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# Triangulating an exotic T quark

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# Triangulating an exotic T quark

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Limits on an exotic heavy quark T are broadly generalized by considering the full range of  $T \to Wb, th$  or tZ branching ratios. We combine results of specific  $T \to tZ$  and  $T \to Wb$  searches with limits on various combinations of decay modes evaluated by re-interpreting other searches. We find strong bounds across the entire space of branching ratios, ranging from  $m_T > 415$  GeV to  $m_T > 557$  GeV at 95% confidence level.

#### PACS numbers: 12.60,-i, 14.65.-Jk

## I. INTRODUCTION

A fourth generation of fermions would be a natural extension to the standard model of particle physics. Direct searches for a chiral fourth-generation (b' and t'), however, have yielded no evidence for a fourth generation of fermions. Specifically, the CMS collaboration has set limits of  $m_{t'} > 557$  GeV if BR( $t' \to Wb$ )=100% [1] and  $m_{b'} > 611$  GeV [2] if BR( $b' \to Wt$ )=100%. Even if these branching ratios are reduced by off-diagonal mixing terms, these analyses have complementary sensitivity which is nearly impossible to escape if the fourth generation is chiral and decays via W-boson emission [3].

A fourth-generation quark, however, may be a vector particle (T) which has exotic decays [4, 5], such as  $T \to tZ$  or th, see Fig. 1. Such a quark is a generic feature [6] of models in which the Higgs boson is a composite state, such as models featuring a "little Higgs" [7-9]. The contribution from the T quark in such models is essential to cancel the contributions to the Higgs mass from the top quark, keeping the Higgs mass at the electroweak scale.

The LHC collaborations have strong sensitivity to such a quark [10]. CMS performed a dedicated search for this T quark in data with 1 fb<sup>-1</sup>, reporting  $m_T > 475$  GeV [11] if BR( $T \to tZ$ )=100%, but no searches have been reported for the  $T \to th$  mode, nor for models with realistic mixtures of decay modes, see Fig. 2.

In previous work, we reinterpreted an ATLAS search for  $b' \to tW$  which has broad sensitivity for other heavy quark modes; we set limits on the T-quark mass in the case of a realistic mixture branching ratios,  $m_T > 419 \text{ GeV } [12]$  if  $\text{BR}(T \to tZ) = 15\%$ ,  $\text{BR}(T \to th) = 35\%$ ,  $\text{BR}(T \to Wh) = 50\%$ .

In this paper, we relax the assumptions which determine the branching ratios as a function of mass and explore the entire space of possible branching ratios, achieving an important generalization of the existing limits.

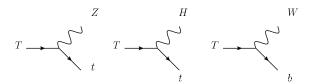


FIG. 1: Decay modes of a heavy exotic quark, T

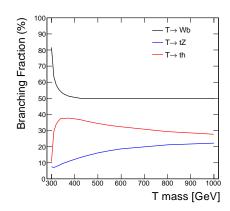


FIG. 2: Branching ratio of T-quark decays to Wb, tZ and th vs  $m_T$ , from the model in Ref. [4].

#### II. DIRECT SEARCHES

The CMS collaboration searched for a heavy chiral fourth-generation t' quark decaying via Wb in the dilepton decay mode using data with 5.0 fb<sup>-1</sup> of luminosity. We assume that this limit,  $m_T > 557$  GeV at 95% CL, is also applicable to the exotic quark T when it decays to Wb; parton-level studies of  $T \to Wb$  and  $t' \to Wb$  show no discernable differences in W boson or b quark momenta or angles. Other analyses have also been performed[13–15], but we only consider the strongest limits

In addition, CMS searched directly for T-quark production specifically in the mode  $T \to tZ$ . Assuming BR(tZ) is 100%, their analysis yields  $m_T > 475$  GeV at 95% CL from data with 1 fb<sup>-1</sup> of integrated luminosity.

The ATLAS collaboration reported a search for heavy fourth-generation down-type chiral quarks (b') using data with 1 fb<sup>-1</sup> [16] of integrated luminosity. The b' decays via tW, leading to a final state with four W bosons and two b quarks. The single-lepton mode is used, leaving three hadronically decaying W bosons. The ATLAS search makes use of a novel technique for tagging boosted W bosons by searching for jet pairs with small angular separation. The analysis variables are the jet multiplicity and W boson multiplicity; it is a counting experiment in nine bins:  $N_{jet} = (6,7,\geq 8) \times N_W = (0,1,\geq 2)$ . This analysis does not directly set limits on T-quark models, but it is clear that if such particles were produced, they would appear as an excess in this analysis.

Note that the precise aim of the original  $b' \to tW$  analysis is not directly relevant to the analysis presented in this paper, as we are reinterpretting it in another context. However, since the decay modes are similar  $(b' \to tW)$  vs  $T \to tZ$  in topology makes the reinterpretation more powerful and robust.

## III. REINTERPRETING $b' \rightarrow tW$

Given the clear sensitivity of the ATLAS b' search to T-quark production and decay, we reinterpret this analysis in the context of T-quark models.

The ATLAS analysis uses a binned likelihood with nine bins in jet and W-boson multiplicity:  $N_{jet} = (6,7,\geq 8) \times N_W = (0,1,\geq 2)$ . Jets are reconstructed using the anti- $k_t$  algorithm with a distance parameter of 0.4 and are required to have  $p_T > 25$  GeV. The W-boson multiplicity is estimated by counting the number of unique jet pairs that simultaneously have invariant mass within 70-100 GeV and are within  $dR \leq 1.0$  of each other.

Until recently, reinterpretation of such a multi-bin analysis was effectively impossible, as it would require publication by the experiment of the complete likelihood details, including bin-to-bin correlations. However, we showed in recent work [12] that if the template for a new signal model can be expressed as a linear combination of the templates for models tested by the experiment, then limits on the new model can be trivially derived. Note that uncertainties in the background model (including correlations) which are included in the original experimental analysis are automatically included in the reinterpretted limits as well. This approach can be understood as an interpolation strategy, and is valid when the dominant systematic uncertainties are due to the background sources, or are similar between the basis templates and the new signal model.

We exploit this approach to derive limits on T-quark production and decay in a variety of decay modes. We generate T production and decay using MADGRAPH [22], use PYTHIA [23] to model showering and hadronization, and use PGS [17] to describe the detector response. To be consistent with the ATLAS b' analysis we use anti-

 $k_t$  jets with a cone size of 0.4 and  $p_T > 25$  GeV. We reconstruct W bosons using the same parameters as the ATLAS analysis. In every case, we use  $m_h = 125$  GeV.

In the following sections, we find mixtures of the  $b' \to tW$  templates which closely approximate templates for exotic T quark decays in tZ, th, (tZ and th), (Wb and th), sor (Wb, th, and tZ) decay modes. In each case, we find the approximations to be imperfect but reasonable. Variations of the template mixtures which have discrepancies of similar magnitude but opposite sign give similar results to those we present, suggesting that the results are not highly sensitive to these discrepancies. Uncertainties due to the imperfect description are assessed.

#### A. Decays to tZ

If the T-quark decays exclusively to tZ, then the final state of  $t\bar{t}ZZ$  closely resembles the  $t\bar{t}WW$  final state of the b' search, as the jet resolution does not allow us to distinguish between  $W\to qq'$  and  $Z\to q\bar{q}$  decays, and the mass window used  $(70-100~{\rm GeV})$  encompasses most hadronic Z-boson decays.

However, to reinterpret the  $t\bar{t}WW$  in terms of the  $t\bar{t}ZZ$ , we do not assume that the kinematics are identical. Instead, we simulate  $TT \to t\bar{t}ZZ$  events as described above, and apply the ATLAS  $t\bar{t}WW$  selection. This allows us to form the prediction for  $t\bar{t}ZZ$  in the nine  $t\bar{t}WW$  bins in jet and W-boson multiplicity:  $N_{jet} = (6,7,\geq 8) \times N_W = (0,1,\geq 2)$ . We then form a linear combination of  $t\bar{t}WW$  templates which match the  $t\bar{t}ZZ$  prediction and use the experimentally reported limits on those  $t\bar{t}WW$  templates to calculate a limit on the  $t\bar{t}ZZ$  prediction, using our technique described above [12].

Hence, we do not require that the tagging efficiency for W and Z bosons be identical, only similar enough that we may represent the  $t\bar{t}ZZ$  templates in terms of  $t\bar{t}WW$  templates. In this section we show that linear combinations of  $b'\to tW$  templates may be used to sufficiently describe  $T\to tZ$  templates.

Table I gives the details of these linear combinations. For each mass point of the T quark, we list the coefficients,  $a_i$ , of b'-quark templates used to derive the limits (also listed) for that T-quark mass. For example, the 550 GeV T-quark template is described by a linear combination of 500, 550 and 600 GeV b'-quark templates scaled to 0.01, 0.06 and 1.14, respectively. The linear combinations are shown along with corresponding original T templates in Fig. 3 for the 450 and 550 GeV T-quark cases. Fig. 4 shows the derived upper limits on the cross-section. If  $BR(T \to tZ)=100\%$ , the reinterpretation of the b' search yields  $m_T > 446$  GeV at 95% CL, comparable to the CMS limit of  $m_T > 475$  GeV derived from a search optimized for this mode.

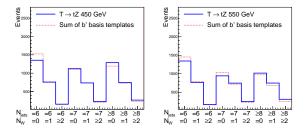


FIG. 3: Jet and W-boson multiplicity, for  $T \to tZ$  events (solid blue). Also shown (dashed red) is the sum of b' basis templates used to derive the limit on the T quark model. Left for  $m_T = 450$  GeV, right for  $m_T = 550$  GeV.

TABLE I: Details of the predicted limit on T pair-production and  $T \to tZ$  decay, using basis templates from  $b' \to tW$  decays at ATLAS.

	$T_{300}$	$T_{350}$	$T_{400}$	$T_{450}$	$T_{500}$	$T_{550}$	$T_{600}$
$a_{b'300}$	0.74	0.13	0	0	0	0	0
$a_{b'350}$	0	0.30	0.07	0	0	0	0
$a_{b'400}$	0	0.25	0.49	0.03	0	0	0
$a_{b'450}$	0	0.01	0.13	0.37	0.10	0	0
$a_{b'500}$	0	0	0	0.14	0.024	0.01	0
$a_{b'550}$	0	0	0	0.28	0.36	0.06	0.01
$a_{b'600}$	0	0	0	0.76	0.97	1.14	0.66
$\sigma_i^{\text{limit}}/\sigma_i^{\text{theory}}$ (pred)	0.22	0.41	0.69	1.02	1.87	3.4	6.47
$\sigma^{\text{limit}'}[\text{pb}] \text{ (pred)}$	1.77	1.31	0.98	0.67	0.62	0.58	0.6

## B. Decays to th

Decays of the T-quark to th would give a  $t\bar{t}hh$  final state. If  $m_H=125$  GeV, the predominant Higgs boson decay mode is  $b\bar{b}$ . As in the case of tZ decays, this gives a final state similar to that used in the b' search, though the larger Higgs mass gives a smaller number of observed W-boson tags due to the 70-100 GeV mass window, and a somewhat larger jet multiplicity.

The templates for  $T \to th$  decay are shown in Fig. 5. It was not possible to find a linear combination of b' basis templates which accurately reproduce the th templates unless we remove th events with more than eight jets. This reduces the sensitivity, as the overall yield is decreased, and thus produces a somewhat conservative limit.

Table II gives the details of the decomposition and Fig. 6 shows the derived upper limits on the cross-section. If BR( $T \to th$ )=100%, the reinterpretation of the b' search yields  $m_T > 423$  GeV at 95% CL, the first limit in this mode.

# C. Decays to tZ and th

To probe a mixed case, we allow both tZ and th decays, which gives  $t\bar{t}ZZ$ ,  $t\bar{t}hh$  modes as well as the mixed mode  $t\bar{t}Zh$ . The relative tZ: th branching ratios are

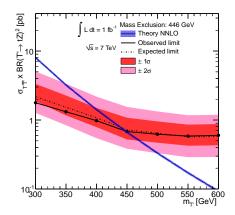


FIG. 4: Upper limits at 95% CL on the cross-section for T-quark pair production in the tZ decay mode.

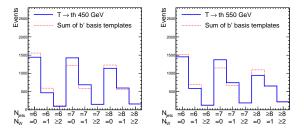


FIG. 5: Jet and W-boson multiplicity, for  $T \to th$  events (solid blue). Also shown (dashed red) is the sum of b' basis templates used to derive the limit on the T quark model. Left for  $m_T = 450$  GeV, right for  $m_T = 550$  GeV.

unchanged, see Fig. 2.

The templates for  $T \to th, tZ$  decay are shown in Fig. 7. Table III gives the details of the decomposition and Fig. 8 shows the derived upper limits on the cross-section. If BR(th or tZ)=100%, the reinterpretation of the b' search yields  $m_T > 419$  GeV at 95% CL, the first limit in this mixed mode.

TABLE II: Details of the predicted limit on T pair-production and  $T \to tH$  decay, using basis templates from  $b' \to Wt$  decays at ATLAS.

	$T_{300}$	$T_{350}$	$T_{400}$	$T_{450}$	$T_{500}$	$T_{550}$	$T_{600}$
$a_{b'300}$	0.62	0.23	0.07	0.02	0	0	0
$a_{b'350}$	0	0	0	0.03	0	0	0
$a_{b'400}$	0	0	0	0.01	0.03	0	0
$a_{b'450}$	0	0	0	0	0	0	0
$a_{b'500}$	0	0.77	1.13	0.62	0.40	0.29	0.17
$a_{b'550}$	0	0	0	0	0	0	0
$a_{b'600}$	0	0.03	0	0	0	0.01	0
$\sigma_i^{\text{limit}}/\sigma_i^{\text{theory}} \text{ (pred)}$	0.26	0.47	0.7	1.32	2.32	3.85	6.75
$\sigma^{\text{limit}}$ [pb] (pred)	2.13	1.52	1	0.88	0.77	0.66	0.62

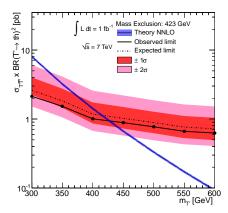


FIG. 6: Upper limits at 95% CL on the cross-section for T-quark pair production in the th decay mode.

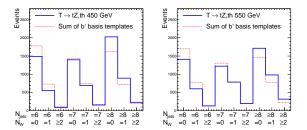


FIG. 7: Jet and W-boson multiplicity, for  $T \to tZ$ , th events (solid blue). Also shown (dashed red) is the sum of b' basis templates used to derive the limit on the T quark model. Left for  $m_T = 450$  GeV, right for  $m_T = 550$  GeV.

#### D. Decays with Wb

We probe two more mixed-decay cases which include Wb decays. The first allows either Wb or th decays; see Fig. 9 for templates, Table IV for decomposition details and Fig. 10 for limits. If BR(th or Wb)=100%, the reinterpretation of the b' search yields  $m_T>415$  GeV at 95% CL, the first limit in this mixed mode.

TABLE III: Details of the predicted limit on T pair-production and  $T \to tZ, tH$  decays, using basis templates from  $b' \to Wt$  decays at ATLAS.

	$T_{300}$	$T_{350}$	$T_{400}$	$T_{450}$	$T_{500}$	$T_{550}$	$T_{600}$
$a_{b'300}$	0.74	0.15	0	0	0	0	0
$a_{b'350}$	0	0.41	0.27	0.05	0	0	0
$a_{b'400}$	0	0	0.15	0.26	0.18	0.05	0.00
$a_{b'450}$	0	0	0	0	0.01	0.10	0.09
$a_{b'500}$	0	0	0	0	0	0	0.00
$a_{b'550}$	0	0	0	0	0	0	0
$a_{b'600}$	0	0	0	0	0	0	0
$\sigma_i^{\text{limit}}/\sigma_i^{\text{theory}}$ (pred)	0.22	0.4	0.75	1.4	2.65	4.42	7.45
$\sigma^{\text{limit}'}[\text{pb}] \text{ (pred)}$	1.79	1.3	1.06	0.93	0.88	0.76	0.69

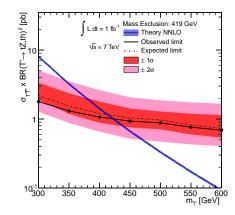


FIG. 8: Upper limits at 95% CL on the cross-section for T-quark pair production in the tZ, tH decay mode.

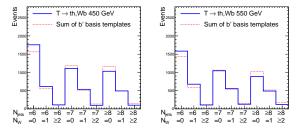


FIG. 9: Jet and W-boson multiplicity, for  $T \to Wb, th$  events (solid blue). Also shown (dashed red) is the sum of b' basis templates used to derive the limit on the T quark model. Left for  $m_T = 450$  GeV, right for  $m_T = 550$  GeV.

The second case allows all three decays (Wb, th or tZ) in the predicted mixture (Fig. 2); see Fig. 11 for templates, Table V for decomposition details and Fig. 12 for limits. If BR(th or tZ or Wb)=100%, the reinterpretation of the b' search yields  $m_T > 419$  GeV at 95% CL as reported previously [12].

In both cases, the T-quark signal has a jet multiplicity which is quite different from the  $b' \to Wt$  basis templates. The most accurate description of T-quark templates uses the low-mass and high-mass b' basis templates (see Tables IV and V) which have the least overlap and so can be varied nearly independently to achieve the desired jet multiplicity of the T-quark signals with Wb decays.

# E. Limits in the branching ratio triangle

Each of the results above place lower limits on the mass of the T quark under specific assumptions about the branching ratios. Table VI summarizes these results. For limits derived from basis templates we estimate the errors by studying the relationship between the  $\chi^2$  of the fit and the predicted limit for various configurations of basis templates (including non-optimal ones). The uncer-

TABLE IV: Details of the predicted limit on T pair-production and  $T \to tH, Wb$  decays, using basis templates from  $b' \to Wt$  decays at ATLAS.

	$T_{300}$	$T_{350}$	$T_{400}$	$T_{450}$	$T_{500}$	$T_{550}$	$T_{600}$
$a_{b'300}$	0.36	0.18	0.07	0.03	0.01	0	0
$a_{b'350}$	0	0	0	0	0	0	0
$a_{b'400}$	0	0	0	0	0	0	0
$a_{b'450}$	0	0	0	0	0	0	0
$a_{b'500}$	0	0	0	0	0	0	0
$a_{b'550}$	0.17	1.18	0.72	0.68	0.15	0.13	0.03
$a_{b'600}$	4.07	3.27	2.16	0.97	0.95	0.51	0.35
$\sigma_i^{\text{limit}}/\sigma_i^{\text{theory}}$ (pred)	0.31	0.42	0.79	1.45	2.79	5	9.37
$\sigma^{\text{limit}'}[\text{pb}] \text{ (pred)}$	2.5	1.37	1.12	0.96	0.92	0.86	0.86

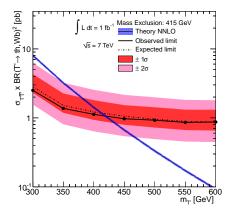


FIG. 10: Upper limits at 95% CL on the cross-section for T-quark pair production in the th, Wb decay mode.

tainty on the mass limits are then derived by extrapolating the optimal-fit  $\chi^2$  to an appropriate threshold. The uncertainty on the mass limits are larger for cases where the fit is worse. For each scenario, we quote the branching ratios at the limit value. Figure 13 shows the results graphically, with interpolated values.

TABLE V: Details of the predicted limit on T pair-production and tZ, tH, Wb decays, using basis templates from  $b' \to tW$  decays at ATLAS.

	$T_{300}$	$T_{350}$	$T_{400}$	$T_{450}$	$T_{500}$	$T_{550}$	$T_{600}$
$a_{b'300}$	0.39	0.20	0.07	0.02	0.01	0	0
$a_{b'350}$	0	0	0	0	0	0	0
$a_{b'400}$	0	0	0	0	0	0	0
$a_{b'450}$	0	0	0	0	0	0	0
$a_{b'500}$	0.01	0	0	0	0	0	0
$a_{b'550}$	0.24	0.94	1.24	0.78	0.39	0.13	0.04
$a_{b'600}$	4.35	2.88	1.46	1.11	0.87	0.68	0.42
$\sigma_i^{\text{limit}}/\sigma_i^{\text{theory}}$ (pred)	0.28	0.43	0.77	1.34	2.38	4.36	8.1
$\sigma^{\text{limit}'}[\text{pb}] \text{ (pred)}$	2.29	1.4	1.09	0.89	0.79	0.75	0.75

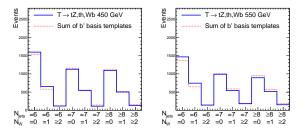


FIG. 11: Jet and W-boson multiplicity, for  $T \to Wb, th, tZ$  events (solid blue). Also shown (dashed red) is the sum of b' basis templates used to derive the limit on the T quark model. Left for  $m_T = 450$  GeV, right for  $m_T = 550$  GeV.

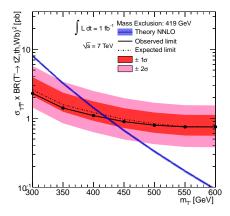


FIG. 12: Upper limits at 95% CL on the cross-section for T-quark pair production in the th, tZ, Wb decay mode.

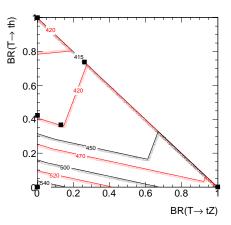


FIG. 13: Lower limits at 95% CL on the mass (GeV) of the exotic T quark for varying assumptions about branching ratios. Black squares show the points at which the limits are calculated; contours are interpolated. Limits at BR(Wb)=100% [1] and BR(tZ)=100% [11] are taken from dedicated searches; other points our reinterpretations (see Table VI).

TABLE VI: Summary of mass limits at 95% CL for various branching ratios. For BR(tZ)=100%, we include for comparison the direct limit from CMS [11] as well as our reinterpretation of the ATLAS [16]  $b' \rightarrow Wt$  search for this case. Uncertainties are due to imperfect description of the signal model via the basis templates.

			Lower limit	
BR(Wb)	) BR( $tZ$	(BR(th))	$m_T [{\rm GeV}]$	Comment
1	0	0	557	Direct search [1]
0	1	0	475	Direct search [11]
0	1	0	$446^{+4}_{-4}$	Reinterpreted here
0	0	1	$423_{-48}^{+23}$	${\bf Reinterpreted\ here}$
0.50	0.13	0.37	$419^{+3}_{-3}$	Reinterpreted [12]
0	0.26	0.74	$419^{+12}_{-18}$	${\bf Reinterpreted\ here}$
0.58	0	0.42	$415^{+6}_{-11}$	${\bf Reinterpreted\ here}$

#### IV. CONCLUSIONS

Searches for an exotic heavy quark T have been previously reported for specific choices of the decay modes: BR(Wb)=100% or BR(tZ)=100%. We consider alternative branching ratio scenarios, scanning the branching ratio triangle. We derive limits for these scenarios by reinterpretting a recent ATLAS search for b'.

We find limits for T at  $m_T > 415$  GeV across the entire triangle, up to  $m_T > 557$  GeV in the case of Wb decay. An optimized experimental search for decays with large th branching ratio would lead to significantly tighter limits across the triangle.

#### V. ACKNOWLEDGMENTS

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