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Testing Einstein Gravity with Cosmic Growth and Expansion

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We test Einstein gravity using cosmological observations of both expansion and structure growth, including the latest data from supernovae (Union2.1), CMB (WMAP7), weak lensing (CFHTLS) and peculiar velocity of galaxies (WiggleZ). We fit modified gravity parameters of the generalized Poisson equations simultaneously with the effective equation of state for the background evolution, exploring the covariances and model dependence. The results show that general relativity is a good fit to the combined data. Using a Padé approximant form for the gravity deviations accurately captures the time and scale dependence for theories like $f(R)$ and DGP gravity, and weights high and low redshift probes fairly. For current observations, cosmic growth and expansion can be fit simultaneously with little degradation in accuracy, while removing the possibility of bias from holding one aspect fixed.

The acceleration of the cosmic expansion was first discovered using the supernovae Type Ia (SNIa) measurements in 1998 [1], and later confirmed by various independent cosmological probes including Cosmic Microwave Background (CMB) [2], Large Scale Structure (LSS) [3], Integrated Sachs-Wolfe (ISW) [4], and so forth. A crucial question is what is the physical origin of the acceleration.

Within the framework of general relativity (GR), a component in the energy budget with substantially negative pressure, dubbed Dark Energy (DE), is needed to drive the acceleration of the universe. The pressure to density ratio, or equation of state parameter, $w(z)$ is an important characteristic to defining the physics responsible; for example, quintessence behavior ($w > -1$) [5], phantom ($w < -1$) [6], and quintom (where w crosses -1 during evolution) [7] properties would each be an important clue.

Another approach to obtain the acceleration is to modify the gravity theory. In this scenario, no dark energy component exists, but the laws of gravity relating spacetime curvature to the material contents are changed in such a way as to drive acceleration; the modified terms can be viewed as an effective dark energy contribution with some effective equation of state.

In terms of only the background expansion, these two approaches are indistinguishable. As well, the growth of structure is suppressed by the acceleration and is further affected by modifying the gravitational laws. These can be offset in such a way that a modified gravity (MG) model and some dark energy (DE) model have the same growth evolution. However, the key point is that they will not in general simultaneously have the same expansion *and* same growth behavior.

This highlights the importance of simultaneously fitting the data both for possible gravitational modifications and the expansion history. However, while much effort in the literature has gone into comparing MG models with the observational data, in almost all cases a ΛCDM background is assumed. This can lead to bias in the gravity parameters derived (if the true cosmology does not have the ΛCDM expansion) by a statistically significant amount, even if $\langle w \rangle = -1$ [8]. Moreover, this can cause further errors since w can be strongly correlated with other cosmological parameters such as neutrino mass, spatial curvature, the tensor perturbations, etc. [9–11].

In this paper, we aim to test GR using the latest observations, including SNIa, CMB, weak lensing (WL), and the peculiar velocity field of galaxies (PV), while specifically fitting the background cosmology simultaneously with the MG parameters. While simultaneous fitting has been considered before, for future data projections, e.g. [12], we use actual data, employ a more comprehensive parametrization of gravity modifications, and study the covariance between MG parameters and the background equation of state in more detail.

In Newtonian gauge, the metric in a perturbed FRW universe reads,

$$ds^2 = -(1+2\Psi)dt^2 + (1-2\Phi)a^2\delta_{ij}dx^i dx^j \,, \quad (1)$$

where Φ and Ψ denote the space curvature perturbation and the gravitational potential respectively, and they are related to the comoving matter density perturbation Δ via [13, 14]

$$k^2\Psi = -4\pi G a^2 \mu(k,a)\rho\Delta, \quad (2)$$

$$k^2(\Phi+\Psi) = -8\pi G a^2 \Sigma(k,a)\rho\Delta, \quad (3)$$

where G is Newton's gravitational constant, ρ is the homogeneous matter density, k is the wavenumber, and a is the expansion scale factor.

The functions μ and Σ are both unity in GR, but in general they can be functions of both scale and time in modified gravity. The quantity μ in the Poisson equation, Eq. (2), determines the modified growth with respect to that in GR, which can be measured using the peculiar velocities of galaxies (or their density field, but this involves a factor of the generally unknown bias factor relating mass to galaxy light). The quantity Σ can be constrained by the weak lensing measurement since it is directly related to the lensing potential $\Phi+\Psi$. Indeed one can view the first equation as governing motion along geodesics of nonrelativistic tracers while the second involves the null geodesics of light. Therefore, the PV and WL measurements are highly complementary to probe for the deviation from GR encoded in the functions μ and Σ [13]. Note that in [15] the variable μ is called $\mathcal{V}$ and $\Sigma=\mathcal{G}$, and a translation table is provided there for other functions in the literature.

One still needs to parametrize the two functions of wavenumber and scale factor. There are many possibilities, including principal component analysis [16], bins [15], etc. In this paper we propose a parametrisation for μ and Σ that covers a wide range of modified gravity models known so far. This parametrisation is based on Brans-Dicke, or more generally scalar-tensor, gravity. Using the quasi-static approximations, the solutions for μ and Σ are obtained as

$$\mu = \frac{G(a)}{G_N}\left(1+\frac{1}{3\frac{a^2M(a)^2}{k^2}+2\omega_{BD}(a)+3}\right), \quad (4)$$

$$\Sigma = \frac{G(a)}{G_N}, \quad (5)$$

where G is the gravitational coupling generalizing Newton's constant G_N measured locally by the Cavendish-type experiments, M is the mass of the scalar field and ω_{BD} is the Brans-Dicke (BD) parameter. Under the quasi-static approximations, G, M and ω_{BD} can weakly depend on time, i.e. $\dot{G}/G, \dot{M}/M, \dot{\omega}_{BD}/\omega_{BD} \ll k/a$. There are two possibilities to recover GR, i.e. $\mu\to 1$, in this Ansatz. One is to have the Compton wavelength of the scalar field smaller than the scales of interest, $k/(aM)\ll 1$, and the other is to consider a large BD parameter $\omega_{BD}\gg 1$.

Since the time variation of the generalized Newton's constant is strongly constrained, we take $G(a)=G_N$ and consider the following two cases for simplicity: (i) the Brans-Dicke parameter vanishes $\omega_{BD}=0$, or (ii) the scalar field is massless $M=0$. Essentially we are considering the sources of modification one by one, and in these cases μ and Σ can be parametrised in a very simple form:

$$\mu = 1+\frac{ca^sk_H^n}{1+3ca^sk_H^n}\,, \quad \Sigma=1\,. \quad (6)$$

Here, in case (i) we have $c=(3M_0^2/H_0^2)^{-1}$, $n=2$, and k_H denotes the dimensionless wavenumber, namely, $k_H\equiv k/H_0$ where k and H_0 are the usual wavenumber and the Hubble constant, respectively; in case (ii) we have $c=1/(2\omega_{BD,0})$ and $n=0$; thus c quantifies the dimensionless amplitude of deviations from GR, with $c\ll 1$ recovering GR, and has the physical interpretation in terms of either the ratio today of the scalar Compton wavelength to the Hubble scale, or the BD parameter. The time variation of either $(aM)^2$ or ω_{BD} is approximated as a power law a^{-s} (so GR is recovered in the early universe for $s>0$), and the spatial variation is set to either k^2 or scale independence. These are motivated by known theories of modified gravity as we discuss below.

Case (i) includes $f(R)$ gravity. For $f(R)$ theory, the deviations depend on the scalaron mass, $M(a)=1/\sqrt{3d^2f/dR^2}$, which defines a Compton length over which the deviations propagate. In general, in the early universe or in high curvature regions, $M\gg H$ and the equations reduce to general relativity. The parametrization $M(a)=M_0a^{-\sigma}$ has been shown to be accurate over the past evolution by [17–19]. The deviation of the MG parameter $\mu(k,a)$ from unity within $f(R)$ theory is [17]

$$\mu(k,a)=1+\frac{1}{3+3(aM/k)^2}\,. \quad (7)$$

Thus within this ansatz for $f(R)$, this leads to Eq. (6) with $s=2(\sigma-1)$.

Case (ii) includes DGP gravity [20]. This has no scale dependence (on cosmological scales, much greater than the Vainshtein scale). For DGP gravity the deviation is given by

$$\mu(k,a)=1-\frac{1}{3}\,\frac{1-\Omega_m^2(a)}{1+\Omega_m^2(a)}\,, \quad (8)$$

where $\Omega_m(a)=\Omega_m a^{-3}/(H/H_0)^2$ is the dimensionless matter density as a function of scale factor, with Ω_m the value today. This corresponds to $2\omega_{BD}(a)=-6/[1-\Omega_m^2(a)]$. Note that in the matter dominated epoch when $\Omega_m(a)\to 1$ then GR is recovered, while in the future when the matter density redshifts away then $\omega_{BD}\to -3$. In general ω_{BD} does not behave as a power law in scale factor: at high redshift $s=3/2$, then it steepens, before evolving toward $s=0$ in the asymptotic future. However, a reasonable fit to the function μ can be achieved by the Padé approximant form of Eq. (6).

For the background expansion, we include an effective dark energy equation of state (EOS) $w(a)=w_0+w_a(1-a)$ known to be highly accurate in describing the background expansion for a wide variety of models [21, 22].

Our full parameter set to be constrained using the current observational data consists of the MG parameters c, s, the expansion parameters w_0, w_a and the other cosmological parameters. Specifically, we parametrise the universe using

$$\mathbf{P}\equiv(\omega_b,\omega_c,\Theta_s,\tau,n_s,A_s,c,s,w_0,w_a,\mathcal{N}), \quad (9)$$

where $\omega_b \equiv \Omega_b h^2$ and $\omega_c \equiv \Omega_c h^2$ are respectively the physical baryon and cold dark matter densities relative to the critical density, Θ_s is the ratio (multiplied by 100) of the sound horizon to the angular diameter distance at decoupling, τ denotes the optical depth to re-ionization, and n_s and A_s are the primordial density power spectral index and amplitude respectively. We also vary, and marginalize over, several astrophysical nuisance parameters denoted by $\mathcal{N}$ when performing our likelihood analysis, including those associated with the galaxy distribution for WL data and the absolute luminosity for supernovae.

The datasets used are of weak lensing (the two point correlation function ξ_E at $\theta > 30'$ from the CFHTLS-Wide survey [23, 24], the same dataset used in Ref. [13]), peculiar velocities ($f\sigma_8$ in four redshift bins in the range of $z \in [0.1, 0.9]$ measured by the WiggleZ team using the redshift space distortion measurement [25])[1], supernovae distances from the Union2.1 compilation including systematic errors [26], the full CMB spectra of WMAP seven year data [2], and baryon acoustic oscillation distance ratios from SDSS DR7 galaxies [3]. We do not directly employ the galaxy density power spectrum so as to avoid uncertainties in galaxy bias, which in principle could give time- and scale-dependent signatures similar to modified gravity.

Given the set of cosmological parameters $\mathbf{P}$ in Eq. (9), we calculate the theoretically expected observables (CMB spectra, luminosity distance, velocity growth factor Θ, and the E-mode component ξ_E of the weak lensing shear) using `MGCAMB` [17]. We then constrain the model parameters using a version of the Markov Chain Monte Carlo (MCMC) package `CosmoMC` [27, 28] modified to include our extra parameters. We impose priors on c, s of $c \geq -1/3$ for $n = 0$ (to avoid the pole in μ, corresponding to excluding $-3/2 < \omega_{BD,0} < 0$), and $c \geq 0$ for $n = 2$, and $s \in [1, 4]$.

The results are summarised in Table I. We run three different types of models: a scale independent case ("DGP"), a scale dependent k^2 case ("$f(R)$"), and a true dark energy case fixing $c = 0$ and including dark energy perturbations in the calculations [29]. The time dependence parameter s cannot be constrained by the data and is marginalized over.

Figure 1 shows the 1D probability distribution functions (PDF) for c. Recall that $c = 0$ is GR, and we see that all cases are consistent with GR. For each case we either also fit s or fix it to $s = 1$ for the scale independent case (mimicking DGP) or to $s = 4$ for the k^2 case (mimicking a particular $f(R)$). Note that when we marginalize over s this can actually tighten the constraints on c because small values of s are then permitted (recall deviations depend on a^s), which strengthens deviations at higher redshifts.

	scale indep. ($n = 0$)		scale dep. ($n = 2$)		GR: $\mu = 1$
	$w = -1$	w_0, w_a float	$w = -1$	w_0, w_a float	w_0, w_a float
c	< 4.0	< 4.1	< 0.002	< 0.002	0
w_0	-1	-0.90 ± 0.19	-1	-0.92 ± 0.20	-0.91 ± 0.19
w_a	0	-0.26 ± 0.78	0	-0.32 ± 0.82	-0.27 ± 0.78

TABLE I: Constraints from current data on the MG parameter c (marginalized over s) and the effective dark energy parameters w_0, w_a for the scale independent, scale dependent MG models and the true dark energy model ($\mu = 1$). For c we quote the 95% CL upper limit, while for w_0 and w_a, we quote the median and 68% CL error. Fitting for the expansion in terms of w_0, w_a rather than fixing $w = -1$ does not degrade the gravity constraint; fitting for gravity in terms of c, s does not degrade the expansion constraint.

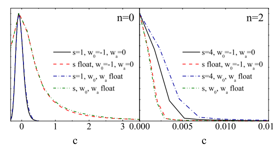

FIG. 1: 1D PDF for the MG amplitude c from the latest observational data. The left panel shows the scale independent case, with s either fixed to 1 or marginalized over, and the background either fixed to ΛCDM or marginalized over w_0, w_a. The right panel shows the analogous curves for the scale dependent (k^2) case. All PDFs are consistent with $c = 0$, corresponding to GR.

Figure 2 contains the 2D joint probability contours for c–s for scale dependent and independent cases. The filled contours represent when the background expansion is fixed to ΛCDM; we see that this does not have a dramatic effect on the results, implying that there is little covariance between the gravity and expansion parameters and that simultaneous fitting is not only desirable but practical.

In Fig 3, we show the reconstructed (effective) $w(z)$ using the constraints on the expansion parameters for both MG models and for the true dark energy case, with c and s marginalized over where appropriate. The consistency of the contours demonstrates that simultaneous fitting of gravity and expansion does not here substantially degrade constraints. Recall that the simultaneous fitting enables avoidance of a possible significant bias if there is any deviation from ΛCDM with GR. Current data is consistent with ΛCDM cosmology even in the presence of possible gravitational modifications.

The lack of covariance between the gravity and expansion parameters is not a general property true for all large scale structure observations; for example [12] found high correlation between the gravitational growth index

[1] We did not use peculiar velocity data when constraining the $n = 2$ models since scale independent growth had already been assumed in extracting the data.

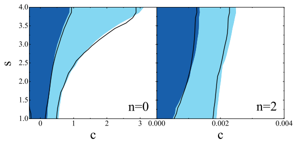

FIG. 2: 68% and 95% CL joint constraints on the MG parameters. Filled and unfilled contours represent the cases when the background cosmology is fixed to ΛCDM, and the effective dark energy equation of state parameters w_0, w_a are allowed to vary, respectively. GR corresponds to $c = 0$, when the value of s is moot. The left (right) panel corresponds to the scale independent (dependent, k^2) case.

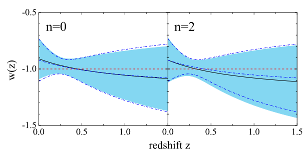

FIG. 3: The reconstructed $w(z)$ with 68% CL error are shown allowing for modified gravity (marginalized over c, s) in the scale independent (left panel) and scale-dependent k^2 (right panel) cases by the filled bands. The reconstruction for true dark energy, with gravity fixed to GR, is shown by the dash-dotted curves, the same in each panel.

γ [30] (closely related to μ) and w_0, w_a when using the galaxy density power spectrum for future data. This is because the density power spectrum involves the integrated growth factor, influenced by both expansion and modified gravity, while the PV field involves the growth rate currently at modest accuracy and current weak lensing probes mostly light deflection. With future data, however, weak lensing will be more sensitive to growth, and galaxy data will probe both growth and growth rate, so we expect that as the constraints tighten they will also become more correlated. Of course both the EOS and MG parameters have covariance with the matter density Ω_m, thus in principle they are correlated indirectly. But for our current datasets the correlation is seen to be very small.

Indeed the constraints on $w(z) = w_0 + w_a(1 - a)$ are nearly independent of the MG parameters. A similar behavior was shown in Fig. 2 of [31] where the joint confidence contours of w_0–w_a were nearly independent of the value of the modified growth index γ. The $w(z)$ behavior reconstructed from our best fit shows the usual "quintom" [7] crossing of -1 at $z \sim 0.4$. Such a pivot point is expected from the strong influence of CMB constraints on the distance to last scattering agreeing with ΛCDM; this induces the "mirage of Λ" [32] where $w_p \equiv w(z \approx 0.4) \approx -1$ even in the presence of time variation in EOS, and is not a consequence of using the w_0, w_a form. As other data gain in leverage relative to this CMB geometric constraint, the crossing may disappear. On the other hand, the effective EOS in modified gravity models can cross -1, so this quintom behaviour, if confirmed by future data to high confidence level, might be a smoking gun of modified gravity.

In summary, to test gravity in a stringent manner, we parametrize the consistent set of gravity field equations through modifying factors in the matter growth (Poisson) equation and light deflection (sum of the metric potentials) equation. The Padé approximant form adopted can cover many different modified gravity theories with scalar degrees of freedom. Note that a simple power law such as $\mu = 1 + \mu_s a^s$ cannot properly weight both high and low redshifts and may bias the results. Simultaneously with fitting for these modifications we also allow the background expansion to deviate from a ΛCDM cosmology through an effective time varying dark energy equation of state. This is important as incorrectly fixing either the gravity side or the expansion side could strongly bias the conclusions.

We then used the most recent observational data – supernova distance data (Union2.1 compilation), CMB (full WMAP-7yr spectra), weak lensing (CFHTLS), and galaxy peculiar velocity (WiggleZ) – to fit these and other cosmological parameters, testing Einstein gravity. The simultaneous fitting of gravity and expansion can be successfully carried out, with little degradation in leverage while avoiding possible bias due to fixing one or the other. In the scale dependent case, the deviation amplitude is constrained to be $c \lesssim 0.002$ corresponding to the constraint on the Compton wavelength $\lambda \lesssim 250\,h^{-1}$Mpc (95% CL), while in the scale independent case cosmological data is not yet precise enough to place strong bounds ($c \lesssim 4.1$ implies $\omega_{BD,0} \gtrsim 0.12$).

General relativity is a good fit with these recent data. Future data will allow more stringent limits, and as growth measurements improve the covariance between gravity and expansion influences should increase, making simultaneous fitting even more necessary. The function Σ entering the light deflection equation will be tested as upcoming large weak lensing surveys deliver data. Next generation data should greatly advance our ability to test gravity and uncover the physical origin of the acceleration of our universe.

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