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“Dark” Z implications for parity violation, rare meson decays, and Higgs physics
Hooman Davoudiasl, Hye-Sung Lee, and William J. Marciano
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I. INTRODUCTION

The existence of cosmic dark matter is now essentially established. It appears to constitute about 22% of the energy-matter budget of the Universe, significantly more than the 4% attributed to visible matter [1]. Nevertheless, the exact nature of dark matter remains mysterious. Is it mainly a new, cosmologically stable, elementary particle that interacts with our visible world primarily through gravity or does it have weak interaction properties that allow it to be detected at high energy accelerators or in sensitive underground cryogenic experiments? Both avenues of exploration are currently in progress. A discovery would revolutionize our view of the Universe and the field of elementary particle physics.

Recently, a possible generic new property of dark matter has been postulated [2] to help explain various astrophysical observations of positron excesses [3]. The basic idea is to introduce a new $U(1)_d$ gauge symmetry mediated by a relatively light $Z_d$ boson that couples to the “dark” charge of hidden sector states, an example of which is dark matter. Such a boson has been dubbed the “dark” photon, secluded or hidden boson, etc [4]. Within the framework adopted in our work, however, we refer to it as the “dark” $Z$ because of its close relationship to the ordinary $Z$ of the Standard Model (SM) via $Z$-$Z_d$ mixing. Consequences of that mixing will be explored in this paper, where after describing the basic characteristics of the dark $Z$, we provide constraints on its properties imposed by low energy parity violating experiments such as atomic parity violation and polarized electron scattering. Future sensitivities are also discussed. We then briefly describe bounds on the mixing currently obtained from rare $K$ and $B$ decays along with the potential for future improvements.

Perhaps the most novel prediction from $Z$-$Z_d$ mixing is its implications for high energy experiments. In particular, it leads to a potentially observable new type of Higgs decay, $H \rightarrow ZZ_d$, with pronounced discovery signatures which we describe [5]. We also discuss a 2 Higgs doublet (2HD) model that exhibits all the features of our general $Z$-$Z_d$ mixing scenario. (Some works of similar spirit, but different contexts can be found in, for example, Refs. [6–10].)

II. SET UP

We begin with what might be called the usual “dark” boson scenario. It is assumed that a new $U(1)_d$ gauge symmetry of the dark matter or any hidden sector interacts with the $SU(3)_C \times SU(2)_L \times U(1)_Y$ of the SM via kinetic mixing between $U(1)_Y$ and $U(1)_d$ [11]. That effect is parametrized by a gauge invariant $B_{\mu\nu}X^{\mu\nu}$ interaction

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{1}{2\cos\theta_W}B_{\mu\nu}X^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad X_{\mu\nu} = \partial_\mu Z_{d\nu} - \partial_\nu Z_{d\mu}$$

(1)

with $\varepsilon$ a dimensionless parameter which is unspecified (the normalization of the term proportional to $\varepsilon$ has been chosen to simplify the notation in the results that follow). At the level of our discussion, $\varepsilon$ is a potentially infinite counter term necessary for renormalization. Its finite renormalized value is to be determined by experiment. In most discussions, $\varepsilon$ is assumed to be $\lesssim O(\text{few} \times 10^{-3})$. It could, of course, be much smaller [12].

After removal of the $\varepsilon$ cross-term by field redefinitions

$$B_\mu \rightarrow B_\mu + \frac{\varepsilon}{\cos\theta_W}Z_{d\mu}$$

(2)

leading to

$$A_\mu \rightarrow A_\mu + \varepsilon Z_{d\mu}$$

$$Z_\mu \rightarrow Z_\mu - \varepsilon \tan\theta_W Z_{d\mu}$$

(3)
for the photon and Z boson fields, one is left with an induced coupling of the $Z_d$ to the usual electromagnetic current (with summation over all charged quarks and leptons)

$$\mathcal{L}_{\text{int}} = -e\varepsilon J_\mu^{em} Z_d\mu$$

$$J_\mu^{em} = \sum_f Q_f f^{\gamma\mu} f + \cdots$$ \hspace{1cm} (4)

where the ellipsis includes $W^\pm$ current terms and $Q_f$ is the electric charge ($Q_e = -1$). (It is generally assumed that $U(1)_d$ is broken and the $Z_d$ becomes massive via a scalar Higgs singlet or a Stueckelberg mass generating mechanism [13, 14, 1].) Note also that the induced coupling of $Z_d$ to the weak neutral current via Eq. (3) is highly suppressed at low energies in the above basic scenario due to a cancellation between $\varepsilon$ dependent field redefinition and $Z$-$Z_d$ mass matrix diagonalization effects induced by $\varepsilon$ (see, for example, Ref. [15] and our appendix). The phenomenology of the interaction in Eq. (4) has been well examined as a function of $m_{Z_d}$ and $\varepsilon$ (e.g. Refs. [16–18]). With the assumption 10 MeV $\lesssim m_{Z_d} \lesssim$ 10 GeV and $\varepsilon \lesssim O(\text{few} \times 10^{-3})$, bounds have been given and new experiments are underway to find the $Z_d$ via its production in high intensity electron scattering [19]. We will consider this same mass range for our phenomenological analysis in this work. The lower bound $m_{Z_d} \gtrsim 10$ MeV is required in order that astrophysical and beam-dump processes do not severely constrain the interactions of dark $Z$ which, as discussed below, develops an axion-like component for $m_{Z_d} \to 0$.

Because of its coupling to our particle world via the small electromagnetic current coupling in Eq. (4), $Z_d$ is often called the “dark” photon (even though that name was originally intended for a new weakly coupled long-range interaction [20]).

Here, we generalize the above $U(1)_d$ kinetic mixing scenario to include $Z$-$Z_d$ mass mixing by introducing the $2 \times 2$ mass matrix

$$M_0^2 = m_{Z_d}^2 \left( \begin{array}{cc} 1 & -\varepsilon_Z \\ -\varepsilon_Z & m_{Z_d}^2/m_{Z}^2 \end{array} \right)$$ \hspace{1cm} (5)

where $m_{Z_d}$ and $m_Z$ (with $m_{Z_d}^2 \ll m_{Z}^2$) represent the “dark” $Z$ and SM $Z$ masses in the limit of no mixing. The $Z$-$Z_d$ mixing is parametrized by

$$\varepsilon_Z = \frac{m_{Z_d}}{m_Z} \delta,$$ \hspace{1cm} (6)

with $\delta$ a small model dependent quantity. We ignore the $\varepsilon$ contribution from Eq. (2) in the mass matrix, since its inclusion would only affect this part of our discussion at $O(\varepsilon^2)$ (see appendix). The assumed off-diagonal $m_{Z_d}$ dependence in Eq. (6) allows smooth $m_{Z_d} \to 0$ behavior for all $\varepsilon_Z$-induced amplitudes involving $Z_d$, even those stemming from non-conserved current interactions. Also, for simplicity, ordinary fermions are assumed to be neutral under $U(1)_d$, i.e. they do not carry any fundamental dark charge. Their only couplings to $Z_d$ are induced through $\varepsilon$ and $\varepsilon_Z$. More general cases are possible and interesting, but beyond the scope of this paper.

So far, $\delta$ is rather arbitrary, although $0 \leq \delta^2 < 1$ is required to avoid an infinite-range or tachyonic $Z_d$. One expects $\delta$ to be small because of the disparity of $m_Z$ and $m_{Z_d}$. We later show that low energy phenomenology actually requires $\delta^2 \lesssim 0.006$, while rare $K$ and $B$ decays have sensitivity to $\delta^2 \lesssim 10^{-4} - 10^{-6}$ for low mass $Z_d$. We will also demonstrate how the form in Eq. (5) naturally emerges in a simple 2HD extension of the SM, the details of which will be discussed in the appendix. However, we emphasize that our general results follow from $Z$-$Z_d$ mixing through a generic mass matrix of the form in Eq. (5) and are not exclusively tied to any specific expanded Higgs sector. That mixing could, for example, potentially arise from loop effects or dynamical symmetry breaking.

Overall, mixing leads to mass eigenstates $Z$ and $Z_d$

$$Z = Z^0 \cos \xi - Z_d^0 \sin \xi$$

$$Z_d = Z^0 \sin \xi + Z_d^0 \cos \xi$$ \hspace{1cm} (7)

where (see appendix)

$$\tan 2\xi \simeq \frac{2m_{Z_d}\delta}{m_Z^2} = 2\varepsilon_Z.$$ \hspace{1cm} (8)

It is expected that $\sin \xi$ is very small (partly due to the assumed smallness of $m_{Z_d}/m_Z$ and partly due to small $\delta$) and does not measurably affect $Z$ pole parameters (such as $m_Z$ and $\Gamma_Z$) because these are shifted fractionally at $O(\varepsilon_Z^2)$, and only require $\varepsilon_Z \lesssim O(0.01)$. However, it can, nevertheless, lead to other interesting new phenomenology which overcomes the $m_{Z_d}/m_Z$ suppression in $\varepsilon_Z$.

As the first example, we consider very low $Q^2$ parity violating effects where the smallness of $m_{Z_d}/m_Z$ in the induced $Z_d$ couplings is offset by the $m_{Z_d}^2/m_Z^2$ enhancement from $Z$ vs $Z_d$ propagators. Then we describe the induced decays $K \to \pi Z_d$ and $B \to K Z_d$, as well as the high energy decay $H \to ZZ_d$, where the small induced coupling factor $m_{Z_d}/m_Z$ is overcome by a $m_K/m_{Z_d}$, $m_B/m_{Z_d}$ and $m_H/m_{Z_d}$ enhancements, respectively, in the longitudinal polarization component of the $Z_d$ production amplitudes.

### III. Atomic Parity Violation and Polarized Electron Scattering

We begin our analysis by writing out the full $Z_d$ coupling to fermions from $\varepsilon$ as well as $\varepsilon_Z$.

$$\mathcal{L}_{\text{int}} = \left( -e \varepsilon J_{\mu}^{em} - \frac{g}{2 \cos \theta_W} \varepsilon_Z J_{\mu}^{NC} \right) Z_d^{\mu}$$ \hspace{1cm} (9)

where $J_{\mu}^{NC}$ is given in Eq. (4) and

$$J_{\mu}^{NC} = \sum_f (T_{3f} - 2Q_f \sin^2 \theta_W f^{\gamma\mu} f - T_{3f} f^{\gamma\mu} \gamma_5 f)$$ \hspace{1cm} (10)
with $T_{3f} = \pm 1/2$ ($T_{3c} = -1/2$) and $\sin^2 \theta_W \simeq 0.23$ is the weak mixing angle of the SM. The inclusion of $Z$-$Z_d$ coupling has introduced parity violation. The $J^\mu_{NC}Z_d^\mu$ coupling is similar to the $J^\mu_{NC}Z^\mu$ coupling of the SM $Z$ but reduced by $\varepsilon_Z$ in magnitude. Hence, the name “dark” $Z$, since it is the $\varepsilon_Z$ induced interactions that we primarily address. Note that the effects of $\varepsilon$ and $\varepsilon_Z$ can be combined into a simple form

$$L_{\text{int}} = -\frac{g}{2 \cos \theta_W} \varepsilon_Z J^\mu_{NC}Z_d^\mu$$

(11)

by the replacement $J^\mu_{NC}(\sin^2 \theta_W) = J^\mu_{NC}(\sin^2 \theta'_W)$

$$\sin^2 \theta'_W = \sin^2 \theta_W - \frac{\varepsilon}{\varepsilon_Z} \cos \theta_W \sin \theta_W$$

(12)

in Eq. (10). In that format, one can judge the relative importance of $\varepsilon$ in low energy $Z_d$ phenomenology. It depends on the size of $(\varepsilon/\varepsilon_Z)(\cos \theta_W / \sin \theta_W)$. For $\varepsilon$ very small, it will have little effect, but will be significant if $\varepsilon \sim \varepsilon_Z$.

The new source of parity violation in Eq. (9) or Eq. (11), is particularly important for experiments at $Q^2 < m_Z^2$, where the $Z_d$ propagator can provide an enhancement due to $m_Z^2 \ll m_Z^2$. The overall effect for parity violating amplitudes $M^\mu_{NC} = (G_F/2\sqrt{2})F(\sin^2 \theta_W)$ in the SM is (in leading order) to replace

$$G_F \rightarrow \rho_d G_F$$

$$\sin^2 \theta_W \rightarrow \kappa_d \sin^2 \theta_W$$

(13)

with [21]

$$\rho_d = 1 + \delta^2 \frac{m_{Z_d}^2}{Q^2 + m_{Z_d}^2}$$

$$\kappa_d = 1 - \frac{\varepsilon}{\varepsilon_Z} \frac{\delta \cos \theta_W}{\sin \theta_W} \frac{m_{Z_d}^2}{Q^2 + m_{Z_d}^2}$$

(14)

or from Eq. (6)

$$\kappa_d = 1 - \frac{\varepsilon}{m_{Z_d}} \frac{\delta \cos \theta_W}{\sin \theta_W} \frac{m_{Z_d}^2}{Q^2 + m_{Z_d}^2}$$

(15)

It is quite plausible that in a more complete theory, $\varepsilon \propto (m_{Z_d}/m_Z)\delta = \varepsilon_Z$. Then, the effects from kinetic mixing and $Z$-$Z_d$ mixing become similar in form and magnitude. Here, we allow $\varepsilon$ to remain a separate independent parameter.

Assuming no accidental cancellation between the $\rho_d$ and $\kappa_d$ in Eq. (14), Cesium atomic parity violation currently provides the best low energy experimental constraint on those parameters over the entire approximate range of interest (10 MeV $\lesssim m_{Z_d} \lesssim 10$ GeV) since $Q^2 \ll m_Z^2$. The nuclear weak charge measured in atomic parity violation (to lowest order in the SM) is given by $Q_W = -N + Z(1 - 4 \sin^2 \theta_W)$ which when compared with experiment probes new physics. There is excellent agreement between the SM prediction for the weak charge of Cesium (including electroweak radiative corrections) [22–24]

$$Q_W^{\text{SM}}(\text{Cs}) = -73.16(5)$$

(16)

and the experimental value [25–27]

$$Q_W^{\text{exp}}(\text{Cs}) = -73.16(35).$$

(17)

Based on the shift due to $\varepsilon$, $\varepsilon_Z$ and $\delta$

$$Q_W \rightarrow -73.16(1 + \delta^2) + 220 \frac{\varepsilon}{\varepsilon_Z} \delta^2 \cos \theta_W \sin \theta_W,$$

the above agreement then implies the following constraints

$$\left| \frac{\delta^2(1 - 1.27 \frac{\varepsilon}{\varepsilon_Z})}{2} \right| \leq 0.006$$

(18)

$$\delta^2 \leq 0.006, \text{ for } \varepsilon \ll \varepsilon_Z.$$  

(19)

For $\varepsilon \approx \varepsilon_Z$, the constraints on $\delta^2$ become dilute and the possibility of cancellation occurs if one tunes $\varepsilon/\varepsilon_Z \approx 0.8$. (We note that the fine tuning $\varepsilon/\varepsilon_Z \approx 0.8$ is similar to a relation employed in Ref. [8] to try and reconcile what appears to be discrepancies in dark matter search scattering experiments on heavy nuclei. However, such a scenario is significantly constrained by the bounds on $\delta$ described below.)

An independent constraint primarily applicable to $\kappa_d$ because of its relative insensitivity to $\rho_d$ comes from parity violating polarized electron-electron Møller scattering asymmetries [28, 29]. Experiment E158 at SLAC [30] measured the low energy value of $\sin^2 \theta_W(Q^2)$ at $Q^2 \approx (0.16 \text{ GeV})^2$ and compared it with expectations based on running the $Z$ pole value $\sin^2 \theta_W(m_Z)$ down to low $Q^2$ [29]. The good agreement with SM loop effects leads to (ignoring the small $\rho_d$ effect)

$$\left| \frac{\varepsilon}{\varepsilon_Z} \frac{m_{Z_d}^2}{(0.16 \text{ GeV})^2 + m_{Z_d}^2} \right| \leq 0.006.$$

(20)

For $m_{Z_d}^2 \gg (0.16 \text{ GeV})^2$ and $\varepsilon_Z \approx \varepsilon$, the constraints in Eqs. (19) and (20) are essentially the same. However, for a light $m_{Z_d} \lesssim 200 \text{ MeV}$, the bound in Eq. (20) can be somewhat diluted. Nevertheless, for some range of $(\varepsilon, m_{Z_d})$ values, Eq. (20) can provide more restrictive bounds on $\delta$. For example, consider $\varepsilon \approx 2 \times 10^{-3}$ and $m_{Z_d} \approx 100 \text{ MeV}$ which lie in the region favored by the current discrepancy between theory and experimental values of the muon anomalous magnetic moment [31]. In that case, Eq. (20) becomes

$$\delta < 0.01$$

(21)

which is considerably tighter than Eq. (19). If the muon anomaly discrepancy is due to a light $Z_d$ and $\varepsilon \sim 10^{-3}$, that boson’s effect on the value of $\sin^2 \theta_W$ extracted from future more precise very low $Q^2$ parity violating experiments [32] could eventually become observable.
The sensitivity in Eqs. (20) and (21) is expected to improve by up to an order of magnitude from ongoing and proposed polarized $ep$ and $ee$ scattering experiments at JLAB [32] as well as proposed $Q^2 \simeq (0.05 \text{ GeV})^2$ $ep$ studies at MESA in Mainz [33]. Our analysis illustrates the complementarity of direct searches at intense electron scattering facilities in JLAB and Mainz for a light vector particle (the “dark” photon coupled through kinetic mixing) produced via electron scattering, with low $Q^2$ measurements of $\sin^2 \theta_W$ in parity violating experiments (that probe $\varepsilon$ and the mass mixing of the “dark” $Z$). We also note that proposed measurements of atomic parity violation for ratios of different nuclear isotopes would eliminate atomic physics uncertainties as well as any dependence on $\rho_d$ [34–37]. They would then be sensitive to $(\varepsilon/\varepsilon_Z)\delta^2$ but with negligible $Q^2$ dependence (since $Q^2 \sim 0$). It is amusing to note that in principle, very low energy measurements of $\sin^2 \theta_W$ in atomic parity violation and low $Q^2$ polarized electron scattering experiments could find different $\sin^2 \theta_W$ results from one another if a very low mass $Z_d$ is contributing to both, because of the $Q^2$ dependence in Eq. (14).

Our conclusion, based on the above discussion, is that currently, $\delta^2 \lesssim 0.006$ is a modest, reasonably reliable constraint for most values of $m_{Z_d}$, although fine tuning of $\varepsilon$ and $\varepsilon_Z$ could loosen the bound. That constraint can be much stronger for $\varepsilon \sim 10^{-3}$ [see Eq. (21)], and could be further improved significantly by future low energy parity violating experiments. For now, the bound $\delta^2 \lesssim 0.006$ provides a starting point for comparison with the sensitivity to $\delta^2$ in rare $K$ and $B$ decays which we next describe.

IV. RARE $K$ AND $B$ DECAYS

Experimental studies of rare flavor changing weak neutral current decays of $K$ and $B$ mesons have proven to be powerful probes of high and low scale “new” physics phenomena. Here, we illustrate the effect of $Z$-$Z_d$ mass mixing on the transition amplitudes $s \to dZ_d$ and $b \to sZ_d$ induced within the framework of CKM (Cabibbo-Kobayashi-Maskawa) charged current mixing (See Fig. 1). Those loop induced couplings can lead to decays such as $K \to \pi Z_d$ and $B \to KZ_d$ or $K^* Z_d$ characterized by the signature $Z_d \to \ell^+ \ell^-$ ($\ell = e$ or $\mu$) with invariant mass $m_{\ell \ell} = m_{Z_d}$ or $Z_d \to$ missing energy where $Z_d$ decays into $\nu \bar{\nu}$ or essentially undetectable light hidden sector particles. In all such 2-body decays, the mono energetic outgoing $\pi$ or $K$ will provide a tight constraint (for a given $m_{Z_d}$) and a very distinct overall signal.

Here, we would like to note that the phenomenology of the $Z_d$ is affected by its lifetime $\tau_{Z_d}$. A sufficiently large value of $\tau_{Z_d}$ will allow $Z_d$ to escape the detector and lead to a missing energy signal. However, for smaller values of $\tau_{Z_d}$, a displaced vertex can provide a distinct signature. In Fig. 2, using representative values of $\delta$ and $\varepsilon$, we have plotted $\tau_{Z_d}$ for $10 \text{ MeV} \leq m_{Z_d} \leq 10 \text{ GeV}$, assuming that $Z_d$ only decays into SM final states. We provide a simple formula for the partial width of $Z_d$ into SM fermions, $\Gamma(Z_d \to f f)$, in the appendix.

Of course, the amplitudes for $dsZ_d$ and $sbZ_d$ being loop induced will in general depend on the details of the complete model considered, including its underlying Higgs flavor symmetry breaking structure. Those details are beyond the scope of this paper where we are primarily interested in the generic effects of $Z$-$Z_d$ mixing parametrized by $\varepsilon_Z = (m_{Z_d}/m_Z)\delta$ in Eq. (6).

A simple illustrative example of a scenario that leads to $Z$-$Z_d$ mixing and CKM induced flavor changing weak neutral currents is the Type-I 2HD model discussed in Sec. VI and detailed in the appendix. There, the underlying $U(1)_{d}$ gauge symmetry naturally forbids tree level flavor changing neutral currents in the scalar and pseudoscalar Higgs sectors. It also yields, through Higgs doublet and singlet vacuum expectation values, a mechanism to provide mass for $Z_d$ and give rise to a small $\delta$ in Eq. (6).

To obtain the induced $Z_d$ flavor changing amplitudes, we can make use of existing CKM loop induced calculations for $\delta L_\nu \mu Z''$ and $\delta L_\mu b_L Z''$ amplitudes [38] and replace $Z \to \varepsilon_Z Z_d$. (See Fig. 1.) (We ignore kinetic mixing induced couplings, since their effects are highly sup-

![FIG. 1: Examples of diagrams contributing to $b \to sZ_d$. Similar diagrams give rise to $s \to dZ_d$.](image)

![FIG. 2: $\tau_{Z_d}$ lifetime with $Z_d$ mass for $\delta^2 = 10^{-4}$ with $\varepsilon = 0$ (solid blue curve) and $\varepsilon = 2 \times 10^{-3}$ (dashed blue curve) cases. We take $\rho$, $\phi$, $J/\psi$, $\Upsilon$ masses as the representative threshold for decays to mesons.](image)
pressed. For example, Ref. [39] found $\text{BR}(B \to KZ_d) \sim 6 \times 10^{-2} \varepsilon^2$ for $m_{Z_d} \approx 1$ GeV. As we demonstrate, mass mixing, $\varepsilon_Z$, induced rates can be much larger and potentially observable.) As an alternative computational strategy, if we are primarily interested in relatively light $Z_d$ bosons compared to $m_K$ and $m_B$, we can employ the Goldstone boson equivalence theorem [40] to obtain amplitudes for longitudinally polarized $Z_d$ bosons from flavor changing axion like pseudoscalar couplings well documented in the literature. For our purpose, the latter approach will suffice; however, the direct $Z$ calculation provides a nice cross-check. Nevertheless, we note that the results discussed below should be viewed as somewhat incomplete and should be taken as approximate.

The relevant $d_L \gamma_\mu s_L \partial^\mu a$ and $\bar{s}_L \gamma_\mu b_L \partial^\mu a$ axion couplings were computed for the 2HD model more than 30 years ago by Hall and Wise [41] and independently by Frere, Vermaseren and Gavela [42]. More recently, they were checked and applied to the decay $B \to K a$, $a \to \ell^+\ell^-$ in Ref. [43]. Here, we use those results to estimate the branching ratios for $K \to \pi Z_d$ (longitudinal) and $B \to K Z_d$ (longitudinal) which should approximate the full $Z_d$ final state rates up to corrections of $O(m_{Z_d}^2/m_K^2)$ and $O(m_{Z_d}^2/m_B^2)$ respectively. Comparison of those estimates with experiments can then be used to constrain $\delta$ for the ranges $m_{Z_d}^2 \ll (m_K - m_\pi)^2$ and $m_{Z_d}^2 \ll (m_B - m_K)^2$ modulo regions not covered because of experimental acceptance cuts on the data (which are beyond the scope of this paper). For example, $m_{Z_d} < 140$ MeV is not covered because of $\pi^0 \to e^+e^\gamma$ Dalitz decay background. Similarly, masses of $Z_d$ near charmonium resonance regions are not covered.

We begin with the predicted branching ratio for $K \to \pi Z_d$ (longitudinal) in the 2HD model. Based on the analysis in Ref. [41], but adjusting for a modern $m_t$ value, since top now dominates the amplitudes in Fig. 1

$$\text{BR}(K^+ \to \pi^+ Z_d)_{\text{long}} \simeq 4 \times 10^{-4} \delta^2,$$ (22)

where the numerical factor in that expression includes QCD suppression effects and depends on the physical charged scalar Higgs mass of the 2HD model. Those uncertainties should be considered part of the overall model dependence of our analysis.

The $Z_d$ produced in Eq. (22) is expected to decay promptly (see, however, Fig. 2) to $\ell^+\ell^-$ pairs with invariant mass $m_{Z_d}$ or to missing energy which might be $\nu\bar{\nu}$ or light hidden sector particles. Those decays would add to the SM predictions and should be part of the experimentally measured branching ratios [1, 44, 45]

$$\text{BR}(K^+ \to \pi^+ e^+ e^-)_{\text{exp}} = 3.00 \pm 0.09 \times 10^{-7} \quad (23)$$
$$\text{BR}(K^+ \to \pi^+ \mu^+ \mu^-)_{\text{exp}} = 9.4 \pm 0.6 \times 10^{-8} \quad (24)$$
$$\text{BR}(K^+ \to \pi^+ \nu\bar{\nu})_{\text{exp}} = 1.7 \pm 1.1 \times 10^{-10} \quad (25)$$

unless eliminated by acceptance cuts which would negate bounds in certain $m_{Z_d}$ regions. For example, the result in Eq. (23) applied a $m_{ee} > 140$ MeV cut while Eq. (25) was obtained with a rather stringent cut on $E_\pi$. Clearly, a new round of bump hunting in the $\ell^+\ell^-$ spectrum is warranted. Toward that end, we note that $Z_d \to \ell^+\ell^-$ decays will have a characteristic polarized spin-1 $\sin^2 \theta$ distribution relative to the longitudinal polarization of the $Z_d$. Unlike the spin-0 axion case, where due to chiral conservation the $a$ preferentially decays to the heaviest fermion possible and the distribution in isotropic, we expect $\text{BR}(Z_d \to \ell^+\ell^-) \simeq \text{BR}(Z_d \to \mu^+\mu^-)$ modulo phase space.

With the above caveats, we compare Eq. (22) with (23), (24), and (25) which agree with SM expectations and find rather tight bounds

$$\delta \lesssim 0.01/\sqrt{\text{BR}(Z_d \to e^+ e^-)} \quad (26)$$
$$\delta \lesssim 0.001/\sqrt{\text{BR}(Z_d \to \text{missing energy})} \quad (27)$$

modulo acceptance cut criteria.

Eqs. (11) and (12) yield [46]

$$\frac{\text{BR}(Z_d \to e^+ e^-)}{\text{BR}(Z_d \to \nu\bar{\nu})} \simeq \frac{1}{6} + \frac{1}{2} \left( \frac{\varepsilon}{\varepsilon_Z} \right)^2,$$ (28)

where $\varepsilon$ from kinetic mixing now comes into play. For $\varepsilon \gg \varepsilon_Z$, the charged lepton decays dominate and Eq. (26) is more applicable. For $\varepsilon \ll \varepsilon_Z$, the tighter constraint in Eq. (27) takes precedence. Of course, both should be used cautiously, given their model and experimental acceptance dependence.

For the case of $B \to K Z_d$ (longitudinal), we can apply a similar approach and find [41–43]

$$\text{BR}(B \to K Z_d)_{\text{long}} \simeq 0.1 \delta^2.$$ (29)

The relatively large coefficient in Eq. (29) results from a factor of $m_t^2$ in the $b \to s Z_d$ loop induced correction from Fig. 1. That factor makes rare $B$ decays a particularly sensitive probe of the $Z_d$. Employing the recent bounds that follow from the discussion of $B \to K a$, with the axion-type particle $a \to \ell^+\ell^-$ in Refs. [39, 43] implies conservatively $\text{BR}(B \to K Z_d \to K\ell^+\ell^-) < 10^{-7}$, while the bound from $B$-decay containing missing energy are based on [1, 47, 48]

$$\text{BR}(B^+ \to K\nu\bar{\nu})_{\text{exp}} < 1.4 \times 10^{-5}.$$ (30)

We then roughly find

$$\delta \lesssim 0.001/\sqrt{\text{BR}(Z_d \to \ell^+\ell^-)} \quad (31)$$
$$\delta \lesssim 0.01/\sqrt{\text{BR}(Z_d \to \text{missing energy})} \quad (32)$$

It has been suggested [39] that even tighter bounds may be obtained from dedicated searches for $\ell^+\ell^-$ pairs in $B$ decays, particularly if displaced vertices result from suppressed decay rates. Nevertheless, even the relatively crude bounds in Eqs. (31) and (32) are very constraining where applicable and are likely to be significantly improved by future dedicated searches.
TABLE I: Standard Model Higgs decay branching ratios for $m_H = 125$ GeV ($\Gamma_H \approx 4.1$ MeV) from Ref. [40].

<table>
<thead>
<tr>
<th>$H$ Decay Channel</th>
<th>Branching Ratio</th>
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<tbody>
<tr>
<td>$b\bar{b}$</td>
<td>0.578</td>
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<tr>
<td>$WW^*$</td>
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<tr>
<td>$gg$</td>
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<td>$c\bar{c}$</td>
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<tr>
<td>$Z\gamma$</td>
<td>$1.5 \times 10^{-3}$</td>
</tr>
<tr>
<td>$H \rightarrow ZZ^* \rightarrow \ell_1^+\ell_1^-\ell_2^+\ell_2^-$</td>
<td>$1.2 \times 10^{-4}$</td>
</tr>
<tr>
<td>$H \rightarrow ZZ^* \rightarrow \ell^+\ell^-\nu\bar{\nu}$</td>
<td>$3.6 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

On the basis of our analysis, it is clear that rare $K$ and $B$ decays provide sensitive windows to $Z-Z_d$ mass mixing and should be further explored in future high intensity experiments. In fact for both cases, a more refined binned analysis of existing data would likely result in tighter bounds than those in Eqs. (26) and (31) or even uncover a hint of the $Z_d$'s presence. Although applicable to a limited range of $m_{Z_d}$ and dependent on $m_{Z_d}$ branching ratios, one can easily conclude $\delta \lesssim 0.01 - 0.001$ over some restricted $m_{Z_d}$ domain. In addition, further improvements are possible and warranted. That constraint on $\delta$ sets a standard for other rare decay studies. As we show in the next section, it is possible that searches for the rare Higgs decay $H \rightarrow ZZ_d$ have the statistical significance to also explore $\delta \lesssim 0.01 - 0.001$ but have the potential advantage of covering a much broader range of $m_{Z_d}$ values including $m_{Z_d} \gtrsim 5$ GeV if backgrounds can be controlled.

V. HIGGS DECAYS

We now address a primary consequence of our paper, the decay $H \rightarrow ZZ_d$ induced by $Z-Z_d$ mass matrix mixing. To put our analysis into a current day perspective, we take $m_H = 125$ GeV, a value roughly suggested by early small excesses at the Large Hadron Collider (LHC) in the expected decay modes $H \rightarrow \gamma\gamma, \, WW^*, \, ZZ^*$ [50, 51]. We note, however, that our findings regarding the sensitivity of Higgs searches for $H \rightarrow ZZ_d$ are fairly independent of the exact value of $m_{H}$.

To set the stage, we estimate that, roughly, one expects each LHC experiment to have about 75000 Higgs bosons in the existing data before cuts (for the integrated luminosity of $4.7 - 4.9$ fb$^{-1}$ with $E_{c.m.} = 7$ TeV) for $m_H = 125$ GeV in the SM. In Table I, we list the expected Higgs decay branching ratios within the context of the SM. Of particular interest for comparison with $H \rightarrow ZZ_d$ are the SM decays (1) $H \rightarrow ZZ^* \rightarrow \ell_1^+\ell_1^-\ell_2^+\ell_2^-$ and (2) $H \rightarrow ZZ^* \rightarrow \ell^+\ell^- \nu\bar{\nu}$ where the * signifies a “virtual,” off mass shell boson and $\ell = e, \mu$. The first of these, even at the BR $\sim 10^{-4}$ level, may have already been seen at the LHC where a handful of candidate events have been reported. If it truly is a Higgs signal, hundreds more 4-lepton $\ell_1^+\ell_1^-\ell_2^+\ell_2^-$ events will be clearly observed in the coming years. The second decay, $H \rightarrow \ell^+\ell^- \nu\bar{\nu}$, is more difficult and to our knowledge has not been experimentally studied.

For the first case, one lepton pair will have an invariant mass of $m_Z \approx 91$ GeV while the second pair will have an invariant mass ranging from 0 to about 34 GeV with a differential decay rate distribution as depicted in Fig. 3. The second mode $H \rightarrow ZZ^* \rightarrow \ell^+\ell^- \nu\bar{\nu}$, with the neutrinos identified by missing energy, while experimentally more challenging should be searched for as well, since it can be used to constrain potentially invisible decays of the $Z_d$, as we subsequently discuss.

As we shall see, the decays $H \rightarrow ZZ_d$ are significantly enhanced beyond naive expectations, even for very small mixing. To appreciate that phenomenon, we remind the reader that for a very heavy Higgs ($m_Z^2 \gg m_V^2, \, m_W^2$) the decay rates for $H \rightarrow W^+W^-$ and $H \rightarrow ZZ$ can become enormous, growing like $\sim g^2 m_H^3 / m_V^6$, $V = W, Z$ with increasing $m_H$. That behavior comes about because the final state W and Z bosons are longitudinally polarized, resulting in a $\sim m_H^2 / m_V^6$ enhancement factor at the decay rate level (for each final state gauge boson).

Such an effect is a manifestation of the Goldstone boson equivalence theorem which states that at high energies ($s \gg m_V^2$), $S$-matrix elements involving $W^\pm$ and $Z$ bosons are equivalent, up to $O(m_V / \sqrt{s})$, to the corresponding amplitudes in the Higgs-Goldstone scalar theory with the Goldstone boson replacing $W_L^\pm, \, Z_L$ (longitudinal components). In the heavy Higgs limit, the $W^+W^-$ and $ZZ$ decay products are essentially longitudinally polarized and behave like their Goldstone boson components. The Higgs coupling to Goldstone bosons is of the form $-i g/2 m_H^2 / m_V$, squaring that coupling and dividing by $1/m_H$ gives the $\Gamma(H \rightarrow WW') \sim g^2 m_H^6 / m_V^4$ exhibited by heavy Higgs decays. We note that the longitudinal polarization of the gauge bosons can be very helpful in identifying a Higgs decay since the subsequent decay $W$ or $Z \rightarrow$ leptons have a characteristic angular distribution $\propto \sin^2 \theta$ relative to the polarization.

Of course, our example of 125 GeV Higgs is too light to decay into $W^+W^-$ or $ZZ$ pairs. It can, however, decay into one real and one virtual boson with the latter directly producing a lepton pair with an invariant mass distribution as illustrated in Fig. 3 [52]. The integrated partial width for $H \rightarrow ZZ^* \rightarrow \ell_1^+\ell_1^-\ell_2^+\ell_2^-$ is, however, suppressed by $\alpha/4\pi$ (from the $Z^*\ell_1^+\ell_2^-$ coupling and 3-body phase space) and the small BR($Z \rightarrow \ell^+\ell^- \nu\bar{\nu}$) $\simeq 2 \times 0.034$ for $\ell = e, \mu$. One finds

$$\Gamma(H \rightarrow ZZ^* \rightarrow \ell_1^+\ell_1^-\ell_2^+\ell_2^-) \simeq 1.8 \times 10^{-6} \frac{G_F}{8\sqrt{2}\pi} m_H m_W^2$$

with no significant sign of enhancement for longitudinal polarization, not surprising, since $m_H / m_Z \approx 1.4$ in our example. Nevertheless, even with the $10^{-8}$ suppression
factor in Eq. (33), it is expected that a SM 125 GeV Higgs should be starting to be seen with about several events per experiment in existing data, after acceptance cuts, and with hundreds more to follow in subsequent years. So, Eq. (33) represents a decay rate standard that is easily discernible if backgrounds are in check. We note that the decay rate for $H \rightarrow ZZ^* \rightarrow \ell^+\ell^-\nu\bar{\nu}$ is expected in the SM to be about 3 times larger than Eq. (33) but more difficult to measure.

Now we come to the decay $H \rightarrow ZZ_d$ due to $Z$-$Z_d$ mixing in our “dark” $Z$ scenario. That mixing, parametrized by $\varepsilon_Z = (m_{Z_d}/m_Z)\delta$, a very small quantity, might naively appear to be negligible since it leads to a tiny $HZZ_d$ coupling $\sim (g/\cos\theta_W)m_{Z_d}\varepsilon_Z$. Consequently, the $H \rightarrow ZZ_d$ decay rate will be suppressed by $\varepsilon^2_Z = (m_{Z_d}/m_Z)^2\delta^2$. However, because of the Goldstone boson equivalence theorem, we gain an enhancement factor of $\sim (m_{H}/m_{Z_d})^2$ in the decay rate for longitudinally polarized $Z_d$ final states (a feature that may also help in identifying their subsequent $Z_d \rightarrow \ell_1^+\ell_2^-$ products via angular distribution if statistics suffice). That enhancement negates the small $m_{Z_d}/m_Z$ factor in the $HZZ_d$ coupling. Also, there is no $\alpha/4\pi$ suppression for $H \rightarrow ZZ_d$, only the small $\text{BR}(Z \rightarrow \ell^+\ell^-) \simeq 2 \times 0.034$ that needs to be included for $Z$ identification. A detailed calculation (see appendix) leads to

$$\Gamma(H \rightarrow ZZ_d \rightarrow \ell_1^+\ell_1^-\ell_2^+\ell_2^-)$$

$$\simeq 7 \times 10^{-3} \frac{G_F m^3_H}{8\sqrt{2}\pi} \delta^2 \text{BR}(Z \rightarrow \ell_1^+\ell_2^-)$$

Note the $m_H^3$ behavior which results from $Z$ and $Z_d$ being produced in their longitudinal polarization modes. A similar formula with $\text{BR}(Z \rightarrow \ell^+\ell^-)$ replaced by $\text{BR}(Z \rightarrow \text{missing energy})$ applies to the case $Z_d \rightarrow \nu\bar{\nu}$ or invisible “dark” particles.

In terms of its branching fraction relative to the SM expected width, one finds

$$\frac{\Gamma(H \rightarrow ZZ_d)}{\Gamma_{SM}^{H}(125 \text{ GeV})} \simeq 16 \times \delta^2 \lesssim 0.1$$

with $\Gamma_{SM}^{H}(125 \text{ GeV}) \approx 4.1 \times 10^{-3}$ GeV [49] and using the low energy bound in Eq. (19). We see that as much as 10% of all LHC Higgs decays could be producing $ZZ_d$. With current statistics, even a 10% loss of SM expectations would not be noticed; but eventually it would be uncovered by precision Higgs production and decay studies.

Taking the ratio of Eqs. (34) and (33) gives

$$\frac{\Gamma(H \rightarrow ZZ_d \rightarrow \ell_1^+\ell_1^-\ell_2^+\ell_2^-)}{\Gamma(H \rightarrow ZZ^* \rightarrow \ell_1^+\ell_1^-\ell_2^+\ell_2^-)}$$

$$\simeq 10^4\delta^2 \text{BR}(Z \rightarrow \ell_2^+\ell_2^-)$$

with a similar expression

$$\frac{\Gamma(H \rightarrow ZZ_d \rightarrow \ell^+\ell^- + \text{missing energy})}{\Gamma(H \rightarrow ZZ^* \rightarrow \ell^+\ell^- + \text{missing energy})}$$

$$\simeq (1/3) \times 10^4\delta^2 \text{BR}(Z \rightarrow \text{missing energy})$$

for invisible $Z_d$ decays. Even for $\delta^2 \simeq 10^{-4}$, well below the atomic parity violation bound of 0.006 in Eq. (19), one would expect $H \rightarrow ZZ_d$ events with $\ell_1^+\ell_1^-\ell_2^+\ell_2^-$ or $\ell^+\ell^- + \text{missing energy}$ to be starting to appear or already present in LHC data. If there are no $Z_d \rightarrow$ dark particles decays, we expect the branching fractions of $Z_d$ into $\ell^+\ell^-$ to be given by Eq. (28). Therefore, in that case, one expects $\text{BR}(Z \rightarrow \ell^+\ell^-)$ to be relatively large, particularly if $(\varepsilon/\varepsilon_Z)^2 \gtrsim 1$. If $Z_d \rightarrow$ dark particles dominates its decay rate and significantly dilutes $\text{BR}(Z \rightarrow \ell^+\ell^-)$, one still has the possibility of seeing $H \rightarrow ZZ_d \rightarrow \ell^+\ell^- + \text{missing energy}$, although this perhaps is more experimentally challenging. Of course, given the original motivation for introducing a $Z_d$ into astrophysics as a way of explaining positron excesses through its decays, a relatively large $\text{BR}(Z \rightarrow \ell^+\ell^-)$ might be expected.

Returning to Eq. (36), we see that even for a somewhat suppressed $\text{BR}(Z \rightarrow \ell^+\ell^-)$, the LHC experiments should be able to search for a $Z_d$ in the $H \rightarrow \ell_1^+\ell_1^-\ell_2^+\ell_2^-$ decay chain down to $\delta^2\text{BR}(Z \rightarrow \ell^+\ell^-) \sim \mathcal{O}(10^{-5})$, depending on backgrounds. (The domain explored by rare $K$ and $B$ decays for some subset of $m_{Z_d}$ values.) The signature, two isolated lepton pairs $\ell_1^+\ell_1^- + \ell_2^+\ell_2^-$ with a total invariant mass of $m_H$ and individual masses of $m_Z$ and $m_{Z_d}$ should stick out as a spike in the invariant mass plot of Fig. 3, as illustrated for $m_{Z_d} = 5$ GeV and $\delta^2\text{BR}(Z \rightarrow \ell^+\ell^-) = 10^{-5}$. In the bin centered at $M_{\ell\ell} = 5$ GeV, the SM expectation from Higgs of $m_H = 125$ GeV is $\sim 6.3 \times 10^{-9}$ GeV, while the signal associated with $H \rightarrow ZZ_d$ is $\sim 4.5 \times 10^{-8}$ GeV. With existing data of $N_{\text{Higgs}} \approx 75000$, no meaningful number of signal or background events are expected, and one would need $N_{\text{Higgs}} \simeq 10^6$ for 3$\sigma$ evidence (beyond the SM $H \rightarrow ZZ^* \rightarrow 4\ell$ channel) at the LHC experiments. However, this simple estimate ignores other reducible and irreducible backgrounds and a more reliable statement requires inclusion of such details. Also, the $\ell_2^+\ell_2^-$ decay pair...
from $Z_d$ should exhibit an angular distribution consistent with its longitudinal polarization. That sensitivity is potentially orders of magnitude below the $\delta^2 < 0.006$ already established by atomic parity violation. We note that while the Higgs decay constraints on $\delta$ may not surpass those derived before from rare $K$ and $B$ decays, they are applicable well beyond the $O(\text{GeV})$ regime of $m_{Z_d}$, relevant for the meson decays. They represent a potentially unique broad capability of the LHC unmatched by low energy experiments.

We should point out that current searches for $H \rightarrow ZZ^* \rightarrow 4\ell$ are likely to miss $H \rightarrow ZZ_d$ because they generally cut out a lighter second lepton pair with $M_{4\ell} \lesssim 15$ GeV, i.e., the range of interest, in order to avoid $Z\gamma^*$ backgrounds. Hopefully, our results will provide some incentive for revisiting the low mass region in search of $Z_d$.

In addition to $Z_d \rightarrow \ell^+\ell^-$, one should mount a search for $H \rightarrow ZZ_d \rightarrow \ell^+\ell^- + \text{missing energy}$. Here, one might be helped by the fact that the missing energy and missing momentum of the $Z_d$ decay pair are nearly equal. A thorough study of LHC capabilities for uncovering that decay mode is clearly warranted. We also add that the Higgs can have a decay mode $H \rightarrow Z_d Z_d$, in our framework. The rate for this decay is proportional to $\delta^4$, so, roughly, it is suppressed compared to the $ZZ_d$ mode by $O(\delta^2)$ which, given our bound in Eq. (19), is a suppression of 0.006 or smaller. The rate for the $Z_d Z_d$ channel could be enhanced if hidden sector scalars that couple directly to $Z_d$ and give it mass are allowed to mix with the the SM sector Higgs scalars.

VI. A 2 HIGGS DOUBLET EXAMPLE

In the preceding discussion, we examined the dark $Z$ phenomenology in a general framework. As mentioned before, the main ingredient we introduced was mass mixing between the SM $Z$ and the $Z_d$ which could be realized in a variety of models. In this section, to demonstrate how our general framework might be realized, we will consider a 2 Higgs doublet extension of the SM. (See Ref. [53] for a recent review on 2HD models.) Here, we assume two $SU(2)_L \times U(1)_Y$ Higgs doublets, $H_1$ and $H_2$, but allow $H_d$ to carry a “dark” charge that couples it directly to $U(1)_d$. Note that the assumption of the $U(1)_d$ in our example is well-motivated, as it allows the model to evade severe constraints from flavor-changing neutral currents that are often addressed through the introduction of a $Z_2$ symmetry in generic 2HD models. We also allow, for generality, a singlet scalar, $H_d$, that also provides part of the $Z_d$ mass through its “dark” sector vacuum expectation value $\nu_d$.

With the above assumptions, $H_d$ does not couple directly to ordinary fermions, but does contribute to $W^\pm$, $Z$ and $Z_d$ masses as well as $Z-Z_d$ mixing through its vacuum expectation value $\nu_d$. Such a setup is akin to what is often called a Type-I 2HD model [54]. Here, we will take $H_2$ to be a SM-like Higgs scalar, identified as $H$ in our preceding general analysis. To keep the discussion simple, we ignore scalar mixing among the $H_1$, $H_2$, and $H_d$ states. The $\nu_1$, $\nu_2$, and $\nu_d$ vacuum expectation values of $H_1$ (the SM doublet), $H_2$, and $H_d$ give rise to $\delta = \sin \beta \sin \beta_d$ where $\tan \beta = \nu_2/\nu_1$ and $\tan \beta_d = \nu_2/\nu_d$, as will be shown in the appendix. The condition of a SM-like $H_1$ can be satisfied, to a good approximation, for $\tan \beta \lesssim 1/3$, and does not require a large hierarchy of scales in the Higgs sector. The constraints on $\delta$ previously discussed will however constrain the product $\sin \beta \sin \beta_d$.

There are many additional features of our 2HD model worth studying. For example, non-zero Higgs scalar mixing (which we set to zero) could give rise to enhancements in $H \rightarrow Z_d Z_d$, as mentioned before, or perhaps $H \rightarrow hh$ ($h$ being a lighter Higgs scalar remnant of $H_2$) [55]. Those possibilities are interesting but more model dependent.

VII. SUMMARY AND CONCLUSION

In this work, we explored the possibility of mass mixing between the $Z$ boson of the SM and a new light vector boson $Z_d$ associated with a hidden or dark sector $U(1)_d$ gauge symmetry. Such a light state has been invoked in discussions of astrophysical anomalies that may originate from cosmic dark matter. We dub this new vector boson the “dark” $Z$, as its properties are analogous to that of the SM $Z$. In particular, the couplings of $Z_d$ can provide new sources of parity violation and measurably affect the decay of the Higgs through novel channels such as $H \rightarrow ZZ_d$. Existing atomic parity violation, polarized $e$ scattering, and rare $K$ and $B$ decay data already place interesting bounds on the degree of $Z-Z_d$ mass mixing, but further improvement is possible and warranted (see Table II).

The presence of kinetic mixing affects the phenomenology of $Z_d$, but much of the main physics discussed in this work persists even in the absence of kinetic mixing. Various experimental efforts are currently devoted to possible signals of the “dark” photon, based solely on the possibility of kinetic mixing between $U(1)_d$ and the SM photon. Here, we want to emphasize the $m_{Z_d}^2/m_{Z_d}^2$ enhancement factor in low energy parity violation and the longitudinal polarization enhancement $E_{Z_d}/m_{Z_d}$, with

\[ 1 \] One could contemplate searching for $Z_d$ effects in precision neutrino neutral current cross section measurements such as $\nu_\mu e \rightarrow \nu_\mu e$ or deep-inelastic $\nu_\mu N \rightarrow \nu_\mu X$. However, to be competitive with anticipated low energy parity violation polarized electron scattering or atomic experiments, those neutrino studies would have to reach $\sim \pm 0.1\%$ statistical and normalization uncertainties, a challenging task that would likely require a high energy neutrino factory (see Ref. [56]). A detailed discussion of $Z_d$ effects on neutrino cross sections will be given in a separate publication.
Table 2: Rough ranges of current (future) constraints on $\delta$ from various processes examined along with commentary on applicability of the bounds. These processes have negligible sensitivity to pure kinetic mixing effects.

<table>
<thead>
<tr>
<th>Process</th>
<th>Current (future) bound on $\delta$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Energy Parity Violation</td>
<td>$\delta \lesssim 0.08 - 0.01$ (0.001)</td>
<td>Fairly independent of $m_{Z_d}$. Depends on $\varepsilon$.</td>
</tr>
<tr>
<td>Rare K Decays</td>
<td>$\delta \lesssim 0.01 - 0.001$ (0.0003)</td>
<td>$m_{\mu}^2 &lt; m_{Z_d}^2 \lesssim m_{K}^2$. Depends on BR($Z_d$).</td>
</tr>
<tr>
<td>Rare B Decays</td>
<td>$\delta \lesssim 0.02 - 0.001$ (0.0003)</td>
<td>$m_{\mu}^2 &lt; m_{Z_d}^2 \lesssim m_{B}^2$. Depends on BR($Z_d$). Some mass gap $\sim 3$ GeV</td>
</tr>
<tr>
<td>$H \rightarrow ZZ_d$</td>
<td>$\delta \lesssim (0.003 - 0.001)$</td>
<td>$m_{Z_d}^2 \lesssim (m_H - m_Z)^2$. Depends on BR($Z_d$). Needs BKG studies.</td>
</tr>
</tbody>
</table>

$E_{Z_d}$ the energy of $Z_d$, in rare meson decays and the Higgs decay $H \rightarrow ZZ_d$. These enhancements make such processes particularly sensitive to very small $Z$-$Z_d$ mixing. In particular, future polarized $e\mu$ and $ee$ scattering experiments can provide further probes of the scenario we have considered in this work. These parity violating probes are sensitive to a wide range of $Z$-$Z_d$ masses, including $m_{Z_d} \lesssim 140$ MeV, where other searches fail due to $\pi^0$ Dalitz decays background and are independent of $Z_d$ branching fractions. The rare $K$ and $B$ decays currently provide some of the most stringent bounds on the degree of $Z$-$Z_d$ mixing, however they depend on the $Z_d$ branching fractions and also do not apply to $m_{Z_d}$ above the meson mass. In addition, there can be gaps in the bounds, for example in the $m_{Z_d}$ charmonium mass region.

In the event of the discovery of a SM-like Higgs at the LHC, say at $\sim 125$ GeV based on current hints, a new front in the search for a dark $Z$ can be established.

The Higgs decay data are particularly unique for $m_{Z_d} \gtrsim 5$ GeV, and hence probe a part of parameter space that is inaccessible to meson data. The reach for this new physics can be extended well beyond the current limits through precise measurements of Higgs decays, as may be done at an $e^+e^-$ or $\mu^+\mu^-$ collider if high statistics are available. We conclude that pushing the above types of experiments as far as possible is strongly motivated, for they could be windows to the “dark side” of particle physics.

Acknowledgments

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Appendix A: Gauge Kinetic Terms

The gauge kinetic terms allowed by the gauge symmetries $SU(2)_L \times U(1)_Y \times U(1)_d$ are

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} + \frac{1}{2} \frac{\varepsilon}{\cos \theta_W} \hat{B}_{\mu\nu} \hat{Z}_{d\mu}^{0\nu} - \frac{1}{4} \hat{Z}_{d\mu}^0 \hat{Z}_{d\mu}^{0\nu}$$

(A1)

with $F_{\mu\nu} = \partial_\mu F_\nu - \partial_\nu F_\mu$. The hatted quantities are fields before the diagonalization of the gauge kinetic terms. The diagonalization is done by the field redefinition known as a $GL(2, R)$ rotation

$$\begin{pmatrix} Z_{d\mu}^0 \\ \hat{B}_\mu \end{pmatrix} = \begin{pmatrix} \sqrt{1 - \varepsilon^2 / \cos^2 \theta_W} & 0 \\ -\varepsilon / \cos \theta_W & 1 \end{pmatrix} \begin{pmatrix} \hat{Z}_{d\mu}^0 \\ \hat{B}_\mu \end{pmatrix}$$

(A2)

after which, $B$ gets a $\hat{Z}_d$ component proportional to $\varepsilon$ while $Z_d$ does not get any $\hat{B}$ component.

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} Z_{d\mu\nu} Z_{d\mu\nu}$$

(A3)

We will take $\hat{Z}_{d\mu}^0 = Z_{d\mu}^0$ and $\hat{B}_\mu = B_\mu + (\varepsilon / \cos \theta_W) Z_{d\mu}^0$ and ignore $O(\varepsilon^2)$ terms from here on. After electroweak mixing with Weinberg angle $\theta_W$

$$\begin{pmatrix} A \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B \\ W_3 \end{pmatrix}$$

(A4)

we get

$$A_\mu = \hat{A}_\mu - \varepsilon \hat{Z}_{d\mu}^0$$

$$Z^0_\mu = \hat{Z}_{d\mu}^0 + \varepsilon \tan \theta_W \hat{Z}_{d\mu}^0$$

$$Z_{d\mu}^0 = \hat{Z}_{d\mu}^0$$

(A5)
as an effect of the gauge kinetic mixing. Thus, $Z_0^0$ is unaffected to $\mathcal{O}(\varepsilon)$ while both $A_\mu$ and $Z_\mu^0$ are shifted by the gauge kinetic mixing followed by the electroweak mixing. However, the bare fields do not take into consideration $Z^0$-$Z_0^0$ mixing via the mass matrix from the Higgs mechanism which we will deal with in the following.

**Appendix B: Scalar Kinetic Terms**

The scalar kinetic term is given by

$$\mathcal{L}_{\text{scalar}} = \sum_i |D_\mu \Phi_i|^2$$  \hspace{1cm} (B1)

where $i$ runs for all Higgs scalars. Considering only neutral components of gauge bosons, we have

$$D_\mu \Phi_i = \left( \partial_\mu + igY[\Phi_i]|\tilde{B}_\mu + igT_3[\Phi_i]|\tilde{W}_{3\mu} + ig_dQ_d[\Phi_i]|\tilde{Z}_d^\mu \right) \Phi_i$$  \hspace{1cm} (B2)

before gauge kinetic diagonalization where $Y$, $T_3$, $Q_d$ are hypercharge, isospin, and dark charge, respectively. After symmetry breaking, the scalars can be written with the vacuum expectation values ($v_i$).

$$\Phi_i = \frac{1}{\sqrt{2}} (H_i + v_i)$$  \hspace{1cm} (B3)

1. **Vector boson mass**

From Eq. (B1), we can get the relevant vector boson mass terms

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2} m_{Z^0}^2 Z^0 Z^0 - \Delta^2 Z^0 Z_0^0 + \frac{1}{2} m_{Z_d^0}^2 Z_0^0 Z_d^0 + \cdots.$$  \hspace{1cm} (B4)

The mixing of two vector bosons is given by

$$\left( \begin{array}{c} Z \\ Z_d \end{array} \right) = \left( \begin{array}{cc} \cos \xi & -\sin \xi \\ \sin \xi & \cos \xi \end{array} \right) \left( \begin{array}{c} Z_0^0 \\ Z_d^0 \end{array} \right)$$  \hspace{1cm} (B5)

with

$$\tan 2\xi = \frac{2\Delta^2}{m_{Z^0}^2 - m_{Z_d^0}^2}.$$  \hspace{1cm} (B6)

**2HD Model Realization:**

We discuss some details in context of the 2HD model example in Sec. VI. We set $U(1)_d$ charges as $Q_d[H_1] = 0$, $Q_d[H_2] = Q_d[H_3] = 1$ for notational convenience. Then the gauge boson mass-squared are given by, with $g_Z = g' / \sin \theta_W = g / \cos \theta_W$,

$$m_{Z_0^0}^2 = \frac{1}{4} g_Z^2 (v_1^2 + v_2^2),$$

$$m_{Z_d^0}^2 = g_d^2(v_d^2 + v_2^2) + \frac{\varepsilon}{\cos \theta_W} g_d g' v_2^2 + \frac{1}{4} \left( \frac{\varepsilon}{\cos \theta_W} \right)^2 g'^2(v_1^2 + v_2^2),$$

$$\Delta^2 = \frac{1}{2} g_d g_Z v_2^2 + \frac{1}{4} \frac{\varepsilon}{\cos \theta_W} g_d g' v_2^2.$$  \hspace{1cm} (B7)

We assume $m_{Z_0^0}^2 \ll m_{Z_d^0}^2$ which will be the case as long as $(g_d^2, \varepsilon g_d, \varepsilon^2) \ll g_Z^2$ and $v_d$ is not exceedingly larger than the electroweak scale. We define $\tan \beta \equiv v_2/v_1$, $\tan \beta_d \equiv v_2/v_d$, and $v^2 \equiv v_1^2 + v_2^2 \simeq (246 \text{ GeV})^2$. Then we have

$$m_Z^2 \simeq m_{Z_0^0}^2 = \frac{1}{4} g_Z^2 v^2,$$

$$m_{Z_d}^2 \simeq m_{Z_0}^2 - \frac{(\Delta^2)^2}{m_{Z_0}^2} = g_d^2(v_d^2 + v_2^2 \sin^2 \beta \cos^2 \beta) = g_d^2 v^2 \sin^2 \beta \sin^2 \beta_d,$$

$$\xi \simeq \frac{\Delta^2}{m_{Z^0}^2} = \frac{2g_d}{g_Z} \sin^2 \beta + \varepsilon \tan \theta_W.$$  \hspace{1cm} (B8)
Gauge kinetic mixing $\varepsilon$ does not contribute to $Z_d$ mass but it affects the $Z-Z_d$ mixing angle $\xi$.

(i) In the $v_2 = 0$ limit (i.e. pure dark photon limit), $Z_d$ mass is entirely from the Higgs singlet $H_d$ and the $Z-Z_d$ mixing angle is provided entirely by $\varepsilon$. We have

$$m_Z^2 \simeq m_{Z_0}^2, \quad m_{Z_d}^2 \simeq g_{Z_d}^2 v_d^2, \quad \xi \simeq \varepsilon \tan \theta_W$$  \hspace{1cm} (B9)

which give

$$M_0^2 \simeq \left( \begin{array}{cc} m_Z^2 & -\varepsilon \tan \theta_W m_Z^2 \\ -\varepsilon \tan \theta_W m_Z^2 & m_{Z_d}^2 + \varepsilon^2 \tan^2 \theta_W m_Z^2 \end{array} \right).$$  \hspace{1cm} (B10)

The mixing induced by the mass matrix cancels the effects due to field redefinition in Eq. (A5) for the $Z_d$ induced neutral current coupling.

(ii) In the $\varepsilon = 0$ limit (i.e. pure dark $Z$ limit),

$$m_{Z_d}^2 \simeq g_{Z_d}^2 v_2^2 \frac{\sin^2 \beta}{\sin^2 \beta_d} (1 - \sin^2 \beta \sin^2 \beta_d)$$

$$\xi \simeq \frac{2 g_{Z_d}}{g_Z} \frac{\sin^2 \beta}{m_Z} \frac{\sin \beta \sin \beta_d}{\sqrt{1 - \sin^2 \beta \sin^2 \beta_d}}.$$  \hspace{1cm} (B11)

Taking $1 - \sin^2 \beta \sin^2 \beta_d \simeq 1$ is valid when $|\Delta^2| \ll m_{Z_0} m_{Z_d}$. In this limit

$$m_Z^2 \simeq m_{Z_0}^2, \quad m_{Z_d}^2 \simeq m_{Z_0}^2, \quad \xi \simeq \varepsilon \xi_Z$$  \hspace{1cm} (B12)

with

$$\varepsilon_Z = \frac{m_{Z_d} \delta}{m_Z} \quad \text{and} \quad \delta = \sin \beta \sin \beta_d.$$  \hspace{1cm} (B13)

2. Higgs-Vector-Vector Couplings

We assume no mixing among Higgs scalars and refer to the SM-like Higgs as $H$. From Eq. (B1), we can get the relevant Higgs coupling to vector bosons.

$$L_{\text{scalar}} = \frac{1}{2} C_{HZZ} HZZ + C_{HZZ_d} HZZ_d + \frac{1}{2} C_{HZdZd} HZdZd + \cdots$$  \hspace{1cm} (B14)

The Feynman rules for coupling of $H$ to two vector bosons $V_1$ and $V_2$ are then given by $ig_{\mu\nu}C_{HV_1V_2}$.

In the 2HD example, we get

$$C_{HZZ} = C_{HZZ_d} = \frac{g_Z^2}{2} \cos \beta (\cos \xi + \varepsilon \tan \theta_W \sin \xi)^2$$

$$C_{HZd} = \frac{g_{Z_d}^2}{2} \cos \beta (\cos \xi + \varepsilon \tan \theta_W \sin \xi)(\sin \xi - \varepsilon \tan \theta_W \cos \xi)$$

$$C_{HZdZd} = \frac{g_{Z_d}^2}{2} \cos \beta (\sin \xi - \varepsilon \tan \theta_W \cos \xi)^2$$  \hspace{1cm} (B15)

with $C_{HZZ_d} = \frac{1}{2} g_Z v$. The ratio of couplings is

$$\Theta = \frac{C_{HZd}}{C_{HZZ}} = \frac{C_{HZdZd}}{C_{HZZ_d}} = \frac{\sin \xi - \varepsilon \tan \theta_W \cos \xi}{\cos \xi + \varepsilon \tan \theta_W \sin \xi}.$$  \hspace{1cm} (B16)

which, with small $|\xi| \ll 1$ from Eq. (B8), yields

$$\Theta \simeq \xi - \varepsilon \tan \theta_W \simeq \frac{2 g_{Z_d}}{g_Z} \sin^2 \beta$$  \hspace{1cm} (B17)

showing that $\Theta$ is not sensitive to $\varepsilon$. 

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The relevant Higgs decay rates, for $m_{Z_d} \ll m_H$, are given by

$$
\Gamma(H \to ZZ) = \frac{1}{128\pi} \frac{m_H^3}{m_Z^2} \sqrt{1 - \frac{4m_Z^2}{m_H^2}} \left(1 - \frac{4m_Z^2}{m_H^2} + \frac{12m_H^4}{m_H^4}\right) (\mathcal{C}_{HZZ})^2
$$

$$
\Gamma(H \to Z_dZ_d) \simeq \frac{1}{64\pi} \frac{m_H^3}{m_{Z_d}^2} \left(1 - \frac{m_{Z_d}^2}{m_H^2}\right)^3 (\Theta \mathcal{C}_{HZZ})^2
$$

$$
\Gamma(H \to Z_dZ_d) \simeq \frac{1}{128\pi} \frac{m_H^4}{m_{Z_d}^4} (\Theta^2 \mathcal{C}_{HZZ})^2
$$

with couplings given in Eq. (B15). Eq. (B18) conveniently shows the effects of phase space and $Z$-$Z_d$ mixing in the Higgs decay rates. The ratio of Higgs decay rates in the $Z_dZ_d$ and $ZZ_d$ channels is

$$
\frac{\Gamma(H \to Z_dZ_d)}{\Gamma(H \to ZZ)} \simeq \frac{\Theta^2}{2} \frac{m_Z^2}{m_{Z_d}^2} \left(1 - \frac{m_{Z_d}^2}{m_H^2}\right)^{-3}
$$

$$
\simeq \frac{1}{2} \sin^2 \beta \sin^2 \beta_d \left(1 - \frac{m_{Z_d}^2}{m_H^2}\right)^{-3} = \frac{1}{2} \delta^2 \left(1 - \frac{m_{Z_d}^2}{m_H^2}\right)^{-3}
$$

where Eqs. (B11) and (B12) have been used in the second line.

**Appendix C: $Z_d$ Decay Width**

Using Eqs. (10) and (11) in the text, we find that the partial decay width of $Z_d$ into the SM fermion pair $f\bar{f}$ is given by, neglecting $m_f/m_{Z_d}$ corrections [46],

$$
\Gamma(Z_d \to f\bar{f}) \simeq \frac{N_C}{48\pi} \frac{\epsilon_Z^2 g_Z^2}{g_{1f}^2 + g_{A_f}^2} m_{Z_d},
$$

where $g'_{1f} = T_{3f} - 2Q_f (\sin^2 \theta_W - (\epsilon/\epsilon_Z) \cos \theta_W \sin \theta_W)$ and $g_{A_f} = -T_{3f}$. Here, $N_C = 3$ for quarks and $N_C = 1$ for leptons.

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