ϵ ($\to 0$). The relative probability $p_{j\alpha}$ is then

$$p_{j\alpha} \propto H_\alpha^q \kappa_{j\alpha} s_\alpha \,, \tag{50}$$

where H_α is the Hubble parameter in the parent vacuum P_α, $\kappa_{j\alpha} \equiv (4\pi/3)\lambda_{j\alpha}H_\alpha^{-4}$ is the dimensionless nucleation rate, and q and s_α are given by the asymptotic behavior of the fraction of comoving volume occupied by a (non-terminal) vacuum X at time t:

$$f_X(t) \to s_X e^{-qt} \,. \tag{51}$$

In the above equations, we have adopted the expressions that apply when we take t to be the scale factor time, although the final result does not depend on the choice of the time variable. The asymptotic behavior of Eq. (51) is obtained by solving the rate equation

$$\frac{df_X}{dt} = \sum_Y M_{XY} f_Y \,, \qquad M_{XY} = \kappa_{XY} - \delta_{XY} \sum_Z \kappa_{ZX} \,, \tag{52}$$

where $\kappa_{XY} = (4\pi/3)\lambda_{XY}H_Y^{-4}$, and X, Y and Z run over all the vacua in the landscape. All nonzero eigenvalues of M_{XY} have negative real parts, and the eigenvalue with the smallest (by magnitude) real part is pure real, and is denoted by $-q$. This eigenvalue controls the asymptotic behavior of f_X and appears in Eq. (51). The vector s_X is proportional to the eigenvector of M_{XY} corresponding to the eigenvalue $-q$, and is determined by

$$\left(\sum_Y \kappa_{YX} - q\right)s_X = \sum_Z \kappa_{XZ}s_Z \,. \tag{53}$$

Equation (50) shows formally that $p_{j\alpha} \propto \kappa_{j\alpha}$, but we need to be careful, because s_α is itself determined by the nucleation rates. We will use Eq. (53) to understand the dependence of s_α on the $\kappa_{j\alpha}$. We first note that the positivity of s_X implies that q is smaller than the decay rate of the slowest decaying vacuum, called the dominant vacuum D: $q \leq \min_X(\sum_Y \kappa_{YX}) \equiv \kappa_D$. In fact, assuming that upward transitions have very small rates, $q \approx \kappa_D$ to a very good approximation (and $s_X \approx \delta_{XD}$ at the leading order) [40]. (Ref. [41] points out that the dominant vacuum could in fact be replaced by a closely spaced system of vacuum states, but that does not affect the conclusions

here.) Since bubble nucleation rates are exponentially sensitive to the parameters of the potential, we expect that the $-q$ term in Eq. (53) is negligible except for $X = D$:

$$\sum_Y \kappa_{YX} s_X = \sum_Z \kappa_{XZ} s_Z \quad \text{for } X \neq D \,. \tag{54}$$

Note that we can regard $\kappa_{YX} s_X$ as a "probability current" associated with the transition $X \to Y$, and then this equation is simply a statement of current conservation, where $X = D$ acts as a source and terminal vacua T, defined by $\sum_Y \kappa_{YT} = 0$, as sinks.

To determine the dependence of s_α on the $\kappa_{i\alpha}$, we can rewrite Eq. (54) with a relabeling of the indices:

$$\kappa_\alpha s_\alpha = \sum_Z \kappa_{\alpha Z} s_Z \,, \tag{55}$$

where $\kappa_\alpha = \sum_j \kappa_{j\alpha}$. In both situations discussed in Section 3.2.1, Figs. 2 and 3, it is reasonable to expect that the history leading to various parent vacua α is statistically independent with that afterwards, e.g. how fast those vacua decay: κ_α. Under this assumption, the right-hand side of Eq. (55) can be taken to be independent of κ_α (at least in the sense that there is no statistical correlation between the right-hand side of Eq. (55) and κ_α), leading to $s_\alpha \propto \kappa_\alpha^{-1}$. Inserting this result into Eq. (50), we see that

$$p_{j\alpha} \propto \frac{\kappa_{j\alpha}}{\kappa_\alpha} \,, \tag{56}$$

which says simply that the probability of observing a transition from parent P_α to child C_j is proportional to the branching ratio for this transition, but is unaffected by the absolute decay rate of the parent P_α. (Note that the right-hand side of Eq. (55) is proportional to W_α as defined above Eq. (29), so we have found that $p_{j\alpha} \propto W_\alpha \times$ branching ratio, motivating the weighting used in Eq. (29).)

The dependence of $p_{j\alpha}$ on branching ratios, but not on absolute decay rates, can also be shown for other measures. For the scale factor cutoff measure, an example of measures in class (III), a calculation of $p_{j\alpha}$ has been performed in Ref. [41], giving $p_{j\alpha} \propto \kappa_{j\alpha} s_\alpha$ where s_α again satisfies Eq. (53). Equation (56) follows immediately by the same argument. Scale factor measure can also be analyzed by recasting it as a local "fat geodesic" measure, as described in Ref. [24], and then the relative numbers of different transitions are clearly determined only by branching ratios that are encountered as the fat geodesic is followed into the future. The same result can be seen for the causal patch measure, also in class (III), using a local formulation analogous to the fat geodesic formulation. Specifically, Bousso [33] has shown that the probabilities $p_{j\alpha}$ in the causal patch measure can be computed by following a single geodesic, so they are determined directly from the branching ratios. The quantum measure of Ref. [16] also gives Eq. (56) (see Appendix A).

Thus, for a wide class of measures, the probabilities $p_{j\alpha}$ depend on the branching ratios of decays, but not on the decay rates themselves, which depend exponentially on the parameters of

the potential energy function. What does this tell us about the dependence of $f(N, \{Q_{i,\text{obs}}\})$ on N?

As we said at the start of this section, if the correlation between the tunneling region and the slow-role region of the potential is very weak, then $B = 1$. But if the correlation is strong, there are two possibilities: faster tunneling rates can correlate with smaller values of N, or larger values.

Consider first the case in which faster tunneling rates correlate with smaller values of N, thereby exerting pressure toward smaller N. We do not know how to estimate the strength of the correlation, but we can bound the effect by considering the most extreme possibility. Suppose, therefore, that for any given parent P_α we identify the nearest neighbors in the landscape, and assume that the decay rates to any other states are negligible. We let K be the number of such neighbors, and for simplicity we will assume that K is the same for all vacua, with a value of perhaps several hundred. For the strongest possible correlations, we can assume that the fastest decay will correspond to the smallest value of N, the second fastest decay will correspond to the second smallest value of N, etc., through the list of all K decay modes. To maximize the magnitude of the effect, we will further assume that the decays are dominated by the fastest, so that all other decay rates are negligible. (In Appendix B we find that this situation is actually quite plausible.) Since the fastest decay is also the one with smallest N, we find that, for any parent P_α, the branching ratio is 1 for the decay with the smallest value of N, and all other branching ratios can be approximated as zero.

If we now look at the transitions $P_\alpha \to C_j$ that contribute to $f_0(N, \{Q_{i,\text{obs}}\})$, as described by Eq. (29), we see that the final distribution $f(N, \{Q_{i,\text{obs}}\})$ is obtained by examining each pair (α, j) and applying a test: if the transition gives the smallest value of N of all K decays of P_α, then its branching ratio is 1, and it is kept. If, however, the value of N for the transition is not the smallest of all decays of P_α, the branching ratio is zero, and it is dropped. Thus, we can obtain an equation for $f(N, \{Q_{i,\text{obs}}\})$ by multiplying each term in Eq. (29) by the probability that the term corresponds to the lowest value of N out of K choices, and then renormalizing. But the new factor is just the probability that the other $K - 1$ values of N are larger, so for this case

$$B(N, \{Q_{i,\text{obs}}\}) = B_1(N, \{Q_{i,\text{obs}}\}) \propto \left[\int_N^\infty dN \, f_0(N, \{Q_{i,\text{obs}}\}) \right]^{K-1}. \tag{57}$$

If $f_0(N, \{Q_{i,\text{obs}}\}) \propto 1/N^p$, then $B(N, \{Q_{i,\text{obs}}\}) \propto 1/N^{(K-1)(p-1)}$, which is a huge suppression for large N. This was of course calculated as the maximum possible effect. Since we do not know how to assess the degree of correlation between tunneling rates and N, we could imagine suppression by any power of N from zero up to $(K-1)(p-1)$.

Now consider the alternative extreme, in which faster tunneling rates correlate with larger values of N, thereby exerting pressure toward larger N. The logic is all the same, but the result is very different. Eq. (57) is replaced by

$$B(N, \{Q_{i,\text{obs}}\}) = B_2(N, \{Q_{i,\text{obs}}\}) \propto \left[\int_0^N dN \, f_0(N, \{Q_{i,\text{obs}}\}) \right]^{K-1}. \tag{58}$$

Only the limits integration are different, but because of the fact that $f_0(N, \{Q_{i,\text{obs}}\})$ strongly favors small N, the quantity in square brackets now has a value very close to 1 for interesting values of N. Raising it to a large power does not produce a big effect. In fact, if we use Eq. (37) as an example, and choose $N = 60$ and $K = 300$, we find $B(N, \{Q_{i,\text{obs}}\}) = 1$ to better than 20 decimal places! For those measures for which only the branching ratios are relevant, correlations between tunneling rates and N cannot drive N to larger values. The reason is simply that if N is near 60, it is almost certainly the largest N among all the decays of the parent, so requiring it to be the largest has no effect.

While it is plausible that a given vacuum can have significant decay rates to only a few hundred nearest neighbors, we would like to also allow for the possibility that this is wrong, and that maybe a significant fraction of the landscape is available as a potentially significant decay channel. In that case we should take K to be 10 to the power of several hundred, and the whole picture changes. Then the powers in Eqs. (57) and (58) become enormous, and the factors in square brackets become completely controlling. As we will see in Appendix B, if we choose the largest or smallest element out of $\sim 10^{500}$ tries, from a normal distribution, the result is expected to be 48 standard deviations away from the mean. If there is a perfect correlation between N and the tunneling rate, as we assumed in the extreme example above, then N would be driven effectively to 0 or infinity, and the situation would have already been ruled out (if N is driven to 0), or else N would be essentially infinite. More realistically, however, we only know that choosing the fastest decay out of something like 10^{500} possibilities will result in a decay rate with an action that is of order 50 standard deviations smaller than the mean, but the strength of the correlation with N is unknown. The probability distribution for N could, therefore, be biased in either direction, and the bias might be weak or strong.

While measures of classes (II) and (III) generically lead to probabilities that depend only on branching ratios, as in Eq. (56), not quite all measures of interest fit this description. In particular, the stationary measure of Ref. [35] does not really fit into our classifications; it has many properties of class (I), while at the same time avoiding the youngness paradox. For measures in classes (II) and (III) we have seen that the factor of $\kappa_{j\alpha}$ appearing in Eq. (50) is accompanied by $s_\alpha \propto \kappa_\alpha^{-1}$, but that happens only when the abundances of the potential parent vacua P_α are determined primarily by their decay rates. In measures of class (I) it is the production rate of a given vacuum that primarily determines its abundance, while the decay rate of P_α has almost no effect on its abundance. Then there is no factor of κ_α^{-1} accompanying $\kappa_{j\alpha}$, and the probability of observing a transition from any parent P_α to any child C_i is proportional to $\kappa_{j\alpha}$. Since decay rates behave exponentially in the parameters of the potential function, for the stationary measure we expect that $B(N, \{Q_{i,\text{obs}}\})$ can be written as

$$B(N, \{Q_{i,\text{obs}}\}) = e^{\beta(N, \{Q_{i,\text{obs}}\})} \,, \tag{59}$$

where $\beta(N, \{Q_{i,\text{obs}}\})$ is a mild, non-exponential function of N. Thus, $f(N, \{Q_{i,\text{obs}}\})$ can have an exponential sensitivity to N. As in the class (II) and (III) cases, however, we could conjecture that the tunneling rates are only very weakly correlated with the slow-roll part of the potential, in which case we have $B = 1$, or equivalently $\beta = 0$, as before. In any case, the final probability $P(N)$ for stationary measure certainly has the slow-roll volume increase factor e^{3N}, i.e. Eq. (1). We are not aware of any measure in which $f(N)$ takes the form of Eq. (59) while $P(N)$ does not depend on the slow-roll volume increase factor, e^{3N}.

3.2.4 $f(N, \{Q_{i,\text{obs}}\})$ for scenario (iii), quantum tunneling: summary of results

We now summarize what we have learned about the probability distribution of N in a bubble universe. According to the discussions in the previous two subsections, the probability density $f(N, \{Q_{i,\text{obs}}\})$ for the *onset* of an episode of slow-roll inflation that is compatible with our observations $\{Q_{i,\text{obs}}\}$ can be written generically as

$$f(N, \{Q_{i,\text{obs}}\}) \propto \frac{1}{N^p} e^{\beta(N)} . \tag{60}$$

If the slow-roll part of the potential is correlated with the part controlling the tunneling, and if the correlation favors small values of N, then we might generically expect

$$p \gg 1 . \tag{61}$$

The effects of correlations between the slow-roll and tunneling parts of the potential for some measures depend crucially on how many significant decay channels compete in the decay of a given vacuum. Perhaps only a few hundred nearest neighbors in the landscape are relevant (small K option), or perhaps a substantial fraction of the landscape is relevant (large K option). If there is only weak correlation between the slow-roll part of the potential and the part controlling the tunneling, or if the correlation favors large values of N and we are considering the small K option, then the power p is determined purely by the statistics for the slow-roll part of the potential; then the analysis of Section 3.2.2 gives

$$p \begin{cases} > 0 \\ = 0 \end{cases} \text{ if the observable inflation occurs } \begin{cases} \text{accidentally} \\ \text{due to some mechanism} . \end{cases} \tag{62}$$

With the large K option, correlations in the potential that favor large N can be very significant; they need not have a power-law behavior.

The exponent $\beta(N)$ has the possibility of being nonzero only for measures, such as the stationary measure, which have the property that the probability of observing a transition from P_α to C_j depends on the tunneling rate for the transition, and not just the branching ratio. For measures of this type, $\beta(N)$ can arise due to correlations between the slow-roll part of the potential and the part

controlling the tunneling; if those correlations are weak, then $\beta(N) \approx 0$. But if the correlations are strong, then $\beta(N)$ can be very significant. Since we know very little about $\beta(N)$, we can use the fact that we are interested in only a small range of N about $N_{\mathrm{obs,min}}$ to expand $\beta(N)$ in a Taylor series: $\beta(N) \approx \beta(N_{\mathrm{obs,min}}) + \{\partial\beta/\partial N(N_{\mathrm{obs,min}})\}(N - N_{\mathrm{obs,min}}) + O(N - N_{\mathrm{obs,min}})^2$. The constant term does not affect the dependence on N, so we can drop it and replace $\beta(N)$ in Eq. (60) by

$$\beta(N) \to \beta' N \,, \tag{63}$$

where $\beta' \equiv \partial\beta/\partial N(N_{\mathrm{obs,min}})$ is a constant that does not depend on N, which can take either sign depending on details of the landscape potential. The magnitude of β' can be as small as zero, but to estimate how large it might be, we recall that it arises from the correlation between N and the tunneling rate $\Gamma \sim e^{-S}$ of the parent-to-child vacuum decay, where S is the bounce action associated with the decay. Suppose the potential barrier separating the parent from child vacuum is characterized by a field distance $\Delta\varphi$, a barrier height ΔV_h, and an energy density difference ΔV_{diff}. Then, the bounce action generically scales as the thin wall limit expression $S \approx \frac{27\pi^2}{2}\Delta\varphi^4 \, \Delta V_h^2/\Delta V_{\mathrm{diff}}^3$ [42]. If the amount of slow-roll inflation N is strongly correlated with the part of the potential energy function relevant for the tunneling, then we might estimate $|\beta'| \sim O(S/N_{\mathrm{obs,min}})$:

$$|\beta'| \sim O\left(\frac{27\pi^2}{2} \frac{\Delta\varphi^4 \, \Delta V_h^2}{N_{\mathrm{obs,min}} \, \Delta V_{\mathrm{diff}}^3}\right) \,, \tag{64}$$

which can easily be much larger than 1, depending on parameters. In our estimation $|\beta'|$ can lie anywhere from zero up to a number of the order shown above, and it can have either sign.

The probability density $P_{\mathrm{obs}}(N)$ of finding ourselves in a region that has undergone N e-folds of slow-roll inflation is then given by Eq. (9), where $M_m(N)$ (defined by Eq. (4)) depends on whether the measure rewards volume increase by slow-roll inflation (class (II)) or not (class (III)). We therefore obtain

$$P_{\mathrm{obs}}(N) \propto \frac{1}{N^p} \, e^{qN} \, \theta(N - N_{\mathrm{obs,min}}) \,, \tag{65}$$

where p is given by Eq. (61) or (62), and $q = 3 + \beta'$ for class (II) measures while $q = \beta'$ for class (III) measures; these values for p and q are summarized in Table 1. This is the expression we will use in our phenomenological analysis in Section 4.

3.3 $f(N, \{Q_{i,\mathrm{obs}}\})$ for scenario (iv): inflation preceded by a prior episode of inflation

We finally consider scenario (iv), the case where there is another episode of inflation just before our last cosmic inflation. In this case, the power spectrum of density fluctuations, $\mathcal{P}(k)$, can show a sharp spike as a function of the momentum scale k. One might, therefore, think that this can

Class (II) measures: rewarding slow-roll volume increase

	Measures not depending on decay rates	Measures depending on decay rates
Weak correlation between slow-roll and tunneling	$p \geq 0; \ q = 3$	$p \geq 0; \ q \simeq 3$
Strong positive correlation between slow-roll and tunneling	$p \geq 0; \ q = 3$	$p \geq 0; \ q = 3 + \beta'$ $\beta' \gtrsim O(1)$
Strong negative correlation between slow-roll and tunneling	$p \gg 1; \ q = 3$	$p \gg 1; \ q = 3 + \beta'$ $-\beta' \gtrsim O(1)$

Class (III) measures: not rewarding volume increase

	Measures not depending on decay rates	Measures depending on decay rates
Weak correlation between slow-roll and tunneling	$p \geq 0; \ q = 0$	$p \geq 0; \ |q| \ll 1$
Strong positive correlation between slow-roll and tunneling	$p \geq 0; \ q = 0$	$p \geq 0; \ q = \beta'$ $\beta' \gtrsim O(1)$
Strong negative correlation between slow-roll and tunneling	$p \gg 1; \ q = 0$	$p \gg 1; \ q = \beta'$ $-\beta' \gtrsim O(1)$

Table 1: Expected values of p and q in Eq. (65) for class (II) and (III) measures. They depend on whether the slow-roll and tunneling parts of the potential are weakly or strongly correlated, and on whether the correlation is positive (favoring large values of N) or negative (favoring small values of N). They also depend on whether the measure predicts that the probability of observing a particular transition depends only on its branching ratio (middle column), or depends on the decay rate (right column). The table is constructed for the small K option (see the text). The large K option would change the behavior of strong positive correlations for measures not depending on decay rates, giving a strong push toward large N which is not necessarily a power law.

provide a nonzero curvature over the visible universe, either positive or negative, by having large fluctuations at a length scale beyond the current horizon. This is, however, not the case. Since low multipoles of CMB temperature fluctuations are sensitive to density fluctuations at scales larger than the horizon (Grishchuk-Zel'dovich effect [43]), the observed size of these low multipoles $\approx 10^{-5}$ does not allow the curvature to extend much beyond $|\Omega_k| \approx O(10^{-5})$ [18].

Therefore, even if the past history of our pocket universe is complicated so that $\mathcal{P}(k)$ has a nontrivial structure, we do not expect to see curvature coming from density fluctuations at a level, e.g., beyond $|\Omega_k| \approx 10^{-4}$.[5] Since we know of no other way that positive curvature can be generated

---

[5]While completing this paper, Ref. [17] has appeared which quantitatively analyzes this issue, finding that the probability of obtaining $|\Omega_k| > 10^{-4}$ from superhorizon density fluctuations in a model consistent with the CMB is

in a multiverse model, we conclude that a future measurement of positive curvature at a level of $\Omega_k \lesssim -10^{-4}$ would exclude the entire framework considered here. Any observation of negative curvature at $\Omega_k \gtrsim 10^{-4}$ would have to be attributed to Coleman-De Luccia tunneling.

4   Expectations for the Number of *e*-folds and Curvature

We now discuss implications of the probability distribution in Eq. (65) for current and future measurements of curvature. Recall that $N_{\mathrm{obs,min}}$ denotes the minimum amount of slow-roll inflation required to satisfy the current observational constraint, $\Omega_k \lesssim 0.01$. (In this section we consider only $\Omega_k > 0$.) Its value depends on the detailed history of our own pocket universe, especially on the reheating temperature, but is in the range $N_{\mathrm{obs,min}} \approx (40 - 60)$. Following FKMS, we assume that the requirement of structure formation provides an anthropic lower bound on the amount of slow-roll inflation:

$$N_{\mathrm{anthropic}} \simeq N_{\mathrm{obs,min}} - 3.0 \,. \tag{66}$$

(Here, we have assumed only the weak requirement that dwarf galaxies form. If we require that typical galaxies form, then 3.0 is replaced by 1.9.) To test the consistency of the current constraint on Ω_k with multiverse probabilities, we use Eqs. (10) with $f(N, \{Q_{i,\mathrm{obs}}\}) M_m(N) = e^{qN}/N^p$ (see Eq. (65)) to express the probability P_{current} that a pocket universe which has undergone $N_{\mathrm{anthropic}}$ *e*-folds of slow-roll inflation will go on to undergo at least $N_{\mathrm{obs,min}}$ *e*-folds of inflation:

$$
\begin{aligned}
P_{\mathrm{current}} &= \int_{N_{\mathrm{obs,min}}}^{\infty} dN \, P_{\mathrm{obs},\not\Omega_k}(N) \\
&= \int_{N_{\mathrm{obs,min}}}^{\infty} dN \, \frac{e^{qN}}{N^p} \Bigg/ \int_{N_{\mathrm{anthropic}}}^{\infty} dN \, \frac{e^{qN}}{N^p} \,.
\end{aligned}
\tag{67}
$$

Figure 4(a) shows which regions of the *p*-*q* plane are excluded by yielding low values of P_{current}, at various levels of confidence, using $N_{\mathrm{obs,min}} = 60$.

Figure 4(a) shows that $q > 0$ is always allowed, but we should keep in mind that this is based on an idealization that is not reliable. It arises from the fact that the probability distribution $P(N) \propto e^{qN}/N^p$ diverges at large N for any $q > 0$, for any value of p. But if q is positive and small, and p is positive and large, then the divergent behavior will not occur until N is very large, at which point the linear approximation that we introduced in Eq. (63) will no longer be valid. Thus, for small positive q and large positive p, a more sophisticated analysis would be needed.

If we assume that there is only a weak correlation between the tunneling and slow-roll parts of the potential function, then measures of class (II), which reward slow-roll volume increases, are clearly allowed by Fig. 4(a). As shown in Table 1, these measures give $q = 3$ or at least $q \simeq 3$. Measures of
___
less than $\approx 10^{-6}$.

Figure 4: The diagram on the left shows the consistency of the current bound $\Omega_k \lesssim 0.01$ with multiverse probabilities. Assuming a probability distribution for the number N of e-folds of slow-roll inflation given by $P(N) \propto e^{qN}/N^p$, we calculate the probability that a pocket universe which has undergone $N_{\text{anthropic}} = 57$ e-folds of inflation will experience at least $N_{\text{obs,min}} = 60$ e-folds. The hypothesis that our pocket universe was drawn from such a probability distribution would be excluded at the 1σ, 2σ, 3σ, or 4σ level if this probability is less than 31.7%, 4.6%, 0.27%, or 0.0063%, respectively. The diagram shows the excluded regions in the p-q plane. Under the same assumptions, the diagram on the right shows the probability that our pocket universe has $\Omega_k > 10^{-4}$. More precisely, it shows the probability that a pocket universe which has undergone 60 e-folds of slow-roll inflation will not inflate by more than another factor of 10 (thereby suppressing Ω_k by no more than another factor of 100).

class (III), which do not reward slow-roll volume increases, are also consistent with P_{current}. These measures give $q = 0$ or very small, so the graph shows that the hypothesis is excluded at the 1σ level only if $p \gtrsim 23$. By contrast, in Section 3.2.2 we found that values in the range of $p = 0$ to $p = 8$ seemed plausible.

If there is a strong, positive correlation (i.e., favoring large N) between the tunneling and slow-roll parts of the potential function, then all the measures shown in Table 1 are again consistent with P_{current}. For measures that do not depend on decay rates, for the small K option (as defined in Section 3.2.4), the situation is identical to that described in the previous paragraph; for the large K option, the pressure toward large N improves the consistency. For those measures that depend on decay rates, q is given a positive contribution β' of order 1, which pushes an already acceptable (p, q) combination further from the excluded regions.

If, however, there is a strong negative correlation (i.e., favoring small N) between the tunneling and slow-roll parts of the potential function, then measures of class (III) (not rewarding volume increases) are very likely excluded, depending on exactly how strong the correlations are. The correlations cause p to become large, and for measures depending on decay rates, q to become negative as well. Only the mildest range of "strong" negative correlations would be consistent. Measures of class (II), which reward slow-roll volume in increase, would still be allowed if they do not depend on decay rates, since they would have $q = 3$. But for those that do depend on decay rates, $q = 3 + \beta'$, where $\beta' < 0$, so it could be allowed or not, depending on the magnitude of β'.

To discuss future measurements, we note that our pocket universe will have a curvature beyond Ω_k if the amount of slow-roll inflation satisfies

$$N < N_{\text{obs,min}} + \frac{1}{2} \ln \frac{\Omega_{k,\text{max}}}{\Omega_k} \equiv N(\Omega_k) \,, \tag{68}$$

where $\Omega_{k,\text{max}} \simeq 0.01$ is the maximum curvature allowed by the current observation. Recalling Eq. (65) for the probability density for the number N of e-folds of slow-roll inflation experienced

Figure 5: The probabilities of finding nonzero curvature in future measurements at the level of $\Omega_k = 10^{-3}$ (dashed) and 10^{-4} (solid) for the multiverse distribution $P(N) \propto 1/N^p$. The probabilities depend on the amount of slow-roll inflation $N_{\text{obs,min}}$ corresponding to the maximum curvature allowed by the current observation, $\Omega_{k,\text{max}} \simeq 0.01$.

by our pocket universe, the probability that $N < N(\Omega_k)$ is given by

$$
\begin{aligned}
P_{\text{future}}(\Omega_k) &= \int_0^{N(\Omega_k)} dN \, P_{\text{obs}}(N) \\
&= \int_{N_{\text{obs,min}}}^{N(\Omega_k)} dN \, \frac{e^{qN}}{N^p} \Bigg/ \int_{N_{\text{obs,min}}}^{\infty} dN \, \frac{e^{qN}}{N^p} \, .
\end{aligned}
\tag{69}
$$

In Fig. 4(b), we show contours in the p-q plane for $P_{\text{future}}(10^{-4})$, using $N_{\text{obs,min}} = 60$. In Fig. 5, we plot the probability for future measurements to find $\Omega_k > 10^{-3}$ (dashed) and 10^{-4} (solid) as a function of p, with $q = 0$, for $N_{\text{obs,min}} = 60$ and 40. We find that for relatively large $p \gtrsim$ a few, there is a reasonable chance that we can observe nonzero curvature larger than $\Omega_k \gtrsim 10^{-4}$. For $p \simeq 10$, the probability can be as high as $\approx 40\%$ for $N_{\text{obs,min}} = 40$, which corresponds to the case of a (very) low reheating temperature.

In the future, the PLANCK satellite and SDSS will be able to probe Ω_k to the level of ≈ 0.005 [44]. The planned Subaru surveys also have the potential to reach a 0.3% level precision: $\sigma(\Omega_k) \approx 0.003$ [45]. In the longer run, a hypothetical cosmic variance-limited CMB experiment together with a measurement of the baryonic acoustic oscillations at the precision expected from the Square Kilometer Array will constrain curvature with a precision of about 5×10^{-4}, which can give weak evidence for nonzero curvature down to the level of $\Omega_k \approx 10^{-3}$ [46]. Furthermore, a future square kilometer array optimized for 21 cm tomography could improve the sensitivity to about $\sigma(\Omega_k) \approx 2 \times 10^{-4}$ [47], approaching the fundamental limit with which one can probe the geometry of the universe given $Q \approx 10^{-5}$ [46]. Therefore, if our own pocket universe was indeed created by bubble nucleation in eternally inflating spacetime, then there is a reasonable chance (of $O(10\%)$) that we can see nonzero negative curvature in future measurements.

5   Conclusions

The eternally inflating multiverse provides a consistent framework for explaining coincidences and fine-tuning in our universe. In particular, it provides the leading explanation for the observed accelerating expansion of the universe: $\Omega_\Lambda \sim \Omega_{\text{matter}}$. Along similar lines, the framework also provides the possibility that the present-day curvature contribution, Ω_k, is not too far below the leading contributions to the total energy budget. Although Ω_k is suppressed exponentially by the

deterministic, slow-roll inflation that has occurred in our past, $\Omega_k \sim e^{-2N}$, there is still a reasonable possibility that Ω_k is larger than $\sim 10^{-4}$, the level we could reach in future observations.

We have studied this possibility, particularly focusing on the question: "If future observations reveal nonzero curvature, what can we conclude?" We have found that whether an observable signal arises or not depends crucially on three issues: the cosmic history just before the observable inflation, the measure adopted to define probabilities in the eternally inflating spacetime, and the properties of the correlation between the tunneling and slow-roll parts of the potential. These strong dependencies would allow us to draw some definite conclusions about these issues, if nonzero Ω_k is found in future experiments.

Our conclusions are as follows. If future measurements reveal positive curvature at the level $\Omega_k \lesssim -10^{-4}$, then ...

- The framework of the eternally inflating multiverse, as currently understood, is excluded with high significance. If no (currently unknown) mechanism can be found to explain a positively curved pocket universe in an eternally inflating multiverse, then we would have to conclude that our universe arose in a different way, e.g. directly by creation from "nothing" [48].

If future measurements instead reveal negative curvature $\Omega_k \gtrsim 10^{-4}$, then ...

- Diffusive (new or chaotic type) eternal inflation is excluded as a phenomenon in our immediate past. In particular, within the context of the eternally inflating multiverse (as currently understood), our pocket universe must have been born by a bubble nucleation. In this paper we justified this conclusion by examining the evolution of Q, the density perturbation amplitude, from the end of diffusive eternal inflation to the time at which the wave numbers visible in the CMB exited the Hubble horizon. We argued that this evolution required more than enough e-folds to suppress any trace of curvature. This conclusion is strengthened further by the fact that if the density perturbation amplitude was large ($\delta\rho/\rho \sim 1$) on the horizon scale at the onset of inflation, then the Grishchuk-Zel'dovich effect requires the amount of inflation to be large, $N - N_{\mathrm{obs,min}} \gtrsim 6$, completely diluting observable curvature effects [43, 49]. The bubble nucleation process avoids this situation by producing, without violating causality, a highly homogeneous space that is curvature dominated.

- Barring the unlikely possibility of a conspiracy between the slow-roll volume increase and tunneling rate ($\beta' \simeq -3$; see Table 1), the probability measure must not reward the slow-roll volume increase e^{3N}. Examples of such measures include the causal patch measure [33], the scale-factor cutoff measure [34], and the quantum measure [16].

- The origin of the observed slow-roll inflation—the last $N \approx (40 - 60)$ e-folds of inflation—must be an accidental feature of the potential, selected by anthropic conditions. In particular, it could not be due to a theoretical mechanism that ensures the flatness of the potential in the

vicinity of our vacuum.

- We do not know how to predict the strength or even the sign of possible correlations between the tunneling and slow-roll parts of the inflaton potential, so we considered all possibilities. We found that a strong negative correlation, one that correlates small N with rapid transitions, could have very strong effects which are already excluded by the fact that Ω_k is smaller than is required by anthropic considerations. If curvature is observed, then the possibility of strong positive correlations (those which favor large N) would be ruled out for those measures, such as the stationary measure, for which the probability of observing a transition depends on the decay rate, and not just the branching ratio. For other measures, the consequence of strong positive correlations depends on our estimate of the number of decay channels of our parent vacuum that can potentially have a significant branching ratio. If the significant decays are limited to a few hundred nearest neighbors in the landscape, then strong positive correlations are allowed, and have no perceptible effects on curvature or anything else. On the other hand, if a substantial fraction of the landscape is accessible with potentially significant rates, then a strong positive correlation would drive a significant increase in N, which would be ruled out if curvature were observed.

If future measurements do not find curvature, $|\Omega_k| \lesssim 10^{-4}$, then we would not learn much about the structure of the multiverse; in particular, it does not support or disfavor the framework.

We also addressed the question of whether the current constraint on $\Omega_k \lesssim 0.01$ is consistent with the predictions of the multiverse picture. We found that the present constraint is consistent, except that for measures that do not reward volume increase, strong negative (favoring small N) correlations between the slow-roll and tunneling part of the potential are ruled out.

In the course of these studies, we were led to consider the characteristics of vacuum decay branching ratios, focusing on the question of whether decays are typically dominated by a single channel. We found that for vacua that are sufficiently long-lived ($S \gtrsim 10^3$ if significant decays are limited to several hundred, or $S \gtrsim 10^6$ if decays can access the landscape, where the decay rate $\Gamma \sim e^{-S}$), it is plausible that a single channel can dominate the decay.

In the next decade or two, we expect to have new data from measurements of the CMB, baryonic acoustic oscillations, 21 cm absorption, and so on, which will allow us to probe the curvature of the universe down to the level of $\Omega_k \sim 10^{-4}$. If nonzero Ω_k is found in these measurements, it would reveal another coincidence in our universe: slow-roll inflation in our past did not last much longer than needed to cross the anthropic threshold. This would provide further evidence for the framework of the multiverse. Moreover, it would give us important information about the probability measure, the cosmic history just before the observable inflation, and the correlations in the inflaton potential function. In particular, it would strongly suggest that the probability measure does not reward volume increase, and that we are living in a bubble universe formed in an eternally inflating

spacetime.

Acknowledgments

A   Volume Increase in the Quantum Measure

In the framework of Ref. [16], the state of the multiverse is described in a fixed reference (local Lorentz) frame associated with a fixed spatial point p. The Hilbert space corresponding to a fixed semi-classical geometry $\mathcal{M}$ takes the form

$$\mathcal{H}_{\mathcal{M}} = \mathcal{H}_{\mathcal{M},\text{bulk}} \otimes \mathcal{H}_{\mathcal{M},\text{horizon}}, \tag{70}$$

where $\mathcal{H}_{\mathcal{M},\text{bulk}}$ and $\mathcal{H}_{\mathcal{M},\text{horizon}}$ represent Hilbert space factors associated with the degrees of freedom inside and on the stretched apparent horizon $\partial\mathcal{M}$. The entire Hilbert space for dynamical spacetime is then given by the direct sum of the Hilbert spaces for different $\mathcal{M}$'s:

$$\mathcal{H} = \bigoplus_{\mathcal{M}} \mathcal{H}_{\mathcal{M}}. \tag{71}$$

The full Hilbert space for quantum gravity, $\mathcal{H}_{\text{QG}}$, also contains the states associated with spacetime singularities, $\mathcal{H}_{\text{QG}} = \mathcal{H} \oplus \mathcal{H}_{\text{sing}}$, but these states in $\mathcal{H}_{\text{sing}}$ do not play an important role in our discussion here.

The multiverse state $|\Psi(t)\rangle$ is in general a superposition of elements in Hilbert space $\mathcal{H}_{\text{QG}}$, and evolves deterministically and unitarily in this Hilbert space. (We take the Schrödinger picture throughout.) The probabilities for any physical questions can then be given by the (extended) Born rule [16]. For example, one can specify a certain "premeasurement" situation A_{pre} (e.g. the configuration of an experimental apparatus before measurement) as well as a "postmeasurement" situation A_{post} (e.g. those after the measurement but without specifying outcome) as $A = \{A_{\text{pre}}, A_{\text{post}}\}$, and then ask the probability of a particular result B (specified, e.g., by a physical configuration of the pointer of the apparatus in A_{post}) to be obtained. The relevant probability $P(B|A)$ is then

$$P(B|A) = \frac{\iint dt_1 dt_2 \, \langle\Psi(0)|\, U(0,t_1)\, \mathcal{O}_{A_{\text{pre}}}\, U(t_1,t_2)\, \mathcal{O}_{A_{\text{post}} \cap B}\, U(t_2,t_1)\, \mathcal{O}_{A_{\text{pre}}}\, U(t_1,0)\, |\Psi(0)\rangle}{\iint dt_1 dt_2 \, \langle\Psi(0)|\, U(0,t_1)\, \mathcal{O}_{A_{\text{pre}}}\, U(t_1,t_2)\, \mathcal{O}_{A_{\text{post}}}\, U(t_2,t_1)\, \mathcal{O}_{A_{\text{pre}}}\, U(t_1,0)\, |\Psi(0)\rangle}. \tag{72}$$

Here, $U(t_1, t_2) = e^{-iH(t_1-t_2)}$ is the time evolution operator (for a fixed time parameterization t), and $\mathcal{O}_X$ is the operator projecting onto states consistent with condition X. This formula can be used to answer questions both regarding global properties of the universe and outcomes of particular experiments, providing complete unification of the eternally inflating multiverse and the many-worlds interpretation of quantum mechanics.

Now, suppose that the probability density for the *onset* of slow-roll inflation is given by $f(N)$. To figure out which class the quantum measure belongs, we want to know if the probability density of finding an observer *at a fixed location with respect to p* has an extra factor e^{3N} or not (see e.g. [50] for relevant discussions). Since each component of $|\Psi(t)\rangle$ describes the system within the horizon as viewed from p, however, it is obvious that this extra factor does not exist, i.e., how long a state stays in the slow-roll inflation phase does not affect the probability defined by Eq. (72), as long as the reheating temperature is fixed. This is because states corresponding to different N look identical after the reheating, except for quantities that depend on initial conditions at the onset of the slow-roll inflation. And since we are made out of baryons which are synthesized after the reheating (i.e. whose density does not depend on the history before the reheating), the probability density of us finding a universe with N e-folds of slow-roll inflation is simply $f(N)$ in a region where the anthropic factor is unity, $N > N_{\text{anthropic}}$ (see Section 2.3). This implies that the measure belongs to class (III), according to the classification in Section 2.2.[6]

A similar argument implies that the probability does not depend on the decay rate of a parent vacuum either. The quantum measure, therefore, gives $q = 0$ in Eq. (65); see Table 1 in Section 3.2.4.

B   Possibility of Single-Channel Dominance in Multiverse Evolution[7]

When a metastable vacuum P_α decays, there are generically a very large number of decay modes. One might assume that the decay products are dominated by vacua that are nearest neighbors to P_α in the landscape, and that the other vacua in the landscape can be neglected. In that case, we would expect perhaps several hundred possible decay modes. On the other hand, it is conceivable that a substantial fraction of the vacua in the landscape have the possibility of being significant decay channels for P_α, and then the number of relevant channels would be something like 10 to the power of several hundred. We will call the number of relevant decay channels K, allowing K to be anywhere from several hundred to 10 to the power of several hundred. In this appendix we will explore the possibility that this large number of decays is dominated by a single channel, finding it

---

[6]Incidentally, if we were made out of relics left over from the era before the inflation, such as the grand unified theory monopole, then the probability of us finding a universe with N e-folds of slow-roll inflation would be $f(N)e^{-3N}$ (without taking into account the dynamics for clustering etc), since the density of such relics is diluted by the inflation.

[7]We particularly thank Larry Guth for his help with this appendix.

much more plausible than one might naively guess, especially for long-lived vacua (i.e., vacua with decay rates $\Gamma \sim e^{-S}$, where $S \gtrsim 10^6$ for large K, of $S \gtrsim 10^3$ for small K). This issue is relevant for Section 3.2.1, in discussing the possibilities for a multiverse described by Fig. 2 or Fig. 3, and also in Section 3.2.3, in estimating the influence of nucleation rates on the probability distribution for N.

We have no real knowledge of the nucleation rates in the landscape, so we will pursue the simple hypothesis that they follow (approximately) the normal distribution:

$$f(S) \approx \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(S-\bar{S})^2}{2\sigma^2}}, \tag{73}$$

where $\bar{S}$ and σ are, respectively, the mean and the standard deviation. We will assume that $\bar{S} \gg \sigma$, so that we can ignore the statistically small possibility that the distribution gives a negative value for S. Later we will briefly discuss the case where σ and $\bar{S}$ are comparable.

We now ask: for a given P_α, what is the typical ratio of the fastest decay rate to the next fastest? Since the number of possible decay modes is very large, one might naively think that this ratio is close to unity; namely, whatever the fastest rate is, there would likely be many other possible decay modes that would have very similar rates. This is, however, not obvious because, although the density of the values for the decay rates is indeed huge near the peak in the distribution, we are interested in the maximum transition rate and the rates that are very near the maximum. These are in the tails of the distribution, so there is no guarantee that the naive thinking applies.

To estimate the minimum value of S, which we call S_1, we define the cumulative probability distribution function

$$\Phi(x) \equiv \int_{-\infty}^{x} f(t)\,dt, \tag{74}$$

which is the probability that a randomly chosen value of S is less than x. We estimate the value of S_1 by requiring

$$\Phi(S_1) = \frac{1}{K}; \tag{75}$$

that is, we imagine drawing K random values $\{S^{(1)}, \ldots, S^{(K)}\}$ from the probability distribution $f(S)$, and insist that the expectation value for the number of $S^{(i)}$'s less than S_1 is equal to one.

In the region of interest, $(\bar{S} - S_1)/\sigma \gg 1$, the left-hand side of Eq. (75) can be replaced by its asymptotic expansion [51]

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{(\bar{S}-S_1)^2}{2\sigma^2}} \left(\frac{\sigma}{\bar{S}-S_1} - \frac{\sigma^3}{(\bar{S}-S_1)^3} + \cdots \right) = \frac{1}{K}, \tag{76}$$

giving

$$S_1 = \bar{S} - \sqrt{2 \ln K}\,\sigma + O\left(\frac{\ln(\sqrt{\ln K})}{\sqrt{\ln K}}\,\sigma \right), \tag{77}$$

35

and

$$f(S_1) = \frac{\sqrt{2\ln K}}{K\sigma} + O\left(\frac{\ln(\sqrt{\ln K})}{K\sqrt{\ln K}\,\sigma}\right). \tag{78}$$

Note that $Kf(S_1)$ is the density of sample points at S_1, so we can estimate a typical difference $\Delta S \equiv S_2 - S_1$, where S_2 is the second smallest action, as

$$\Delta S \approx \frac{\sigma}{\sqrt{2\ln K}}\,. \tag{79}$$

The density grows arbitrarily large with K, but only as the square root of the logarithm! As we will now see, for reasonable examples this is not nearly enough to allow the second fastest decay mode to compete with the fastest one.

As an alternative estimate of ΔS, one could estimate S_2 directly by setting $\Phi(S_2) = \frac{2}{K}$, which has the effect of replacing K by $\frac{1}{2}K$ in Eq. (77). The result for ΔS is then equal to the result in Eq. (79) multiplied by $\ln 2$.

It is hard to know what a typical tunneling action is, because various calculations have given values over a huge range. Some of these calculations are summarized in Ref. [41]. For example, a calculation of the decay of an uplifted anti de Sitter vacuum in Ref. [52] gives an action

$$S \sim \frac{8\pi^2 M_{\mathrm{Pl}}^2}{m_{3/2}^2}\,, \tag{80}$$

which the authors estimate as $S \lesssim 10^{34}$ using $m_{3/2} \gtrsim 10^2$ GeV. Freivogel and Lippert [53] concluded that any vacuum capable of supporting life must decay with an action

$$S \lesssim 10^{40\pm20} \tag{81}$$

to avoid overproducing Boltzmann Brains, and then showed that KKLT [54] vacua decay with actions less than 10^{22}. In Ref. [55], however, the authors argue that the vast majority of flux vacua with small cosmological constant undergo rapid decay, with tunneling actions of order one.

As sample numbers to use here, we consider a transition for which the field excursion $\Delta\varphi$ is of order M_{Pl}, while the barrier height ΔV_h and the energy density difference ΔV_{diff} are each of $O(M_{\mathrm{unif}}^4)$, where $M_{\mathrm{unif}} \approx 10^{16}$ GeV is the (supersymmetric) unification scale. A small hierarchy between M_{unif} and M_{Pl} ensures that metastable minima of the potential are long-lived, since the natural size for the action is given by $S \approx \frac{27\pi^2}{2}\Delta\varphi^4\,\Delta V_h^2/\Delta V_{\mathrm{diff}}^3$ [42]. This estimate gives $S \sim O(10^{10})$, and we choose a relatively small σ, $\sigma \sim O(10^8)$. (Such a narrow distribution of S might arise from a structure of the landscape [56].) We begin by considering $K \sim O(10^{500})$, a number appropriate for considering decays to a substantial fraction of the landscape. For actions near the peak of the probability distribution, the density of sample points per unit of S would then be of order $K/\sigma \sim O(10^{492})$, so for every decay channel there would typically be many more that would have

the same action to hundreds of decimal places. Nonetheless, at the tail of the distribution where the fastest two decays are to be found, the density of sample points is only $\sqrt{2\ln K}/\sigma$, and $\sqrt{2\ln K}$ is only $\simeq 48$. Thus, for our toy numbers the density of sample points in the tail is only $\approx 5 \times 10^{-7}$. This means that the two smallest points for S are likely to be separated by $\Delta S \approx 2 \times 10^6$, which means that the leading nucleation rate dominates over the second place nucleation rate by a factor of $e^{\Delta S} \sim e^{2\times 10^6}$. Of course if we used K of order a few hundred, the situation would become even more extreme. For $K \simeq 200$, for example, $\sqrt{2\ln K} \approx 3.3$, so the density of sample points in the tail is only $\approx 3 \times 10^{-8}$, and the leading nucleation rate will dominate over the second place rate by a factor of about $e^{\Delta S} \sim e^{3\times 10^7}$.

An important caveat of this analysis is the arbitrariness of choosing a normal distribution for the values of S. Something resembling a normal distribution is plausible, but the actual distribution could be very different. Furthermore, the normal distribution clearly has to be modified for cases where it predicts a negative value for S. For $K \sim O(10^{500})$, Eq. (77) implies that the dominating value of S is about 48 standard deviations below the mean, so clearly the whole approach would break down if $\bar{S} - 48\sigma$ were not positive. Thus the approach is viable only if $\bar{S} \gtrsim 50\sigma$. To obtain strong dominance of the leading decay we need $\sigma/50 \gtrsim 10^2$, so the argument presented here can lead to the conclusion of single-channel dominance only if $\bar{S} \gtrsim 10^6$. For $K \simeq O(200)$ this is less of a problem, because we need only insist that $\bar{S} - 3.3\sigma$ is positive, so a similar argument shows that we need only require that $\bar{S} \gtrsim 10^3$.

If one is interested in parameters for which the normal distribution gives negative values of S, one could explore the possibility of using a probability distribution which is positive by construction. A probability distribution that is often used as a model for positive-definite quantities is the gamma distribution,

$$f_\Gamma(S) = \frac{\lambda^{p+1}}{\Gamma(p+1)} S^p e^{-\lambda S} \theta(S) , \qquad (82)$$

where $p > 0$ and $\lambda > 0$ are parameters to be chosen. Since we are interested only in the low-S tail, we can explore a simpler distribution

$$f_{\text{simple}}(S) = \frac{(p+1)S^p}{S_{\text{max}}^{p+1}} \theta(S)\,\theta(S_{\text{max}} - S) , \qquad (83)$$

where $p > 0$ and $S_{\text{max}} > 0$ are to be chosen. Applying the same analysis as above, we find that the density of sample points at S_1, the lowest value of S out of K samples, is given by

$$K f_{\text{simple}}(S_1) \approx \frac{p+1}{S_{\text{max}}} K^{\frac{1}{p+1}} . \qquad (84)$$

If we insist that $K f_{\text{simple}}(S_1) \lesssim 10^{-2}$ to lead to single-channel domination, then with $K \simeq O(200)$ we find that $p = 2$ allows $S_{\text{max}} \sim 1800$, $p = 3$ allows $S_{\text{max}} \sim 1500$, while $p = 4$ allows $S_{\text{max}} \sim 1450$. Thus, the new probability distribution does not allow us to extend the argument below $\bar{S} \sim 10^3$,

so we conclude that single-channel dominance is not likely to occur for actions this small. If $K \sim O(10^{500})$, then this distribution will never give single-channel dominance.

In summary, these considerations suggest that decays of vacua for which the typical action is $\gtrsim 10^6$ if $K \sim O(10^{500})$, or $\gtrsim 10^3$ if $K \simeq O(200)$, are plausibly dominated by a single channel. This allows for the possibility that the entire multiverse is dominated by a single channel. For example, in the scale-factor cutoff measure, the spacetime volume is typically dominated by a very slowly-decaying, presumably very low energy density vacuum, called the dominant vacuum. An upward tunneling is required to access the high-energy part of the multiverse. In deciding whether the upward tunneling is dominated by a single channel, one should keep in mind that most of the action appearing in this calculation is associated with the initial state, and will apply to all final states; so only a small part of the action is relevant for estimating the spread of the values for the action. Nonetheless, it is conceivable that this upward tunneling is dominated by a single channel, and that a single pathway of subsequent tunnelings dominates the multiverse, as depicted in Fig. 3. It is also possible, however, that this is not the case, and that Fig. 2 is a more accurate description of the multiverse.

References

[1] A. H. Guth, *"The inflationary universe: a possible solution to the horizon and flatness problems,"* Phys. Rev. D **23**, 347 (1981);
A. D. Linde, *"A new inflationary universe scenario: a possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems,"* Phys. Lett. B **108**, 389 (1982);
A. Albrecht and P. J. Steinhardt, *"Cosmology for grand unified theories with radiatively induced symmetry breaking,"* Phys. Rev. Lett. **48**, 1220 (1982).

[2] E. Komatsu *et al.* [WMAP Collaboration], *"Five-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: cosmological interpretation,"* Astrophys. J. Suppl. **180**, 330 (2009) [arXiv:0803.0547 [astro-ph]]; *"Seven-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: cosmological interpretation,"* Astrophys. J. Suppl. **192**, 18 (2011) [arXiv:1001.4538 [astro-ph.CO]].

[3] S. W. Hawking, *"The development of irregularities in a single bubble inflationary universe,"* Phys. Lett. B **115**, 295 (1982);
A. A. Starobinsky, *"Dynamics of phase transition in the new inflationary universe scenario and generation of perturbations,"* Phys. Lett. B **117**, 175 (1982);
A. H. Guth and S.-Y. Pi, *"Fluctuations in the new inflationary universe,"* Phys. Rev. Lett. **49**, 1110 (1982);

J. M. Bardeen, P. J. Steinhardt and M. S. Turner, *"Spontaneous creation of almost scale-free density perturbations in an inflationary universe,"* Phys. Rev. D **28**, 679 (1983).

[4] See, e.g., C. J. Hogan, *"Why the universe is just so,"* Rev. Mod. Phys. **72**, 1149 (2000) [astro-ph/9909295];
L. J. Hall and Y. Nomura, *"Evidence for the multiverse in the standard model and beyond,"* Phys. Rev. D **78**, 035001 (2008) [arXiv:0712.2454 [hep-ph]].

[5] S. Weinberg, *"Anthropic bound on the cosmological constant,"* Phys. Rev. Lett. **59**, 2607 (1987);
see also, T. Banks, *"T C P, quantum gravity, the cosmological constant and all that...,"* Nucl. Phys. B **249**, 332 (1985);
A. D. Linde, *"The inflationary universe,"* Rept. Prog. Phys. **47**, 925 (1984).

[6] A. G. Riess *et al.* [Supernova Search Team Collaboration], *"Observational evidence from super-novae for an accelerating universe and a cosmological constant,"* Astron. J. **116**, 1009 (1998) [arXiv:astro-ph/9805201];
S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], *"Measurements of Ω and Λ from 42 high-redshift supernovae,"* Astrophys. J. **517**, 565 (1999) [arXiv:astro-ph/9812133].

[7] A. H. Guth and E. J. Weinberg, *"Could the universe have recovered from a slow first order phase transition?,"* Nucl. Phys. B **212**, 321 (1983).

[8] A. Vilenkin, *"The birth of inflationary universes,"* Phys. Rev. D **27**, 2848 (1983).

[9] A. D. Linde, *"Eternally existing self-reproducing chaotic inflationary universe,"* Phys. Lett. B **175**, 395 (1986); *"Eternal chaotic inflation,"* Mod. Phys. Lett. A **1**, 81 (1986).

[10] R. Bousso and J. Polchinski, *"Quantization of four-form fluxes and dynamical neutralization of the cosmological constant,"* JHEP **06**, 006 (2000) [arXiv:hep-th/0004134];
S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, *"De Sitter vacua in string theory,"* Phys. Rev. D **68**, 046005 (2003) [arXiv:hep-th/0301240];
L. Susskind, *"The anthropic landscape of string theory,"* arXiv:hep-th/0302219;
M. R. Douglas, *"The statistics of string / M theory vacua,"* JHEP **05**, 046 (2003) [arXiv:hep-th/0303194].

[11] A. Vilenkin and S. Winitzki, *"Probability distribution for Ω in open universe inflation,"* Phys. Rev. D **55**, 548 (1997) [arXiv:astro-ph/9605191].

[12] J. Garriga, T. Tanaka and A. Vilenkin, *"The density parameter and the anthropic principle,"* Phys. Rev. D **60**, 023501 (1999) [arXiv:astro-ph/9803268].

[13] B. Freivogel, M. Kleban, M. Rodriguez Martinez and L. Susskind, *"Observational consequences of a landscape,"* JHEP **03**, 039 (2006) [arXiv:hep-th/0505232].

[14] M. Tegmark, *"What does inflation really predict?,"* JCAP **04**, 001 (2005) [arXiv:astro-ph/0410281];

J. March-Russell and F. Riva, *"Signals of inflation in a friendly string landscape,"* JHEP **07**, 033 (2006) [arXiv:astro-ph/0604254];

B. Bozek, A. Albrecht and D. Phillips, *"Curvature constraints from the causal entropic principle,"* Phys. Rev. D **80**, 023527 (2009) [arXiv:0902.1171 [astro-ph.CO]];

R. Bousso and S. Leichenauer, *"Predictions from star formation in the multiverse,"* Phys. Rev. D **81**, 063524 (2010) [arXiv:0907.4917 [hep-th]];

R. Bousso, B. Freivogel, S. Leichenauer and V. Rosenhaus, *"Geometric origin of coincidences and hierarchies in the landscape,"* arXiv:1012.2869 [hep-th];

A. De Simone and M. P. Salem, *"The distribution of Ω_k from the scale-factor cutoff measure,"* Phys. Rev. D **81**, 083527 (2010) [arXiv:0912.3783 [hep-th]];

P. W. Graham, R. Harnik and S. Rajendran, *"Observing the dimensionality of our parent vacuum,"* Phys. Rev. D **82**, 063524 (2010) [arXiv:1003.0236 [hep-th]];

J. Frazer and A. R. Liddle, *"Exploring a string-like landscape,"* JCAP **02**, 026 (2011) [arXiv:1101.1619 [astro-ph.CO]];

D. Yamauchi, A. Linde, A. Naruko, M. Sasaki and T. Tanaka, *"Open inflation in the landscape,"* arXiv:1105.2674 [hep-th].

[15] For reviews, see e.g. A. H. Guth, *"Inflation and eternal inflation,"* Phys. Rept. **333**, 555 (2000) [arXiv:astro-ph/0002156];

A. Vilenkin, *"A measure of the multiverse,"* J. Phys. A **40**, 6777 (2007) [arXiv:hep-th/0609193];

S. Winitzki, *"Predictions in eternal inflation,"* Lect. Notes Phys. **738**, 157 (2008) [arXiv:gr-qc/0612164];

A. Linde, *"Inflationary cosmology,"* Lect. Notes Phys. **738**, 1 (2008) [arXiv:0705.0164 [hep-th]].

[16] Y. Nomura, *"Physical theories, eternal inflation, and the quantum universe,"* JHEP **11**, 063 (2011) [arXiv:1104.2324 [hep-th]]; *"Quantum mechanics, spacetime locality, and gravity,"* arXiv:1110.4630 [hep-th].

[17] M. Kleban and M. Schillo, *"Spatial curvature falsifies eternal inflation,"* arXiv:1202.5037 [astro-ph.CO].

[18] M. Kleban, Private communication, later explained more fully in Ref. [17].

[19] S. Coleman and F. De Luccia, *"Gravitational effects on and of vacuum decay,"* Phys. Rev. D **21**, 3305 (1980).

[20] J. R. Gott III, *"Creation of open universes from de Sitter space,"* Nature **295**, 304 (1982);

M. Bucher, A. S. Goldhaber and N. Turok, *"An open universe from inflation,"* Phys. Rev. D **52**, 3314 (1995) [hep-ph/9411206];

K. Yamamoto, M. Sasaki and T. Tanaka, *"Large angle CMB anisotropy in an open universe in the one bubble inflationary scenario,"* Astrophys. J. **455**, 412 (1995) [astro-ph/9501109].

[21] J. Silk and M. S. Turner, *"Double inflation,"* Phys. Rev. D **35**, 419 (1987).

[22] A. D. Linde, *"Hybrid inflation,"* Phys. Rev. D **49**, 748 (1994) [astro-ph/9307002].

[23] L. Randall, M. Soljacic and A. H. Guth, *"Supernatural inflation: inflation from supersymmetry with no (very) small parameters,"* Nucl. Phys. B **472**, 377 (1996) [hep-ph/9512439];
M. Kawasaki, T. Takayama, M. Yamaguchi and J. Yokoyama, *"Power spectrum of the density perturbations from smooth hybrid new inflation Model,"* Phys. Rev. D **74**, 043525 (2006) [hep-ph/0605271];
D. H. Lyth, *"The hybrid inflation waterfall and the primordial curvature perturbation,"* arXiv:1201.4312 [astro-ph.CO];
M. Kawasaki, A. Kusenko and T. T. Yanagida, *"Primordial seeds of supermassive black holes,"* arXiv:1202.3848 [astro-ph.CO].

[24] R. Bousso, B. Freivogel and I.-S. Yang, *"Properties of the scale factor measure,"* Phys. Rev. D **79**, 063513 (2009) [arXiv:0808.3770 [hep-th]].

[25] R. Bousso, *"Complementarity in the multiverse,"* Phys. Rev. D **79**, 123524 (2009) [arXiv:0901.4806 [hep-th]];
R. Bousso and I.-S. Yang, *"Global-local duality in eternal inflation,"* Phys. Rev. D **80**, 124024 (2009) [arXiv:0904.2386 [hep-th]].

[26] A. Linde and A. Mezhlumian, *"Stationary universe,"* Phys. Lett. B **307**, 25 (1993) [arXiv:gr-qc/9304015];
A. Linde, D. Linde and A. Mezhlumian, *"From the big bang theory to the theory of a stationary universe,"* Phys. Rev. D **49**, 1783 (1994) [arXiv:gr-qc/9306035];
A. Vilenkin, *"Predictions from quantum cosmology,"* Phys. Rev. Lett. **74**, 846 (1995) [arXiv:gr-qc/9406010].

[27] A. H. Guth, in Ref. [15];
M. Tegmark, in Ref. [14];
R. Bousso, B. Freivogel and I.-S. Yang, *"Boltzmann babies in the proper time measure,"* Phys. Rev. D **77**, 103514 (2008) [arXiv:0712.3324 [hep-th]].

[28] D. N. Page, *"Is our universe likely to decay within 20 billion years?,"* Phys. Rev. D **78**, 063535 (2008) [arXiv:hep-th/0610079];
R. Bousso and B. Freivogel, *"A paradox in the global description of the multiverse,"* JHEP **06**, 018 (2007) [arXiv:hep-th/0610132].

[29] B. Feldstein, L. J. Hall and T. Watari, *"Density perturbations and the cosmological constant from inflationary landscapes,"* Phys. Rev. D **72**, 123506 (2005) [arXiv:hep-th/0506235];

J. Garriga and A. Vilenkin, *"Anthropic prediction for Λ and the Q catastrophe,"* Prog. Theor. Phys. Suppl. **163**, 245 (2006) [arXiv:hep-th/0508005].

[30] A. Vilenkin, *"Freak observers and the measure of the multiverse,"* JHEP **01**, 092 (2007) [arXiv:hep-th/0611271].

[31] A. D. Linde and V. Mukhanov, *"The curvaton web,"* JCAP **04**, 009 (2006) [arXiv:astro-ph/0511736];
L. J. Hall, T. Watari and T. T. Yanagida, *"Taming the runaway problem of inflationary landscapes,"* Phys. Rev. D **73**, 103502 (2006) [arXiv:hep-th/0601028].

[32] J. Garriga, D. Schwartz-Perlov, A. Vilenkin and S. Winitzki, *"Probabilities in the inflationary multiverse,"* JCAP **01**, 017 (2006) [arXiv:hep-th/0509184];
R. Easther, E. A. Lim and M. R. Martin, *"Counting pockets with world lines in eternal inflation,"* JCAP **03**, 016 (2006) [arXiv:astro-ph/0511233].

[33] R. Bousso, *"Holographic probabilities in eternal inflation,"* Phys. Rev. Lett. **97**, 191302 (2006) [arXiv:hep-th/0605263].

[34] A. De Simone, A. H. Guth, M. P. Salem and A. Vilenkin, *"Predicting the cosmological constant with the scale-factor cutoff measure,"* Phys. Rev. D **78**, 063520 (2008) [arXiv:0805.2173 [hep-th]];
R. Bousso, B. Freivogel and I.-S. Yang, in Ref. [24];
see also, A. Linde, D. Linde and A. Mezhlumian, in Ref. [26].

[35] A. Linde, *"Towards a gauge invariant volume-weighted probability measure for eternal inflation,"* JCAP **06**, 017 (2007) [arXiv:0705.1160 [hep-th]];
A. Linde, V. Vanchurin and S. Winitzki, *"Stationary measure in the multiverse,"* JCAP **01**, 031 (2009) [arXiv:0812.0005 [hep-th]].

[36] A. R. Liddle and D. H. Lyth, *"Cosmological Inflation and Large-Scale Structure,"* (Cambridge University Press, Cambridge, 2000), p. 186.
The quantities called $\mathcal{P}_{\mathcal{R}}$ and δ_H in this book correspond, respectively, to the quantities $\Delta_{\mathcal{R}}^2$ and Q used here.

[37] Ref. [36], p. 188.

[38] S. W. Hawking and I. G. Moss, *"Supercooled phase transitions in the very early universe,"* Phys. Lett. B **110**, 35 (1982);
E. J. Weinberg, *"Hawking-Moss bounces and vacuum decay rates,"* Phys. Rev. Lett. **98**, 251303 (2007) [hep-th/0612146].

[39] P. Batra and M. Kleban, *"Transitions between de Sitter minima,"* Phys. Rev. D **76**, 103510 (2007) [hep-th/0612083].

[40] D. Schwartz-Perlov and A. Vilenkin, *"Probabilities in the Bousso-Polchinski multiverse,"* JCAP **06**, 010 (2006) [arXiv:hep-th/0601162].

[41] A. De Simone, A. H. Guth, A. D. Linde, M. Noorbala, M. P. Salem and A. Vilenkin, *"Boltzmann brains and the scale-factor cutoff measure of the multiverse,"* Phys. Rev. D **82**, 063520 (2010) [arXiv:0808.3778 [hep-th]].

[42] S. Coleman, *"The fate of the false vacuum: semiclassical theory,"* Phys. Rev. D **15**, 2929 (1977) [Erratum-ibid. D **16**, 1248 (1977)];
M. J. Duncan and L. G. Jensen, *"Exact tunneling solutions in scalar field theory,"* Phys. Lett. B **291**, 109 (1992).

[43] L. P. Grishchuk, Y. .B. Zeldovich, *"Long-wavelength perturbations of a Friedmann universe, and anisotropy of the microwave background radiation,"* Astron. Zh. **55**, 209 (1978) [Sov. Astron. **22**, 125 (1978)].

[44] D. J. Eisenstein, W. Hu and M. Tegmark, *"Cosmic complementarity: joint parameter estimation from CMB experiments and redshift surveys,"* Astrophys. J. **518**, 2 (1999) [arXiv:astro-ph/9807130].

[45] M. Takada, Private communication.

[46] M. Vardanyan, R. Trotta and J. Silk, *"How flat can you get? A model comparison perspective on the curvature of the universe,"* Mon. Not. Roy. Astron. Soc. **397**, 431 (2009) [arXiv:0901.3354 [astro-ph.CO]].

[47] Y. Mao, M. Tegmark, M. McQuinn, M. Zaldarriaga and O. Zahn, *"How accurately can 21 cm tomography constrain cosmology?,"* Phys. Rev. D **78**, 023529 (2008) [arXiv:0802.1710 [astro-ph]];

[48] A. D. Linde and A. Mezhlumian, *"Inflation with $\Omega \neq 1$,"* Phys. Rev. D **52**, 6789 (1995) [astro-ph/9506017].

[49] A. Kashlinsky, I. I. Tkachev and J. Frieman, *"Microwave background anisotropy in low-Ω_0 inflationary models and the scale of homogeneity in the universe,"* Phys. Rev. Lett. **73**, 1582 (1994) [astro-ph/9405024];
P. G. Castro, M. Douspis and P. G. Ferreira, *"Scale of homogeneity of the universe from WMAP,"* Phys. Rev. D **68**, 127301 (2003) [astro-ph/0309320].

[50] G. Larsen, Y. Nomura and H. L. L. Roberts, *"The cosmological constant in the quantum multiverse,"* Phys. Rev. D **84**, 123512 (2011) [arXiv:1107.3556 [hep-th]].

[51] See, e.g., M. Abramowitz and I. A. Stegun, *"Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables,"* National Bureau of Standards Applied Mathematics

Series – 55, Tenth Printing (1972), Eq. (7.1.23), available at `http://people.maths.ox.ac.uk/~macdonald/aands/index.html`

[52] A. Ceresole, G. Dall'Agata, A. Giryavets, R. Kallosh and A. D. Linde, *"Domain walls, near-BPS bubbles, and probabilities in the landscape,"* Phys. Rev. D **74**, 086010 (2006) [hep-th/0605266].

[53] B. Freivogel and M. Lippert, *"Evidence for a bound on the lifetime of de Sitter space,"* JHEP **12**, 096 (2008) [arXiv:0807.1104 [hep-th]].

[54] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, in Ref. [10].

[55] M. Dine, G. Festuccia, A. Morisse and K. van den Broek, *"Metastable Domains of the Landscape,"* JHEP **06**, 014 (2008) [arXiv:0712.1397 [hep-th]].

[56] N. Arkani-Hamed, S. Dimopoulos and S. Kachru, *"Predictive landscapes and new physics at a TeV,"* arXiv:hep-th/0501082.
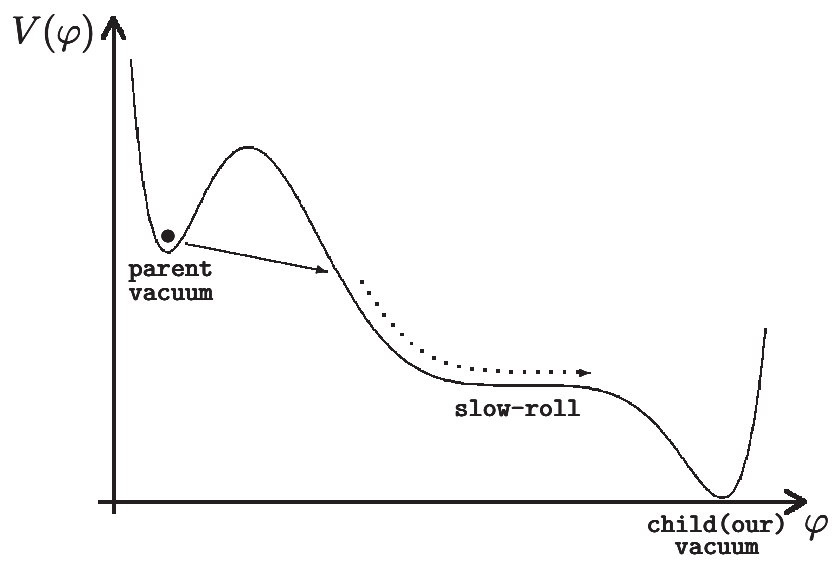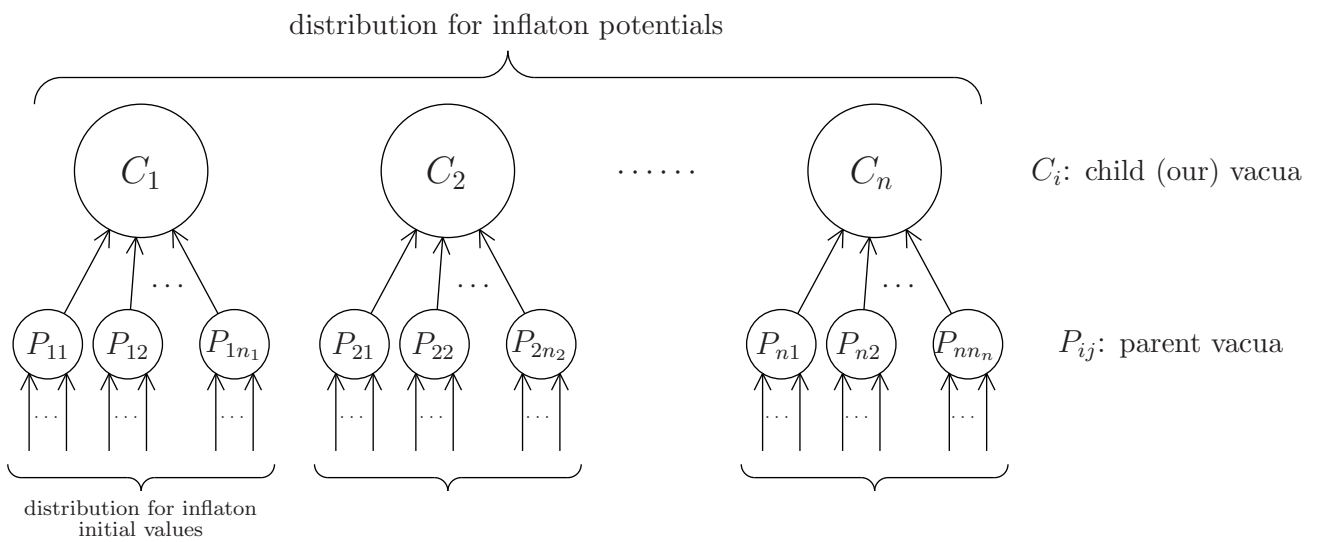
Figure 1   21APR12

Figure 2   21APR12

C_*: our vacuum

C_1 C_2 C_* C_n

C_i: child vacua

Each C_i has different inflaton potential and initial value.

P_1 P_2 P_* P_n

P_i: parent vacua

This path dominates the others, but we don't know what it actually is $\implies$ probability
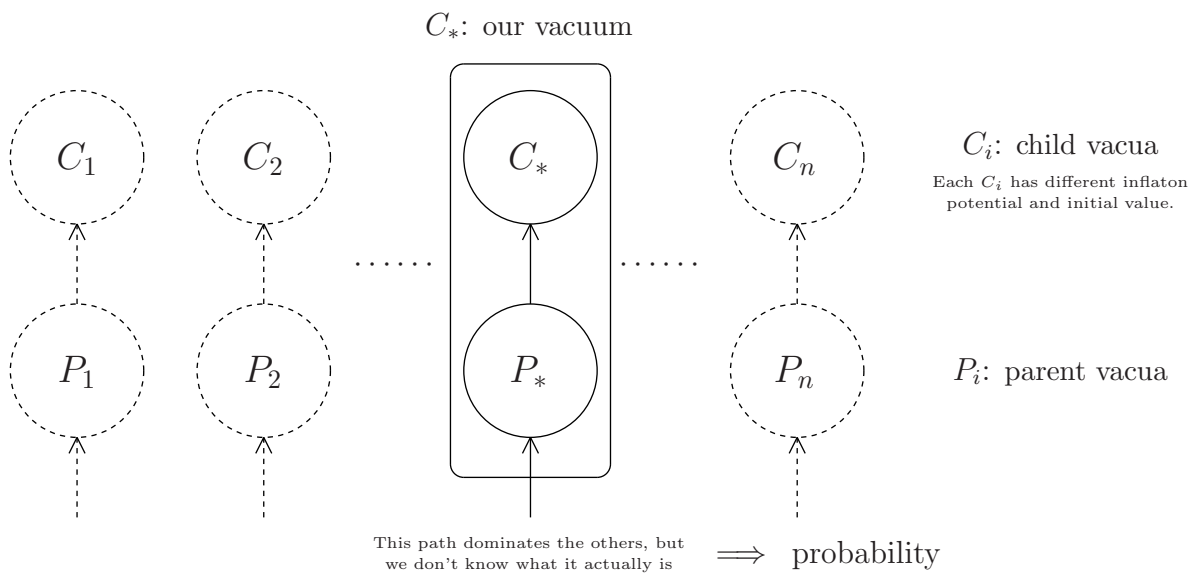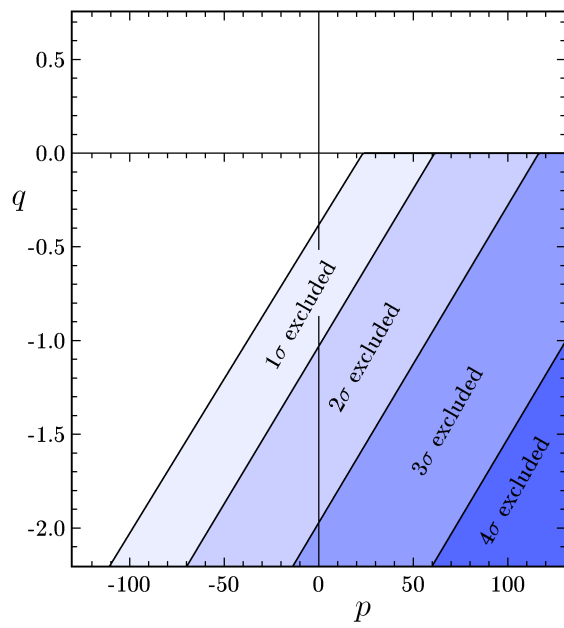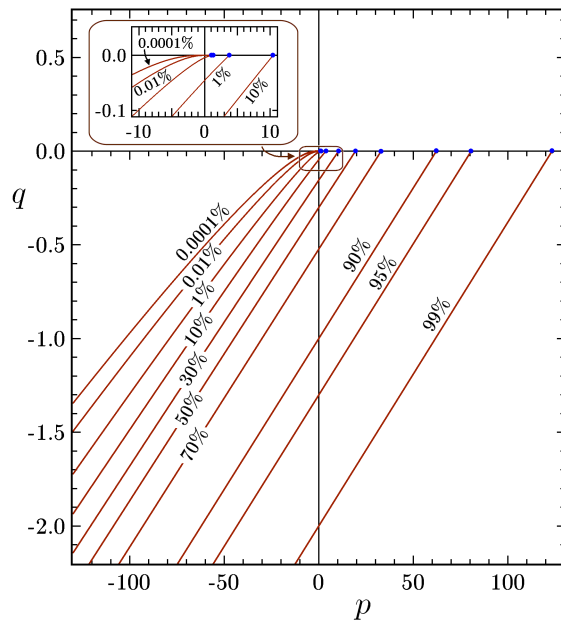
Figure 3   21APR12

Figure 4a   21APR12

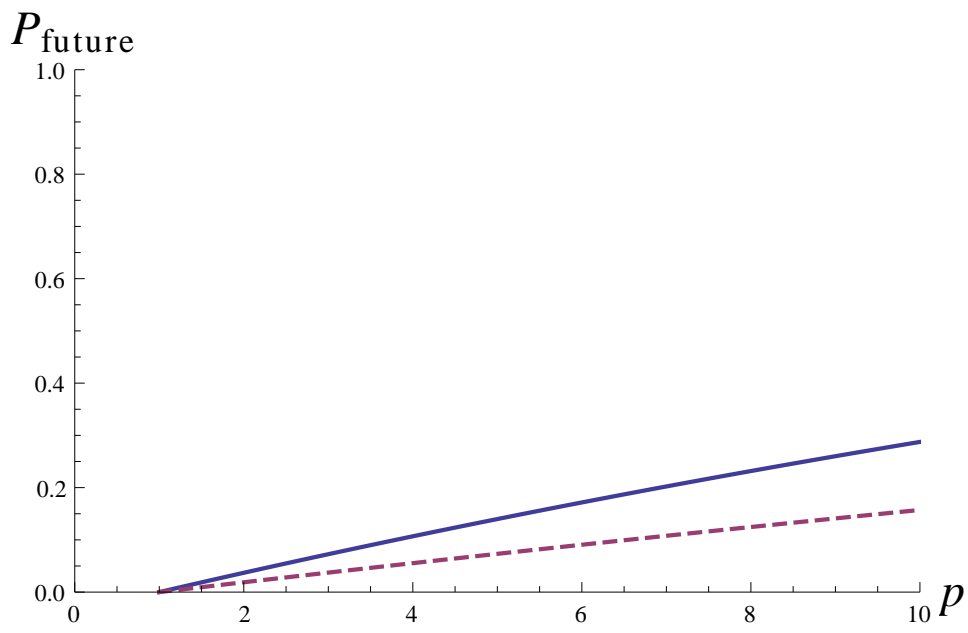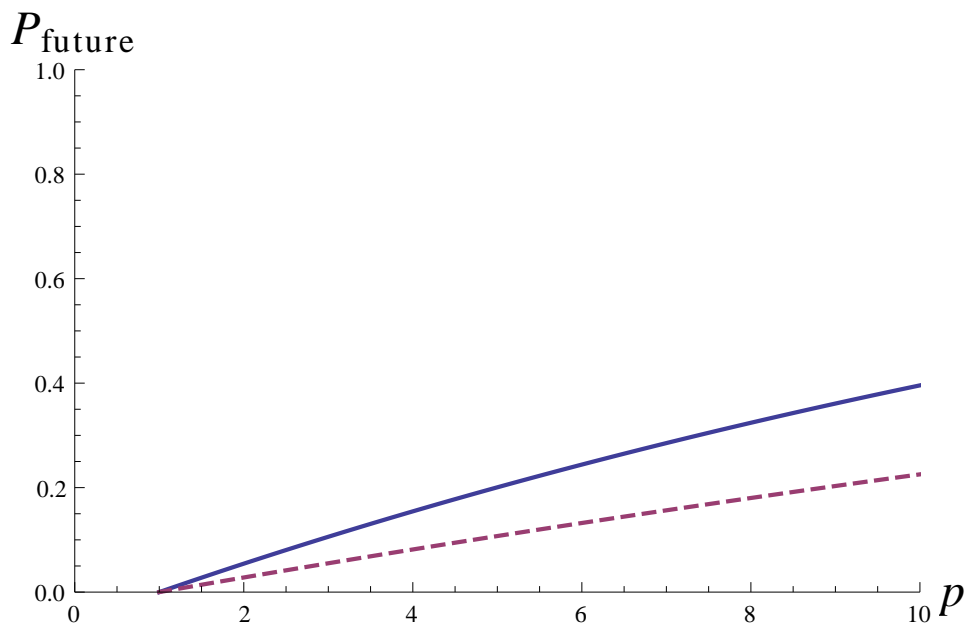Figure 4b   21APR12

Figure 5a   21APR12

Figure 5b   21APR12