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Testing General Relativity Using the Evolution of Linear Bias

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We investigate the cosmic evolution of the linear bias in the framework of a flat FLRW spacetime. We consider metric perturbations in the Newtonian gauge, including Hubble scale effects. Making the following assumptions, (i) scale independent current epoch bias b_0, (ii) equal accelerations between tracers and matter, (iii) unimportant halo merging effects (which is quite accurate for $z < 3$), we analytically derive the scale dependent bias evolution. The identified scale dependence is only due to Hubble scale evolution GR effects, while other scale dependence contributions are ignored. We find that up to galaxy cluster scales the fluctuations of the metric do not introduce a significant scale dependence in the linear bias. Our bias evolution model is then used to derive a connection between the matter growth index γ and the observable value of the tracer power spectrum normalization $\sigma_8(z)$. We show how this connection can be used as an observational test of General Relativity on extragalactic scales.

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1. INTRODUCTION

The distribution of matter on large scales, based on different extragalactic objects, can provide important constraints on models of cosmic structure formation. However, a serious problem that hampers such a straightforward approach is our limited knowledge of how luminous matter traces the underlying mass distribution. In particular, the concept of the so called *biasing* between different classes of extragalactic objects and the background matter distribution was put forward by Kaiser [1] and Bardeen et al. [2] in order to explain the higher amplitude of the 2-point correlation function of clusters of galaxies with respect to that of galaxies themselves.

Such a biasing is statistical in nature; with galaxies and clusters being identified as high peaks of an underlying, initially Gaussian, random density field, while it is assumed to be linear and scale-independent[1]. Formally, the linear bias factor, b, is defined as the ratio of the extragalactic mass tracer fluctuations, $\delta_{\rm tr}$, to those of the underlying mass, δ_m:

$$\delta_{\rm tr} = b\delta_m \ . \tag{1.1}$$

Since the two-point correlation function in a continuous density field is defined as $\xi(r) = \langle\delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{r})\rangle$, one can write the bias factor as the square root of the ratio of the two-point correlation function of the tracers to the underlying mass:

$$b = \left(\frac{\xi_{\rm tr}}{\xi_m}\right)^{1/2} \ . \tag{1.2}$$

in which case one considers the large-scale correlation function, ie., scales corresponding roughly to the so-called halo-halo term of the dark matter (hereafter DM) halo correlation function (see for example [4]). Furthermore, since the variance of a density field, smoothed

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[1] However, in the non-linear scales of clustering, ie., mostly below 5 h^{-1} Mpc, a scale dependence of the bias has been observed (see for example [3]).

at some scale R, is the correlation function at zero lag ($\sigma_R^2 = \xi_R(0) = \langle \delta_R^2(\mathbf{x}) \rangle$) one can also write the linear bias factor as the ratio of the variances of the tracer and underlying mass density fields, smoothed at some linear scale, traditionally taken to be 8 h^{-1} Mpc (at which scale the variance is of order unity):

$$b = \frac{\sigma_{8,\mathrm{tr}}}{\sigma_{8,m}} \,. \tag{1.3}$$

We refer the reader to a thorough discussion of the methods to estimate the tracer correlation function and also of the corresponding determination of the tracer bias values, at different redshifts, that appears in Papageorgiou et al. ([5]; and references therein).

The bias factor may have many dependencies; even assuming that it is scale independent, it necessarily depends on the type of the mass tracer as well as on the epoch z, since the fluctuations evolve with time as gravity draws together galaxies and mass. It is evident, therefore, that the bias factor should also depend on the different dark energy models (hereafter DE), including those of modified gravity [7]. It is the redshift evolution of bias, $b(z)$, which is very important in order to relate observations with models of structure formation and it has been shown to be a monotonically increasing function of redshift [8]-[21].

In the literature there are two basic families of analytic bias evolution models. The first, called the *galaxy merging* bias model, is based on the Press-Schechter [8] formalism, on the peak-background split [2] and on the spherical collapse model [9], and reproduces relatively well the results of numerical simulations, although differences have been found especially at the high and low DM halo mass range. These differences have lead to modifications of the original model to include the effects of ellipsoidal collapse [10]; to new values of the bias model parameters [11]; to new forms of the bias model fitting function [12] or even to a non-Markovian extension of the excursion set theory [13].

The second family of bias evolution models assumes a continuous mass-tracer fluctuation field, proportional to that of the underlying mass, and the tracers act as "test particles". In this context, the hydrodynamic equations of motion and linear perturbation theory are used. An original suggestion, named *galaxy conserving* bias model used the continuity equation and the assumption that tracers and underlying mass share the same velocity field [14–17], while the bias evolution is provided by the solution of a 1st order differential equation as: $b(z) = 1 + (b_0 - 1)/D(z)$, with b_0 the bias factor at the present time and $D(z)$ the growing mode of density perturbations. However, this bias model suffers from two fundamental problems: *the unbiased problem* ie., the fact that an unbiased set of tracers at the current epoch remains always unbiased in the past, and *the low redshift problem* ie., the fact that this model represents correctly the bias evolution only at relatively low redshifts $z \lesssim 0.5$ [18]. Note that [19] has extended this model to also include an evolving mass tracer population in a ΛCDM cosmology.

An attempt to derive a bias evolution model free of the above mentioned problems, utilized all three hydrodynamical equations of motion, linear perturbation theory and the fact that mass-tracers and underlying mass share the same gravity field, but not necessarily the same velocity field and that the linear bias is scale independent. This resulted in a second order differential equation in b, the solution of which provided the evolution of the linear and scale-independent bias (see [6, 20] and [21]). We would like to stress here that the provided solutions apply to cosmological models, within the framework of general relativity.

In the context of a scale-independent bias factor, an extension of the previous model, valid for all DE and modified gravity cosmologies, was recently proposed by Basilakos, Plionis & Pouri [7]. This extension provides a tool, using the evolution of bias, to put constraints on those DE models which adhere to general relativity, as well as to investigate whether the DE reflects on the nature of gravity ("geometrical dark energy").

Overall, the scope of the present article is (a) to extend the original Basilakos et al. [7] bias solution, by taking into account possible contributions from the metric fluctuations, and (b) to propose new tools that can be used in order to test the validity of general relativity on cosmological scales.

The structure of our paper is as follows. The basic theoretical elements of the problem are presented in section II, where we introduce [for a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) geometry] the basic cosmological equations. The issue related with the linear bias is discussed in section III. In this section we also present the general bias solution in the framework, by taking into account metric perturbations for the Newtonian gauge. Finally, the main conclusions are summarized in section V.

2. SCALE DEPENDENT MATTER AND TRACER DENSITY PERTURBATIONS

Let us derive the basic equations that govern the evolution of the mass density contrast as well as of the extragalactic tracers, modeled here as a presurreless fluid ($p_m = p_{\mathrm{tr}} = 0$). Note that the perturbed FLRW spacetime in the Newtonian gauge is given by

$$ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Phi)a^2(t)d\mathbf{x}^2 \tag{2.1}$$

where Φ is the Newtonian potential, $a(t)$ is the scale factor (normalized to unity at the present time) and $d\mathbf{x}^2$ is the flat spatial metric. In the current paper we assume a slowly varying gravitational potential Φ. Note that considering the unperturbed spacetime one can easily derive the background equations

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_{de}) \ , \tag{2.2}$$

$$\dot{\rho} + 3H(\rho + p_{de}) = 0 \ . \tag{2.3}$$

In the above set of differential equations, an over-dot denotes derivative with respect to time, ρ_m and ρ_{de}, are the matter and dark energy densities with $\rho = \rho_m + \rho_{de}$, $H = \dot{a}/a$ is the Hubble parameter, whereas $p_{de} = w_{de}\rho_{de}$, corresponds to the pressure assuming non-clustering dark energy. Note that for $w_{de}(z) = -1$ we recover the concordance ΛCDM model.

A. Matter density perturbations

In this section, we discuss the basic equation which governs the evolution of the matter perturbations up to horizon scales and within the framework of any DE model. Following the notations of Dent et al. [22] the perturbed (anisotropic stress-free) equations in the Newtonian gauge take the form

$$\ddot{\Phi} = -4H\dot{\Phi} + 8\pi G\rho_{de}w_{de}\Phi \tag{2.4}$$

$$\dot{\delta}_m = 3\dot{\Phi} + \frac{k^2}{a^2}v_{f,m} \tag{2.5}$$

$$\dot{v}_{f,m} = -\Phi \tag{2.6}$$

with constraint equations

$$3H(H\Phi + \dot{\Phi}) + \frac{k^2}{a^2}\Phi = -4\pi G\rho_m\delta_m \tag{2.7}$$

$$(H\Phi + \dot{\Phi}) = -4\pi G\rho_m v_{f,m} \tag{2.8}$$

where $v_{f,m} \equiv -v_m a$ (v_m is the velocity potential for matter). In this context, the combination of the relativistic equations (2.4)-(2.8) obtains the basic differential equation

$$\ddot{\delta}_m + 2H\dot{\delta}_m + \frac{k^2}{a^2}\Phi = 0 \ . \tag{2.9}$$

A solution of the above equation provides the evolution of the matter fluctuations in the linear regime.

On the other hand, the linear matter overdensity $\delta_m \equiv \delta\rho_m/\rho_m$ is written as a function of the gravitational potential Φ and the background variables as follows [23]:

$$-4\pi G\rho_m\delta_m = \frac{k^2}{a^2}\Phi + 3H^2\Phi + 3H\dot{\Phi} \tag{2.10}$$

where G denoting Newton's gravitational constant. In the sub-Hubble (small scale) approximation ($\frac{k^2}{a^2} \gg H^2$) equation (2.10) takes the form

$$-4\pi G\rho_m\delta_m = \frac{k^2}{a^2}\Phi \tag{2.11}$$

which is the usual Poisson equation. In this context, inserting the Poisson equation (2.11) into eq.(2.9) we derive the well known scale independent equation

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\rho_m\delta_m = 0 \ . \tag{2.12}$$

Now, for any type of dark energy an efficient parametrization of the matter perturbations is based on the growth rate of clustering [24]

$$f_0(a) = \frac{d\ln\delta_m(a)}{d\ln a} = \Omega_m^{\gamma}(a) \tag{2.13}$$

where γ is the so called growth index (see Refs. [25–29]) and $\Omega_m(a) = \Omega_m a^{-3}/E^2(a)$.[2] Integrating eq.(2.13) we obtain an approximate solution of eq.(2.12) which is valid for any type of dark energy[3]:

$$\delta_m(a) = a\exp\left[\int_{a_i}^{a}\frac{dx}{x}\ (\Omega_m^{\gamma}(x) - 1)\right] \tag{2.14}$$

where a_i is the scale factor of the universe at which the matter component dominates the cosmic fluid (here we use $a_i \simeq 10^{-2}$). Following standard lines we have $\delta_m(a) \propto D(a)$, where $D(a)$ is the linear growing mode, usually scaled to unity at the present epoch $D(a) = \delta_m(a)/\delta_m(1)$. It is interesting to mention that measuring the growth index could provide an efficient way to discriminate between modified gravity models and DE models which adhere to general relativity. Indeed it was theoretically shown that for DE models inside general relativity the growth index γ is well fit by $\gamma_{\rm GR} \approx 6/11$ (see [28, 29]).

For the benefit of the reader we point here that for the traditional Λ cosmology it has been found, by some of us [22, 35], that the linear matter fluctuation field starts to become scale-dependent, due to metric perturbations in Newtonian gauge, on scales larger than about $\sim 50 - 100h^{-1}$Mpc ($k < 0.01 - 0.02h$Mpc^{-1}). Therefore, for large scales we have to use the generalized Poisson equation (2.10) which is valid up to horizon scales. Notice that on dimensional grounds we may approximate the quantity $3H\dot{\Phi}$ in Eq. (2.10) as $3H\dot{\Phi} \simeq 3H^2\Phi$ (see also [22]). This is justified on the basis of Eq. 2.4 since [given also that $4\pi G\rho_{de} = O(H^2)$] the only timescale that determines the evolution of Φ is the Hubble scale H. It is therefore a good approximation to assume that $\dot{\Phi} \simeq H\Phi$. Using the latter condition and inserting Eq.(2.10) into Eq.(2.9) one can easily find that

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{eff}\rho_m\delta_m = 0 \tag{2.15}$$

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[2] Ω_m is the density parameter at the present time and $E(a) = H(a)/H_0$ is the normalized Hubble function. For the usual Λ cosmology we have $E(a) = (\Omega_m a^{-3} + 1 - \Omega_m)^{1/2}$.

[3] Since the pure matter universe (Einstein de-Sitter) has the solution of $\delta_{ES,m} = a$, we normalize our DE models to get $\delta_m \simeq a$ at large redshifts, which should hold due to the dominance of the non-relativistic matter component.

where

$$G_{eff}(a,k) = \frac{G}{1+\xi_k(a,k)} \tag{2.16}$$

and

$$\xi_k(a,k) = \frac{3a^2H^2(a)}{c^2k^2} \; . \tag{2.17}$$

For many DE models, it is convenient to study the growth evolution in terms of the expansion scale a rather than t. If we change the variables from t to a ($\frac{d}{dt} = aH\frac{d}{da}$) then the time evolution of the mass density contrast (see Eq. (2.15)) takes the following form

$$\frac{d^2\delta_m}{da^2} + A(a)\frac{d\delta_m}{da} - B(a,k)\delta_m = 0 \tag{2.18}$$

where

$$A(a) = \frac{d\ln E}{da} + \frac{3}{a} \tag{2.19}$$

and

$$B(a,k) = \frac{3\Omega_m}{2a^5E^2(a)} \; [(1+\xi_k(a,k)]^{-1} \; . \tag{2.20}$$

We would like to end this section with a discussion on the evolution of the scale dependent growth rate of clustering. Obviously, it becomes important to construct a scale-dependent parametrization that is analogous to Eq.(2.13) and solves (approximately) Eq.(2.18) for all scales k. In order to construct such a parametrization we focus on the matter dominated era when most of the growth occurs and express $\xi_k(a,k)$ as [22]

$$\xi_k(a,k) = \frac{3H_0^2\Omega_m}{ac^2k^2} \; . \tag{2.21}$$

In this context, Dent et al. [22] proposed that the scale-dependent growth rate $f(a,k)$ may be expressed in terms of the scale-independent growth rate $f_0(a)$ in the form:

$$f(a,k) = \frac{d\ln\delta_m(a,k)}{d\ln a} = \frac{f_0(a)}{1+\xi_k(a,k)} = \frac{\Omega_m^\gamma(a)}{1+\xi_k(a,k)} \; . \tag{2.22}$$

Thus from Eq.(2.22) we simply get

$$\delta_m(a,k) = a\exp\left[\int_{a_i}^{a} \frac{dx}{x}\left(\frac{\Omega_m^\gamma(x)-1}{1+\xi_k(x,k)}\right)\right] \; . \tag{2.23}$$

Due to $\delta_m(a,k) \propto D(a,k)$ the normalized growth factor becomes

$$D(a,k) = \frac{\delta_m(a,k)}{\delta_m(1,k)} = \frac{\delta_m(z,k)}{\delta_m(0,k)} \tag{2.24}$$

where $a = (1+z)^{-1}$.

B. Tracer density perturbations

Now we use the same formalism as before but for the tracers. We would like to spell out what are the basic assumptions here (see also [36]). First of all we consider that the mass tracer population is conserved with time i.e. that the effects of hydrodynamics (merging, feedback mechanisms etc) do not significantly alter the population mean. However, the effects of merging have been phenomenologically modeled and it has been found (using N-body simulations) that they are important only for $z \gtrsim 2.5-3$ [21] as far as the bias factor is concerned. In what follows we also treat tracers with $p_{\rm tr} = 0$ which implies that galaxies (or clusters of galaxies) are collisionless. Thus the tracer density evolves as

$$\dot{\rho}_{\rm tr} + 3H\rho_{\rm tr} = 0 \; . \tag{2.25}$$

Due to the same gravity field the corresponding equations (2.4) and (2.7) are also valid here. On the other hand we have

$$\begin{aligned} \dot{\delta}_{\rm tr} &= 3\dot{\Phi} + \frac{k^2}{a^2}v_{f,{\rm tr}} && (2.26)\\ \dot{v}_{f,{\rm tr}} &= -\Phi && (2.27)\\ (H\Phi + \dot{\Phi}) &= -4\pi G\rho_{\rm tr}v_{f,{\rm tr}} && (2.28) \end{aligned}$$

where $v_{f,{\rm tr}} \equiv -v_{\rm tr}a$ and $v_{\rm tr}$ is the velocity potential of the tracers (in general different with that of matter $v_{\rm tr} \neq v_m$), $\rho_{\rm tr}$ is the tracer density, ρ_m is the mass density and Φ is the gravitational potential.

Since the tracers and the underlying matter share the same gravitational field, this implies that the generalized Poisson equation (2.10) remains practically the same. In other words we could have different velocity fields ($v_{\rm tr} \neq v_m$) but the corresponding accelerations ($\dot{v}_{\rm tr} = \dot{v}$) are the same. Again, by taking the approximation $3H\dot{\Phi} \simeq 3H^2\Phi$ and using Eqs. (2.25), (2.4), (2.7) and Eqs.(2.26)-(2.28), we can obtain after some algebra the time evolution equation for the tracer fluctuation field

$$\ddot{\delta}_{\rm tr} + 2H\dot{\delta}_{\rm tr} - 4\pi G_{eff}\rho_m\delta_m = 0 \tag{2.29}$$

or

$$\frac{d^2\delta_{\rm tr}}{da^2} + A(a)\frac{d\delta_{\rm tr}}{da} - B(a,k)\delta_m = 0 \; . \tag{2.30}$$

C. Gauge Dependence

The Newtonian gauge used in the above calculations is physically interesting because it corresponds to a time slicing of isotropic expansion. However, the matter density perturbation $\delta_m(t,k)$ is a gauge dependent quantity and therefore it is important to clarify how our results change in alternative gauges, and what their connection is with observable gauge invariant quantities.

Scalar metric perturbations around a spatially flat background can be written in the following general form [34]

$$ds^2 = a^2\{(1+2\Phi)d\tau^2 - 2B_{|i}dx^i d\tau - [\delta_{ij} - 2(\Psi\delta_{ij} - E_{|_{ij}})]dx^i dx^j\ \}, \tag{2.31}$$

where a and τ are the conformal cosmic expansion scale factor and the conformal cosmic time; "$_|$" denotes the background three-dimensional covariant derivative. The corresponding perturbed energy-momentum tensor T^μ_ν has the form

$$\begin{aligned} T^0_{\ 0} &= \rho_m(1+\delta_m)\ , \\ T_0^{\ i} &= \rho_m U_{|i}\ , \\ T^0_{\ i} &= -\rho_m(U-B)_{|i}\ , \\ T^i_{\ j} &= -\rho_m \Sigma_{|ij}\ , \end{aligned} \tag{2.32}$$

where ρ_m is the unperturbed pressureless matter density; U and Σ determine velocity perturbation and anisotropic shear perturbation.

Gauge choices simplify the above expressions by setting various quantities to 0. For example we have $\Psi=\Phi=B=0$ in the Synchronous Gauge (SG), $B=E=0$ in the Newtonian Gauge (NG) and $U=B=0$ in the Comoving Time-orthogonal Gauge (CTG).

In the special case of the synchronous gauge, which corresponds to a time slicing obtained by the matter local rest frame everywhere in space (the free falling observer frame), the line element of the perturbed spacetime is given by

$$ds^2 = a^2(\tau)[-d\tau^2 + (\delta_{ij}+h_{ij})dx^i dx^j] \tag{2.33}$$

It is straightforward to derive the growth equation for $\delta_m \equiv \delta\rho_m/\rho_m$ in a matter dominated universe in the synchronous gauge to obtain [23, 35]

$$\ddot{\delta}_m^{SG} + 2H\dot{\delta}_m^{SG} - 4\pi G\rho_m\delta_m^{SG} = 0 \tag{2.34}$$

This growth equation is exact in the synchronous gauge in the case of matter domination and involves no scale dependence as in the case of equation (2.15) of the Newtonian gauge. This scale independence is an artifact of the particular time slicing of the synchronous gauge which is a good approximation on small scales but is unable to capture the horizon scale effects modifying the growth function on large scales.

Nevertheless equations (2.15) and (2.34) clearly agree on small scales where $\xi_k \to 0$. Therefore, for larger scales ($k < 0.01hMpc$) the question that arises is the following: *What is the proper gauge to use when comparing with observations?*

This question has been addressed in Ref. [30] where a gauge invariant observable replacement was obtained for δ_m. This observable $\delta_m^{obs}(t,k)$ involves the matter density perturbation $\delta_m(t,k)$ corrected for redshift distortions due to peculiar velocities and gravitational potential. It also includes volume and position corrections. The final expression however is complicated and makes the theoretical predictions based on it not easy to implement and manipulate. However, in Ref. [31] it was pointed out that the Newtonian gauge matter perturbation δ_m^{NG} is a good approximation to the observable gauge invariant perturbation $\delta_m^{obs}(t,k)$ even on very large scales (comparable to the Hubble scale). This result will also be justified in the remaining part of this section.

The cosmological perturbations evolution is well described by the gauge-invariant (GI) approach, pioneered by Bardeen[32]. This approach may be used to identify physical observables as gauge invariant quantities (e.g., Refs.[33, 34]). A gauge-invariant matter density perturbation may be constructed as [32, 33]

$$\delta_m^{GIS} \equiv \delta_m + 3\frac{\dot{a}}{a}(U-B)\ , \tag{2.35}$$

δ_m^{GIS} coincides with the density perturbation δ_m^{SG} in the Synchronous Gauge for the pressureless matter system. Thus δ_m^{GIS} corresponds to the density perturbation relative to the observers everywhere comoving with the matter. These *free falling* observers do not experience the isotropic expanding background of the universe because the peculiar velocity of matter is distinct from the Hubble flow. Thus δ_m^{SGI} has physical significance only for perturbations on scales small compared to the Hubble scale.

In addition to δ_m^{GIS}, it is straightforward to construct alternative gauge invariant quantities related to the matter overdensity as evaluated in different gauges. Such a gauge-invariant variable closely related to the matter overdensity in the Newtonian gauge is of the form [32–34],

$$\delta_m^{GIN} \equiv \delta_m + \frac{\dot{\rho}}{\rho}(B-\dot{E}) = \delta_m - 3\frac{\dot{a}}{a}(B-\dot{E})\ . \tag{2.36}$$

and has important advantages over δ_m^{GIS} discussed in what follows. δ_m^{GIN} coincides with the density perturbation δ_m^{NG} in the Newtonian Gauge (NG), in which $B=E=0$.

It is also straightforward to construct two gauge-invariant scalar potentials ϕ and ψ, which reduce to the gravitational potential in the Newtonian limit: [34]:

$$\begin{aligned} \phi &\equiv \Phi - \frac{\dot{a}}{a}(B-\dot{E})\ , \\ \psi &\equiv \Psi + \frac{1}{a}\frac{d}{d\tau}[(B-\dot{E})a]\ . \end{aligned} \tag{2.37}$$

The gauge invariant gravitational potential ϕ obeys the Poisson equation[32, 33] sourced by δ_m^{GIS} with no appearance of the Hubble scale:

$$\bigtriangledown^2\phi = -k^2\phi = 4\pi G\rho a^2\delta_m^{GIS}. \tag{2.38}$$

where k is the (comoving) wavenumber of the Fourier mode. The Poisson equation is valid only for scales small compared to the Hubble radius $1/H$ while on scales larger than the Hubble scale the growth of matter density perturbations is frozen. Hence, δ_m^{GIS} can not be regarded as the observable matter density perturbation on scales comparable to the Hubble scale. Therefore, the observable density perturbation on both the small-scale and the large-scale modes can not be described *directly* by δ_m^{GIS} even though it is a gauge-invariant quantity.

The other gauge invariant perturbation δ_m^{GIN} has some important attractive features with respect to observability, not shared by δ_m^{GIS}. These are summarized as follows:

- It reduces to the Newtonian gauge perturbation δ_m^{NG}, *i.e.* it corresponds to a frame which respects the isotropic expansion of the universe and is therefore more appropriate for description of large scale perturbations. This reduction also simplifies the calculation of this perturbation.

- It drives a scale dependent modification of the Poisson equation for the gauge invariant potential ϕ. Indeed, the time-time part of the linearized Einstein equation gives [23, 34]

$$\bigtriangledown^2\phi - 3\frac{\dot{a}}{a}(\frac{\dot{a}}{a}\psi + \dot{\phi}) = 4\pi G\rho a^2 \delta_m^{GIN}. \qquad (2.39)$$

  Thus, the anticipated scale dependence on scales comparable to the Hubble scale is picked up by the perturbation δ_m^{GIN}.

- It is gauge invariant as anticipated for any observable quantity.

Thus, the gauge invariant δ_m^{GIN} and the Newtonian gauge variable δ_m^{NG} to which it reduces, constitute an attractive choice for making theoretical calculations to obtain the gravitational potential and the matter density perturbation that can be directly compared with observations on large scales. However, these theoretically obtained quantities need to also be corrected for bias, redshift distortions (due to gravitational potential and peculiar velocities), lensing magnification and volume distortion[30].

3. THE EVOLUTION OF BIAS

Clearly, due to the metric perturbations, equations (2.15) and (2.29) involve a scale k dependence of bias in contrast to the small scale approximate equation (2.12) which is scale-invariant. Therefore, due to Eq.(1.1) one would expect that the bias factor must inherit a similar dependence to that of the density fluctuations namely $b = b(z,k)$. Within this framework, we can distinguish three possible bias evolution cases:

Case 1: Tracers and Mass share same velocity field: Here we use the assumption of Tegmark & Peebles [16] (see also [15]), that the tracers and the underlying mass distribution share the same velocity field. Using the latter and Eqs.(2.5), (2.26) we have

$$\dot{\delta}_{\rm tr} - \dot{\delta}_m = 0 \; . \qquad (3.1)$$

Now since we assume linear biasing, i.e. Eq.(1.1), we obtain:

$$\delta_m \frac{db}{dt} + (b-1)\frac{d\delta_m}{dt} = 0 \Rightarrow \frac{d(y\delta_m)}{dt} = 0 \qquad (3.2)$$

where $y = b - 1$ and $\delta_m \propto D$. An integration of Eq.(3.2) provides:

$$b(z,k) = 1 + y(z,k) = 1 + \frac{b_0 - 1}{D(z,k)} \qquad (3.3)$$

where b_0 is the bias factor at the present time.

This model is known to suffer from the so-called *unbiased* and the *low redshift* problems, by which the bias of mass tracers which either obey $b_0 < 1$ or are located at relative large redshifts, $z > 0.5$, cannot be modeled by Eq.(3.3)

Case 2: Tracers and Mass share same acceleration field: Now we consider that both the tracers and the underlying mass distribution share the same gravitational field but different velocity fields [6]. Inserting Eq.(1.1) into Eq.(2.15) and using simultaneously Eq.(2.29), we obtain a second order differential equation which describes the evolution of the linear bias factor, b, between the background matter and the mass-tracer fluctuation field:

$$\ddot{y}\delta_m + 2(\dot{\delta_m} + H\delta_m)\dot{y} + 4\pi G_{\rm eff}\rho_m\delta_m y = 0 \; , \qquad (3.4)$$

where $y = b - 1$. [4] Transforming equation (3.4) from t to a, we simply derive the evolution equation of the function $y(a,k)$ [where $y(a,k) = b(a,k) - 1$] which has some similarity with the form of eq.(2.18) as expected. Indeed, we have:

$$\frac{d^2y}{da^2} + \left[A(a) + \frac{2f(a,k)}{a}\right]\frac{dy}{da} + B(a,k)y = 0 \; . \qquad (3.5)$$

In Basilakos & Plionis [6, 20], we have provided an approximate solution of Eq.(3.4), using $f(z,k) = f_0(z) \sim 1$ (which is valid at relatively large redshifts), for cosmological models in the framework of general relativity, which contain quintessence (or phantom) dark energy. Here our aim is to provide an exact bias solution taking into account metric perturbations, namely $b(z,k)$, for all possible dark energy cosmologies (for the case with no metric perturbations see [7]).

[4] The current theoretical approach does not treat the possibility of having interactions in the dark sector. Also discussions beyond the linear biasing regime can be found in [37] (and references therein).

Inserting now $y(a,k) = g(a)/D(a,k)$ into Eq.(3.5) and using simultaneously equation (2.18) and the first equality of equation (2.22), we obtain:

$$\frac{d^2g}{da^2} + A(a)\frac{dg}{da} = 0 \tag{3.6}$$

a general solution of which is

$$g(a) = C_1 + C_2 \int \frac{da}{a^3 E(a)} \tag{3.7}$$

where C_1 and C_2 are the integration constants. Utilizing now $a = (1+z)^{-1}$, $b = y + 1 = (g/D) + 1$ and Eq.(3.7), we finally obtain the functional form which provides our general solution for all possible types of DE models, as:

$$b(z,k) = 1 + \frac{b_0 - 1}{D(z,k)} + C_2 \frac{J(z)}{D(z,k)} \tag{3.8}$$

where

$$J(z) = \int_0^z \frac{(1+x)dx}{E(x)} \; . \tag{3.9}$$

An extension of the above model to include the effects of halo merging processes, which introduces one further component in Eq.(3.8), has been phenomenologically modeled in [7, 21] and it was found, using cosmological N-body simulations, that such effects are important only for $z \gtrsim 2.5 - 3$. Therefore, in the light of currently available "growth of structure" data (which reach $z \sim 1$; *WiggleZ* [38]), the merging term in the bias evolution model has been neglected.

Notice that the dependence of our bias evolution model on the different cosmologies enters through the different behavior of $D(z,k)$, which is affected by γ (see equations 2.23, 2.24), and of $E(z) = H(z)/H_0$. Since different halo masses result in different values of b_0, one should expect that the constants of integration $C_1 = b_0 - 1$ and C_2 should be functions of the mass of dark matter halos (see [21]), assuming that the extragalactic mass tracers are hosted by a DM halo of a given mass. It is interesting to mention here that our bias model, similarly to most others proposed in the literature, relate a mass tracer, being a galaxy, an AGN or a cluster of galaxies, with a host dark matter halo within which the mass tracer forms and evolves. The models themselves follow the linear evolution of the host halo and not the internal evolution of the astrophysical processes of the tracer. Thus the assumption is that the effects of nonlinear gravity and hydrodynamics (merging, feedback mechanisms, etc.) can be ignored in the linear-regime (see [15, 16]).

Comparing our solution of Eq.(3.8) with that of *Case 1*, $b(z,k) = 1 + (b_0 - 1)/D(z,k)$, it becomes evident that the latter misses one of the two components of the full solution simply because the assumption of equal velocities leads to a first order homogeneous differential equation for the bias (3.2). While the assumption of equal accelerations (with $v_{\rm tr} \neq v_m$) involves the full linear perturbation analysis and thus it produces a second order homogeneous differential equation (3.5), independent solutions of which are $1/D(z,k)$ and $J(z)/D(z,k)$.

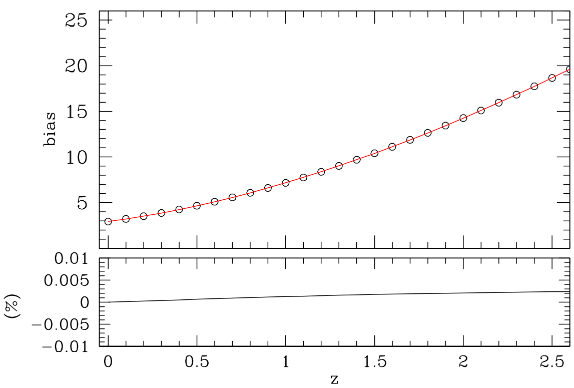

FIG. 1: The scale-dependent bias z-evolution (*upper panel*) at galaxy cluster scales, $k \simeq 0.05h\mathrm{Mpc}^{-1}$ (open symbols), while the scale-independent prediction is shown as the solid line. In the lower panel we present the fractional difference with respect to the scale-independent bias model. Note that we use $\Omega_m = 0.273$, $w_{de}(z) = -1$, $\sigma_{8,m}(0) = 0.81$ and $\gamma = 0.55$.

The further component in the bias solution, provided by the above model, solves the the known *unbiased* and the *low redshift* problems, by which the Tegmark & Peebles [16] (*Case 1*) model suffers.

Case 3: Tracers and Mass do not share same velocity field: Here we obtain a similar to *Case 2* bias model but using the nomenclature of Tegrmark and Peebles [16]. We now drop the main assumption used in *Case 1*, that the mass-tracers and the underlying mass distribution share the same velocity fields, by allowing some sort of relation between the two (matter and mass-tracers) velocity fields. We obtain again the corresponding equation (3.2), starting only from the continuity equations (2.5, 2.26) and introducing an additional time-dependent term, $v_{f,\rm tr}(a) - v_{f,m}(a)$, which we associate with the effects of the different velocity fields. We also use the same notation, as in our original formulation, that the tracers and the underlying mass distribution share the same gravity (accelerations) field. Then we obtain:

$$\dot{\delta}_{\rm tr} - \dot{\delta}_m = \frac{k^2}{a^2}\left[v_{f,\rm tr}(a) - v_{f,m}(a)\right] \tag{3.10}$$

or

$$\frac{d(y\delta_m)}{dt} = \frac{k^2}{a^2}\left[v_{f,\rm tr}(a) - v_{f,m}(a)\right] \tag{3.11}$$

a general solution of which is

$$y(a,k)\delta_m(a,k) = C_1 + \int \frac{k^2 da}{a^3 H(a)}\left[v_{f,\rm tr}(a) - v_{f,m}(a)\right] \; . \tag{3.12}$$

Thus the evolution of bias becomes:

$$b(z,k) = 1 + \frac{b_0 - 1}{D(z,k)} + \frac{I(z,k)}{D(z,k)} \tag{3.13}$$

where

$$I(z,k) = \int_0^z \frac{(1+x)}{E(x)} \frac{k^2}{\delta_m(0,k)H_0} \left[v_{f,\mathrm{tr}}(x) - v_{f,m}(x)\right] dx \tag{3.14}$$

and $b_0 - 1 = C_1/\delta_m(0,k)$. Obviously, if $v_{f,\mathrm{tr}} = v_{f,m}$ then Eq.(3.13) boils down to Eq.(3.3) as it should. Although we do not have a fundamental theory to model the time-dependent $v_{f,\mathrm{tr}}(a) - v_{f,m}(a)$ function, it appears to depend on the Hubble constant H_0 as well as on the scale due to the fact that the quantity $I(z,k)$ of Eq.(3.14) has to be unit-less. With the above in mind, we thus observe that from Eqs.(3.8), (3.9) and (3.13), (3.14) we obtain:

$$v_{f,\mathrm{tr}}(a) - v_{f,m}(a) = \frac{C_2 H_0 \delta_m(a=1,k)}{k^2} \tag{3.15}$$

implying the following scaling of the velocity potentials with the scale factor:

$$v_{\mathrm{tr}}(a) - v_m(a) = -\frac{C_2 H_0 \delta_m(a=1,k)}{ak^2}\ . \tag{3.16}$$

In general the above difference between the velocity potentials could have had a different form with respect to the scale factor, but it must always be $\propto H_0/k^2$ in order for the $I(z,k)$ integral to be unit-less.

Finally, in Figure 1 (*upper panel*) we compare the scale-dependent (symbols) and scale-independent (solid curve) bias evolution $b(z,k)$ at galaxy cluster scales, $k \simeq 0.05h\mathrm{Mpc}^{-1}$ ($r \simeq 20h^{-1}$Mpc with $M \simeq 5\times10^{14}h^{-1}M_\odot$). As it is expected, the biasing is a monotonically increasing function of redshift. In the *lower panel* we show the fractional difference between the two bias evolution results and it becomes quite evident that the fluctuations of the FLRW metric do not affect the bias evolution.

4. TESTING GRAVITY AT COSMOLOGICAL SCALES

Nowadays, the issue of testing the validity of general relativity at cosmological scales is considered one of the most fundamental and challenging problems on the interface uniting Astronomy, Cosmology and Particle Physics and indeed in the last decade there have been theoretical debates among cosmologists regarding the methods that researchers have to develop in order to make this achievement. Briefly, it is interesting to mention that measuring the growth index could provide an efficient way to discriminate between modified gravity models and DE models which adhere to general relativity. Indeed it was theoretically shown that for DE models inside general relativity the growth index γ is well fitted by $\gamma_{\mathrm{GR}} \approx 6/11$ (see [28],[29]). Notice, that in the case of the braneworld model of Dvali, Gabadadze & Porrati [39] we have $\gamma \approx 11/16$ (see also [28]), while for the $f(R)$ gravity models we have $\gamma(z) \approx 0.41 - 0.21z$ for $\Omega_m = 0.273$ [40].

Recently, it has been proposed (see for example [41]) that an efficient avenue to constrain the γ parameter is by

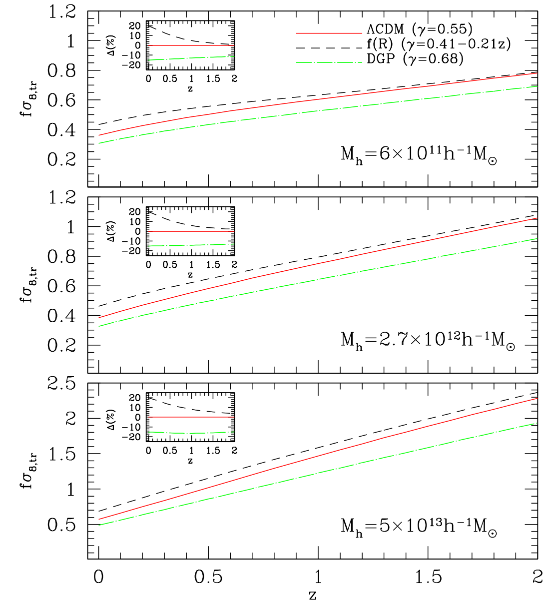

FIG. 2: The predicted growth history of the Universe for different flat cosmological models (*upper panel*: $M_h = 6\times10^{11}h^{-1}M_\odot$, *middle panel*: $M_h = 2.7\times10^{12}h^{-1}M_\odot$) and *bottom panel*: $M_h = 5\times10^{13}h^{-1}M_\odot$) and their fractional difference with respect to the ΛCDM model (see *insert panels*). The cosmological models shown are: $f(R)$ (dashed line) with $\gamma(z) = 0.41 - 0.21z$, concordance ΛCDM (solid line) with $\gamma = 0.55$ and DGP (dot-dashed) with $\gamma = 0.68$. Note that we use $\Omega_m = 0.273$, $\sigma_{8,m}(0) = 0.81$.

determining observationally the redshift-dependent linear growth of perturbations. Other methods have also been proposed in the literature, such as redshift space distortions in the galaxy power spectrum, weak lensing and the growth rate of massive galaxy clusters (see for example [42] and references therein). Indeed, it has been shown [43] that a good test to discriminate among "classical" DE models and modified gravity models is to compare the expected combination parameter of the growth rate of structure, $f(z)$, and the redshift-dependent rms fluctuations of the linear density field, $\sigma_8(z)$, with that measured observationally (from large redshift surveys, like the *WiggleZ*; [38, 47] and references therein).

Armed, with our bias evolution model it is straightforward to obtain theoretically a model-independent way of expressing the parameter combination $f(z)\sigma_{8,\mathrm{tr}}(z)$. Since the metric fluctuations do not significantly affect the evolution of bias and thus the formation of large scale structures at the scales of interest, we utilize: $\xi_k(z,k) \equiv 0$. Therefore, using the solution of the bias evolution eq.(1.3) and eq.(2.22) we find that the growth history of the Universe is given by:

$$f(z)\sigma_{8,\mathrm{tr}(z)}(M_h,z) = \Omega_m^\gamma(z) b(M_h,z) \sigma_{8,m}(z) \tag{4.1}$$

where $\sigma_{8,m}(z) = \sigma_{8,m}(0)D(z)$ and $b(M_h,z)$ is given by eq(3.8). Our proposed gravity test consists in comparing the above expectation with observationally determined values of $f(z)\sigma_{8,\mathrm{tr}}(z)$, estimated by using existing or future redshift catalogs of extragalactic mass tracers (large red galaxies, optical or X-ray QSO, clusters of galaxies, etc). Note, that such data are already available in the literature for the case of bright emission-line galaxies [38].

Evidently, the essential cosmological parameters that enter in the theoretical expectation of Eq.(4.1) are: $\Omega_m, w_{de}(z), \sigma_{8,m}(0)$ and γ. Note however that:

- there is only a weak dependence of γ on $w(z)$, as has been found in Linder & Cahn [28], which implies that one can separate the background expansion history, $E(z) = H(z)/H_0$, constrained by a large body of cosmological data (SNIa, BAO, CMB), from the fluctuation growth history, given by γ, and
- the value of $\sigma_{8,m}(0)$ remains relatively constant ($\sigma_{8,m}(0) \in [0.77, 0.81]$) for a range of dark energy equations of state (ΛCDM, quintessence, CPL), as shown by a recent analysis of SDSS Large Red Galaxies [48].

In particular, the aim of our proposed method is to constrain, for a given expansion history, the value of γ, and test whether there are deviations from the GR expectations. In order to visualize the redshift and γ dependence of the $f\sigma_{8,\mathrm{tr}}$, we compare in Figure 2 for different masses of dark matter halos, three flat cosmological models, in which we impose $\Omega_m = 0.273$ and $\sigma_{8,m}(0) = 0.81$. In particular, we consider the following cases:
(a) the $f(R)$ model with $\gamma(z) = 0.41 - 0.21z$ (dashed line),
(b) the concordance ΛCDM ($\gamma = 0.55$, solid line), and
(c) the DGP model with $\gamma = 0.68$ (dot-dashed line).

The inset panels of Figure 2 show the relative difference of the $f(R)$ or DGP model $f\sigma_8$ parameter combination with respect to that of the ΛCDM. Interestingly, the $f(R)$ models show the largest deviations ($\geq 10\%$) at the lower redshift end ($z \leq 0.5$), while the DGP model shows large deviations also at the higher redshift end. It should be mentioned that qualitatively and quantitatively the relative differences among the models are quite similar, independently of the DM halo mass used, a fact which indicates that any extragalactic mass tracer can be used with the same efficiency to perform this cosmological test.

We will elaborate on the details of our proposed method in a forthcoming paper, but it is important to realize that the parameters (C_2, b_0) of our bias model depend: (a) on the characteristic DM halo mass, within which the mass tracer is embedded (see Appendix A), and (b) on the values of Ω_m and $\sigma_{8,m}(0)$ (see for example, [5]). In any case we expect that the variation of the bias model (b_0, C_2) parameters, within a physically acceptable range of $\Omega_m, \sigma_{8,m}(0)$ values, should be quite small. For example, the fact that $b_0[\propto 1/\sigma_{8,m}(0)]$ with $\sigma_{8,m}(0) \in [0.77, 0.81]$ (as indicated in [48]) implies that b_0 remains mostly unaffected as far as its dependence on $\sigma_{8,m}(0)$ is concerned.

5. CONCLUSIONS

In the current work we provide a general bias evolution model, based on linear perturbation theory, which takes into account also metric fluctuations. We find that the metric fluctuations do not affect the evolution of bias and thus the formation of large scale structures. We argue that the evolution of the mass-tracer fluctuations, quantified by their variance on some smoothed scale, can be used to test the validity of general relativity on cosmological scales. We would like to remind the reader that in Basilakos et al. [7] paper we have derived the evolution of bias within the context of scale-independent bias. The combination of the latter and current works provides a complete investigation of the linear bias evolution issue in cosmological studies. We show in our current work that the use of the combination parameter of the growth rate of structure and the rms fluctuations of the linear density field could provide an efficient avenue to discriminate among "geometrical" (modified gravity) dark energy models and those that adhere to general relativity.

It is however important to spell out clearly which are the basic assumptions of our model, which are common also to many other bias models in the literature: (a) Hubble scale GR effects taken into account in the fluctuation growth (b) the Newtonian gauge approach is employed (c) the mass tracers and the underlying mass share the same gravity field but different velocity fields, in contrast to the bias model proposed by Tegmark & Peebles [16] in which they proposed the they share the same velocity field. (d) the biasing is linear on the scales of interest, and (e) that each DM halo is populated by one extragalactic mass tracer, which is an assumption that enters, at the present development of our model, only in the comparison of our model with observational bias data and not in the derivation of its functional form. Finally, we assume unimportant halo merging effects which is quite accurate for $z < 3$.

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Appendix A: Parametrizing the Bias Evolution Model using N-body Simulations

Our analytical solution Eq.(3.8) gives a family of dark matter (DM) halo bias curves with two unknown parameters (b_0, C_2), which depend on the halo mass as well as on the cosmological background (see [20] and the Appendix in [5]). One can determine the behavior of the linear bias factor as a function redshift and halo mass, evaluating these constants using, for example, N-body simulations. To this end we use the results of the high resolution, collisionless, WMPA7 ΛCDM simulation of [5]. Here we only present the basic information regarding this N-body simulation.

The simulation is a random realization of the concordance ΛCDM model [44] with a volume of a 500 h^{-1} Mpc cube and 512^3 particles. The adopted cosmological parameters are: $\Omega_m = 0.273$, $\Omega_\Lambda = 1 - \Omega_m$, $h = 0.704$ and $\sigma_8 = 0.81$, while the particle mass is $7.07 \times 10^{10} h^{-1} M_\odot$ comparable to the mass of a single galaxy. The simulation was performed with an updated version of the parallel Tree-SPH code GADGET2 [46].

Furthermore, the details of the method used to identify the DM halos and estimate their bias, as a function of redshift, with respect to the underlying mass distribution have been presented elsewhere (eg., [45] and references therein). We only mention here that the DM halos are defined using a Friends of Friends algorithm with a linking length $l = 0.17\langle n\rangle^{-1/3}$, where $\langle n\rangle$ is the mean particle density and that the DM halo bias is estimated by measuring the ratio of the variance of the DM halo fluctuations to that of the underlying DM in spheres of 8 h^{-1} Mpc radius, while its uncertainty is based on the bootstrap re-sampling technique.

For our present analysis we use 10 different redshift snapshots of the N-body simulation, spanning the range: $0 \le z \le 5$, while the DM halo catalogs are determined for five different halo mass intervals (as in [45]).

In order therefore the estimate the constants b_0 and C_2 within the context of the specific cosmological model used, we fit the DM halo bias results of the N-body simulation to the corresponding theoretical bias formula (3.8) and find accurate fitting formulas for both parameters. These are

$$b_0(M_h) = C_\beta \left[1 + \left(\frac{M_h}{10^{14} h^{-1} M_\odot}\right)^\beta\right] \qquad \text{(A.1)}$$

with $C_\beta = 0.857 \pm 0.021$ and $\beta = 0.55 \pm 0.06$. and

$$C_2(M_h) = C_\mu \left(\frac{M_h}{10^{14} h^{-1} M_\odot}\right)^\mu \qquad \text{(A.2)}$$

with $C_\mu = 1.105 \pm 0.018$ and $\mu = 0.255 \pm 0.005$. In figure 3 we show the simulation based values of the b_0 and C_2 parameters as a function of DM halo mass together with the best fitted functions, provided in eq. (A.1) and (A.2).

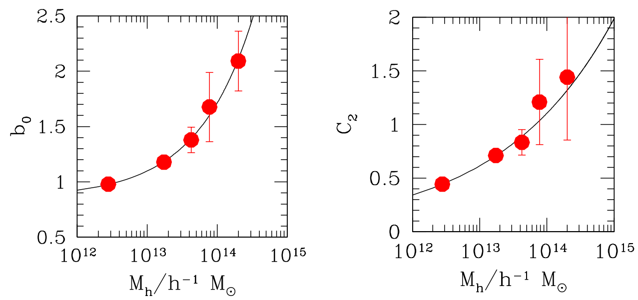

FIG. 3: The values of C_2 and b_0 as a function of DM halo mass and the corresponding best fit curves given by eq.(A.1) and (A.2).

The cosmological dependence of these parameters is the subject of work in progress.

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