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Phys. Rev. D **85**, 122005 — Published 19 June 2012

DOI: 10.1103/PhysRevD.85.122005

Light scalar field constraints from gravitational-wave observations of compact binaries

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Scalar-tensor theories are among the simplest extensions of general relativity. In theories with light scalars, deviations from Einstein's theory of gravity are determined by the scalar mass m_s and by a Brans-Dicke-like coupling parameter $\omega_{\rm BD}$. We show that gravitational-wave observations of nonspinning neutron star-black hole binary inspirals can be used to set lower bounds on $\omega_{\rm BD}$ and upper bounds on the combination $m_s/\sqrt{\omega_{\rm BD}}$. We estimate via a Fisher matrix analysis that individual observations with signal-to-noise ratio ρ would yield $(m_s/\sqrt{\omega_{\rm BD}})(\rho/10) \lesssim 10^{-15}$, 10^{-16} and 10^{-19} eV for Advanced LIGO, ET and eLISA, respectively. A statistical combination of multiple observations may further improve these bounds.

I. INTRODUCTION

Scalar-tensor theories, in which gravity is mediated by a tensor field as well as a nonminimally coupled scalar field, are popular and simple alternatives to Einstein's general relativity [1–3]. Generic scalar-tensor theories are of interest in cosmology, and under certain conditions they can be shown to be equivalent to $f(R)$ theories [4, 5]; they have also been investigated in connection with inflation and cosmological acceleration (see e.g. [6, 7]).

Light, massive scalars are a generic prediction of the low-energy limit of many theoretical attempts to unify gravity with the Standard Model of particle physics. For example, string theory suggests the existence of light scalars ("axions") with masses m_s that could be as small as the Hubble scale ($\sim 10^{-33}$ eV). For a coherent scalar field to be cosmologically distinct from cold dark matter, or to play a quintessence-like role, it must be very light: 10^{-33} eV $< m_s < 10^{-18}$ eV. In this "string axiverse" scenario, cosmic microwave background observations, galaxy surveys and even astrophysical measurements of black hole spins may offer exciting experimental opportunities to set constraints on the masses and couplings of the scalar fields [8–10]. Therefore, astrophysical and cosmological observations may provide important clues on the relation between gravity and the other forces.

In this paper, building on previous work [11], we explore the possibility of constraining the mass and coupling of a scalar field via gravitational-wave observations of compact binary inspirals with Earth-based or space-based interferometers, such as Advanced LIGO/Virgo [12–14], the Einstein Telescope [15, 16], eLISA/NGO [17] or Classic LISA [18]. Until recently, calculations of gravitational radiation in scalar-tensor theories focused on massless scalar fields (see e.g. [19–25]). When $m_s = 0$, deviations from general relativity can be parametrized in terms of a single coupling constant, that for historical reasons is usually chosen to be the Brans-Dicke parameter $\omega_{\rm BD}$. The best constraint on this parameter to date ($\omega_{\rm BD} > \omega_{\rm Cass} = 40,000$) comes from Cassini measurements of the Shapiro time delay [26]. For massless scalar-tensor theories, gravitational-wave observations of mixed binaries can provide constraints comparable to (or marginally better than) the Cassini bound [27–29]. Recent studies explored some of the implications that light, massive scalar fields may have in the context of gravitational-wave phenomenology. For example, resonant superradiant effects induced by light massive scalars may produce "floating orbits" when small compact objects spiral into rotating black holes, leaving a distinct signature in gravitational waves [30].

The general action of scalar-tensor theories with a single scalar field can be cast in the form

$$S = \frac{1}{16\pi}\int\left[\phi R - \frac{\omega(\phi)}{\phi} g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + M(\phi)\right](-g)^{\frac{1}{2}} d^4x + \int \mathcal{L}_M(g^{\mu\nu}, \Psi)\,, \qquad (1)$$

where $\mathcal{L}_M$ is the matter Lagrangian and Ψ collectively denotes the matter fields. In this paper we consider a scalar-tensor theory with a constant coupling function $\omega(\phi) = \omega_{\rm BD}$ and a mass term, as in [11]. Expanding the potential $M(\phi)$ about a (cosmologically determined) background value of the scalar field $\phi = \phi_0$, under the assumption of asymptotic flatness, we have

$$M(\phi) \simeq \frac{1}{2} M''(\phi_0)(\phi - \phi_0)^2\,. \qquad (2)$$

Then the equation of motion for the scalar field has the schematic form [11]

$$\Box_g \phi - m_s^2(\phi - \phi_0) = (\text{source terms})\,, \qquad (3)$$

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where

$$m_s^2 = -\frac{\phi_0}{3+2\omega_{\rm BD}} M''(\phi_0)\,. \quad (4)$$

We note that for $m_s = 0$ this theory reduces to the well-known Brans-Dicke theory [31].

Throughout the paper we use geometrical units $G = c = 1$, so all quantities can be expressed in (say) seconds: for example, the mass of the sun $M_\odot = 4.926 \times 10^{-6}$ s. We assume a standard ΛCDM cosmological model with $H_0 = 72\ {\rm km\,s^{-1}Mpc^{-1}}$, $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$. Given the scalar field mass $\tilde{m}_s$ (in eV), we find it convenient to define a quantity $m_s = \tilde{m}_s/\hbar$ with dimensions of inverse length (or inverse time, since $c = 1$), as this is the quantity that would appear in the flat-space Klein-Gordon equation $(\Box - m_s^2)\varphi = 0$. To convert between $\tilde{m}_s$ and m_s, it is sufficient to note that $\hbar = 6.582 \times 10^{-16}$ eV s. We will use $\tilde{m}_s$ and m_s interchangeably in the rest of the paper; the units should be clear from the context.

Our study is the first to explore bounds on massive scalar tensor theories using gravitational-wave observations of compact binaries. We only consider quasicircular, nonspinning binaries for two reasons: (1) resonant effects in the context of gravitational-wave detection were considered in [32], and they would produce large enough dephasing to prevent detection using general relativistic templates; (2) in the absence of resonant effects, the introduction of aligned spins would increase parameter measurement errors by about one order of magnitude. However, spin precession and eccentricity would reduce the errors by a comparable amount [29, 33]. For these reasons, our bounds for quasicircular, nonspinning binaries should be comparable to bounds resulting from more realistic (and complex) waveform models, as long as resonant effects don't play a role.

In this paper we will focus on quasicircular neutron star-black hole binaries. The main reason is that, according to our previous investigation [11], only *mixed binaries* (i.e., binaries whose members have different gravitational binding energies) can produce significant amounts of scalar gravitational radiation. Under the assumption of asymptotic flatness, dipole radiation is produced due to violations of the strong equivalence principle when the binary members have unequal "sensitivities": $s_1 \neq s_2$. The sensitivities are related to the gravitational binding energies of each binary member (labeled by $a = 1\,,2$): $s_a = 1/2$ for a Schwarzschild black hole, $s_a \sim 0.2$ for a neutron star, and $s_a \sim 10^{-4}$ for a white dwarf [20, 34]. Roughly speaking, dipole radiation is produced when the system's center of mass is offset with respect to the center of inertia, so mixed and eccentric binaries are the best observational targets to constrain scalar-tensor theories. A second reason to consider neutron star-black hole binaries is that dipolar radiation should not be emitted in black hole-black hole systems because of the no-hair theorem, i.e. the fact that black hole solutions in scalar-tensor theories are the same as in GR (see [35] and references therein). Recently, building on earlier work by Jacobson [36], Horbatsch and Burgess pointed out that scalar fields that vary on cosmological timescales may violate the no-hair theorem, so that even black hole-black hole binaries may produce dipolar radiation [37]. The existence of dipolar radiation in extended theories of gravity and the investigation of possible bounds on dipolar radiation are active research topics [38–40].

In summary, here we will make the conservative assumption that only mixed binaries generate dipolar radiation, and we will investigate bounds on the mass and coupling of the scalar field coming from gravitational-wave observations of black hole-neutron star binaries. For simplicity we will set the mass of the neutron star to be $M_{\rm NS} = 1.4\ M_\odot$, and we will focus on nonrotating black holes. This rules out by construction the possibility of floating orbits of the kind studied in [30, 32].

We will consider the bounds that could be obtained using different gravitational-wave detectors. For Advanced LIGO we use the fit to the expected power spectral density given in Table I of [41] (henceforth "AdLIGO") as well as the zero-detuning, high power configuration, as given analytically in Eq. (4.7) of [42] ("AdLIGO ZDHP"). In both cases we assume the power spectral density to be infinite below a seismic noise cutoff frequency of 20 Hz. For the Einstein Telescope we use the analytical fit presented in Table I of [41], assuming a lower cutoff frequency of 10 Hz. For Classic LISA we use the analytical Barack–Cutler expression [43], as corrected in [28]. For eLISA/NGO we use the noise model (inclusive of galactic background noise) discussed in [17].

In general, the gravitational radiation from a binary in massive scalar-tensor theories depends on the scalar field mass m_s and on the coupling constant $\xi \sim 1/\omega_{\rm BD}$ (see [11] and Eq. (7) below for a more precise definition). Matched-filtering detection of gravitational radiation from compact binaries relies primarily on an accurate knowledge of the wave phasing. In any theory that differs from general relativity, the gravitational-wave phasing will be modified by a slight amount that depends on the parameters of the theory. In massless scalar-tensor theories the dominant (dipolar) contribution scales like $1/\omega_{\rm BD}$. If we measure a waveform consistent with general relativity we can set upper bounds on deviations of the dipolar term from zero, and therefore we can set lower limits on $\omega_{\rm BD}$ [27–29]. By computing the gravitational-wave phase in the stationary-phase approximation (SPA; cf. Eq. (16) below) we will see that the scalar mass always contributes to the phase in the combination $m_s^2\xi \sim m_s^2/\omega_{\rm BD}$, so that gravitational-wave observations of nonspinning, quasicircular inspirals can only set upper limits on $m_s/\sqrt{\omega_{\rm BD}}$. For large signal-to-noise ratio (SNR) ρ, the constraint is inversely proportional to ρ. It is useful to plot constraints that result from observations with SNR $\rho = 10$: this corresponds to a minimal threshold for detection of the corresponding binary system, and in this sense it places an upper limit on the achievable bounds on m_s (and a lower limit on the achievable bounds on $\omega_{\rm BD}$).

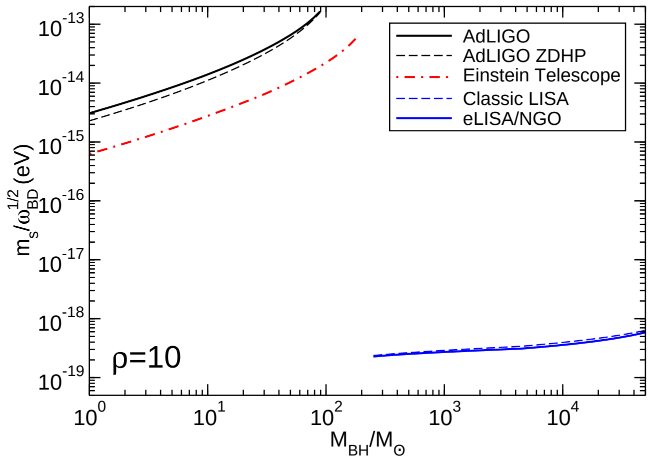

FIG. 1. Bounds on $m_s/\sqrt{\omega_{\rm BD}}$ with AdLIGO, AdLIGO ZDHP, ET, Classic LISA and eLISA/NGO at fixed SNR $\rho = 10$.

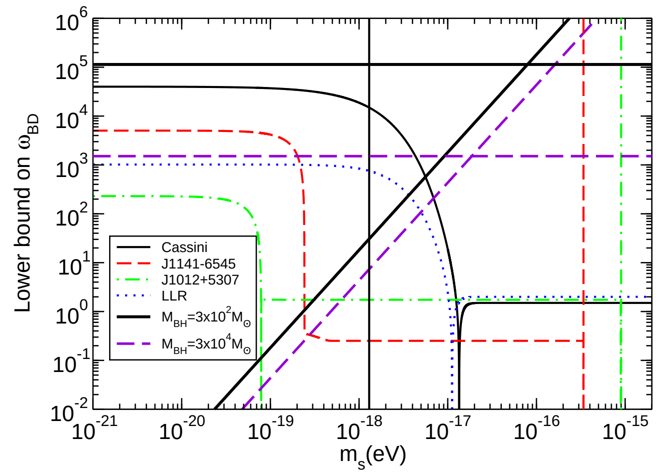

FIG. 2. Bounds from eLISA/NGO at SNR $\rho = 10$, compared to Solar System and binary pulsar bounds.

In Fig. 1 we plot the upper bound on $m_s/\sqrt{\omega_{\rm BD}}$ resulting from neutron star-black hole binary observations with SNR $\rho = 10$, as a function of the black hole mass $M_{\rm BH}$, for different detectors. The order of magnitude of these bounds is essentially set by the lowest frequency accessible to each detector, and it can be understood by noting that the scalar mass and gravitational-wave frequency are related (on dimensional grounds) by $m_s({\rm eV}) = 6.6 \times 10^{-16} f({\rm Hz})$, or equivalently $f({\rm Hz}) = 1.5 \times 10^{15}\, m_s({\rm eV})$. For eLISA, the lower cutoff frequency (imposed by acceleration noise) $f_{\rm cut} \sim 10^{-5}$ Hz corresponds to a scalar of mass $m_s \simeq 6.6 \times 10^{-21}$ eV. For Earth-based detectors the typical seismic cutoff frequency is ~ 10 Hz, corresponding to $f_{\rm cut} \sim 6.6 \times 10^{-16}$ eV. These lower cutoff frequencies set the order of magnitude of scalar masses probed by space-based and Earth-based detectors.

The best bounds are obtained from the intermediate mass-ratio inspiral of a neutron star into a black hole of mass $M_{\rm BH} \lesssim 10^3\ M_\odot$, as observed by a space-based instrument such as eLISA or Classic LISA. In summary, we conclude that the most competitive bounds would come from space-based gravitational wave detectors, and that they would be of the order

$$\left(\frac{m_s}{\sqrt{\omega_{\rm BD}}}\right)\left(\frac{\rho}{10}\right) \lesssim 10^{-19}\ {\rm eV}. \qquad (5)$$

In Fig. 2 we plot current bounds on $\omega_{\rm BD}$ as a function of m_s coming from (i) Cassini measurements of the Shapiro time delay (solid black line), (ii) Lunar Laser Ranging bounds on the Nordtvedt effect (dotted blue line), and (iii) measurements of the orbital period derivative of two binary pulsar systems. We refer the reader to [11] for a detailed derivation of these bounds.

On top of the existing bounds we plot, for comparison, future bounds from eLISA observations of neutron star-black hole binaries where the black hole has mass $M_{\rm BH} = 300\ M_\odot$ (solid lines) and $M_{\rm BH} = 3 \times 10^4\ M_\odot$ (dashed lines). In both cases the bounds refer to observations with SNR $\rho = 10$.

For each system, as discussed above, gravitational wave observations provide *two* constraints: a lower limit on $\omega_{\rm BD}$ (corresponding to horizontal lines in the plot) and an upper limit on $m_s/\sqrt{\omega_{\rm BD}}$ (corresponding to the straight diagonal lines). For each of the two neutron star-black hole systems, a gravitational-wave observation would exclude the complement of a trapezoidal region on the top left of the plot. We see that bounds from space-based gravitational-wave observations will improve over the Cassini, Lunar Laser Ranging and binary pulsar bounds bound only for neutron star-black hole systems with $M_{\rm BH} \lesssim 10^3\ M_\odot$. In a sense, these predictions are conservative: for example, for a gravitational-wave observation with SNR $\rho = 100$ both bounds (the horizontal and diagonal lines) would improve by an order of magnitude. Therefore, a single high-SNR observation (or the statistical combination of several observations, see e.g. [44]) may yield bounds on massive scalar-tensor theories that are significantly better than Solar System or binary pulsar bounds.

The plan of the paper is as follows. In Section II we compute the gravitational-wave phase for massive scalar-tensor theories in the stationary phase approximation. In Section III we use these results to compute the Fisher information matrix in these theories. Section IV presents the bounds obtainable with Earth- and space-based interferometers. Section V discusses possible directions for future research.

II. STATIONARY PHASE APPROXIMATION IN MASSIVE SCALAR-TENSOR THEORIES

As discussed in Section I, we focus on scalar-tensor theories with constant coupling and a massive scalar field, following the treatment and notation of [11]. We consider the inspiral of a binary system composed of two compact objects with masses m_1 and m_2. For consistency with the notation of [28], here and below all masses m_i are measured in the detector frame; they are related to the masses $m_i^{(0)}$ in the source frame by $m_i = (1+z)m_i^{(0)}$. Let $m = m_1 + m_2$ be the total mass of the binary, $\mu = \eta m = (m_1 m_2)/(m_1+m_2)$ the reduced mass (with η the symmetric mass ratio), and $\mathcal{M} = \mu^{3/5} m^{2/5} = \eta^{3/5} m$ the so-called chirp mass[1]. We will denote by f the gravitational-wave frequency of the binary.

The dominant contributions to gravitational radiation in scalar-tensor theories are dipolar and quadrupolar. In the adiabatic approximation, the time derivative of the gravitational-wave frequency (denoted by an overdot) reads [11]

$$\frac{\dot f}{f} = -\frac{\dot P}{P} = \frac{8}{5}\frac{\mu m^2}{r^4}\kappa_1 + \frac{\mu m}{r^3}\mathcal{S}^2 \kappa_D \,, \tag{6}$$

where

$$\begin{aligned} \xi &= \frac{1}{2+\omega_{\rm BD}}\,, \\ \mathcal{G} &= 1 - \xi(s_1 + s_2 - 2 s_1 s_2)\,, \\ \Gamma &= 1 - 2\frac{s_1 m_2 + s_2 m_1}{m}\,, \\ \mathcal{S} &= s_2 - s_1\,, \\ \kappa_1 &= \mathcal{G}^2\left[12 - 6\xi + \xi\Gamma^2\left(1 - \frac{m_s^2}{4\pi^2 f^2}\right)^2 \Theta(2\pi f - m_s)\right]\,, \\ \kappa_D &= \mathcal{G}\left[2\xi\left(1 - \frac{m_s^2}{\pi^2 f^2}\right)\right]\Theta(\pi f - m_s)\,, \end{aligned} \tag{7}$$

and $\Theta(x)$ is the Heaviside function.

The gravitational-wave and orbital frequencies are related by $f = 2 f_{\rm orb}$, where (see e.g. Eq. (2.25c) in [20])

$$\Omega_{\rm orb} = 2\pi f_{\rm orb} = \sqrt{\frac{\mathcal{G} m}{r^3}}\,. \tag{8}$$

Following [45, 46], we can compute the phase $\psi(f)$ of the gravitational waveform as a function of the wave frequency f in the stationary phase approximation (SPA):

$$\psi(f) = 2\pi f t(f) - \phi(f) - \frac{\pi}{4}\,, \tag{9}$$

where

$$t(f) = \int^f \frac{df'}{\dot f'}\,, \tag{10}$$

$$\phi(f) = 2\pi \int^f \frac{f'}{\dot f'} df'\,. \tag{11}$$

For light scalars ($m_s \lesssim 10^{-19}$ eV) we know that $\omega_{\rm BD}$ must be larger than the Cassini bound $\omega_{\rm Cass} = 40{,}000$ (see Fig. 2), therefore we are justified in assuming that $\xi \ll 1$ and we can linearize in ξ. For scalar masses $m_s \gtrsim 10^{-18}$ eV, strong couplings ($\xi \sim 1$ or larger) are not experimentally ruled out; however our focus here is on constraining light scalars of cosmological relevance. A straightforward calculation linearizing in ξ yields

$$\begin{aligned} \dot f = \frac{96}{5}\pi^{8/3}\mathcal{M}^{5/3} f^{11/3}\Bigg\{ & 1 - \frac{2}{3}\xi(s_1+s_2-2s_1s_2) - \frac{1}{2}\xi \\ & + \frac{\xi\Gamma^2}{12}\left(1 - \frac{m_s^2}{4\pi^2 f^2}\right)^2 \Theta(2\pi f - m_s) \\ & + \frac{5}{48}\xi\left(1 - \frac{m_s^2}{\pi^2 f^2}\right)\mathcal{S}^2(\pi m f)^{-2/3}\Theta(\pi f - m_s) + \dots \Bigg\}\,, \end{aligned}$$

where dots denote contributions of higher post-Newtonian order. Let us define the quantity

$$\nu \equiv m_s^2 m^2\,. \tag{12}$$

This quantity is dimensionless, since (as discussed above) the scalar mass m_s has dimensions of inverse length (or time), and in geometrical units the total mass of the binary system has dimensions of length (or time).

Let $m_{20} = 10^{-20}$ eV be the typical scalar mass below which we get bounds on the scalar coupling from Solar System experiments [11, 47], and let $M_\odot$ be the mass of the Sun. We get the scaling

$$\nu = 5.60 \times 10^{-21}\left(\frac{m_s}{m_{20}}\right)^2\left(\frac{m}{M_\odot}\right)^2\,, \tag{13}$$

so that a binary with $m = 10^6 M_\odot$ (a typical target for eLISA) could have at most $\nu \sim 5.60 \times 10^{-9}$ when $m_s = m_{20}$. With this definition we can write

$$\begin{aligned} \dot f = \frac{96}{5}\pi^{8/3}\mathcal{M}^{5/3} f^{11/3}\Bigg\{ & 1 - \frac{2}{3}\xi(s_1+s_2-2s_1s_2) - \frac{1}{2}\xi \\ & + \left[\frac{\xi\Gamma^2}{12}\left(1 - \frac{\nu}{2(\pi m f)^2} + \frac{\nu^2}{16(\pi m f)^4}\right)\Theta(2\pi f - m_s)\right. \\ & \left. + \frac{5\xi\mathcal{S}^2}{48}\left(\frac{1}{(\pi m f)^{2/3}} - \frac{\nu}{(\pi m f)^{8/3}}\right)\Theta(\pi f - m_s)\right] + \dots \Bigg\}\,. \end{aligned} \tag{14}$$

Furthermore, we note that (dubbing either $f_0 = m_s/(2\pi)$ or $f_0 = m_s/\pi$)

$$\begin{aligned} \int_{-\infty}^f df'(f')^n \Theta(f' - f_0) &= \Theta(f - f_0)\int_{f_0}^f df'(f')^n \\ &= \Theta(f - f_0)\frac{f^{n+1}}{n+1} + \text{constant}\,, \end{aligned} \tag{15}$$

where the constant can be absorbed in "time at coalescence" and "phase at coalescence" integration constants

[1] Notice that Refs. [24, 27] introduce a slightly different definition of the chirp mass in the context of scalar-tensor theories.

(t_c, ϕ_c). The final result for the SPA phase reads

$$
\begin{aligned}
\psi(f) = {} & 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128(\pi\mathcal{M}f)^{5/3}} \times \\
& \times \Big\{ 1 + \zeta \\
& + \frac{20}{9} A \eta^{-2/5} (\pi\mathcal{M}f)^{2/3} - 16\pi\eta^{-3/5}(\pi\mathcal{M}f) + \ldots \\
& + \xi\Gamma^2\nu \left[\frac{5}{462}\eta^{6/5}(\pi\mathcal{M}f)^{-2} - \frac{\nu}{1632}\eta^{12/5}(\pi\mathcal{M}f)^{-4} \right] \\
& \times \Theta(2\pi f - m_s) \\
& + \xi\mathcal{S}^2 \left[\frac{25\nu}{1248}\eta^{8/5}(\pi\mathcal{M}f)^{-8/3} - \frac{5}{84}\eta^{2/5}(\pi\mathcal{M}f)^{-2/3} \right] \\
& \times \Theta(\pi f - m_s) \Big\} \,, \qquad (16)
\end{aligned}
$$

where we defined

$$
\zeta = \frac{2}{3}\xi\left(s_1 + s_2 - 2 s_1 s_2\right) + \frac{\xi}{2} - \frac{\xi\Gamma^2}{12}\Theta(2\pi f - m_s) \,. \quad (17)
$$

Adding higher-order corrections in the post-Newtonian velocity parameter $v = (\pi m f)^{1/3} = (\pi\mathcal{M}f)^{1/3}\eta^{-1/5}$ to the standard GR phase is trivial. Eq. (2.2) of [28] lists all contributions up to 2PN (including spin-orbit and spin-spin interactions). Here we ignore spins, but we do include all nonspinning contributions up to 3.5PN order: cf. Eq. (3.3) and (3.4) of [48].

Arun recently argued on general grounds that dipolar emission in generic extensions of general relativity should always introduce a term proportional to v^{-2} in the SPA [39]. Our calculation shows that a nonzero mass of the scalar introduces additional structure in the waveform: in particular, Eq. (16) contains terms proportional to ν (and hence to m_s^2) that scale like v^{-8} and v^{-6}, and terms proportional to ν^2 that scale like v^{-12}. Arun's argument to constrain dipolar radiation [39], while correct when $m_s = 0$, is not general enough to encompass all scalar-tensor theories (let alone theories whose action contains quadratic or higher-order terms in the curvature, such as Gauss-Bonnet or Chern-Simons modified gravity [38, 40]). While these large negative powers of v could in principle be strongly dominant over "standard" quadrupolar radiation at small frequencies, the presence of the Heaviside functions protects the SPA phase from a possible "infrared divergence" in the limit $v \to 0$.

Arun pointed out that scalar-tensor theories cannot be easily incorporated within the original parametrized post-Einsteinian framework because of additional terms that should appear as amplitude corrections to the waveform [39]. This issue was recently addressed in [40], where the parametrized post-Einsteinian framework was extended to allow for amplitude corrections. The problem here is of a different nature, since we work in the restricted post-Newtonian approximation and therefore we ignore amplitude corrections "by construction". What happens instead is that some *phase* corrections are multiplied by Heaviside functions $\Theta(\pi f - m_s)$ and $\Theta(2\pi f - m_s)$ that reflect the dipolar or quadrupolar nature of the various contributions to the flux. This is a conceptual difficulty with power-law parametrizations of deviations from general relativity, but it is not an obstacle in practice, because we know from Solar System observations that m_s must be small. Therefore in our parameter estimation calculations we will assume that $m_s = 0$, and look for deviations of m_s from zero. Under this approximation the condition $f \geq m_s/\pi$ is always satisfied, and all Heaviside functions can be set equal to one.

In massive scalar-tensor theories, corrections to general relativity depend on two small parameters, ν and ξ. The structure of the phasing function (16) has a definite hierarchy when seen as a multivariate Taylor series in ξ and ν. However, the term proportional to v^{-12} (which is proportional to $\xi\nu^2$ and formally of "third order" in the small parameters) will produce sizeable corrections whenever $\nu \sim v^6$. More generally, it is possible that some extensions of general relativity will contain additive corrections to the phasing of the form (say) $\alpha v^{-a} + \beta v^{-b}$, with $\alpha \ll 1$, $\beta \ll 1$, $a > 0$ and $b > 0$. The parametrized post-Einsteinian framework relies on identifying a leading-order correction to a general relativistic waveform. In such cases it may be hard to identify a single leading-order term: a simple power counting in v is not sufficient, unless we have independent means of constraining the relative magnitude of the small parameters α and β.

III. FISHER MATRIX IN MASSIVE SCALAR-TENSOR THEORIES

Following the notation and terminology of [28], we work in the angle-averaged approximation and we write the gravitational waveform in the restricted post-Newtonian approximation (where only the leading-order term is kept in the wave *amplitude*) as

$$
\tilde{h}(f) = \gamma\mathcal{A}f^{-7/6}e^{\mathrm{i}\psi(f)} \,. \quad (18)
$$

Here $\mathcal{A} = \frac{1}{\sqrt{30}\pi^{2/3}}\frac{\mathcal{M}}{D_L}$, D_L is the luminosity distance to the source and γ is a geometrical correction factor: $\gamma = 1$ for Earth-based detectors, and $\gamma = \sqrt{3}/2$ for Classic LISA and eLISA (cf. [28]). When the binary members are nonspinning the gravitational waveform depends on seven parameters: $\{\ln\mathcal{A}, t_c, \phi_c, \ln\mathcal{M}, \ln\eta, \xi, \nu\}$.

An estimate of the accuracy in determining these parameters can be obtained using the Fisher matrix formalism. The calculation of the Fisher matrix requires explicit expressions for the derivative of the waveforms with respect to the various parameters. We refer the reader to [28, 46] for a detailed overview of the Fisher matrix calculation in the present context, and to Appendix A for some subtleties in estimating the error on m_s given the error on ν. For theories involving a massive scalar, Eqs. (2.13) and (2.14) of [28] must be modified as follows. Eq. (2.13f) becomes

$$\frac{\partial \tilde{h}}{\partial \ln \mathcal{M}} = -\mathrm{i}\frac{5}{128(\pi \mathcal{M} f)^{5/3}}\tilde{h} \times \left\{ \xi \Gamma^2 \nu \left[\frac{1}{42} v^{-6} - \frac{\nu}{480} v^{-12} \right] \Theta(2\pi f - m_s) + \xi \mathcal{S}^2 \left[\frac{5\nu}{96} v^{-8} - \frac{1}{12} v^{-2} \right] \Theta(\pi f - m_s) + 1 + \zeta + A_4 v^2 + B_4 v^3 + C_4 v^4 \right\}, \quad (19)$$

and Eq. (2.13g) becomes

$$\frac{\partial \tilde{h}}{\partial \ln \eta} = -\mathrm{i}\frac{1}{96(\pi \mathcal{M} f)^{5/3}}\tilde{h} \times \left\{ \xi \Gamma^2 \nu \left[-\frac{9}{308} v^{-6} + \frac{9\nu}{2720} v^{-12} \right] \Theta(2\pi f - m_s) + \xi \mathcal{S}^2 \left[-\frac{15\nu}{208} v^{-8} + \frac{3}{56} v^{-2} \right] \Theta(\pi f - m_s) + A_5 v^2 + B_5 v^3 + C_5 v^4 \right\}. \quad (20)$$

Note that only terms proportional to v^{-2} survive as $m_s \to 0$ (or equivalently as $\nu \to 0$), and they correspond to the K_4 and K_5 terms in [28]. In the absence of spins our waveform now depends on seven parameters (rather than six, as in the massless case), so we have two equations replacing (2.13e) of [28]:

$$\frac{\partial \tilde{h}}{\partial \xi} = \mathrm{i}\frac{3}{128(\pi \mathcal{M} f)^{5/3}}\tilde{h} \times \left\{ \Gamma^2 \left[\frac{5\nu}{462} v^{-6} - \frac{\nu^2}{1632} v^{-12} - \frac{1}{12} \right] \Theta(2\pi f - m_s) + \mathcal{S}^2 \left[\frac{25\nu}{1248} v^{-8} - \frac{5}{84} v^{-2} \right] \Theta(\pi f - m_s) + \frac{2}{3}(s_1 + s_2 - 2 s_1 s_2) + \frac{1}{2} \right\}, \quad (21)$$

$$\frac{\partial \tilde{h}}{\partial \nu} = \mathrm{i}\frac{3}{128(\pi \mathcal{M} f)^{5/3}}\tilde{h} \times \left\{ \xi \Gamma^2 \left[\frac{5}{462} v^{-6} - \frac{\nu}{816} v^{-12} \right] \Theta(2\pi f - m_s) + \xi \mathcal{S}^2 \left[\frac{25}{1248} v^{-8} \right] \Theta(\pi f - m_s) \right\}. \quad (22)$$

We remark again that we are interested in deviations of m_s and ξ from zero. Following the procedure outlined after Eq. (2.15) in [28], when computing the Fisher matrix and its inverse we set $m_s = \nu = 0$ in the derivatives listed above (of course, all Heaviside functions are then equal to one). We verified the validity of this assumption by repeating the calculation with $m_s = 10^{-20}$ eV (a mass for which we know from Solar System constraints that the coupling must be small [11]): our bounds are essentially unaffected by this modification. However it should be remarked that it is *not* possible to set $\xi = 0$. The reason is apparent from Eq. (22): $\frac{\partial \tilde{h}}{\partial \nu}$ is proportional to ξ, and therefore the Fisher matrix becomes singular in the limit $\xi \to 0$ ($\omega_{\rm BD} \to \infty$).

By inspection we see that when ν contributes to the SPA phase in Eq. (16) it is always multiplied by the coupling parameter ξ, so we can only constrain the product $\xi\nu$, or in other words the combination $\delta\nu/\omega_{\rm BD}$, where $\delta\nu$ measures variations of ν from zero. In practice we can estimate the accuracy in estimating $\delta\nu$ by setting $\xi = (2 + \omega_{\rm BD})^{-1}$ equal to some small but nonzero value compatible with Solar System bounds. For the calculations in this paper we chose $\omega_{\rm BD} = 4 \times 10^5$, but we verified that the measurement error on $\delta\nu$ scales linearly with $1/\xi$ for small coupling parameters. Therefore we present our results in terms of the combination $\delta\nu/\omega_{\rm BD}$, which is independent of the assumed value of $\omega_{\rm BD}$ in the small-coupling limit.

Appendix A shows that once we know the statistical error on ν, say σ_ν, the statistical error σ_{m_s} on m_s can be estimated as

$$\sigma_{m_s} \simeq \frac{\sigma_\nu^{1/2}}{m}. \quad (23)$$

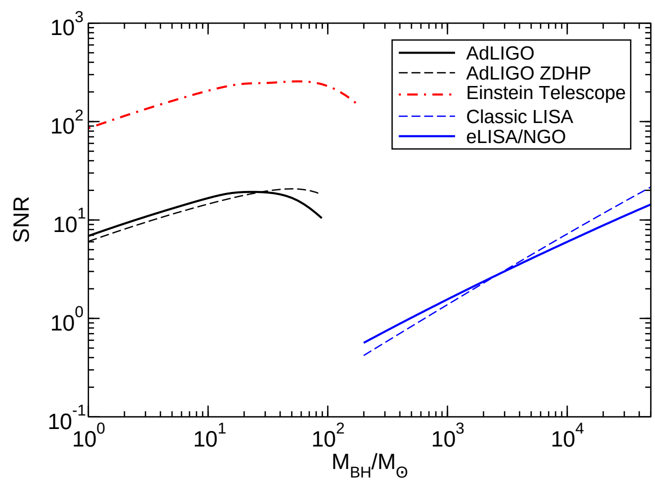

FIG. 3. SNR for observations of neutron star-black hole binaries as a function of the black hole mass $M_{\rm BH}$ for various detectors (AdLIGO, AdLIGO ZDHP, ET, Classic LISA and eLISA/NGO) at luminosity distance $D_L = 200$ Mpc.

IV. BOUNDS WITH EARTH- AND SPACE-BASED INTERFEROMETERS

For each of the five detectors (AdLIGO, AdLIGO ZDHP, the Einstein Telescope, Classic LISA and eLISA) we present "conservative" estimates of the bounds by considering binaries with fixed SNR $\rho = 10$. There are two reasons for this choice: one is conceptual (Fisher matrix calculations are unreliable for black hole masses such that $\rho \lesssim 10$) and one is practical (using $\rho = 10$ allows direct comparison with previous work, e.g. [28]).

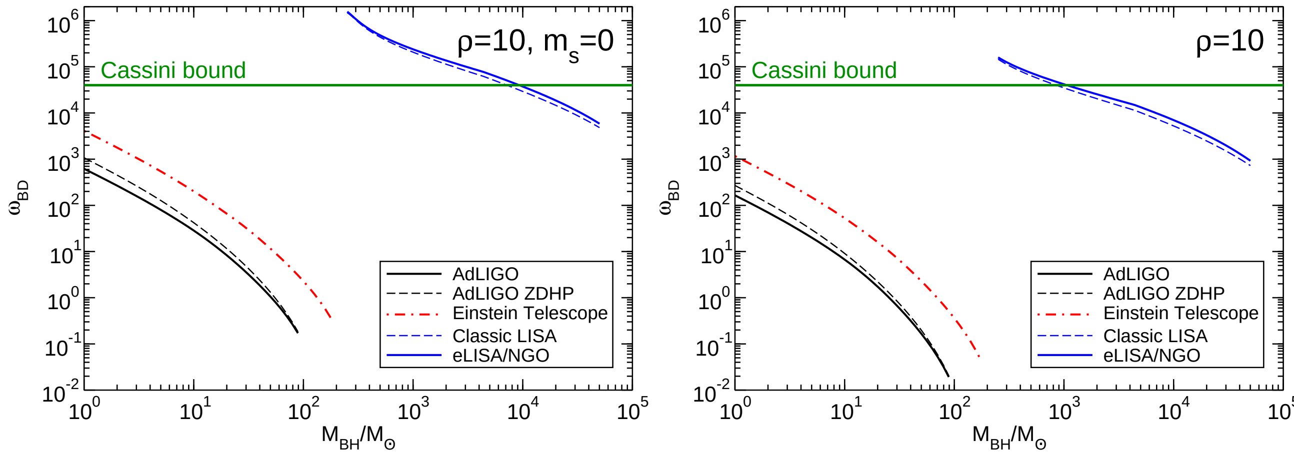

FIG. 4. Bound on $\omega_{\rm BD}$ obtained and considering a six-parameter Fisher matrix at fixed SNR $\rho = 10$. Left: we set $m_s \propto \nu = 0$ and consider a six-parameter Fisher matrix. Right: same, but for a seven-parameter Fisher matrix with $\nu \neq 0$.

We consider neutron star-black hole binaries where the neutron star has mass $M_{\rm NS} = 1.4 M_\odot$, and we vary the black hole mass in a range depending on the optimal sensitivity window of each detector. Bounds are inversely proportional to the SNR, which in turn is inversely proportional to the luminosity distance of the binary ($\rho \sim 1/D_L$). To facilitate rescaling of our results, in Fig. 3 we plot the SNR of neutron star-black hole binaries at luminosity distance $D_L = 200$ Mpc as a function of the black hole mass $M_{\rm BH}$, for all five detectors[2]. Note that AdLIGO and AdLIGO ZDHP are very similar in terms of SNR. The same applies to Classic LISA and eLISA: in fact, eLISA has slightly larger SNR when $M_{\rm BH} \lesssim 2000\ M_\odot$, mainly because the eLISA "armlength" is a factor of five smaller with respect to Classic LISA (see [17]).

The bounds that can be placed on $m_s/\sqrt{\omega_{\rm BD}}$ were discussed in Section I. In Fig. 4 we complement those results by plotting the bounds on $\omega_{\rm BD}$, estimated as in [28]. The left panel shows the bounds on $\omega_{\rm BD}$ that we would obtain if we considered *massless* scalar tensor theories, as in [28]. In the right panel we show that when $m_s \neq 0$ the bounds get worse by about one order of magnitude. This is expected: we are adding one additional, highly correlated parameter to the waveform, and this reduces parameter estimation accuracy on *all* intrinsic parameters of the binary by roughly one order of magnitude. This degradation of the bounds is analogous to what happens when we add spin-orbit terms to the SPA: cf. the discussion around Table IV of [28]. As we could expect from the SNR plots, AdLIGO bounds are very similar to AdLIGO ZDHP bounds, and eLISA does slightly better than LISA for small black hole masses. Whether we consider massive or massless scalar-tensor theories, the best bounds (competitive with the Cassini bound) would come from observations of the intermediate mass-ratio inspiral of a neutron star into a black hole of mass $M_{\rm BH} \lesssim 10^3\ M_\odot$, as observed by a space-based instrument such as eLISA or Classic LISA.

[2] The neutron star-black hole binaries of interest here have large mass ratio. For large mass-ratio systems the SNR is proportional to $M_{\rm NS}/M_{\rm BH}$. Bounds on the scalar mass and on $\omega_{\rm BD}$ scale linearly with the SNR, so changing the mass of the neutron star has the trivial effect of rescaling the bounds by a constant factor which is very close to unity.

V. CONCLUSIONS AND OUTLOOK

In this paper we have studied the bounds on massive scalar tensor theories with constant coupling from gravitational-wave observations of quasicircular, non-spinning neutron star-black hole binary inspirals in the restricted post-Newtonian approximation. We found that neutron star-black hole systems will yield bounds $(m_s/\sqrt{\omega_{\rm BD}})(\rho/10) \lesssim 10^{-15}$, 10^{-16} and 10^{-19} eV for Advanced LIGO, ET and Classic LISA/eLISA, respectively. We also found that the best bounds on $\omega_{\rm BD}$ would come from space-based observations of the intermediate mass-ratio inspiral of a neutron star into a black hole of mass $M_{\rm BH} \lesssim 10^3\ M_\odot$.

It would be interesting to drop the restricted post-Newtonian approximation and to consider amplitude corrections in the context of massive scalar-tensor theories (cf. [39, 40] for recent work in this direction). Furthermore, in our analysis we have neglected time delay effects which arise because the massive scalar modes propagate slower than the massless tensor modes; while presumably small, it is worthwhile to investigate how these effects would change our bounds. Another obvious extension of this work would be to include spin precession, orbital eccentricity and the merger/ringdown waveform. All of these effects could improve our conservative estimate of the bounds on m_s and $\omega_{\rm BD}$. Finally, it would be interesting to estimate the improvement on the bounds that would result from observing *several* neutron star-black hole systems with one or more detectors (see e.g. [44, 49] for preliminary studies in a slightly different context).

Acknowledgments. E.B. was supported by NSF Grant No. PHY-0900735 and NSF CAREER Grant No. PHY-1055103. We also acknowledge support from the NRHEP–295189 FP7–PEOPLE–2011–IRSES Grant and by FCT projects PTDC/FIS/098032/2008 and PTDC/FIS/098025/2008. We are grateful to K. G. Arun, Giulio D'Agostini, Paolo Pani, Nicolàs Yunes and Clifford Will for discussions. We are particularly grateful to Michele Vallisneri for clarifying some issues related to error estimates.

Appendix A: Some subtleties in the calculation of parameter estimation errors

The inversion of the Fisher matrix yields the statistical errors on the seven parameters chosen to parametrize the waveform, namely $\{\ln\mathcal{A}, t_c, \phi_c, \ln\mathcal{M}, \ln\eta, \xi, \nu\}$, and the associated correlation coefficients. Our goal is to get a bound on m_s, assuming that the background value of m_s is zero.

In order to evaluate the statistical error of the scalar field mass m_s, we first find the statistical error of the variable

$$m_s^2 = \frac{\nu}{m^2} = \frac{\nu\eta^{6/5}}{\mathcal{M}^2} \tag{A1}$$

as a function of the statistical errors of $\nu, \mathcal{M}, \eta$.

The error on a parameter θ^a can be obtained by inverting the Fisher matrix

$$\Gamma_{ab} = \left(\frac{\partial h}{\partial\theta^a}\Big|\frac{\partial h}{\partial\theta^b}\right) . \tag{A2}$$

If we consider the logarithm of a parameter, say $\theta^{\bar{a}} = \ln\theta^a$, as we do here for $\mathcal{M}$ and η, then obviously

$$\Gamma_{\bar{a}b} = \left(\frac{\partial h}{\partial\ln\theta^a}\Big|\frac{\partial h}{\partial\theta^b}\right) = \theta^a\Gamma_{ab} . \tag{A3}$$

So switching to the logarithm of a parameter corresponds to a rescaling of the corresponding matrix element by θ_a. For the elements of the variance-covariance matrix Σ^{ab} we have

$$\Sigma^{\bar{a}\bar{a}} = \Gamma^{-1}_{\bar{a}\bar{a}} = (\theta^a)^{-2}\Gamma^{-1}_{aa} , \quad \Sigma^{\bar{a}b} = (\theta^a)^{-1}\Gamma^{-1}_{ab} . \tag{A4}$$

The transformation of the variance-covariance matrix under a change of variables is given (e.g.) in Appendix A of [50]. Suppose we are given functional relations $y_i(x_j)$, and introduce the Jacobian $D_x^{ij} = \frac{\partial y_i}{\partial x_j}$. Following the notation of that paper, the result reads

$$\Sigma_x = (D_x)^{-1}\Sigma_y((D_x)^T)^{-1} . \tag{A5}$$

For reasons explained in Appendix A of [50], by considering the diagonal components of the error matrix in the new variables y we get a conservative estimate of the errors. In conclusion, if $x_i = (\nu\,, \ln\mathcal{M}\,, \ln\eta)$ we can write

$$\begin{aligned}\sigma^2_{m_s^2} &= \Sigma_{m_s^2\, m_s^2} = \sum_{ij}\frac{\partial m_s^2}{\partial x_i}\frac{\partial m_s^2}{\partial x_j}\Sigma_{ij} = \\ &= \frac{\eta^{12/5}}{\mathcal{M}^4}\left[\Sigma_{\nu\nu} + \frac{36}{25}\nu^2\Sigma_{\ln\eta\,\ln\eta} + 4\nu^2\Sigma_{\ln\mathcal{M}\,\ln\mathcal{M}}\right. \\ &\left. + \frac{12}{5}\nu\Sigma_{\nu\,,\ln\eta} - 4\nu\Sigma_{\nu\,\ln\mathcal{M}} - \frac{24}{5}\nu^2\Sigma_{\ln\eta\,\ln\mathcal{M}}\right] .\end{aligned} \tag{A6}$$

Because we assume that $\nu = 0$ in the background, this expression reduces to

$$\sigma_{m_s^2} = \frac{\sqrt{\Sigma_{\nu\nu}}}{m^2} = \frac{\sigma_\nu}{m^2} . \tag{A7}$$

Following common use in the gravitational-wave literature, we will ignore subtle issues associated with the fact that $m_s > 0$ and simply use $\sigma_{m_s} = \sqrt{\sigma_{m_s^2}}$ to obtain the conservative estimate given in Eq. (23) of the main text: if there is a 68% probability of finding $m_s^2 < \sigma_{m_s^2}$, then there is the same probability of finding $m_s < \sqrt{\sigma_{m_s^2}}$.

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