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Abstract

We revisit the transverse single spin asymmetry in the angular distribution of a Drell-Yan dilepton pair. We study this asymmetry by using twist-3 collinear factorization, and we obtain the same result both in covariant gauge and in the light-cone gauge. Moreover, we have checked the electromagnetic gauge invariance of our calculation. Compared to previous calculations we properly treat the transverse momentum expansion, and as a consequence our final expression for the asymmetry differs from all the previous results given in the literature. The overall sign of this asymmetry is as important as the sign of the Sivers asymmetry in Drell-Yan.

1 Introduction

The observation of transverse single spin asymmetries (SSAs) in various hard scattering processes has stimulated new remarkable developments both on the theoretical and the experimental side. As a consequence, the study of SSAs currently represents a very active field of research [1–3]. The interest in such effects is essentially twofold: first, SSAs allow one to address the parton structure of the nucleon beyond the collinear parton model approximation. Second, SSAs are ideal observables in order to further explore in which cases the machinery of QCD factorization still applies and in which cases, in its simplest form, it breaks down (see [4] and references therein).

For what concerns the parton structure of the nucleon, in the present work we focus on collinear twist-3 quark-gluon-quark correlations. To be more precise, the central non-perturbative correlator is the so-called ETQS (Efremov-Teryaev-Qiu-Sterman) matrix element [5–7] T_F — and its chiral-odd partner $T_F^{(\sigma)}$ — which typically appears when describing transverse SSAs in the context of collinear higher-twist factorization. The machinery of collinear twist-3 factorization was pioneered already in the early 1980's [5, 8, 9], and in the meantime frequently applied to transverse spin effects in hard semi-inclusive reactions (see, e.g., Refs. [6, 7, 10–12]).

In this paper, we revisit the transverse single spin asymmetry in the angular distribution of a Drell-Yan dilepton pair. This asymmetry is defined as the difference of two spin dependent cross sections with opposite directions of transverse polarization divided by their sum,

$$A_N = \left(\frac{d\sigma(S_T)}{d\Omega dQ^2} - \frac{d\sigma(-S_T)}{d\Omega dQ^2}\right) \Big/ \left(\frac{d\sigma(S_T)}{d\Omega dQ^2} + \frac{d\sigma(-S_T)}{d\Omega dQ^2}\right), \tag{1}$$

where $d\Omega = d\cos\theta d\phi_S$ is a solid angle element of the leptons in a dilepton rest frame, and the azimuthal angle ϕ_S is measured relative to the transverse spin vector. Note that the transverse momentum Q_T of the dilepton pair is integrated out, and we emphasize that integrating over Q_T is essential for applying the collinear factorization approach in the present case.

The asymmetry A_N was already studied in several previous articles, and various different results were obtained. The first calculation, carried out in the light-cone gauge, can be found in Ref. [13]. The authors obtained [1]

$$A_N^{(HTS)} = -\frac{1}{Q}\,\frac{\sin 2\theta \sin\phi_S}{1+\cos^2\theta}\,\frac{\sum_q e_q^2 \int dx \left(T_F^q(x,x) - x\frac{d}{dx}T_F^q(x,x)\right) f_1^{\bar{q}}(x')}{\sum_q e_q^2 \int dx\, f_1^q(x) f_1^{\bar{q}}(x')}\,, \tag{2}$$

where f_1^q is the standard unpolarized twist-2 parton distribution for quark flavor q. The momentum fraction x' is given by $x' = Q^2/(xS)$, with $S = (P+\bar{P})^2$ denoting the square of the cm energy of the process. Later on the presence of the derivative term in the numerator of (2) was doubted, and it was corrected as [14, 15]

$$A_N^{(BMT)} = -\frac{1}{Q}\,\frac{\sin 2\theta \sin\phi_S}{1+\cos^2\theta}\,\frac{\sum_q e_q^2 \int dx\, T_F^q(x,x) f_1^{\bar{q}}(x')}{\sum_q e_q^2 \int dx\, f_1^q(x) f_1^{\bar{q}}(x')}\,. \tag{3}$$

Afterwards, in Ref. [16] A_N was considered in the collinear twist-3 approach, and the result of that study agreed with the expression in (3). Then A_N was computed by using factorization in terms of transverse momentum dependent correlators [17]. The final outcome of that work neither agreed with (2) nor with (3). More recently, A_N was again considered in Ref. [18], where the authors claimed that the spin-dependent hadronic tensor should be multiplied by a factor of 2 compared to previous work [13–16].

This somewhat unclear situation motivated us to revisit this topic. We computed A_N in (1) by means of twist-3 collinear factorization and came up with yet another result. To be specific, our result is just half of the one quoted in Eq. (3), and a quarter of that obtained in Ref. [18]. In order to gain further confidence we checked our calculation in a few different ways. Technically, the most important difference in comparison to previous work is that, when performing the collinear expansion in the twist-3 formalism, we take into account the dependence on transverse parton motion (k_T-dependence) not only in the hadronic tensor but also in the lepton tensor. We emphasize that the contribution from the k_T expansion of the lepton tensor has been overlooked in [14–16, 18]. One symptom of this shortcoming is that existing results obtained in the light-cone gauge [14–16, 18] depend on the boundary condition for the gluon field.

The rest of the paper is organized as follows. In the next section, we introduce our notation, and give some details about the kinematics. In Section 3, we derive the asymmetry in a covariant gauge as well as in the light cone gauge, and the two results agree with each other. In addition, we have checked the electromagnetic gauge invariance by explicit calculation. We summarize the paper in Section 4.

2 Kinematics and notation

We focus on lepton pair production in hadronic scattering which comes from the decay of a virtual photon, $H_a + H_b \to \gamma^* + X \to \ell^+ + \ell^- + X$. The 4-momenta of the leptons are l_1 and l_2, and $q = l_1 + l_2$ denotes the momentum of the virtual photon. The invariant mass of the dilepton pair is Q with $Q^2 = q^2$. For the following calculation we need to introduce the vector $R = l_1 - l_2$. In any dilepton rest frame, R reads

$$R = Q\left(0,\, \sin\theta\cos\phi,\, \sin\theta\sin\phi,\, \cos\theta\right), \tag{4}$$

where the numerical values of θ and ϕ depend on the frame. The correlation associated with A_N in (1) is $\varepsilon_{\mu\nu\rho\sigma}P^\mu \bar{P}^\nu S^\rho R^\sigma$, while the asymmetry usually associated with the Sivers effect [19] is related to

[1] To shorten the notation we suppress throughout terms where quarks and antiquarks are interchanged.

the correlation $\varepsilon_{\mu\nu\rho\sigma}P^\mu \bar{P}^\nu S^\rho q^\sigma$. The latter requires to measure the transverse momentum Q_T of the dilepton pair.

A convenient way of sorting out the different angular dependences of the Drell-Yan cross section is to decompose the lepton tensor in terms of individual independent orthogonal tensors [20–22],

$$L^{\mu\nu} = \Big((q+R)^\mu (q-R)^\nu + (q+R)^\nu (q-R)^\mu - 2Q^2 g^{\mu\nu}\Big) = \sum_i^9 L_i V_i^{\mu\nu}\,. \tag{5}$$

(A discussion of the general structure of the polarized Drell-Yan cross section can be found in Ref. [23].) The L_i represent the angular structures, and the basis tensors $V_i^{\mu\nu}$ can be constructed from a set of (4-dimensional) basis vectors T^μ, X^μ, Y^μ, Z^μ, which are mutually orthogonal to each other and are normalized according to $T^2 = 1$, $X^2 = Y^2 = Z^2 = -1$. For the case of A_N, the relevant angular structures appear in the terms associated with $V_3 = -\frac{1}{2}(Z^\mu X^\nu + Z^\nu X^\mu)$ and $V_8 = -\frac{1}{2}(Z^\mu Y^\nu + Z^\nu Y^\mu)$,

$$L^{\mu\nu} = Q^2 \sin 2\theta \cos\phi \; V_3^{\mu\nu} + Q^2 \sin 2\theta \sin\phi \; V_8^{\mu\nu} + \;\ldots\,. \tag{6}$$

As in the case of (4), the decomposition (6) holds in any dilepton rest frame.

For $Q_T = 0$ (in the hadronic *cm* frame), we choose the following dilepton rest frame: z-axis along the direction of the polarized hadron, and x-axis along the direction of the polarization vector S_T. To be fully specific, the 4-dimensional basis vectors are given by

$$\begin{aligned}
T^\mu &= \frac{q^\mu}{\sqrt{Q^2}}\,, \\
Z^\mu &= \frac{1}{Q}\left(xP^\mu - x'\bar{P}^\mu\right), \\
X^\mu &= S_T^\mu\,, \\
Y^\mu &= \varepsilon^{\mu\nu\rho\sigma} T_\nu Z_\rho X_\sigma\,.
\end{aligned} \tag{7}$$

Because of the specific definition of Z^μ, this frame can actually be considered as partonic *cm* frame.

If $Q_T \neq 0$, one may work in the Collins-Soper frame [24] for which the basis vectors read

$$\begin{aligned}
T^\mu &= \frac{q^\mu}{\sqrt{Q^2}}\,, \\
Z^\mu &= \frac{2}{\sqrt{Q^2 + Q_T^2}}\left(q_{\bar{p}}\tilde{P}^\mu - q_p \tilde{\bar{P}}^\mu\right), \\
X^\mu &= -\frac{Q}{Q_T}\frac{2}{\sqrt{Q^2 + Q_T^2}}\left(q_{\bar{p}}\tilde{P}^\mu + q_p \tilde{\bar{P}}^\mu\right), \\
Y^\mu &= \varepsilon^{\mu\nu\rho\sigma} T_\nu Z_\rho X_\sigma\,.
\end{aligned} \tag{8}$$

In (8) we use the further definitions $\tilde{P}^\mu = [P^\mu - (P\cdot q)/q^2 q^\mu]/\sqrt{S}$, $\tilde{\bar{P}}^\mu = [\bar{P}^\mu - (\bar{P}\cdot q)/q^2 q^\mu]/\sqrt{S}$, with $q_p = P\cdot q/\sqrt{S}$, $q_{\bar{p}} = \bar{P}\cdot q/\sqrt{S}$. At tree level, Q_T is equal to the sum of the intrinsic transverse momenta of the two incoming partons. This implies that for $Q_T \neq 0$ a k_T-dependence is sitting in the unit vectors X^μ, Y^μ and T^μ. (The k_T-dependence of Z^μ is of the order k_T^2 and therefore irrelevant for our twist-3 calculation.) As a result, the terms containing $\cos\phi$ and $\sin\phi$ are k_T-dependent. This k_T-dependence must be taken into account when performing the collinear expansion.

For $Q_T \neq 0$, instead of using the Collins-Soper frame, one can alternatively perform the calculation, for instance, in the Gottfried-Jackson frame [25]. Keeping track of all k_T-dependent terms in the Gottfried-Jackson frame is more involved. Nevertheless, we carried out the calculation, and our final result agrees with what we find in the Collins-Soper frame.

3 Calculation in twist-3 collinear factorization

In order to calculate A_N in Eq. (1) one needs both the unpolarized cross section (in the parton model) and the spin-dependent cross section. The former is well-known and given by

$$\frac{d\sigma}{dQ^2 d\Omega} = \frac{4\pi\alpha_{em}^2}{9Q^2} \sum_q e_q^2 \int dx\, dx'\, f_1^q(x)\, f_1^{\bar{q}}(x') \left[\frac{3}{16\pi} (1+\cos^2\theta)\, \delta\Big(Q^2 - xx'S\Big) \right]. \tag{9}$$

The polarized cross section is a twist-3 effect and depends on quark-gluon-quark correlations, which contain interesting physics beyond the parton model. In fact, such twist-3 correlations associated with both hadrons can give rise to A_N leading to the generic expression [14]

$$A_N \propto \frac{1}{Q} \frac{T_F(x,x) \otimes f_1(x') + h_1(x) \otimes T_F^{(\sigma)}(x',x')}{f_1(x) \otimes f_1(x')}, \tag{10}$$

where h_1 is the transversity distribution. The second (chiral-odd) term in the numerator, which we have not included in Eqs. (2) and (3), was first considered in Ref. [14]. In our calculation we treat both the chiral-even and the chiral-odd contribution to A_N.

The ETQS matrix element T_F and its chiral-odd partner $T_F^{(\sigma)}$ are defined as [2]

$$\begin{aligned} T_F(x,x_1) &= \int \frac{dy^- dy_1^-}{4\pi}\, e^{-ixP^+y^- + i(x_1-x)P^+y_1^-} \langle PS|\bar{\psi}(y^-)\,\gamma^+\,\varepsilon_T^{\nu\mu}\,S_{T\nu}\,gF^+{}_\mu(y_1^-)\,\psi(0)|PS\rangle\,, \\ T_F^{(\sigma)}(x,x_1) &= \int \frac{dy^- dy_1^-}{4\pi}\, e^{-ixP^+y^- + i(x_1-x)P^+y_1^-} \langle PS|\bar{\psi}(y^-)\,\sigma^{\mu+}\,gF^+{}_\mu(y_1^-)\,\psi(0)|PS\rangle\,, \end{aligned} \tag{11}$$

where a summation over color is implicit, and gauge links has been suppressed. In the following two subsections we compute the hard coefficients associated with these matrix elements both in covariant gauge and in the light-cone gauge. It is worthwhile to mention that $T_F(x,x)$ and $T_F^{(\sigma)}(x,x)$ are related to particular k_T-moments of the transverse momentum dependent Sivers function [19] and Boer-Mulders function [26], respectively [17,27].

3.1 Asymmetry derived in covariant gauge

In covariant gauge, the leading contribution of the gluon field is from the component parallel to the direction of its momentum. If one considers P^+ (with P being the momentum of the polarized nucleon) and $\bar{P}^-$ as the large light-cone momenta, then the dominant component of the gluon field for the diagrams shown in Fig. 1 is A^+. Before making the collinear expansion, the incoming partons carry a transverse momentum k_{iT}, which is much smaller than the dominant longitudinal momentum. In order to extract the twist-3 contributions from the diagrams with one-gluon-exchange, one needs to get one power of k_{iT} from the hard scattering part and combine k_{iT} with A^+ in order to convert the gluon field in the matrix element into the corresponding part of the field strength tensor $(\partial_T A^+)$ [7]. As stated above, the k_T-flow may go through the lepton lines via the virtual photon. Therefore, we have to expand the hadronic tensor as well as the lepton tensor in terms of k_{iT} around $k_{iT} = 0$.

Only the two diagrams in Fig. 1 contribute to A_N in covariant gauge. To be more precise, these two diagrams provide the chiral-even T_F part of the asymmetry. In order to get the chiral-odd contribution one has to consider the corresponding two diagrams for which the gluon is associated with the unpolarized hadron. As an example, for the left cut-diagram in Fig. 1 we have the following

[2]For a generic 4-vector v, we define light-cone coordinates according to $v^\pm = (v^0 \pm v^3)/\sqrt{2}$ and $\vec{v}_T = (v^1, v^2)$.

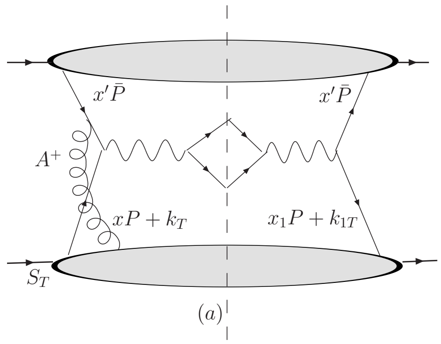

Figure 1: Diagrams contributing to A_N in covariant gauge. The gluon attached to the hard scattering part is longitudinally polarized. In order to extract the twist-3 contribution, one has to expand in k_T and k_{1T}, and to pick up the linear terms.

expansion,

$$\begin{aligned} & H^{\mu\nu,\rho}(xp + k_T, x_1 p + k_{1T}, S_T)\, P_\rho\, L_{\mu\nu}(q = x_1 p + k_{1T} + x'\bar{p}, R) \\ = \;& H^{\mu\nu,\rho}(xp, x_1 p, S_T)\, P_\rho\, L_{\mu\nu}(q = x_1 p + x'\bar{p}, R) \\ & + Q^2 \sin 2\theta \left[\frac{\partial\Big(\cos\phi\, V^{CS}_{3,\mu\nu}\, H^{\mu\nu,\rho}\, P_\rho\Big)}{\partial k^\sigma_{1T}} + \frac{\partial\Big(\sin\phi\, V^{CS}_{8,\mu\nu}\, H^{\mu\nu,\rho}\, P_\rho\Big)}{\partial k^\sigma_{1T}} \right]_{k_T = k_{1T} = 0} k^\sigma_{1T} \\ & + Q^2 \sin 2\theta \left[\sin\phi_S\, V^{CM}_{8,\mu\nu} \frac{\partial H^{\mu\nu,\rho}\, P_\rho}{\partial k^\sigma_T} \right]_{k_T = k_{1T} = 0} k^\sigma_T + \ldots\,, \end{aligned} \tag{12}$$

where the superscripts CM and CS refer to the partonic cm frame and the Collins-Soper frame specified in (7) and in (8), respectively. The azimuthal angle ϕ is understood in the Collins-Soper frame, while the azimuthal angle in the cm frame is just what we defined above as ϕ_S, namely the angle between R_T and S_T. There is no need to distinguish between the polar angle θ in the two frames when expanding around $k_{iT} = 0$ and keeping only the linear terms. For the left cut-diagram in Fig. 1, the lepton tensor is independent of k_T, but it depends on k_{1T}. The used tensor decomposition of the lepton tensor is rather convenient in order to treat this k_{1T}-dependence. This dependence is sitting in three parts: the angular dependences $\cos\phi$ and $\sin\phi$, the tensors $V^{CS}_{3,\mu\nu}$ and $V^{CS}_{8,\mu\nu}$, and the hadronic tensor $H^{\mu\nu,\rho}\, P_\rho$.

The first term of the Taylor expansion in (12) corresponds to the eikonal line contribution to the twist-2 quark distribution, which does not contribute to the asymmetry. One can extract the desired twist-3 term by picking up the terms linear in k_T (and k_{1T}) from the above expansion. Note that in $H^{\mu\nu,\rho}\, P_\rho$ also a delta function of the form $\delta(Q^2 - (xp + k_{1T} + x'\bar{p})^2)$ is hidden. It is easy to see that this delta-function cannot provide a term linear in k_{1T}, and therefore its k_{1T}-dependence is irrelevant for the calculation of A_N. This is actually the reason why the derivative term of T_F, which we briefly discussed in the Introduction, does not show up in A_N. Note that the second term on the r.h.s. of (12) has been overlooked in the previous calculations [14–16]. This term gives rise to half the contribution of the third term, however with a minus sign. This is the reason why our final result for A_N is just half of the one obtained in Refs. [14–16]. In general, the collinear expansion enables one to integrate out three of the four components of the parton loop momenta, and as a result the non-perturbative part can be expressed through the collinear twist-3 correlations T_F and $T_F^{(\sigma)}$.

The strong interaction phase necessary for having a nonzero SSA arises from the partonic scattering amplitude with an extra gluon. As is evident from the diagrams in Fig. 1, this amplitude interferes

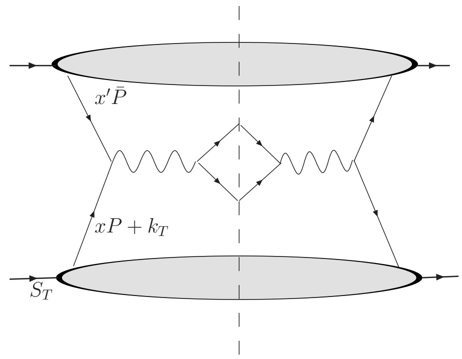

Figure 2: Contribution from the k_T-expansion in the light cone gauge. The k_T-flow goes also through the lepton lines via the virtual photon propagator.

with the real scattering amplitude without a gluon. The imaginary part is due to the pole of the quark (antiquark) propagator and arises when integrating over the longitudinal gluon momentum fraction x_g. In the present case, one has a pole for $x_g = 0$ ("soft gluon pole" from initial state interaction), while there is no contribution from so-called hard gluon poles or soft fermion poles. We extract the imaginary part of the pole by using the formula

$$\mathrm{Im}\,\frac{1}{x_g \pm i\epsilon} = \mp\, i\pi\delta(x_g). \tag{13}$$

Collecting all the pieces we finally arrive at the following polarized differential cross section,

$$\begin{aligned}\frac{d\sigma(S_T)}{dQ^2 d\Omega} \;&=\; \frac{4\pi\alpha_{em}^2}{9Q^2}\sum_q e_q^2 \int dx\, dx' \Big(T_F^q(x,x)\, f_1^{\bar{q}}(x') + h_1^q(x)\, T_F^{(\sigma)\,\bar{q}}(x',x')\Big) \\ &\qquad \times \frac{1}{Q}\left[\frac{3}{32\pi}\,(-\sin 2\theta \sin\phi)\,\delta\Big(Q^2 - xx'S\Big)\right].\end{aligned} \tag{14}$$

This provides the asymmetry

$$A_N = -\frac{1}{2Q}\,\frac{\sin 2\theta \sin\phi_S}{1+\cos^2\theta}\,\frac{\sum_q e_q^2 \int dx\, \Big(T_F^q(x,x)\, f_1^{\bar{q}}(x') + h_1^q(x)\, T_F^{(\sigma)\,\bar{q}}(x',x')\Big)}{\sum_q e_q^2 \int dx\, f_1^q(x) f_1^{\bar{q}}(x')}\,, \tag{15}$$

which, as already stated above, is just half of the result (3) obtained in Refs. [14–16].

3.2 Asymmetry derived in the light-cone gauge

To test the color gauge invariance of our result, we derived the asymmetry also in the color light-cone gauge. In general, in the light-cone gauge both the first order k_T-expansion of the born diagram (see Fig. 2) and the diagrams with one additional exchange of a transversely polarized gluon (see Fig. 3) contribute to the spin dependent cross section at the twist-3 level. The associated twist-3 non-perturbative parts are the matrix elements for which the operators $\bar{\psi}\partial_T\psi$ and $\bar{\psi}A_T\psi$ are sandwiched between the hadron state [8]. Apparently, these two correlators are not QCD gauge invariant. However, if one entirely fixes the light-cone gauge, i.e., if one carries out the calculation using a specific boundary condition for the transverse gluon field at the light-cone infinity, then the two matrix elements can be uniquely related to the gauge invariant quark-gluon-quark correlators T_F and $T_F^{(\sigma)}$ [12, 29].

There exist three frequently used boundary conditions: the retarded boundary condition, the advanced boundary condition, and the anti-symmetric boundary condition. For the Drell-Yan process

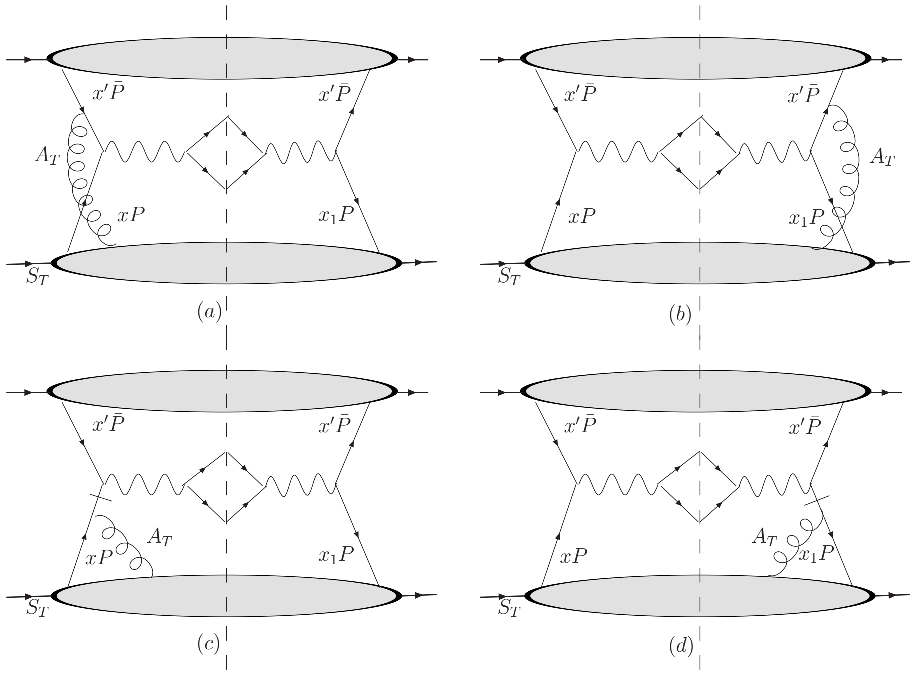

Figure 3: Feynman diagrams with one-gluon-exchange relevant for the calculation of A_N in the light-cone gauge.

the retarded boundary condition $A_T(-\infty^-) = 0$ is the most convenient choice [30]. Exploiting this particular boundary condition, the operators $\bar{\psi}\partial_T\psi$ and $\bar{\psi}A_T\psi$ can be readily rewritten in a gauge invariant form. For example, one has [14, 27]

$$\int \frac{dy^-}{4\pi}\, e^{ixP^+y^-} \langle PS|\bar{\psi}(0)\,\gamma^+\,\varepsilon_T^{\nu\mu}\,S_{T\nu}\,i\partial_{T\mu}\,\psi(y^-)|PS\rangle_{\rm ret} = T_F(x,x)\,, \tag{16}$$

as well as

$$\begin{aligned}&\int \frac{dy^- dy_1^-}{4\pi} P^+ e^{ixP^+y^-}\, e^{i(x-x_1)P^+y_1^-} \langle PS|\bar{\psi}(0)\,\gamma^+\,\varepsilon_T^{\nu\mu}\,S_{T\nu}\,gA_{T\mu}(y_1^-)\,\psi(y^-)|PS\rangle_{\rm ret} \\ &= \frac{i}{x-x_1+i\epsilon}\int \frac{dy^- dy_1^-}{4\pi}\, e^{ixP^+y^-}\, e^{i(x-x_1)P^+y_1^-} \langle PS|\bar{\psi}(0)\,\gamma^+\,\varepsilon_T^{\nu\mu}\,S_{T\nu}\,gF^+{}_\mu(y_1^-)\,\psi(y^-)|PS\rangle\,. \end{aligned}\tag{17}$$

One has to organize the contributions associated with $\bar{\psi}\partial_T\psi$ and $\bar{\psi}A_T\psi$ in a different way when using different boundary conditions [12, 29]. Though the final result is independent of the boundary condition, the calculation of the hard part associated with $\bar{\psi}A_T\psi$ in Drell-Yan is much more involved for both the advanced and the anti-symmetric boundary condition. For example, if one carries out the calculation in the advanced boundary condition, the above two equations should be replaced by [14, 27]

$$\int \frac{dy^-}{4\pi}\, e^{ixP^+y^-} \langle PS|\bar{\psi}(0)\,\gamma^+\,\varepsilon_T^{\nu\mu}\,S_{T\nu}\,i\partial_{T\mu}\,\psi(y^-)|PS\rangle_{\rm adv} = -T_F(x,x)\,, \tag{18}$$

$$\begin{aligned}&\int \frac{dy^- dy_1^-}{4\pi} P^+ e^{ixP^+y^-}\, e^{i(x-x_1)P^+y_1^-} \langle PS|\bar{\psi}(0)\,\gamma^+\,\varepsilon_T^{\nu\mu}\,S_{T\nu}\,gA_{T\mu}(y_1^-)\,\psi(y^-)|PS\rangle_{\rm adv} \\ &= \frac{i}{x-x_1-i\epsilon}\int \frac{dy^- dy_1^-}{4\pi}\, e^{ixP^+y^-}\, e^{i(x-x_1)P^+y_1^-} \langle PS|\bar{\psi}(0)\,\gamma^+\,\varepsilon_T^{\nu\mu}\,S_{T\nu}\,gF^+{}_\mu(y_1^-)\,\psi(y^-)|PS\rangle\,. \end{aligned}\tag{19}$$

One notices that the relation between the operator $\bar{\psi}\partial_T\psi$, the gluonic pole part of the operator $\bar{\psi}A_T\psi$ and the correlator $T_F(x,x)$ differs from that in the retarded boundary condition by a minus sign. Therefore, the individual contributions from $\bar{\psi}\partial_T\psi$ and $\bar{\psi}A_T\psi$ change, while the sum of these two contributions matches with the result in the retarded boundary condition. However, in the advanced boundary condition one has to compute the hard part associated with the operator $\bar{\psi}A_T\psi$ with extreme care, as the k_T dependence needs to be kept throughout the calculation in order to generate the correct gluonic pole structure. Corresponding arguments apply in the case of the anti-symmetric boundary condition. For a general discussion about these issues and some more technical details we refer the interested reader to Refs. [12, 29] and a forthcoming paper [31].

The calculation of the hard part associated $\bar{\psi}A_T\psi$ follows the standard twist-3 approach. There are a total of four diagrams contributing to this hard part, which are illustrated in the Fig. 3 (compare also Ref. [16]). The diagrams (c) and (d) represent the contribution from the so-called special fermion propagator introduced in Ref. [28]. Note that the special propagator actually provides the same contribution to the hard coefficients of both the operator $\bar{\psi}\partial_T\psi$ and the operator $\bar{\psi}A_T\psi$ [28]. Because of this, after relating these operators to T_F as explained above, these two contributions exactly cancel each other. This argument applies to both the retarded and the advanced boundary condition. In the case of the anti-symmetric boundary condition, the relation $\bar{\psi}\partial_T\psi = 0$ [27] and the absence of an imaginary part from the A_T contribution (because of the corresponding principal value prescription) give rise to a vanishing result for these two diagrams.

For the chiral-even contribution, the generalized factorization formula takes the form

$$
\begin{aligned}
\frac{d\sigma(S_T)}{dQ^2 d\Omega} \;\propto\; & \frac{\alpha_{em}^2}{12Q^2}\sum_q e_q^2 \int dx\, dx'\, T_F^q(x,x)\, f_1^{\bar{q}}(x')\, \varepsilon_T^{\rho\sigma} S_{T\rho} \\
& \times \Bigg[\frac{\partial}{\partial k_T^\sigma}\Big(H^{\mu\nu}_{Born}(xp+k_T, x'\bar{p})\, V^{CS}_{3,\mu\nu}\, \sin 2\theta \cos\phi \\
& \qquad + H^{\mu\nu}_{Born}(xp+k_T, x'\bar{p})\, V^{CS}_{8,\mu\nu}\, \sin 2\theta \sin\phi \Big)_{k_T=0} \\
& \quad + \frac{1}{\pi}\int dx_1\, \frac{i}{x-x_1+i\epsilon}\, H^{\mu\nu}_\sigma(xp, x_1 p, x'\bar{p})\, V^{CM}_{8,\mu\nu}\, \sin 2\theta \sin\phi_S \Bigg] ,
\end{aligned}
\tag{20}
$$

where, in the end, only the k_T-expansion of the hadronic tensor contracted with the tensor $V^{CS}_{3,\mu\nu}$ contributes to the asymmetry, while the corresponding expression associated with $V^{CS}_{8,\mu\nu}$ vanishes due to parity conservation. This point is exactly reversed in the case of the chiral-odd part related with $T_F^{(\sigma)}$. We point out that the third term on the r.h.s. of Eq. (20) has been taken into account in the previous calculations [14–16], while the other two terms are missing. The latter lead to just half the contribution of the third term, but comes with a relative minus sign. Note that the required imaginary part in the hard term coupled with the operator $\bar{\psi}A_T\psi$ can arise from the (artificial) pole $1/(x-x_1+i\epsilon)$, which is generated by partial integration in Eq. (17). Moreover, the diagrams with a special propagator contribute to the hard parts resulting from both the k_T-expansion and the gluon-exchange. However, these two contributions cancel each other. The perturbative calculation is rather straightforward. The final result for A_N of the calculation in the light-cone gauge exactly matches with the final result (15) we found in covariant gauge.

4 Summary

In summary, we recalculated the transverse single spin asymmetry A_N in the angular distribution of a Drell-Yan dilepton pair by using twist-3 collinear factorization. Compared to previous work on this topic, we payed particular attention to the k_T-dependence of the lepton tensor when making the collinear expansion. Our final result for A_N in Eq. (15) differs from all the previous results given in the literature. For instance, we find an asymmetry which is just half of what was obtained in Refs. [14–16]. As a side-remark we would like to point out that the k_T-dependence of the lepton tensor might also influence twist-4 ($1/Q^2$) corrections to the unpolarized cross section in Eq. (9). In view of this, we consider it worthwhile to carefully revisit the collinear twist-4 framework for the unpolarized Drell-Yan process [33].

We made various checks in order to gain further confidence in our calculation. First, we verified QCD gauge invariance by performing the calculation both in covariant gauge and in the light-cone gauge. Second, we tested the electromagnetic gauge invariance by recalculating the asymmetry in two specific QED light-cone gauges. Third, we computed the NLO real emission corrections in the leading-log approximation. In a certain sense, this calculation is more straightforward than the lowest order treatment, since the k_T-flow can go through the unobserved parton line. The outcome of this study fully supports our result for A_N presented in the present work. A complete NLO analysis will be presented elsewhere.

At this point we would like to briefly comment on the result obtained in [18]: It may be tempting to assume that the factor of 2 found in [18] (relative to the result in Refs. [14–16]), together with the factor $1/2$ in the present work, could perhaps be combined and ultimately lead back to the result of [14–16]. We consider such a scenario as very unlikely, also in view of the aforementioned NLO result for the real gluon emission. Moreover, as far as we understand, the authors of [18] merely recalculate the special propagator contribution treated in [16], but emphasize the need for a specific boundary condition for the gluon field. While particular boundary conditions may be convenient for doing the calculation, the final result should not depend on them. Why the result of [18] actually differs from [16] is therefore not really obvious to us.

It is important to notice that measuring the sign of A_N can be considered to be equally important as checking the predicted sign reversal of the Sivers effect in Drell-Yan [32]. In either case the physics of initial state gluon interactions would be tested.

The formalism developed in this paper can be extended in order to study similar observables which represent a correlation between the transverse spin and the relative transverse momentum of final state particles. For instance, transverse SSAs for dihadron production in semi-inclusive DIS can, in principle, be treated along the same lines. We plan to address this point in a future work.

Note added: After we finished the calculation a new paper appeared on the very same topic, where parton states are used for the target [34]. The authors find exact agreement with our result in Eq. (15). The treatment in [34] is restricted to the term in (15) which involves the chiral-even functions.

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