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$U(2)^5$ flavor symmetry and lepton universality violation in $W \rightarrow \tau \bar{\nu}_\tau$

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The seeming violation of universality in the τ lepton coupling to the W boson suggested by LEP II data is studied using an Effective Field Theory (EFT) approach. Within this framework we explore how this feature fits into the current constraints from electroweak precision observables using different assumptions about the flavor structure of New Physics, namely $[U(2) \times U(1)]^5$ and $U(2)^5$. We show the importance of leptonic and semileptonic tau decay measurements, giving 3 – 4 TeV bounds on the New Physics effective scale at 90% C.L. We conclude under very general assumptions that it is not possible to accommodate this deviation from universality in the EFT framework, and thus such a signal could only be explained by the introduction of light degrees of freedom or New Physics strongly coupled at the electroweak scale.

I. INTRODUCTION

One of the features of the Standard Model (SM) of particle physics is the universality of the lepton couplings, i.e. the fact that the coupling of the W^\pm to the leptons does not depend on their flavor. However the experimental results from LEP-II on this issue [1–5] showed a slight deviation from universality coming from the third family, giving [6]:

$$R_{\tau\ell}^W = \frac{2 \text{BR}(W \rightarrow \tau \bar{\nu}_\tau)}{\text{BR}(W \rightarrow e \bar{\nu}_e) + \text{BR}(W \rightarrow \mu \bar{\nu}_\mu)} = 1.055(23), \quad (1)$$

resulting in 2.4 standard deviations¹ (all correlations included) from the SM prediction $R_{\tau\ell}^W|_{\text{SM}} = 0.999$ [8], which uncertainty is negligible compared with the experimental error. Recalling also the following ratio:

$$R_{\mu e}^W = \text{BR}(W \rightarrow \mu \bar{\nu}_\mu) / \text{BR}(W \rightarrow e \bar{\nu}_e) = 0.983(18), \quad (2)$$

and the correspondent SM prediction $R_{\mu e}^W|_{\text{SM}} = 1.000$, it can be concluded that the two lightest families seem to attach to the universality principle. Although the discrepancy in Eq. (1) is not large, it is a result that has been around for already seven years and it is worth studying its possible implications on the present experimental scenario. The confirmation or refutation of this measurement is obviously very important, since such a violation by the third family would be a clear indication of New Physics (NP) [9, 10]. However, it will not be easy for the LHC to reach such a precision in this observable, given the theoretical uncertainties associated to a hadronic machine. For this reason it is interesting to check indirectly this anomaly through its interplay with other related measurements.

Precision electroweak observables (EWPO), as well as other precise low energy measurements, provide constraints

on new models looking for deviations that could foresee the NP structure. We study in this article if it is possible to accommodate the apparent discrepancy on the $W \rightarrow \tau \bar{\nu}_\tau$ channel within the present situation provided by EWPO, where essentially no disagreements have been found. In particular, lepton universality has been tested successfully at the per-mil level in $Z \rightarrow \ell^+ \ell^-$ [6] and $\tau \rightarrow \nu_\tau \ell \nu_\ell$ decays (see e.g. Table 3 in Ref. [11]), what makes very challenging to find a NP explanation for the large anomaly shown in Eq. (1). Just for the sake of illustration, we show the values obtained in leptonic Z decays [6]:

$$\begin{aligned} \text{BR}(Z \rightarrow \mu^+ \mu^-) / \text{BR}(Z \rightarrow e^+ e^-) &= 1.001(3), \\ \text{BR}(Z \rightarrow \tau^+ \tau^-) / \text{BR}(Z \rightarrow \mu^+ \mu^-) &= 1.001(3), \end{aligned} \quad (3)$$

in good agreement with the SM predictions, 1.000 and 0.998 respectively.

Instead of adhering to a specific model we will follow an Effective Field Theory approach, where NP is parameterized by a tower of higher-dimensional operators [12–14]. All NP theories which spectrum does not contain new light mass physical states (in comparison to those of the SM), that are weakly coupled at the electroweak scale and invariant under the SM gauge symmetries reduce at lower energies to the same effective Lagrangian, feature that makes this EFT approach very appealing. Guided by the above-mentioned experimental data on lepton universality, we will consider different frameworks where the New Physics does not affect operators involving first and second generation fermions. As we will explain, this can be implemented through the adoption of specific flavor symmetries.

For the numerical analysis, we greatly benefit from Ref. [15], where constraints on these effective operators were obtained via a global fit to precision electroweak data. We modify the associated fitting code to introduce additional observables and operators². These fit procedures are a powerful

¹ The result given in Eq. (1) is obtained from the PDG fit to the branching ratios of the W [6], that uses LEP2 and $p\bar{p}$ colliders data. It is worth mentioning that considering only LEP2 data the discrepancy grows to 2.8σ [5] (2.6σ using only published data [7]), all correlations included.

² The code can be freely downloaded at the web page <http://ific.uv.es/lhcfpheno/>.

tool to analyze the impact of current constraints on different models.

We apply this method to study the possible NP effects in leptonic W decays allowed by electroweak precision data. We will emphasize the role that the leptonic tau decay and the exclusive channel $\tau^- \rightarrow \pi^- \nu_\tau$ play in constraining specific directions of the parameter space of our theory, and the need to include these observables in this kind of analyses. We will see that the observed departure from universality cannot be accommodated within the current experimental scenario under quite general assumptions. Thus in order to be able to explain the observed deviation from lepton universality as a genuine NP effect, it seems to be necessary to resort to a different description of NP that could involve the introduction of new light degrees of freedom or a strongly interacting sector.

A closely related issue driven by the $W\ell\nu_\ell$ vertex is the ratio of widths involving the leptonic decays of pseudoscalar heavy mesons, $P \rightarrow \ell \bar{\nu}_\ell$. Accordingly, if any violation of universality is at work it also should be exposed in ratios of these decays into different charged leptons. Similarly, any modification of the SM coupling of W with the tau lepton could show up, due to gauge symmetry, in the anomalous magnetic moment of the tau. We will comment how our results translate into these subjects.

In the next section the EFT framework is introduced, along with different flavor symmetries and the relevant effective operators. In Sec. III we identify the operators that can generate a lepton universality violation in the third family, whereas in Sec. IV we analyze through a global fit the bounds on these operators from EWPO and other low-energy measurements. Sec. V is devoted to study the sensitivity of the leptonic decays of heavy mesons to the lepton universality violation, and Sec. VI contains our conclusions. An Appendix collects several theoretical expressions not included in the main text.

II. THE EFFECTIVE FIELD THEORY FRAMEWORK

Effective Field Theories embody the features, and particularly the dynamics, of the underlying theory. The astonishing performance of the SM suggests that whatever theory we find at higher energies has to reduce, upon integration of the relevant heavier degrees of freedom, to the key properties of the SM: symmetries and fields, that become its EFT. It is clear, though, that this approach breaks down if the underlying new physics contains physical states with mass $M \ll 1$ TeV, possibility that we do not consider in the present analysis. In this case the appropriate EFT should include that spectrum and its dynamics. In order to properly define our EFT setting we need moreover to assume that the new theory above the SM is weakly coupled at the weak scale, so that the gauge $SU(2)_L \times U(1)_Y$ symmetry is linearly realized.

The trail left in the procedure of integrating out heavier degrees of freedom is a Lagrangian with higher dimensional op-

erators that respect its symmetry and content [12–14]:

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} \sum_a \hat{\alpha}_a^{(5)} \mathcal{O}_a^{(5)} + \frac{1}{\Lambda^2} \sum_a \hat{\alpha}_a^{(6)} \mathcal{O}_a^{(6)} + \dots, \quad (4)$$

where \mathcal{L}_{SM} is the SM Lagrangian, Λ is the NP energy scale and $\mathcal{O}_a^{(n)}$ are $SU(2)_L \times U(1)_Y$ gauge-invariant operators of dimension n built with SM fields (including the standard Higgs boson). Finally $\hat{\alpha}_a^{(n)}$ are the dimensionless Wilson coefficients that carry the information of the underlying dynamics at the Λ scale and are expected to be of $\mathcal{O}(1)$.

The only gauge-invariant operator of dimension five violates lepton number, and thus it can be safely neglected under the assumption that the violation of that symmetry occurs at scales much higher than $\Lambda \sim 1$ TeV. Then the first order corrections to the SM predictions come from dimension-six operators. The contribution from these operators involve terms proportional to v^2/Λ^2 , vE/Λ^2 and E^2/Λ^2 , where $v \approx 174$ GeV is the vacuum expectation value of the Higgs field and E is the energy scale of the process considered. In order to be consistent with the truncation of the effective Lagrangian (4) we work at linear order in the above ratios, i.e. keeping only the contributions coming from the interference of the SM and dimension-six operators.

In this article we consider the study of the apparent violation of universality in the couplings of W to leptons within the above EFT framework, with the goal of finding out if the observed deviation can be explained in terms of NP effects once constraints from precise electroweak observables are taken into account. Motivated by the data and for the sake of simplicity, we will assume two different flavor symmetries that we introduce in the next subsections.

To set the stage for this discussion, we explain first the simpler case of $U(3)^5$ flavor symmetry. In the absence of Yukawa couplings, the SM Lagrangian shows a $U(3)^5 = U(3)_q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_e$ flavor symmetry, corresponding to the independent rotation of each SM fermion field: the quark and lepton doublets q and ℓ and the up-quark, down-quark and charged lepton singlets u , d and e . We can also decompose this symmetry group in the following way:

$$SU(3)^5 \times U(1)_L \times U(1)_B \times U(1)_Y \times U(1)_{PQ} \times U(1)_e \quad (5)$$

where the five global $U(1)$ symmetries can be identified with the total lepton and baryon number, the hypercharge, the Peccei-Quinn symmetry and a remaining global symmetry that we choose to be the rotation of the charged lepton singlet. In the presence of Yukawa couplings this flavor symmetry breaks down to the subgroup $G = U(1)_L \times U(1)_B \times U(1)_Y$.

Requiring that the higher dimensional operators respect the $U(3)^5$ flavor symmetry reduces significantly their number, suppresses undesired Flavor Changing Neutral Current effects and leads to the Minimal Flavor Violation (MFV) framework after the introduction of the Yukawa spurions [16]. The complete list of the twenty-one dimension-six $U(3)^5$ invariant operators can be found in Refs. [17, 18], where this flavor symmetry was assumed in the context of an EFT analysis of electroweak precision data. As an example we show here the three

operators that do not contain fermions:

$$\begin{aligned} O_{WB} &= (h^\dagger \tau^a h) W_{\mu\nu}^a B^{\mu\nu}, \quad O_h^3 = |h^\dagger D_\mu h|^2, \\ O_W &= \epsilon_{abc} W_\mu^{a\nu} W_\nu^{b\lambda} W_\lambda^{c\mu}, \end{aligned} \quad (6)$$

where we follow, with minor modifications, the notation and conventions of Ref. [13]: h is the Higgs boson doublet; τ^a are the Pauli matrices; $W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g \varepsilon_{ijk} W_\mu^j W_\nu^k$, $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and the covariant derivative reads $D_\mu = \partial_\mu - i \frac{g}{2} \tau^i W_\mu^i - i g' Y B_\mu$, with hypercharge $Y(h) = 1/2$.

It is clear that in this special framework it is impossible to generate any departure from lepton universality, as the $U(3)^5$ symmetry allows only for flavor independent NP contributions. For this reason we will relax this symmetry group to smaller groups where the third family is singled out.

A. $[U(2) \times U(1)]^5$ flavor symmetry

Motivated by the experimental observations shown in Eqs. (1) and (2), it is an interesting possibility to assume the flavor symmetry $[U(2) \times U(1)]^5$, that singularizes the third family with respect to the light ones, allowing for different NP contribution to the processes involving the heavy fermions: top, bottom and, in particular, τ and ν_τ .

This framework was indeed studied in Ref. [15], and we will use the same notation, in which q_p , ℓ_p , u_p , d_p and e_p ($p = 1, 2$) represent only the two first generations of fermions, whereas Q , L , t , b and τ represent the third family fields. The new notation makes clear which combinations of flavor indices are allowed by the flavor symmetry. The operators that do not involve fermions are the same as in the $U(3)^5$ case, whereas those involving one or two fermion bilinears split in several operators; for instance:

$$\begin{aligned} O_{he} &= i(h^\dagger D^\mu h)(\bar{e}\gamma_\mu e) \rightarrow O_{he} = i(h^\dagger D^\mu h)(\bar{e}\gamma_\mu e), \\ O_{h\tau} &= i(h^\dagger D^\mu h)(\bar{\tau}\gamma_\mu \tau). \end{aligned} \quad (7)$$

The list of invariant operators is much longer than in the $U(3)^5$ symmetric case, but not all the operators affect the EWPO. For this reason, and following Ref. [15], we do not include in our numerical analyses (i) operators involving top quarks; (ii) operators involving only third-generation fermions; or (iii) operators involving light quarks and third generation leptons³. Moreover, motivated by the experimental result shown in Eq. (2) and for the sake of simplicity we will assume that 2- and 4-fermion operators that only have light generation fermions can be neglected. In this way we are left with the following six operators with one fermion bilinear:

$$O_{hf_1} = i(h^\dagger D^\mu h)(\bar{f}_1 \gamma_\mu f_1) + \text{h.c.}, \quad (8)$$

$$O_{hf_2}^1 = i(h^\dagger D^\mu h)(\bar{f}_2 \gamma_\mu f_2) + \text{h.c.}, \quad (9)$$

$$O_{hf_2}^3 = i(h^\dagger D_\mu \tau^i h)(\bar{f}_2 \gamma^\mu \tau^i f_2) + \text{h.c.}, \quad (10)$$

where $f_1 = \tau, b$ and $f_2 = L, Q$. We also have the following four-fermion operators [15]:

$$\begin{aligned} O_{Lq}^3 &= (\bar{L}\gamma^\mu \tau^i L)(\bar{q}\gamma_\mu \tau^i q), \quad O_{\ell Q}^3 = (\bar{\ell}\gamma^\mu \tau^i \ell)(\bar{Q}\gamma_\mu \tau^i Q), \\ O_{\ell Q}^1 &= (\bar{\ell}\gamma^\mu \ell)(\bar{Q}\gamma_\mu Q), \quad O_{Qe} = (\bar{Q}\gamma^\mu Q)(\bar{e}\gamma_\mu e), \\ O_{eb} &= (\bar{e}\gamma^\mu e)(\bar{b}\gamma_\mu b), \quad O_{\ell b} = (\bar{\ell}\gamma^\mu \ell)(\bar{b}\gamma_\mu b), \\ O_{\ell L}^1 &= (\bar{\ell}\gamma^\mu \ell)(\bar{L}\gamma_\mu L), \quad O_{\ell L}^3 = (\bar{\ell}\gamma^\mu \tau^i \ell)(\bar{L}\gamma_\mu \tau^i L), \\ O_{Le} &= (\bar{L}\gamma^\mu L)(\bar{e}\gamma_\mu e), \quad O_{\ell\tau} = (\bar{\ell}\gamma^\mu \ell)(\bar{\tau}\gamma_\mu \tau), \\ O_{e\tau} &= (\bar{e}\gamma^\mu e)(\bar{\tau}\gamma_\mu \tau). \end{aligned} \quad (11)$$

B. $U(2)^5$ flavor symmetry

In the limit of vanishing Yukawa couplings the SM Lagrangian is invariant under the $[U(2) \times U(1)]^5$ group symmetry considered in the previous section. We can work with a more realistic scenario keeping the third family Yukawas $\mathcal{L} = y_t \bar{Q} h t + y_b \bar{Q} h b + y_\tau \bar{L} h \tau$ and neglecting only those of the two lightest generations. In this case the flavor symmetry breaks down to $U(2)^5 \times U(1)^3$, that we will just call $U(2)^5$, since the three $U(1)$ subgroups are simply the Lepton and Baryon number and Hypercharge of the third generation⁴.

Among the new operators that appear due to the reduction of the symmetry group, only the following four chirality-flipping operators will affect EWPO:

$$\begin{aligned} O_{\tau B}^t &= (\bar{L} \sigma^{\mu\nu} \tau) h B_{\mu\nu} + \text{h.c.}, \\ O_{bB}^t &= (\bar{Q} \sigma^{\mu\nu} b) h B_{\mu\nu} + \text{h.c.}, \\ O_{\tau W}^t &= (\bar{L} \sigma^{\mu\nu} \tau^i \tau) h W_{\mu\nu}^i + \text{h.c.}, \\ O_{bW}^t &= (\bar{Q} \sigma^{\mu\nu} \tau^i b) h W_{\mu\nu}^i + \text{h.c.} \end{aligned} \quad (12)$$

Their chirality-flipping structure translates, in the processes of our interest here, into contributions proportional to the fermion masses, i.e. suppressed by the factor m_f/v with respect to other NP contributions from dimension-six operators. Given that we focus here on the $W \rightarrow \tau \bar{\nu}_\tau$ decay we will not consider in the following the operators O_{bB}^t and O_{bW}^t .

III. $W \rightarrow \tau \bar{\nu}_\tau$ DECAY IN THE EFT FRAMEWORK

When the $U(2)^5$ flavor symmetry is assumed, the SM term and dimension-six operators contributing to the $W \rightarrow \tau \bar{\nu}_\tau$ decay are:

$$\mathcal{L}_{\text{EFT}} = i \bar{L} \not{D} L + \frac{1}{\Lambda^2} \{ \hat{\alpha}_{hL}^3 O_{hL}^3 + \hat{\alpha}_{\tau W}^t O_{\tau W}^t + \text{h.c.} \} \quad (13)$$

³ We noticed that the operator O_{Lq}^3 in Eq. (11) can be strongly constrained by the experimental value of the $\tau \rightarrow \pi \nu_\tau$ process and it is consequently included in our analysis. Ref. [15], not considering this observable, didn't include O_{Lq}^3 .

⁴ A recent analysis of the implications of current flavor data for the quark-sector component of this symmetry, i.e. $U(2)^3$, suitably broken by spurions à la MFV, can be found in Ref. [19].

$$\supset \frac{g}{\sqrt{2}} \left[\left(1 + 2\alpha_{hL}^3\right) \bar{\tau}_L \gamma^\mu \nu_\tau W_\mu^- + \frac{2}{gv} \alpha_{\tau W}^t \bar{\tau}_R \sigma^{\mu\nu} \nu_\tau W_{\mu\nu}^- \right],$$

where $W_\mu^- = (W_\mu^1 + iW_\mu^2)/\sqrt{2}$, and we have introduced the normalized couplings $\alpha \equiv \frac{v^2}{\Lambda^2} \hat{\alpha}$, that we assume to be real hereafter. Working at linear order in the α coefficients, the full decay width reads:

$$\Gamma(W \rightarrow \tau \bar{\nu}_\tau) = \frac{G_F M_W^3}{6\sqrt{2}\pi} (1 - w_\tau^2)^2 \times \left\{ (1 + 4\alpha_{hL}^3) \left(1 + \frac{w_\tau^2}{2}\right) + 6\sqrt{2} w_\tau \alpha_{\tau W}^t \right\}, \quad (14)$$

where $w_\tau = m_\tau/M_W$ and G_F is the tree level Fermi coupling constant defined by $G_F/\sqrt{2} = g^2/(8M_W^2)$. The new contributions to the decay width have the following features:

- There are only two dimension-six operators contributing to this process: \mathcal{O}_{hL}^3 and $\mathcal{O}_{\tau W}^t$. This can be seen if the equations of motion are properly used to reduce the number of operators in the effective basis, as done in Ref. [14], instead of using directly all the operators appearing in the original list of Ref. [12, 13].
- The lepton universality feature of the SM implies that $g_\tau = g$. The operator \mathcal{O}_{hL}^3 simply shifts the SM result in such a way that its effect can be encoded in the following redefinition:

$$g_\tau \equiv g(1 + \delta g_\tau) = g(1 + 2\alpha_{hL}^3). \quad (15)$$

This operator is allowed in the two flavor symmetries that we consider.

- The magnetic operator $\mathcal{O}_{\tau W}^t$ provides a new structure not present in the SM [20–22]. Contrarily to \mathcal{O}_{hL}^3 this is a chirality flipping operator and it gives a contribution suppressed by m_ℓ/M_W due to the derivative dependence. Assuming the $[U(2) \times U(1)]^5$ flavor symmetry this term vanishes.

In what follows we will consider the universality ratios $R_{\ell\ell'}^W = \Gamma(W \rightarrow \ell \nu_\ell)/\Gamma(W \rightarrow \ell' \nu_{\ell'})$, instead of the simple decay rate, in such a way that we do not have to worry about the NP corrections associated to the experimental determination of the Fermi constant G_F , since they cancel in the ratio.

IV. FIT PROCEDURE AND RESULTS

Once we have identified in the previous section the effective operators that can contribute to $R_{\tau\ell}^W$, generating a deviation from lepton universality, we study now the constraints that can be derived on these operators from EWPO and low-energy measurements.

Looking for example at the experimental result (3) it can be understood that one single operator will not be able to explain simultaneously the EWPO and the anomaly in the $W\tau\nu$ vertex shown in Eq. (1), due to the gauge symmetry that connects W and Z bosons. However, when several operators are

Classification	Std. Notation	Measurement
Atomic parity violation	$Q_W(Cs)$ $Q_W(Tl)$	Weak charge in Cs Weak charge in Tl
DIS	g_L^2, g_R^2 R^ν κ $g_V^{\nu e}, g_A^{\nu e}$	ν_μ -nucleon scattering (NuTeV) ν_μ -nucleon scatt. (CDHS, CHARM) ν_μ -nucleon scatt. (CCFR) ν -e scatt. (CHARM II)
Z-pole	Γ_Z σ_0 $R_{f=e,\mu,\tau,b,c}^0$ $A_{FB}^{0,f=e,\mu,\tau,b,c}$ $A_{f=e,\mu,\tau,s,b,c}$ $\sin^2 \theta_{eff}^{lep,t}$	Total Z width e^+e^- hadronic cross section Ratios of decay rates FB asymmetries Polarized asymmetries Hadronic charge asymmetry
LEP II fermion production	$\sigma_{f=q,c,b,\mu,\tau}$ $A_{FB}^{f=c,b,\mu,\tau}$ $d\sigma_e/d\cos\theta$	Total cross sections for $e^+e^- \rightarrow f\bar{f}$ FB asymmetries for $e^+e^- \rightarrow f\bar{f}$ $e^+e^- \rightarrow e^+e^-$ diff. cross section
W pair	$d\sigma_W/d\cos\theta$ M_W	$e^+e^- \rightarrow W^+W^-$ diff. cross section W mass
V_{CKM} unitarity	Δ_{CKM}	V_{ud} and V_{us} extractions [18]
τ decays	$\tau \rightarrow \nu_\tau \ell \bar{\nu}_\ell$ $\tau \rightarrow \nu_\tau \pi$	Leptonic τ decay ($\ell = e, \mu$) [6] Exclusive hadronic τ decay [6]
Anomalous magnetic moment	a_τ	$e^+e^- \rightarrow e^+e^-\tau^+\tau^-$ cross section [23]

TABLE I: Measurements included in this analysis. See Ref. [17] and references therein for detailed descriptions. References are shown only for the new observables.

present one can have cancellations between them and a careful numerical analysis is needed.

With that purpose we updated and modified the Mathematica code developed in Ref. [15], that included electroweak observables at the Z line and at higher energies and other low energy measurements. In addition we include the leptonic tau decay and the exclusive channel $\tau \rightarrow \pi \nu_\tau$, that have an experimental error well below the 1% level and a theoretical error under control. We consider also the anomalous magnetic moment of the tau lepton that, despite its very large experimental uncertainty, is able to constrain the magnetic operators poorly bounded by other observables. The associated formulas are collected in Appendix A and the complete list of the observables used in our analysis can be found in Table I.

We included in the program also the contribution to the different observables coming from the magnetic operators $\mathcal{O}_{\tau W}^t$ and $\mathcal{O}_{\tau B}^t$, not included in Ref. [15] since the $U(2)^5 \times U(1)^5$ symmetry was assumed in that work. The formulas for the Z decay rate can be found in Appendix A 1, whereas the formulas for $e^+e^- \rightarrow \tau^+\tau^-$ cross section have been taken from Ref. [22].

The leptonic decays of heavy pseudoscalar mesons (B^\pm , D^\pm , D_s^\pm) could in principle be considered in order to constrain NP effects in leptonic W decays, but they have not reached yet the necessary experimental precision: the relative

error of the current data on the decays into tau are approximately $\mathcal{O}(6\%)$ for D_s decays and $\mathcal{O}(20\%)$ for B decays, and some of the decays into muon and electron have not been seen yet, preventing a complete analysis of the lepton universality ratios. For these reasons, these observables have not been included in the fit. We will comment on them in Sec. V.

Concerning LHC measurements, the natural channels to analyze for the purpose of this paper are $pp \rightarrow \tau \bar{\nu} X$ and $pp \rightarrow \tau^+ \tau^- X$, where possible modifications of the $W\tau\nu$ vertex and its gauge counterpart $Z\tau^+\tau^-$ can be probed, but unfortunately there is no data available for these particular channels yet. On the theoretical side, the contribution to these processes coming from effective operators has been worked out in Refs. [24, 25] for first generation leptons. In any case, as we will see, the list of observables included in our fit is exhaustive enough to reach a solid answer to the possible lepton universality violations.

With the above-mentioned observables O^i , we build a standard χ^2 function as:

$$\chi^2(\alpha) = \sum_i [O_{\text{th}}^i(\alpha) - O_{\text{exp}}^i] [\sigma_o^2]_{ij}^{-1} [O_{\text{th}}^j(\alpha) - O_{\text{exp}}^j], \quad (16)$$

where the error matrix σ_o^2 includes the experimental error and the uncertainty on the SM prediction combined in quadrature. The theoretical value O_{th}^i contains the up-to-date SM prediction and the contribution of higher dimensional operators through interference with SM vertices, i.e. linear in the α_a couplings.

As a result of this fit we determine the value of the different Wilson coefficients α_a , with their relative errors and the corresponding correlations, or in other words the bounds on the different NP effective operators. In particular we are interested in the bounds associated to the two operators that could generate a lepton universality violation in the W decay (see Eq. (13)), and finally in the determination of the universality ratio $R_{\tau\ell}^W$ extracted from our fit, to be compared with the experimental determination given in Eq. (1).

A. (Semi)leptonic τ decays as precise electroweak observables

In a general analysis involving a big number of operators (free parameters in the fit) it is possible to encounter flat directions, i.e. directions in the parameter space that are not bounded by the experimental data. This means that some operators appear always in the same combination throughout all the observables considered in the fit and then only that combination can be constrained, and not each operator separately. In Ref. [15] four flat directions were identified in the particular fit we are using in this work. However, we show now how the addition of the leptonic tau decay to the list of EWPO included in the fit removes one of these flat directions.

In the limit of $U(2)^5$ flavor symmetry the rate for the lep-

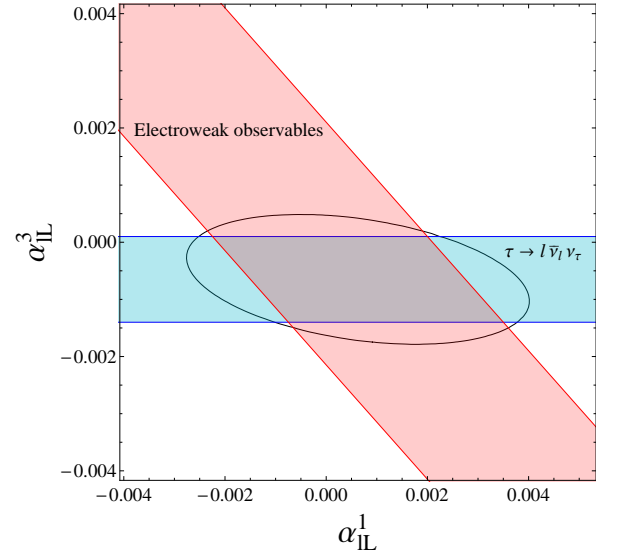


FIG. 1: Phenomenological constraints on the operators $O_{\ell L}^1$ and $O_{\ell L}^3$ from electroweak observables (red diagonal band) and leptonic tau decay rate (blu horizontal band). See Table I for the complete list of observables considered. The black ellipse is the 1σ C.L. region when considering all observables together.

tonic decay of the τ lepton reads⁵:

$$\Gamma_{\tau \rightarrow \nu_\tau \ell \bar{\nu}_\ell} = \frac{G_F^2 m_\tau^5}{192 \pi^3} \left\{ \left[1 + 4\alpha_{hL}^3 + 4\alpha_{h\ell}^3 - 4\alpha_{\ell L}^3 \right] \times \right. \\ \left. \times f\left(\frac{m_\ell^2}{m_\tau^2}\right) + 2\sqrt{2}\alpha_{\tau W}^t \frac{m_\tau}{M_W} g\left(\frac{m_\ell^2}{m_\tau^2}\right) \right\} (1 + \delta_{RC}), \quad (17)$$

where $\ell = e, \mu$, δ_{RC} contains the radiative corrections to the SM contribution [26] and

$$f(x) = 1 - 8x - 12x^2 \ln x + 8x^3 - x^4, \quad (18) \\ g(x) = 1 - 6x + 18x^2 - 10x^3 + 12x^3 \ln x - 3x^4.$$

In order to show the constraining power of the tau decays let us consider the simple situation in which only the operators $O_{\ell L}^1$ and $O_{\ell L}^3$ are not vanishing. As shown in Fig. 1 the electroweak observables, and in particular the $e^+e^- \rightarrow \tau^+\tau^-$ cross section and the forward-backward asymmetry, are able to constrain only the combination $O_{\ell L}^1 + O_{\ell L}^3$. The inclusion of the leptonic tau decay into the fit allows to reduce the one sigma C.L. region to the black ellipse. The two operators are then constrained at the 0.4% and 0.2% level, corresponding to an effective NP scale $\Lambda > 2.7$ TeV and $\Lambda > 4.1$ TeV (90% C.L.) respectively: very strong bounds that show the importance of leptonic tau decays as electroweak precision observable.

⁵ The operator corresponding to $\alpha_{h\ell}^3$ is defined in analogy with O_{hL}^3 in Eq. (10). This self-explanatory notation will be adopted hereafter for operators involving only light fermions. Although we neglect these operators in the subsequent numerical analysis we keep them in the analytic expressions for the sake of completeness.

A similar role is played by the pionic τ decay, where experimental results and SM calculations are also below the per-mil level of precision. The expression for the $\tau \rightarrow \pi \nu_\tau$ decay rate within our $U(2)^5$ flavor symmetric EFT framework is the following:

$$\Gamma_{\tau^- \rightarrow \pi^- \nu_\tau} = \frac{G_F^2 F_\pi^2}{8\pi} |V_{ud}|^2 m_\tau^3 \left(1 - \frac{M_\pi^2}{m_\tau^2}\right)^2 (1 + \delta'_{RC}) \times (1 + 4\alpha_{hL}^3 + 4\alpha_{hq}^3 - 4\alpha_{Lq}^3), \quad (19)$$

where F_π denotes the pion decay constant and δ'_{RC} radiative corrections [27]. It is convenient to work once again with a normalized ratio, namely:

$$R_{\tau/\pi} \equiv \frac{\Gamma_{\tau^- \rightarrow \pi^- \nu_\tau}}{\Gamma_{\pi^- \rightarrow \mu^- \bar{\nu}_\mu}} = \frac{m_\tau^3}{2m_\mu^2 M_\pi} \left(\frac{1 - \frac{M_\pi^2}{m_\tau^2}}{1 - \frac{m_\mu^2}{M_\pi^2}} \right)^2 (1 + \delta_{\tau/\pi}) \times (1 + 4(\alpha_{hL}^3 - \alpha_{h\ell}^3) - 4(\alpha_{Lq}^3 - \alpha_{\ell q}^3)). \quad (20)$$

where $\delta_{\tau/\pi} = 0.0016(14)$ denotes the radiative corrections to the SM contributions [28]. As we can see, this observable represents another probe of the α_{hL}^3 coefficient, and moreover it represents the only observable in our analysis sensitive to the α_{Lq}^3 coefficient. Comparing the experimental value of $R_{\tau/\pi}$ [6, 29] and its SM prediction we get a bound of $\Lambda > 3.1$ TeV (90% C. L.) on the NP effective scale for the four Wilson coefficients appearing in Eq. (20).

B. $[U(2) \times U(1)]^5$ symmetric case: results

In order to study if the $R_{\tau\ell}^W$ anomaly of Eq. (1) can be accommodated in our EFT framework as a genuine New Physics effect and not just a statistical fluctuation, we start with a single operator analysis where only the α_{hL}^3 is present and all the observables of Table I are included. In this case we obtain the expected strong bound:

$$R_{\tau\ell}^W = 0.9997 \pm 0.0015, \quad (21)$$

in good agreement with the SM prediction. As shown in Fig. 2, the very precise measurements of leptonic Z and τ decays dominate our fit, and makes impossible to accommodate the $R_{\tau\ell}^W$ anomaly.

Once we include additional operators, things become less intuitive because cancellations between operators are possible, opening the possibility to explain the $R_{\tau\ell}^W$ anomaly and the leptonic Z and τ decays at the same time.

As a first global analysis, we assume the $[U(2) \times U(1)]^5$ flavor symmetry and we include the 17 operators given in Eqs. (6) and (8-11). It is worth repeating that in order to simplify the discussion and given that the experimental data show no sign of NP related to the light families of fermions, we have assumed that the operators involving only light fermions can be neglected.

Somehow surprisingly we find that even with so many operators, the constraint on α_{hL}^3 is very strong, namely $-3.6 \times 10^{-3} \leq \alpha_{hL}^3 \leq -0.5 \times 10^{-3}$ at 90% C.L. Interestingly enough, this value is two sigmas away from zero, giving the following bound on the universality ratio:

$$R_{\tau\ell}^W = 0.991 \pm 0.004, \quad (22)$$

where we quoted the error at 1σ level in order to be comparable with the experimental result in Eq. (1). Thus we find the curious result that our fit is indeed able to accommodate a violation of lepton universality in the W decays, but *in the opposite direction* than the direct experimental measurement. The explanation for this fact is simple: when we take into account the effect of many higher-dimension operators, the fit introduces some non-zero Wilson coefficients in order to alleviate the small tensions between the experimental values and the SM predictions (see e.g. σ_{had}^0 in Fig. 2). These non-zero contributions then cancel in those observables where the agreement with the SM is perfect. And it turns out that one of these non-zero NP coefficients is α_{hL}^3 . This is exactly why global analyses are interesting: they can find regions on the parameter space where these cancellations between operators take place, offering new possibilities not accessible in single-operator analyses, and difficult to foresee in a naive analysis. Obviously the inclusion of $R_{\tau\ell}^W$ as an additional observable in our fit will reduce this “tension” moving the value of α_{hL}^3 closer to zero, giving a 90% C.L. bound:

$$-3.2 \times 10^{-3} \leq \alpha_{hL}^3 \leq -0.08 \times 10^{-3} \quad (23)$$

The conclusion is once more that we cannot accommodate the $R_{\tau\ell}^W$ along with our long list of precision observables, and thus we are forced to consider it a mere statistical fluctuation. Unlike the single operator case where it could be naively expected, this represents a non-trivial result in a fit with seventeen free parameters.

For the sake of completeness, let us mention that in a truly global $[U(2) \times U(1)]^5$ fit, where operators only involving light fermions (like e.g. $\mathcal{O}_{h\ell}^3$) are also included, the NP bounds become extremely weak and the current experimental value of $R_{\tau\ell}^W$ cannot be excluded anymore.

C. $U(2)^5$ symmetric case: results

Reducing the symmetry group to $U(2)^5$ introduces the chirality-flipping operators $\mathcal{O}_{\tau W}^t$ and $\mathcal{O}_{\tau B}^t$, offering additional NP contributions to the observables and higher cancellations between operators.

From the associated global fit⁶ with 19 free parameters, we get the following 90% bounds on the two operators involved in the W decays: $-3.7 \times 10^{-3} \leq \alpha_{hL}^3 \leq -0.6 \times 10^{-3}$ and

⁶ We do not include in this $U(2)^5$ -symmetry fit the leptonic polarization asymmetries A_ℓ , since they have been extracted assuming only vector and axial-vector couplings.

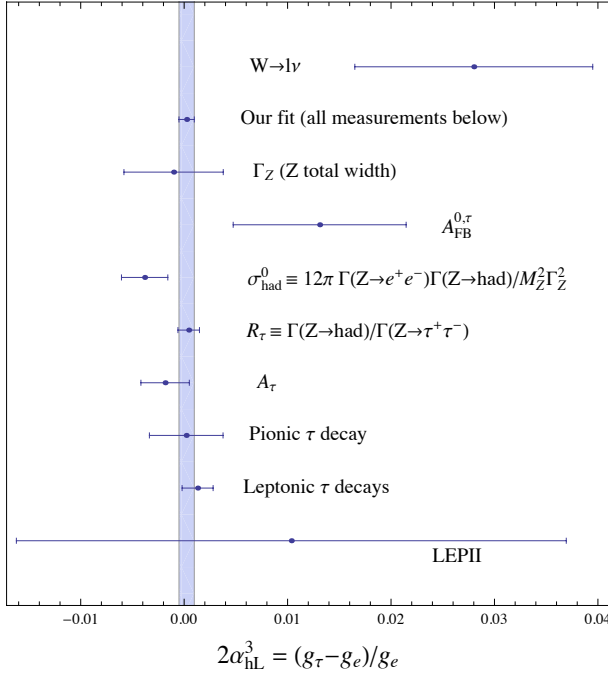


FIG. 2: Bounds obtained for the NP coefficient α_{hL}^3 from the different set of measurements included in our fit. Equivalently, these are the bounds on the deviation from lepton universality in the electroweak coupling to third generation leptons g_τ (see Eq. (15)). For comparison we also show the value obtained, using the experimental data in Eq. (1), from leptonic W decays (not included in our fit). $A_{FB}^{0,\tau}$ is the forward-backward asymmetry measured at LEP1 for tau pairs, A_τ includes the SLD measurement and the LEP1 total τ polarization and the LEP2 bound comes from τ pair cross sections and asymmetries. See PDG [6], chapter 10, for more details.

$0.04 \times 10^{-3} \leq \alpha_{\tau W}^t m_\tau / M_W \leq 5.0 \times 10^{-3}$, where we have explicitly shown the m_τ / M_W suppression that multiplies the $\alpha_{\tau W}^t$ coefficient in the observables. From these values we calculate the prediction for the universality ratio at 1σ level:

$$R_{\tau\ell}^W = 1.01 \pm 0.01. \quad (24)$$

While the constraints on α_{hL}^3 are very similar to the previous case, the presence of a second contribution from the magnetic operator increases the error (and the central value) of $R_{\tau\ell}^W$. This increase is however not enough to nicely accommodate the experimental value of $R_{\tau\ell}^W$ shown in Eq. (1).

V. LEPTONIC DECAYS OF HEAVY MESONS

The leading SM contribution to the $P^- \rightarrow \ell^- \bar{\nu}_\ell$ decays is given by the W exchange and hence it is interesting to point out how these decays get modified by possible deviations from family universality in the $W\ell\nu$ coupling. In particular we are interested in the D , D_s and B decays because they are heavy enough to decay into the tau lepton. Although two dimension-six operators modify the vertex of the W gauge boson with leptons, namely $\mathcal{O}_{h\ell}^3$ and \mathcal{O}_{eW}^t , only the first contributes to

the leptonic decay of heavy mesons, due to the fact that the tensor coupling has no spin-0 component.

In order to get rid of the hadronic uncertainties and the NP corrections to the Fermi constant or the CKM elements appearing in the individual decay widths, we will focus again on the ratio between the tau channel and a light lepton channel:

$$R_{\tau\ell}^P = \frac{\text{BR}(P \rightarrow \tau \bar{\nu}_\tau)}{\text{BR}(P \rightarrow \ell \bar{\nu}_\ell)}, \quad (25)$$

where $\ell = e, \mu$. The effective Lagrangian that mediates these decays, including linear corrections in the α coefficients, can be found in Eq. (34) of [18]. Assuming the $U(2)^5$ flavor symmetry we find the following expressions for the ratios⁷:

$$R_{\tau\ell}^{D(s)} = \frac{h_{D(s)}(m_\tau)}{h_{D(s)}(m_\ell)} \{1 + 4(\alpha_{hL}^3 - \alpha_{h\ell}^3) - 4(\alpha_{LQ}^3 - \alpha_{\ell Q}^3)\},$$

$$R_{\tau\ell}^B = \frac{h_B(m_\tau)}{h_B(m_\ell)} \{1 + 4(\alpha_{hL}^3 - \alpha_{h\ell}^3) - 4(\alpha_{LQ}^3 - \alpha_{\ell Q}^3)\}, \quad (26)$$

where $h_P(m) = m^2(1 - m^2/M_P^2)^2$.

As expected, we find that the α_{hL}^3 coefficient modifies these ratios. However, the bound on this coefficient from our analysis of EWPO and low-energy measurements is below the percent level (see Sec. IV), a precision very far from current experimental results in these decays. The only ratio where we actually have a value, and not just an upper or lower limit, is $R_{\tau\mu}^{D_s} = 9.2(7)$ [6]. We can compare this $\sim 8\%$ experimental error with the $\sim 0.7\%$ determination of the $R_{\tau/\pi}$ ratio, where exactly the same linear combination of NP couplings is probed, as shown in Eq. (20). This level of precision can actually be considered a benchmark sensitivity for future D and D_s meson experiments to become competitive in the NP search within our EFT framework.

On the other hand the leptonic B decays are interesting since they probe a different linear combination of NP coefficients, and therefore are complementary to other observables.

VI. CONCLUSIONS

In the SM the coupling of leptons to the gauge bosons is flavor blind, a property that has been tested successfully in several different observables and experiments, sometimes even at the per-mil level of precision. The latest results from the LEP2 experiment in 2005 showed however a quite sizable deviation ($\sim 5\%$) from universality in the $W\ell\nu_\ell$ coupling of more than two sigmas when comparing the third leptonic family with the two light ones, as shown in Eq. (1).

We have considered in this article the possibility that this deviation represents a real NP effect. We have performed an

⁷ In the B decays we neglect the contribution from a new $U(2)^5$ -invariant operator $\mathcal{O}_{Qb\tau} = (\bar{L}\tau)(\bar{b}Q) + \text{h.c.}$, since this operator does not affect the EWPO included in our fit.

Effective Field Theory analysis where the NP effects are parameterized by a series of Wilson coefficients α_i , that appear in the effective Lagrangian multiplying dimension-six operators. In order to reduce the number of unknown coefficients and motivated by the possible deviation from lepton universality in the $W\tau\nu_\tau$ vertex, we have assumed different flavor symmetries where the third family plays a special role.

Within this framework we have analyzed if it is possible to accommodate the $R_{\tau\ell}^W$ anomaly of Eq. (1) as a real NP effect without spoiling the nice agreement between SM predictions and EWPO observables. As expected, it is not possible to do such a thing with just one effective operator at play, due mainly to the very precise Z and τ leptonic decays, as nicely shown in Fig. 2. More surprisingly we have found that EWPO are such strong constraints that not even in a global analysis where all the operators affecting the third family are present one can accommodate the $R_{\tau\ell}^W$ anomaly.

Should this departure from universality be confirmed by new data, then our analysis disfavor the possibility of explaining it through a weakly coupled theory standing at the TeV scale, unless a quite non-trivial flavor structure occurs. Instead, it would be necessary to resort to a different description of NP that could involve the introduction of new light degrees of freedom or a strongly interacting sector with flavor dependent couplings to leptons. For example previous studies of this deviation from universality in W decays have focused on the possibility that pair production of light charged Higgs bosons, almost degenerate with the W and decaying largely into heavy fermions, could mimic $W \rightarrow \tau \bar{\nu}_\tau$ decays [30, 31]. Modifications on the electroweak gauge group in order to singularize the third family have also been considered [32].

Last but not least we have shown the importance of the current measurements in leptonic and semileptonic τ decays as New Physics constraining observables that probe new directions in the parameter space of our EFT framework, and we have analyzed the sensitivity of the leptonic decays of pseudoscalar mesons to the violations of lepton universality.

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Appendix A: Theoretical expressions

We collect here several theoretical results of observables that have been employed in our analyses and that we do not include in the main text.

1. Leptonic Z decays

The effective Lagrangian contributing to the $Z \rightarrow \tau^+ \tau^-$ decay in the flavour $U(2)^5$ symmetry group is given by:

$$\mathcal{L}_{\text{EFT}} = i \bar{L} \not{D} L + \frac{1}{\Lambda^2} \{ \hat{\alpha}_{hL}^1 \mathcal{O}_{hL}^1 + \hat{\alpha}_{hL}^3 \mathcal{O}_{hL}^3 + \hat{\alpha}_{h\tau} \mathcal{O}_{h\tau} + \hat{\alpha}_{\tau W}^t \mathcal{O}_{\tau W}^t + \hat{\alpha}_{\tau B}^t \mathcal{O}_{\tau B}^t + \text{h.c.} \}, \quad (\text{A.1})$$

and the corresponding decay width is:

$$\begin{aligned} \Gamma(Z \rightarrow \tau^+ \tau^-) &= \frac{G_F M_Z^3}{24 \sqrt{2} \pi} \sqrt{1 - 4z_\tau^2} \\ &\times \left\{ (1 - 4v^-) (1 - 4z_\tau^2) - 24 \sqrt{2} z_\tau t^Z (4s_W^2 - 1) \right. \\ &\left. + [(4s_W^2 - 1) - 4v^+] (4s_W^2 - 1) (1 + 2z_\tau^2) \right\}, \quad (\text{A.2}) \end{aligned}$$

where:

$$\begin{aligned} v^- &= \alpha_{h\tau} - \alpha_{hL}^1 - \alpha_{hL}^3, \\ v^+ &= \alpha_{h\tau} + \alpha_{hL}^1 + \alpha_{hL}^3, \\ t^Z &= c_W \alpha_{\tau W}^t + s_W \alpha_{\tau B}^t, \end{aligned} \quad (\text{A.3})$$

being s_W and c_W the sine and cosine of the weak angle θ_W respectively, and $z_\tau = m_\tau/M_Z$. As discussed in relation with Eq. (14) it can be noticed that the linear contribution of the tensor operators $\mathcal{O}_{\tau W}^t$ and $\mathcal{O}_{\tau B}^t$ is suppressed by the lepton mass over the Z mass. Moreover operators $\mathcal{O}_{h\tau}$, \mathcal{O}_{hL}^1 and \mathcal{O}_{hL}^3 simply modify the weight of the SM vertices.

2. Anomalous magnetic moment of the tau lepton

The tensor operators $\mathcal{O}_{\tau W}^t$ and $\mathcal{O}_{\tau B}^t$ provide a local contribution to the anomalous magnetic moment of the tau lepton. The generic $\gamma \tau \bar{\tau}$ vertex is given by $-i e \varepsilon^\mu(q) \bar{u}(p') V_\mu u(p)$ where:

$$\begin{aligned} V_\mu &= F_1(q^2) \gamma_\mu + i F_2(q^2) \sigma_{\mu\nu} \frac{q^\nu}{2m_\tau} \\ &+ F_3(q^2) \gamma^5 \sigma_{\mu\nu} \frac{q^\nu}{2m_\tau}, \end{aligned} \quad (\text{A.4})$$

and the anomalous magnetic moment of the τ lepton is given by $a_\tau = (g_\tau - 2)/2 = F_2(0)$. By using \mathcal{L}_{EFT} in Eq. (4) we find the following expression for the τ lepton anomalous magnetic moment:

$$a_\tau = a_\tau^{\text{SM}} + \frac{2\sqrt{2}}{s_W} \frac{m_\tau}{M_W} (c_W \alpha_{\tau B}^t - s_W \alpha_{\tau W}^t), \quad (\text{A.5})$$

where the first term in the right-hand side is the SM contribution: $a_\tau^{\text{SM}} = 1.17721(5) \times 10^{-3}$ [33]. The current experimen-

tal result is given by $-0.052 < a_\tau < 0.013$ at 95% C.L. [23], though other analyses establish more stringent limits [22].

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