# Forward hadronic scattering at 8 TeV: Predictions for the LHC

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The Large Hadron Collider (LHC) recently started operating at 8 TeV. In this note, we update our earlier LHC forward hadronic scattering predictions [1–3], giving new predictions, including errors, for the $pp$ total and inelastic cross sections, the $\rho$-value, the nuclear slope parameter $B$, $d\sigma_{\rm el}/dt$, and the large gap survival probability at 8 TeV.

## I. INTRODUCTION

After a long successful run at 7 TeV, the LHC recently started extensive running at 8 TeV. We had made hadronic forward scattering predictions for the full design energy (14 Tev) for the LHC; for details see the review article by M. Block [1]. More recently, we had calculated forward scattering parameters at 7 TeV [2, 3]. The purpose of this note is to make predictions for 8 TeV, including errors due to model uncertainties. Two separate models were combined to make these predictions, the first being the analyticity-constrained analytic amplitude model of Block and Halzen [4] that saturates the Froissart bound [5] and the second being the "Aspen Model", a revised version of the eikonal model of Block, Gregores, Halzen and Pancheri [7] that now incorporates analyticity constraints. Although self-contained, we purposely keep explanations very brief; for more details, see Ref. [1–3].

## II. MODELS USED FOR PREDICTIONS

### A. The analytic amplitude model, $\sigma_{\rm tot}$ and $\rho$

We make the most accurate predictions of the forward $pp$ scattering properties,

$$\sigma_{\rm tot} \;\equiv\; \frac{4\pi}{p}{\rm Im}f(\theta_L = 0) \tag{1}$$

$$\rho \;\equiv\; \frac{{\rm Re}f(\theta_L = 0)}{{\rm Im}f(\theta_L = 0)}, \tag{2}$$

using the analyticity-constrained analytic amplitude model of Block and Halzen [4] that saturates the Froissart bound [5]. By saturation of the Froissart bound, we mean that the total cross section defined in Eq. (1) rises as $\ln^2 s$, where $s$ is the square of the cms energy. In Eq. (1) and Eq. (2), $f(\theta_L)$ is the $pp$ laboratory scattering amplitude as a function of $\theta_L$, the laboratory scattering angle and $p$ is the laboratory momentum. In Fig. 1 of Ref. [3], where we showed that the proton asymptotically is a black disk of gluons whose radius goes as $R_0 \ln(s)$, the upper dashed line (for $\sqrt{(s)} > 100$ GeV) is the total $pp$ ($\bar{p}p$) cross section as a function of the center-of-mass energy, $\sqrt{s}$, given by

$$\sigma_{\rm tot}^0(\nu) \;\equiv\; 37.10\left(\frac{\nu}{m}\right)^{-0.5} + 37.32 - 1.440\ln\left(\frac{\nu}{m}\right) + 0.2817\ln^2\left(\frac{\nu}{m}\right) \;{\rm mb}, \tag{3}$$

where $\nu$ is the laboratory energy and $m$ is the proton mass; thus $\nu/m \approx s/2m^2$. Our use of analyticity constraints—employing new Finite Energy Sum Rules (FESR) [6]—allows us to use *very accurate low energy cross section measurements to act as an anchor* that accurately fixes our high energy cross section predictions. At 8 TeV, we find that $\sigma_{\rm tot} = 97.6 \pm 1.1$ mb; see Ref. [3]. The same set of parameters predict $\sigma_{\rm tot} = 107.3 \pm 1.2$ mb at 14 TeV [1]. Further, using Ref. [3], we predict that $\rho_{pp} = 0.134 \pm 0.001$ at 8 TeV.

## B. The "Aspen" Model: an eikonal model for $pp$ scattering

The "Aspen" model uses an unconventional definition of the eikonal $\chi(b, s)$ in impact parameter space $b$, so that

$$\sigma_{\text{tot}}(s) = 2 \int \left[ 1 - e^{-\chi_I(b,s)} \cos\left(\chi_R(b,s)\right) \right] d^2\vec{b}, \tag{4}$$

$$\rho(s) = \frac{\int e^{-\chi_I(b,s)} \sin(\chi_R(b,s)) \, d^2\vec{b}}{\int \left[ 1 - e^{-\chi_I(b,s)} \cos(\chi_R(b,s)) \right] d^2\vec{b}}, \tag{5}$$

$$B(s) = \frac{1}{2} \frac{\int |e^{-\chi_I(b,s)+i\chi_R(b,s)} - 1| b^2 \, d^2\vec{b}}{\int |e^{-\chi_I(b,s)+i\chi_R(b,s)} - 1| \, d^2\vec{b}}, \tag{6}$$

$$\frac{d\sigma_{\text{el}}}{dt} = \pi \left| \int J_0(qb) \left[ e^{-\chi_I(b,s)+i\chi_R(b,s)} - 1 \right] b \, db \right|^2, \tag{7}$$

$$\sigma_{\text{el}}(s) = \int \left| e^{-\chi_I(b,s)+i\chi_R(b,s)} - 1 \right|^2 d^2\vec{b}, \tag{8}$$

$$\sigma_{\text{inel}}(s) \equiv \sigma_{\text{tot}}(s) - \sigma_{\text{el}}(s) = \int \left( 1 - e^{-2\chi_I(b,s)} \right) d^2\vec{b}, \tag{9}$$

where $\sigma_{\text{inel}}(s)$ is the total inelastic cross section. The even eikonal profile function $\chi^{even}$, which is the only surviving term at the high energies considered here, receives contributions from quark-quark, quark-gluon and gluon-gluon interactions, and can be written in the factorized form

$$\begin{aligned} \chi^{even}(s, b) &= \chi_{qq}(s,b) + \chi_{qg}(s,b) + \chi_{gg}(s,b) \\ &= i \left[ \sigma_{qq}(s)W(b; \mu_{qq}) + \sigma_{qg}(s)W(b; \sqrt{\mu_{qq}\mu_{gg}}) + \sigma_{gg}(s)W(b; \mu_{gg}) \right], \end{aligned} \tag{10}$$

where $\sigma_{ij}$ is the cross sections of the colliding partons, and $W(b; \mu)$ is the overlap function in impact parameter space, parameterized as the Fourier transform of a dipole form factor. The parameters $\mu_{qq}$ and $\mu_{gg}$ are masses which describe the "area" occupied by the quarks and gluons, respectively, in the colliding protons. In this model hadrons asymptotically evolve into black disks of gluons [3]. For details of the parameterization of the model, see Ref. [1].

## C. Total inelastic cross section, $\sigma_{\text{inel}}$

From Eq. (9) and Eq. (4), we calculate the *ratio* $r(s) = \sigma_{\text{inel}}(s)/\sigma_{\text{tot}}(s)$, because most errors due to parameter uncertainties cancel in the ratio. We then multiply $r(s)$ by the (more accurate) total cross section using Eq. (1) (the analytic amplitude model) to obtain the inelastic cross section shown in Fig. 1 of Ref. [3], as the lower (red) curve, given by

$$\sigma_{\text{inel}}^0(\nu) \equiv 62.59 \left( \frac{\nu}{m} \right)^{-0.5} + 24.09 + 0.1604 \ln \left( \frac{\nu}{m} \right) + 0.1433 \ln^2 \left( \frac{\nu}{m} \right) \text{ mb}. \tag{11}$$

At 8 TeV, we find $\sigma_{\text{inel}} = 70.4 \pm 1.3$ mb.

## D. Elastic scattering, $B$ and $d\sigma_{\text{el}}/dt$

From Eq. (6) we find that the nuclear slope parameter $B$, the logarithmic derivative of the elastic cross section with respect to $t$ at $t = 0$, is given by $B = 18.47 \pm 0.12$ $(\text{GeV}/c)^{-2}$ at 8 TeV, where $t$ is the squared momentum transfer.

At 8 TeV, using Eq. (7), we plot the differential elastic scattering cross section $d\sigma_{\text{el}}/dt$, in mb/$(\text{GeV}/c)^2$, against $|t|$, in $(\text{GeV}/c)^2$ as the solid (black) curve in Fig. 1. Also shown is the approximation, $\frac{d\sigma}{dt}|_{t=0} e^{-B|t|}$, valid for small $|t|$, which is the dashed (red) curve. The agreement is striking for small $t$.

A few remarks are in order about Fig. 1. It is clear from inspection that

$$\sigma_{\text{el}} \equiv \int_0^\infty \frac{d\sigma_{\text{el}}}{d|t|} d|t| \tag{12}$$

$$\approx \int_0^\infty \frac{d\sigma_{\text{el}}}{d|t|}\bigg|_{t=0} e^{-B|t|} d|t| \tag{13}$$

$$= \frac{1}{B} \frac{d\sigma_{\text{el}}}{d|t|}\bigg|_{t=0}, \tag{14}$$

a result that clearly does not depend on the details of where the dip in $|t|$ is located, nor on the value or shape of the high $|t|$ portion of $d\sigma_{\mathrm{el}}/d|t|$, since these $|t|$ regions contribute negligible amounts to the integral in Eq. (12).

We here point out that the height of the first dip, i.e. the value of $d\sigma_{\mathrm{el}}/d|t|$ at $|t| \approx 0.55$ GeV$^2$, is difficult to calculate because it results from the *interference* of the real and imaginary parts of the scattering amplitude. The scattering amplitude at very high energies is dominated by its imaginary part; if there were *no* real part, there would be *no* interference and $d\sigma_{\mathrm{el}}/d|t|$ at the dip would vanish, reflecting a pure diffraction pattern given by the Bessel function $J_0(qb)$ in Eq. (7). As a result a minimal change in the real part at the dip introduces a very large fractional change in the value of $d\sigma_{\mathrm{el}}/d|t|$, as well as slightly changing its location in $|t|$. This is illustrated by the fact that none of the models shown by the Totem collaboration [9] at 7 TeV accurately predict both the magnitude and the location of the first minimum; further, their anticipated value of $d\sigma_{\mathrm{el}}/d|t|$ for the shoulder at larger $|t|$ varies widely, with none fitting the data. However, they are successful in reproducing the slope $B$, as well as finding the approximate location of the first dip.

## E.   Rapidity gap survival probability

As shown in Ref. [1], the survival probability $< |S| >^2$ of *any* large rapidity gap is given by

$$< |S|^2 >= \int W(b\,;\mu_{\mathrm{qq}})\, e^{-2\chi_{\mathrm{I}}(s,b)} d^2\, \vec{b}, \tag{15}$$

which is the differential probability density in impact parameter space $b$ for *no* subsequent interaction (the exponential suppression factor) multiplied by the quark probability distribution in $b$ space from Eq. (10)), which is then integrated over $b$. It should be emphasized that Eq. (15) is the probability of *survival* of a large rapidity gap and *not* the probability for the production and survival of large rapidity gaps, which is the quantity observed experimentally. The energy dependence of the survival probability $< |S|^2 >$ is through the energy dependence of $\chi_{\mathrm{I}}$, the imaginary portion of the eikonal given in Eq. (10). A plot of $< |S|^2 >$ as a function of $\sqrt{s}$, the cms energy in GeV, was given in Fig. 3 of Ref. [2]. At 8 TeV, we find the gap survival probability to be $< |S|^2 >= 15.0 \pm 0.05$ %.

## III.   SUMMARY

We summarize our 8 TeV $pp$ forward scattering parameters for the LHC in Table I, comparing them to our 14 TeV predictions taken from Ref. [1].

## Acknowledgments

[1] M. M. Block, Phys. Reports **436**, 71 (2006).
[2] M. M. Block and F. Halzen, Phys. Rev. D **83**, 077901 (2011).
[3] M. M. Block and F. Halzen, Phys. Rev. Lett. **107**, 212002 (2011).
[4] M. M. Block and F. Halzen, Phys. Rev. D **72**, 036006 (2005); M. M. Block and F. Halzen, Phys. Rev. D **73**, 054022 (2006).
[5] M. Froissart, Phys. Rev. **123**, 1053 (1961).
[6] M. M. Block, Eur. Phys. J. C. **47**, 697 (2006).
[7] M. M. Block, E.Gregores, F. Halzen and G. Pancheri, Phys. Rev. D **60** , 054024 (1999).
[8] G. Antchen et al. (Totem Collaboration), EPL **96**, 21002 (2011).
[9] G. Antchen et al. (Totem Collaboration), EPL **95**, 41001 (2011).

## Figures

FIG. 1: The 8 TeV $pp$ differential elastic scattering cross section , $d\sigma_{\text{el}}/dt$, in mb/(GeV/c)$^2$, vs. $|t|$, in (GeV/c)$^2$ is the solid (black) curve. The dashed (red) curve is the small $|t|$ approximation, $\frac{d\sigma}{dt}|_{t=0} e^{-B|t|}$.

**Tables**

TABLE I: Values of forward scattering parameters for the LHC, at 8 and 14 TeV.

| $\sqrt{s}$ (TeV) | $\sigma_{\text{tot}}$ mb | $\sigma_{\text{inel}}$ mb | $\rho$ | $B$ (GeV/c)$^{-2}$ | $<|S|^2>$ % |
|---|---|---|---|---|---|
| 8 | $97.6 \pm 1.1$ | $70.3 \pm 1.3$ | $0.134 \pm 0.001$ | $18.47 \pm 0.12$ | $15.0 \pm 0.05$ |
| 14 | $107.3 \pm 1.2$ | $76.3 \pm 1.4$ | $0.132 \pm 0.001$ | $19.39 \pm 0.13$ | $12.6 \pm 0.06$ |

$d\sigma/dt$, for pp, at 8 TeV,

Dashed Red:$(d\sigma/dt)|_{t=0}\ e^{-Bt}$,

$\dfrac{d\sigma}{dt}\big|_{t=0} = 495.8$ mb/(GeV/c)$^2$

Figure 1        DS11182    06Jun2012