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More on QCD Ghost Dark Energy

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Abstract: The difference between vacuum energy of quantum fields in Minkowski space and in Friedmann-Robterson-Walker universe might be related to the observed dark energy. The vacuum energy of the Veneziano ghost field introduced to solve the $U(1)_A$ problem in QCD is of the form, $H + \mathcal{O}(H^2)$. Based on this, we study the dynamical evolution of a phenomenological dark energy model whose energy density is of the form $\alpha H + \beta H^2$. In this model, the universe approaches to a de Sitter phase at late times. We fit the model with current observational data including SnIa, BAO, CMB, BBN, Hubble parameter and growth rate of matter perturbation. It shows that the universe begins to accelerate at redshift $z \sim 0.75$ and this model is consistent with current data. In particular, this model fits the data of growth factor well as the ΛCDM model.

1. Introduction

The accelerating expansion is still a mystery of modern cosmology since its discovery in 1998 [1], and a new energy component called dark energy (DE) is needed to explain this acceleration expansion within the framework of general relativity. The simplest model of DE is the cosmological constant, which is consistent with all observational data, but it faces with the fine tuning problem [2]. Instead, many alternative DE models have also been proposed [3, 4, 5, 6, 7, 8, 9], but almost all of them explain the acceleration expansion either by introducing new degree(s) of freedom or by modifying general relativity.

Recently the so-called QCD ghost dark energy has been proposed in [10, 11, 12]. The Veneziano ghost field plays a crucial role in the resolution of the $U(1)_A$ problem in QCD [13]. The ghost field has no contribution to the vacuum energy density in Minkowski spacetime, but in a curved spacetime, it gives rise to a vacuum energy density proportional to $\Lambda^3_{QCD}H$ [14, 15, 16, 17], where Λ_{QCD} is QCD mass scale and H is Hubble parameter. Note that in this ghost dark energy model, there are no unwanted features such as violation of gauge invariance, unitarity, causality etc. [10, 11, 14, 15, 16, 17]. In fact, the description in terms of the Veneziano ghost is just a matter of convenience to describe very complicated infrared dynamics of strongly coupled QCD. The Veneziano ghost is not a physical propagating degree of freedom, one can describe the same dynamics using some other approaches (e.g. direct lattice simulations) without using the ghost. Therefore the Veneziano ghost field is quite different from those ghost fields in some dark energy models in the literature, there those ghost fields are real physical degrees of freedom, introduced in order to have the equation of state of dark energy to cross -1. On the other hand, the vacuum energy is generally expected exponentially suppressed because QCD is a theory with a mass gap. However, this issue is elaborated in some details in [14, 15, 16, 17], there it has been convincingly argued that the complicated topological structure of strongly coupled QCD will lead to a vacuum energy density with an inverse linear scale of manifold size in a nontrivial background. The power law behavior is also supported by recent lattice result [15, 18]. Very recently it has been shown that this behavior is also got supported from the holographic description of gauge field [19].

Because this model is totally embedded in standard model and general relativity, one needs not to introduce any new degrees of freedom or to modify Einstein's general relativity. In this model, the energy density of DE is roughly of order $\Lambda^3_{QCD}H$, with $\Lambda_{QCD} \sim 100MeV$ and $H \sim 10^{-33}eV$, so $\Lambda^3_{QCD}H$ gives the right order of observed DE energy density. This numerical coincidence is impressive and also means that this model gets rid of fine tuning problem [10, 11]. The model parameters have been fitted recently by observational data including SnIa, BAO, CMB, BBN and Hubble parameter data [12]. It shows that this model is consistent with those observational data.

On the other hand, it is convincingly argued that the contribution of zero-point fluctuations of quantum field to the total energy density should be computed by subtracting the Minkowski space result from the computation in a FRW space-time [20, 21]. Usually the difference, $H^2\Lambda_c^2$, between the vacuum energies in Minkowski space and in the FRW space-time, is absorbed into a renormalization of Newton's constant G, here H is the Hubble constant of the FRW universe and Λ_c is the cutoff. However, it is true only under the assumption that the vacuum expectation value of the energy-momentum tensor is conserved in isolation [21]. In that reference the authors investigated the role of this term as early dark energy in the evolution of the universe.

Notice the fact that the vacuum energy from the Veneziano ghost field in QCD is of the form $H + \mathcal{O}(H^2)$, see for example, [19], while in the previous works on the QCD ghost dark energy model, the leading term H has been considered only. Having considering the study in [21], one may expect that the subleading term H^2 in the ghost dark energy model might play a crucial role in the early evolution of the universe, acting as the early dark energy.

Based on the QCD ghost dark energy, in this work we therefore investigate a phenomenological model with energy density $\rho_{DE} = \alpha H + \beta H^2$. For other motivations to consider this form see [22], there it is argued that this form of varying cosmological constant could be a possible candidate to solve the two fundamental cosmological puzzles. We study the cosmological evolution of this DE model, fit this model with current observational data and give constraints on the model parameters. Besides, it is worth noticing that a varying DE should have some effect on the evolution of the matter perturbation, so we study the first order perturbation to the matter density and fit this model with the data of linear growth factor.

The paper is organized as follows. In Section 2 we study the dynamical evolution of the DE model. In Section 3, we fit this model with current observational data and discuss the fitting results. The data used are Union II SnIa sample [23], BAO data from SDSS DR7 [24], CMB data (R, l_a, z_*) from WMAP7 [25], 12 Hubble evolution data [26, 27] and Big Bang Nucleosynthesis (BBN) [28, 29], and we also study the effect of the DE on the linear perturbation of matter density. We summarize our work and give some discussions in Section 4.

2. Dynamics of QCD Ghost Dark Energy

To study the dynamics of the DE model, we consider a flat FRW universe with three energy components: matter, DE and radiation. In this ghost DE model, the energy density of DE is given by $\rho_{DE} = \alpha H + \beta H^2$, where α is a constant with dimension $[energy]^3$, roughly of order of Λ_{QCD}^3 where $\Lambda_{QCD} \sim 100\text{MeV}$ is QCD mass scale, and β is another constant with dimension $[energy]^2$. For convenience, we define $\gamma \equiv 1 - \frac{8\pi G}{3}\beta$ and use this throughout the paper.

Arming with this DE density, the Friedman equation reads

$$H^2 = \frac{8\pi G}{3\gamma}\left(\alpha H + \rho_m + \rho_r\right), \tag{2.1}$$

where ρ_m is energy density of matter, whose continuity equation gives

$$\dot{\rho}_m + 3H\rho_m = 0 \implies \rho_m = \rho_{m0}a^{-3}. \tag{2.2}$$

and ρ_r is energy density of the radiation, whose continuity equation gives

$$\dot{\rho}_r + 4H\rho_r = 0 \implies \rho_r = \rho_{r0}a^{-4}. \tag{2.3}$$

We have set $a_0 = 1$ and the subscript 0 stands for the present value of some quantities. Solving the Friedman equation, we have

$$H_{\pm} = \frac{4\pi G}{3\gamma}\alpha \pm \sqrt{\left(\frac{4\pi G}{3\gamma}\alpha\right)^2 + \frac{8\pi G}{3\gamma}\rho_{m0}a^{-3} + \frac{8\pi G}{3\gamma}\rho_{r0}a^{-4}}. \tag{2.4}$$

There are two branches, H_+ represents an expansion solution, while H_- a contraction one. We neglect the latter since it goes against the observation, and for simplicity, write H_+ as H in what follows.

Expressed with fraction energy density of matter and radiation, Ω_{m0}, Ω_{r0}, Equation 2.4 gives an important constraint among these parameters:

$$(\gamma - \Omega_{m0} - \Omega_{r0})H_0 = \frac{8\pi G}{3}\alpha. \tag{2.5}$$

Further, Equation 2.4 can be rewritten as

$$H(z) = H_0\left(\kappa + \sqrt{\kappa^2 + \frac{\Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4}{\gamma}}\right). \tag{2.6}$$

where $\kappa = (1 - (\Omega_{m0} + \Omega_{\gamma 0})/\gamma)/2$ and z is the redshift, $z = 1/a - 1$. We can see from (2.6) that the universe approaches to a de Sitter phase with Hubble parameter $2\kappa H_0$ at late times, while it is dominated by matter and radiation terms at early times.

Note that there exist many papers focusing on the coupling between the time-dependent vacuum energy and matter [30]. But we do not consider here such coupling in our analysis. Namely in our discussion, the DE, matter and radiation are separately conserved. In that case, the corresponding time evolution equation for the matter density contrast $D \equiv \delta\rho_m/\rho_m$ is given by:

$$\ddot{D} + 2H\dot{D} - 4\pi G\rho_m D = 0, \tag{2.7}$$

where the over dot denotes the derivative with respect to the cosmic time. In terms of the growth factor [31], Equation 2.7 can be rewritten as

$$-(1+z)H(z)^2\frac{df}{dz}+2H(z)^2f+H(z)^2f^2-(1+z)H(z)\frac{dH(z)}{dz}f=\frac{3\Omega_{m0}(1+z)^3}{2}, \quad (2.8)$$

where the growth factor f is defined as $f=-(1+z)\frac{d\ln D}{dz}$. In general, there is no analytical solution to Equation 2.8, and we need to solve it numerically. But it is very interesting that the solution of the equation can be approximated as [32]

$$f=\Omega_m(z)^\lambda, \quad (2.9)$$

and the growth index λ can be obtained for some general models as

$$\lambda=\frac{3}{5-\frac{w}{1-w}}+\frac{3}{125}\frac{(1-w)(1-3w/2)}{(1-6w/5)^3}(1-\Omega_m(z)). \quad (2.10)$$

where w is the equation of state of DE. For the case with $1-\Omega_m(z)$ being between zero and 0.8, the accuracy is better than 1%. For the ΛCDM model, the approximation $f(z=0)=\Omega_{m0}^{0.6}+\Omega_{\Lambda 0}(1+\Omega_{m0}/2)/70$ can be made [33]. But in our analysis, instead of parametrization of λ [34], we will solve the Equation 2.8 numerically, by setting the initial condition $f(z=0)=f_0$, where f_0 is a free parameter to be constrained by observational data.

3. Data Fitting

3.1 Model

In order to fit the model with current observational data, we consider a more realistic model which includes DE, Cold Dark Matter, radiation and baryon in a flat FRW universe in this section. In this case, the dimensionless Hubble parameter can be written as,

$$E\equiv\frac{H}{H_0}=\kappa+\sqrt{\kappa^2+\frac{\Omega_{m0}(1+z)^3+\Omega_{r0}(1+z)^4}{\gamma}}, \quad (3.1)$$

where the energy density of baryon and Cold Dark Matter are always written together as $\Omega_{DM0}+\Omega_{b0}=\Omega_{m0}$, and $\Omega_{DM0},\Omega_{b0},\Omega_{r0}$ are present values of dimensionless energy density for Cold Dark Matter, baryon and radiation, respectively. The energy density of radiation is the sum of those of photons and relativistic neutrinos

$$\Omega_{r0}=\Omega_{\gamma 0}\left(1+0.2271N_n\right),$$

where $N_n=3.04$ is the effective number of neutrino species and $\Omega_{\gamma 0}=2.469\times 10^{-5}h^{-2}$ for $T_{cmb}=2.725K$ ($h=H_0/100\,Mpc\cdot km\cdot s^{-1}$).

We will choose h, γ, Ω_{b0} and Ω_{m0} (and also f_0 when we consider the growth factor) as free parameters of the model in the following data fitting. This relation Equation 2.5 implies that there exists a strong degeneracy among h, γ and Ω_{m0}.

3.2 Observational Datasets

We fit our model by employing some observational data including SnIa, BAO, CMB, Hubble evolution data, BBN and the data of growth factor.

The data for SnIa are the 557 Uion II sample [23]. χ^2_{sn} for SnIa is obtained by comparing theoretical distance modulus $\mu_{th}(z) = 5\log_{10}[(1+z)\int_0^z dx/E(x)] + \mu_0$ ($\mu_0 = 42.384 - 5\log_{10} h$) with observed μ_{ob} of supernovae:

$$\chi^2_{sn} = \sum_i^{557} \frac{[\mu_{th}(z_i) - \mu_{ob}(z_i)]^2}{\sigma^2(z_i)}. \tag{3.2}$$

To reduce the effect of μ_0, we expand χ^2_{sn} with respect to μ_0 [35] :

$$\chi^2_{sn} = A + 2B\mu_0 + C\mu_0^2 \tag{3.3}$$

where

$$\begin{aligned} A &= \sum_i \frac{[\mu_{th}(z_i;\mu_0=0) - \mu_{ob}(z_i)]^2}{\sigma^2(z_i)}, \\ B &= \sum_i \frac{\mu_{th}(z_i;\mu_0=0) - \mu_{ob}(z_i)}{\sigma^2(z_i)}, \\ C &= \sum_i \frac{1}{\sigma^2(z_i)}. \end{aligned}$$

(3.3) has a minimum as

$$\widetilde{\chi}^2_{sn} = \chi^2_{sn,min} = A - B^2/C, \tag{3.4}$$

which is independent of μ_0. In fact, it is equivalent to performing an uniform marginalization over μ_0, the difference between $\widetilde{\chi}^2_{sn}$ and the marginalized χ^2_{sn} is just a constant [35]. We will adopt $\widetilde{\chi}^2_{sn}$ as the goodness of fit between theoretical model and SnIa data.

The second set of data is the Baryon Acoustic Oscillations (BAO) data from SDSS DR7 [24], the datapoints we use are

$$d_{0.2} = \frac{r_s(z_d)}{D_V(0.2)}$$

and

$$d_{0.35} = \frac{r_s(z_d)}{D_V(0.35)},$$

where $r_s(z_d)$ is the comoving sound horizon at the baryon drag epoch [36], and

$$D_V(z) = \left[\left(\int_0^z \frac{dx}{H(x)}\right)^2 \frac{z}{H(z)}\right]^{1/3}$$

encodes the visual distortion of a spherical object due to the non Euclidianity of a FRW spacetime. The inverse covariance matrix of BAO is

$$C_{M,bao}^{-1} = \begin{pmatrix} 30124 & -17227 \\ -17227 & 86977 \end{pmatrix}.$$

The χ^2 of the BAO data is constructed as:

$$\chi^2_{bao} = Y^T C_{M,bao}^{-1} Y, \tag{3.5}$$

where

$$Y = \begin{pmatrix} d_{0.2} - 0.1905 \\ d_{0.35} - 0.1097 \end{pmatrix}.$$

The third set of data we use are CMB datapoints (R, l_a, z_*) from WMAP7 [25]. z_* is the redshift of recombination [37], R is the scaled distance to recombination

$$R = \sqrt{\Omega_{m0}} \int_0^{z_*} \frac{dz}{E(z)},$$

and l_a is the angular scale of the sound horizon at recombination

$$l_a = \pi \frac{r(a_*)}{r_s(a_*)},$$

where $r(z) = \int_0^z dx/H(x)$ is the comoving distance and $r_s(a_*)$ is the comoving sound horizon at recombination

$$r_s(a_*) = \int_0^{a_*} \frac{c_s(a)}{a^2 H(a)} da,$$

where the sound speed $c_s(a) = 1/\sqrt{3(1+\overline{R}_b a)}$ and $\overline{R}_b = 3\Omega_b^{(0)}/4\Omega_\gamma^{(0)}$ is the photon-baryon energy density ratio. The χ^2 of the CMB data is constructed as:

$$\chi^2_{cmb} = X^T C_{M,cmb}^{-1} X, \tag{3.6}$$

where

$$X = \begin{pmatrix} l_a - 302.09 \\ R - 1.725 \\ z_* - 1091.3 \end{pmatrix}$$

and the inverse covariance matrix

$$C_{M,cmb}^{-1} = \begin{pmatrix} 2.305 & 29.698 & -1.333 \\ 29.698 & 6825.270 & -113.180 \\ -1.333 & -113.180 & 3.414 \end{pmatrix}.$$

The fourth set of observational data is 12 Hubble evolution data from [26] and [27]. Its χ^2_H is defined as

$$\chi^2_H = \sum_{i=1}^{12} \frac{[H(z_i) - H_{ob}(z_i)]^2}{\sigma_i^2}. \tag{3.7}$$

z	f_{obs}	σ	Ref
0.15	0.51	0.11	[39]
0.22	0.60	0.10	[40]
0.32	0.654	0.18	[41]
0.35	0.70	0.18	[42]
0.41	0.50	0.07	[40]
0.55	0.75	0.18	[43]
0.60	0.73	0.07	[40]
0.77	0.91	0.36	[44]
0.78	0.70	0.08	[40]
1.4	0.90	0.24	[45]
3.0	1.46	0.29	[46]

Table 1: Currently available data for linear growth rate f_{obs} used in our analysis. z is redshift; σ is the 1σ uncertainty of the growth rate data.

Note that the redshift of these data falls in the region $z \in (0, 1.75)$.

The Big Bang Nucleosynthesis (BBN) data we use here are from [28, 29], whose χ^2 is

$$\chi^2_{bbn} = \frac{(\Omega_{b0}h^2 - 0.022)^2}{0.002^2}. \tag{3.8}$$

And finally for the growth factor data, we define

$$\chi^2_f = \sum_{i=1}^{11} \frac{[f(z_i) - f_{ob}(z_i)]^2}{\sigma_i^2}. \tag{3.9}$$

The 11 data of growth factor are summarized in Table 1 [38].

3.3 Fitting Results

The best fitting values and errors of the model parameters are summarized in Table 2, where we also list the best fitting values of the corresponding parameters of ΛCDM model for comparison. The best fitting values of Ω_{m0} and h are slightly smaller than corresponding ones in the ΛCDM model and the best fitting values of Ω_{b0} are larger than corresponding ones in the ΛCDM model. We also can see from Table 2 that adding the data of growth factor dose not have much impact on the values of the parameters, both at 1σ confidence level and 2σ confidence level, which may mean that this model is not sensitive to the linear growth rate of matter. In addition, we find that $\gamma < 1$ is excluded at 2σ confidence level, which means that in the ghost dark energy model, $\beta < 0$. Furthermore, we see that the subleading term H^2 of the dark energy density, the early dark energy, could have a fraction energy density around 10%. In Figure 1 and Figure 2, we plot the 1D marginalized distribution probability of each parameter using the full datasets.

parameter	SN+BAO+CMB+H+BBN	SN+BAO+CMB+H+BBN+F	ΛCDM
h	$0.642^{-0.017,\,-0.025}_{+0.012,\,+0.023}$	$0.642^{-0.015,\,-0.027}_{+0.010,\,+0.021}$	0.708
Ω_{m0}	$0.250^{-0.014,\,-0.025}_{+0.014,\,+0.026}$	$0.251^{-0.014,\,-0.025}_{+0.013,\,+0.026}$	0.266
Ω_{b0}	$0.052^{-0.002,\,-0.003}_{+0.002,\,+0.003}$	$0.052^{-0.002,\,-0.003}_{+0.002,\,+0.003}$	0.045
γ	$1.114^{-0.035,\,-0.062}_{+0.029,\,+0.058}$	$1.105^{-0.028,\,-0.056}_{+0.035,\,+0.063}$	\
f_0	\	$0.473^{-0.018,\,-0.029}_{+0.012,\,+0.024}$	0.485

Table 2: The best fitting values within 1σ and 2σ errors for h, Ω_{m0}, Ω_{b0}, γ and f_0 for the dark energy model. The second column shows the results using the datasets without the data of growth factor, and the third column shows the results fitted with full datasets. The last column shows the best fitting results of ΛCDM model using the full datasets for comparison.

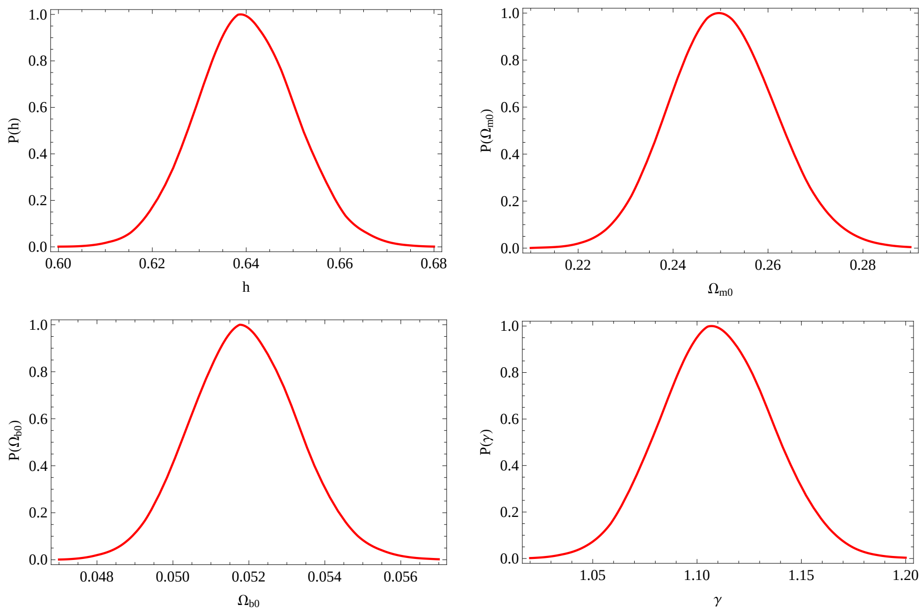

Figure 1: 1D marginalized distribution probability of h, Ω_{m0}, Ω_{b0} and γ using the full datasets.

In Figure 3 we plot the evolution behaviors of the equation of state $w(z)$ of DE and the deceleration parameter $q(z)$, with the best fitting values of the model and the ΛCDM model. In the calculation, we employ the following relations:

$$w(z) = -1 + \frac{(1+z)}{3H(z)}\frac{dH(z)}{dz}, \tag{3.10}$$

$$q(z) = -1 + \frac{(1+z)}{H(z)}\frac{dH(z)}{dz}. \tag{3.11}$$

The results show that in the QCD ghost dark energy model, the universe transits from early matter dominant phase to the de-Sitter phase in the future, as expected. The accelerating expansion begins at $z = 0.75$, which is earlier than what the ΛCDM

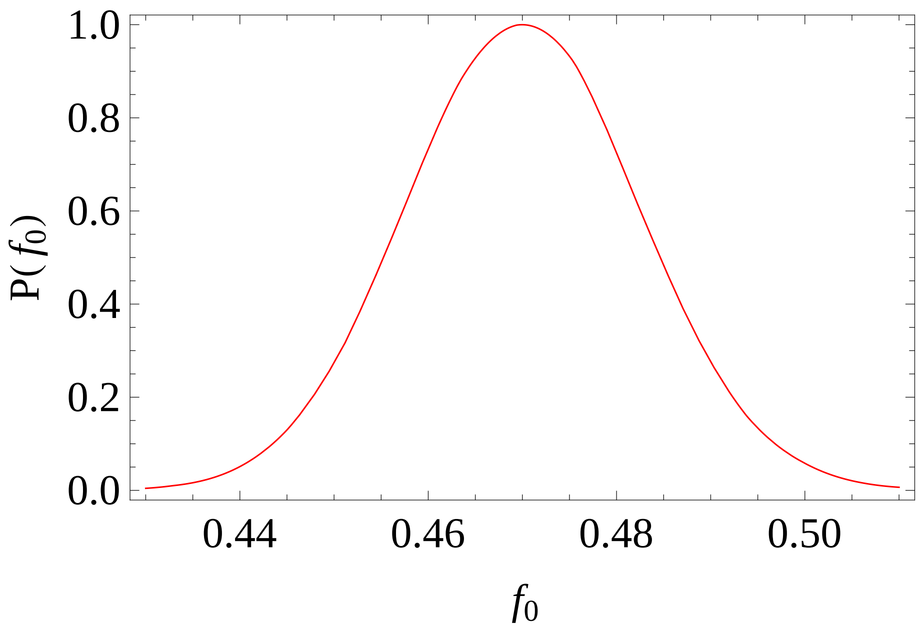

Figure 2: 1D marginalized distribution probability of f_0 using the full datasets.

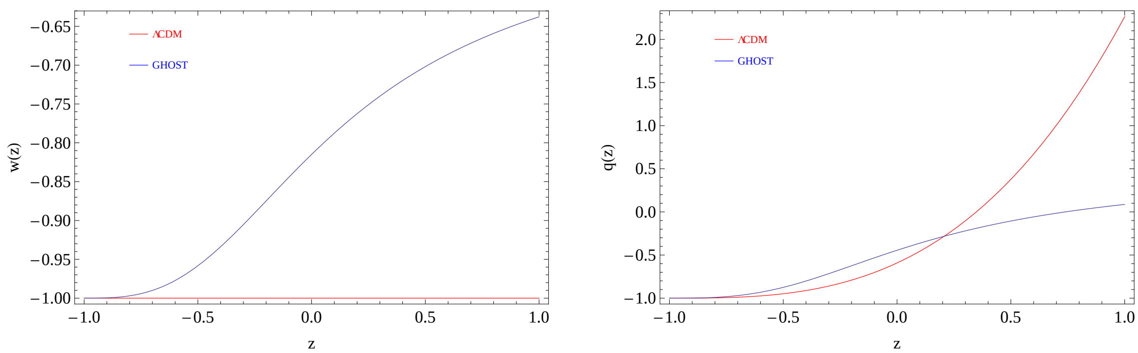

Figure 3: Evolution behaviors of the equation of state of DE and the deceleration parameter for the QCD ghost dark energy model and the ΛCDM model.

model predicts. $w(z)$ varies from $w > -1$ to $w = -1$ which is similar to freezing quintessence model [47].

The total χ^2 of the best fitting values of this model using the full datasets is $\chi^2_{min} = 589.422$ for $dof = 586$. The reduced χ^2 equals to 1.006, which is acceptable, but χ^2_{min} is a little larger than the one for the ΛCDM model, $\chi^2_{\Lambda CDM} = 558.890$. A similar conclusion is also reached by other authors using different data set in [48]. That work studies the dynamics of varying vacuum energy as a cosmological constant. That is, the equation of state of the vacuum energy is always kept as $w = -1$. In that case, there must exist interaction between matter and the vacuum energy. In Figure 4, we plot the evolution of the growth factor for the QCD ghost dark energy model and the ΛCDM model, it shows that the ghost dark energy model can not be

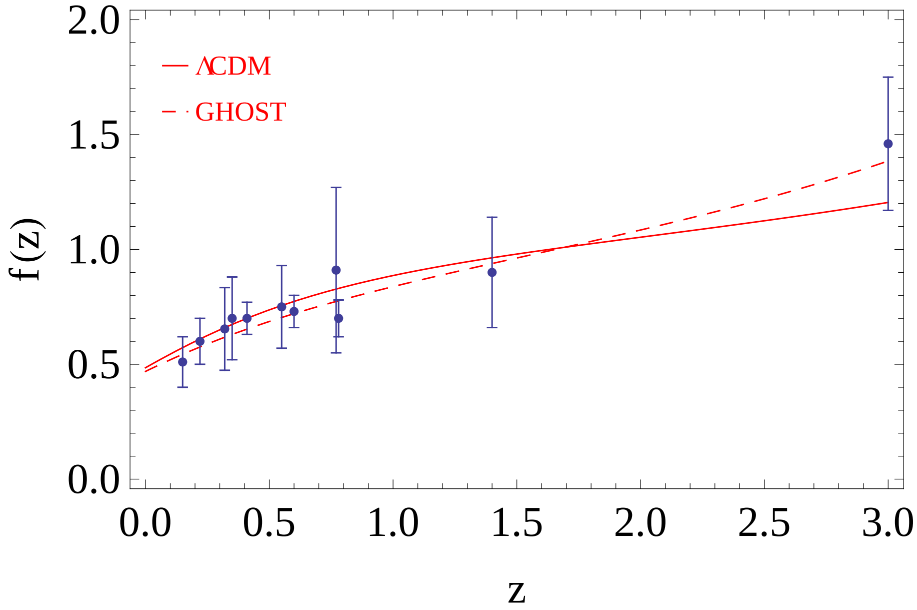

Figure 4: Evolution behaviors of the growth factor for the QCD ghost dark energy model and the ΛCDM model.

discriminated by the data, and that both of these two models fit the data very well, even the ghost dark energy model looks fitting the data better.

4. Conclusion and Discussion

The accelerating expansion (dark energy) of the universe must be closely related to the vacuum energy of quantum fields. It is believed that the difference between the vacuum energies in Minkowski space and in FRW universe might be the origin of observed dark energy. However, the naive estimate indicates that the difference should be of the form $H^2\Lambda_c^2$ [20, 21]. Such a term is too small and cannot derive the universe to accelerating expansion. But this term may play an important role in the early evolution of the universe, acting as an early dark energy.

On the other hand, the vacuum energy difference from the Veneziano ghost field introduced in order to solve the so-called $U(1)_A$ problem in QCD has the exact form, $\alpha H + \beta H^2$, where $\alpha \sim \Lambda_{QCD}^3 \sim (100MeV)^3$. The leading term gives exactly the order of the observed dark energy. Therefore the QCD ghost dark energy model is very attractive in the sense that this model needs not introduce new degrees of freedom or modify Einstein's general relativity, to explain the accelerating expansion of the universe observed today.

In this paper, based on the vacuum energy of QCD ghost field, we investigated a DE model whose energy density has the form $\alpha H + \beta H^2$. We studied the dynamical evolution of the QCD ghost dark energy model and fitted this model with observational data including SnIa, BAO, CMB, BBN, Hubble parameter and the growth factor. The best fitting results show that the subleading term of the energy density makes a negative contribution to the total energy density. In this model, the universe transits from early matter dominant phase to a de-Sitter phase in the future, and the accelerating expansion begins at $z = 0.75$, which is earlier than that of ΛCDM model. The equation of state of DE varies from $w > -1$ to $w = -1$ like a freezing quintessence model.

The total χ^2 of the best fitting values of this model is $\chi^2_{min} = 589.422$ for the full datasets with $dof = 586$. The reduced χ^2 is 1.006, which is acceptable, but χ^2_{min} is a little larger than the one for the ΛCDM model, $\chi^2_{\Lambda CDM} = 558.890$, for the same datasets. We further studied the cosmological dynamics of the model by considering the effect on the growth rate of matter. The ghost dark energy model can not be discriminated by the data, and both of this model and the ΛCDM model fit the data very well.

Finally before ending this paper, we would like to stress that in fact there have not been any precise calculations showing that the vacuum energy density of the Veneziano ghost of QCD in a FRW universe is of the form, $\alpha H + \beta^2$, because the vacuum energy calculation of the Veneziano ghost is quite difficult in both flat and curved spacetimes due to the intrinsic difficulties of QCD and strongly interacting fields in general [10]. But the vacuum energy calculations of the Kogut-Susskind ghost in 2d QED (which is the direct analogue of the Veneziano ghost in QCD), in 2d topological nontrivial spacetime and curved space [10, 16], and the vacuum energy calculations of the effective Veneziano ghost of QCD in 4d Rindler spacetime [11] indeed indicate such a power-law behavior. Note that in those calculations, the kinetic contribution is not included, it is expected that the kinetic contribution is of the same order of magnitude of the potential. While the dynamics of the effective scalar field model for the Veneziano ghost of QCD in FRW universe is discussed in the fourth reference in [10], here we have further made an approximation that the size scale L of some nontrivial manifold is replaced by the Hubble size of the FRW universe, which becomes time-dependent. In principle, in such an extension, one has to consider the kinetic contribution of the time-dependent scale to the vacuum energy density. However, we assume that such a contribution is subdominant due to slowly evolution of the scale, compared to the potential term, and absorb some uncertainties into the two coefficients α and β, because at the moment one could not make an explicit calculation to take into account the effect. Of course, the kinetic contribution of the Veneziano ghost of QCD and the effect in a time-dependent spacetime should be seriously studied if the QCD ghost dark energy model studied in this paper can fit well with the observational data. Clearly at the moment one just can review this model at a phenomenological level.

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