Cosmic coincidence and asymmetric dark matter in a Stueckelberg extension

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Cosmic Coincidence and Asymmetric Dark Matter in a Stueckelberg Extension

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Abstract

We discuss the possibility of the cosmic coincidence generating the ratio of baryon asymmetry to dark matter in a Stueckelberg $U(1)$ extension of the standard model and of the minimal supersymmetric standard model. For the $U(1)$ we choose $L_\mu - L_\tau$ which is anomaly free and can be gauged. The dark matter candidate arising from this extension is a singlet of the standard model gauge group but is charged under $L_\mu - L_\tau$. Solutions to the Boltzmann equations for relics in the presence of asymmetric dark matter are discussed. It is shown that the ratio of the baryon asymmetry to dark matter consistent with the current WMAP data, i.e., the cosmic coincidence, can be successfully explained in this model with the depletion of the symmetric component of dark matter from resonant annihilation via the Stueckelberg gauge boson. For the extended MSSM model it is shown that one has a two component dark matter picture with asymmetric dark matter being the dominant component and the neutralino being the subdominant component (i.e., with relic density a small fraction of the WMAP cold dark matter value). Remarkably, the subdominant component can be detected in direct detection experiments such as SuperCDMS and XENON-100. Further, it is shown that the class of Stueckelberg models with a gauged $L_\mu - L_\tau$ will produce a dramatic signature at a muon collider with the $\sigma(\mu^+\mu^- \to \mu^+\mu^-, \tau^+\tau^-)$ showing a detectable Z' resonance while $\sigma(\mu^+\mu^- \to e^+e^-)$ is devoid of this resonance. Within the above frameworks we discuss several broad classes of models both above and below the electroweak phase transition temperature. Asymmetric dark matter arising from a $U(1)_{B-L}$ Stueckelberg extension is also briefly discussed. Finally, in the models we propose the asymmetric dark matter does not oscillate and there is no danger of it being washed out from oscillations.

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1 Introduction

One of the outstanding puzzles in particle physics and cosmology relates to the so called cosmic coincidence, i.e., the apparent closeness of the amount of baryon asymmetry to the amount of dark matter in the Universe. Thus the WMAP-7 result, with RECFAST version 1.5 to calculate the recombination history [1], gives the baryonic relic density to be $100\Omega_{\rm B}h_0^2 = 2.255 \pm 0.054$ and the dark matter relic density to be $\Omega_{\rm DM}h_0^2 = 0.1126 \pm 0.0036$, which leads to

$$\frac{\Omega_{\rm DM}h_0^2}{\Omega_{\rm B}h_0^2} = 4.99 \pm 0.20\,. \tag{1}$$

The closeness of $\Omega_{\rm DM}h_0^2$ and $\Omega_{\rm B}h_0^2$ points to the possibility that the baryonic matter and dark matter may have a common origin; a possibility that has been noted for some time [2]. In this manuscript we analyze this issue in the framework of a Stueckelberg $U(1)$ extension of the standard model (SM) as well as a Stueckelberg $U(1)$ extension of the minimal supersymmetric standard model (MSSM) [3–9]. There are two main constraints in building models with asymmetric dark matter (AsyDM). First, we need a mechanism for transferring a $B-L$ asymmetry produced in the early universe to dark matter. Second, we must have a mechanism for depleting the symmetric component of dark matter generated via thermal processes.

The above issues have been discussed in the literature in a variety of works [10–19] (for a review see [21]). The models based on the Stueckelberg extensions we discuss in this work are different from the ones considered previously both in terms of the mechanism for depletion of the symmetric component of dark matter as well as regarding the implications for dark matter and signatures at colliders. Specifically, we consider a $U(1)_X$ extension of the standard model gauge group which is anomaly free. Further, we consider dark matter candidates which will carry lepton number but not a baryon number, and are singlets of the standard model gauge group. Now in the leptonic sector it is known [22] that for the standard model case we may choose one of the linear combinations $L_e - L_\mu$, $L_\mu - L_\tau$, $L_e - L_\tau$ to be anomaly free and can be gauged. The gauged $L_e - L_\mu$ has been discussed previously in the context of PAMELA positron excess and multi-component dark matter [8] and $L_\mu - L_\tau$ in the context of muon anomalous moment [23] and in the context the PAMELA positron excess [24]. Here we consider a gauged $L_\mu - L_\tau$ in the discussion of asymmetric dark matter as this choice is the more appropriate one for the analysis here. Specifically, we will consider a $U(1)_X$, $X = L_\mu - L_\tau$ Stueckelberg extension of the standard model as well as of the MSSM. As is well-known, the MSSM supplemented by supergravity soft breaking gives the neutralino as the lowest supersymmetric particle and with R parity a candidate for dark matter. Thus for the AsyDM to work in the MSSM extensions it is necessary to have the neutralino as a subdominant component. This issue will be addressed as well as the question if such a subdominant component may still be detectable in experiments for the direct detection of dark matter. It is found that the Stueckelberg models with a gauged $L_\mu - L_\tau$ can produce dramatic signatures at a muon collider leading to a detectable Z' resonance in the $\mu^+\mu^- \to \mu^+\mu^-, \tau^+\tau^-$ cross section while the $\mu^+\mu^- \to e^+e^-$ cross section exhibits no such resonance. Finally we consider the possibility of an asymmetric dark matter in the Stueckelberg extension of $B-L$.

An important issue regarding asymmetric dark matter concerns the possibility that such matter can undergo oscillations [19]. Thus, for example, consider a model which allows for a Majorana mass term $\mathcal{L} = -m_M XX + h.c.$, where X is the dark particle. The presence of such a term along with other mass terms allows for the oscillation of X to its anti-particle $\bar{X}$. Detailed analysis show that in this circumstance over the age of the Universe the asymmetric dark matter would produce a symmetric component which would lead to pair annihilation. Such processes could completely wipe out the asymmetric dark matter generated in the early universe, and could render such models largely invalid [19]. In the models we consider mass terms that can generate oscillations are forbidden because of gauge invariance, either $U(1)_{L_\mu - L_\tau}$ or $U(1)_{B-L}$. Thus dark matter oscillations are absent in the class of models we consider. However, we wish to add a further explanation here. In general there are two alternative possibilities for the Stueckelberg mechanism to arise. One possibility is that it is a low energy remnant of a Higgs mechanism which contains a scalar field S that would in general allow a term of the type XXS^n and thus generate a Majorana mass term for X as a consequence of VEV formation of the scalar field S. This would lead to dark matter oscillations once again. The second possibility, which is the view point adopted in this work, is that the Stueckelberg mechanism arises from the Green-Schwarz term in string theory (see, e.g., [20]). In this case there is no fundamental Higgs field which develops a VEV and thus Majorana mass term would not be generated and there would be no dark matter oscillation.

We will discuss six broad classes of models. Three of these will be anchored in extensions of the standard model, one in extension of the two Higgs doublet model, and two in extensions of the minimal supersymmetric standard model. We will consider cases where the asymmetry transfer interaction may lie above or below the electroweak phase transition scale, i.e., the scale where the Higgs boson gets its VEV. In the supersymmetric case we will consider the case where all of the sparticles are in the thermal bath at temperatures where the asymmetry transfer takes place as well as the case where the first two generations of squarks are heavy and are Boltzmann suppressed in the thermal bath.

The outline of the rest of the paper is as follows: In Section 2 we give a brief introduction to cosmic coincidence as well as asymmetric dark matter and the technique for the computation of the ratio of dark matter density to the baryonic matter density in the Universe and of the dark matter mass. Here we describe six broad classes of models which we will discuss in detail later. In Section 3 we carry out an explicit computation of these quantities in extensions of the standard model and of the two Higgs doublet model. In this section we also consider the case with inclusion of right-handed neutrinos. In Section 4 the analysis is redone for models in extensions of MSSM. In Appendix A we give a master formula for the computation of the asymmetric dark matter mass which is valid for temperatures above the electroweak phase transition scale. Here we show that the results of models discussed in previous sections can be deduced as limiting cases. In Section 5 we consider an explicit Stueckelberg extension of the standard model which generates asymmetric dark matter. We discuss solutions to the Boltzmann equations for relics in the presence of asymmetric dark matter, and show that the symmetric component of dark matter can be depleted from resonant annihilation via the Z' pole. In Section 6 we give a Stueckelberg extension of MSSM. Here a similar resonant annihilation of the symmetric component of dark matter is valid. There are several additional particles that arise in this case which include an extra scalar particle (the ρ) from the Stueckelberg sector. The decay width of this particle is computed in Appendix B and it is shown that it decays rapidly and is removed from the relativistic plasma. There are also additional neutralinos which we assume lie above the lightest MSSM neutralino and thus the lightest MSSM neutralino continues to be the lightest supersymmetric particle (LSP). In Section 7 we show that the MSSM neutralino is a subdominant component and thus does not interfere with the AsyDM mechanism. It is also shown here that the subdominant component can produce a spin-independent cross section which lies within reach of future experiments for the direct detection of dark matter. In Section 8 we discuss the signatures of the models at a muon collider. A $(B-L)$ Stueckelberg extension is discussed in Section 9 which also produces asymmetric dark matter. Conclusions are given in Section 10. In Appendix C, we compute the $\mu^+\mu^- \to e^+e^-$ at the loop level via $Z'-\gamma$, $Z'-Z$ exchange and show that the corresponding production cross section is too small to be discernible.

2 Cosmic coincidence and asymmetric dark matter

In the analysis here we assume that a $B-L$ asymmetry has been generated in the early universe. We do not speculate on how this asymmetry comes about as it could be by any number of different processes such as baryogenesis or leptogenesis [25]. Thus, for example, baryon asymmetry, specifically a non-vanishing $B-L$, can arise in the early universe by decay of super-heavy particles in some grand unified models [26, 27] consistent with experimental proton decay limits [28]. A $(B-L)$ asymmetry of this type will not be washed out by sphaleron processes which preserve $B-L$. Such an asymmetry is then transferred to the dark sector at high temperatures via an interaction of the form [12]

$$\mathcal{L}_{\text{asy}} = \frac{1}{M^n_{\text{asy}}} \mathcal{O}_{\text{DM}} \mathcal{O}_{\text{asy}} \,, \tag{2}$$

where M_{asy} is the scale of this interaction,[1] and $\mathcal{O}_{\text{asy}}$ is an operator constructed from SM/MSSM fields which carries a non-vanishing $B-L$ quantum number while $\mathcal{O}_{\text{DM}}$ carries the opposite $B-L$ quantum number. This interaction would decouple at some temperature greater than the dark matter mass. As the Universe cools, the dark matter asymmetry freezes on order of the baryon asymmetry, which explains the observed relation between baryon and dark matter densities.

At the temperature where Eq. (3) is operational, and using the fact that the chemical potential of particles and anti-particles are different, the asymmetry in the particle and antiparticle number densities is given by

$$\begin{aligned} n_i - \bar{n}_i &= \frac{g_i}{2\pi^2} \int_0^\infty \mathrm{d}q\, q^2 \left[(e^{(E_i(q)-\mu_i)/T} \pm 1)^{-1} - (e^{(E_i(q)+\mu_i)/T} \pm 1)^{-1} \right] \\ &\equiv \frac{g_i T^3}{6} \times \begin{cases} \beta\mu_i c_i(b) & \text{bosons}\,, \\ \beta\mu_i c_i(f) & \text{fermions}\,, \end{cases} \end{aligned} \tag{4}$$

where n_i and $\bar{n}_i$ denote the equilibrium number density of particle and antiparticle respectively, g_i counts the degrees of freedom of the particle, $E_i(q) = \sqrt{q^2 + m_i^2}$ where m_i is the mass of particle i, μ_i is the chemical potential of the particle ($-\mu_i$ is the chemical potential of the antiparticle), and $+1\,(-1)$ in the denominator are for case when the particle is a fermion (boson). In the ultra relativistic limit ($T \gg m_i$) the mass of the particle can be dropped. In our analysis below we use the approximation of a weakly interacting plasma where $\beta\mu_i \ll 1$, and $\beta \equiv 1/T$ and one has

$$n_i - \bar{n}_i \ \sim \ \frac{g_i T^3}{6} \times \begin{cases} 2\beta\mu_i + \mathcal{O}\big((\beta\mu_i)^3\big) & \text{bosons}\,, \\ \beta\mu_i + \mathcal{O}\big((\beta\mu_i)^3\big) & \text{fermions}\,. \end{cases} \tag{5}$$

[1] In the radiation-dominated era, the Hubble expansion rate is given by $H \sim T^2/M_{\text{Pl}}$, where $M_{\text{Pl}} = 2.435 \times 10^{18}$ GeV is the reduced Plank mass. For an interaction suppressed by a factor $1/M^n_{\text{asy}}$, the interaction rate at temperature T is $\Gamma(T) \sim T^{2n+1}/M^{2n}_{\text{asy}}$. Thus, the interaction will decouple if $\Gamma < H$, i.e., when

$$M^{2n}_{\text{asy}} > M_{\text{Pl}} T^{2n-1} \,. \tag{3}$$

In the limit where Eq. (5) holds we have $c_i(b) = 2, c_i(f) = 1$. This limit is a useful approximation as it simplifies the analysis of the chemical potentials that are needed in the generation of dark matter. However, full analysis can be easily done by using the exact expression of Eq. (4). We will discuss inclusion of these in Appendix A. There the functions $c_i(b)$ and $c_i(f)$ introduced in Eq. (4) will be found useful. The mass of the dark matter is constrained by the experimental ratio of dark matter to baryonic matter given in Eq. (1). Defining B to be the total baryon number in the Universe and X to be the total dark matter number, we obtain

$$\frac{\Omega_{\rm DM}}{\Omega_{\rm matter}} = \frac{X \cdot m_{\rm DM}}{B \cdot m_{\rm B}} \approx 5\,, \tag{6}$$

so that the dark particle mass is given by

$$m_{\rm DM} \approx 5 \cdot \frac{B}{X} \cdot 1 \text{ GeV}\,. \tag{7}$$

Applying the general thermal equilibrium method [29] (see also [11]), it is not difficult to express B and X in terms of the chemical potentials and then find their ratio. We note one subtlety is that while X and $B-L$ (where by $B-L$ we mean the $B-L$ in the standard model sector) are conserved after the interaction in Eq. (2) decouples, B is not. Thus, for example, the top quark would drop out from the thermal bath at some temperature T_t and one must solve the new set of μ equations at $T < T_t$ which would affect the computation of B although $B - L$ is conserved. Typically one takes T_t to be M_t but it could be somewhat lower. Specifically, as the temperature drops below $M_t \sim 173$ GeV, the top quark becomes semi-relativistic but could still be involved in the thermal equilibrium constraints. A precise determination of T_t is out of the scope of this paper, and here we simply assume that T_t lies below M_t. Further, as the temperature falls below the temperature where sphaleron processes decouple, B and L would be separately conserved down to the current temperatures. Thus the relevant B to compute the dark matter mass in Eq. (7) would be the baryon number below the sphaleron temperature which we label $B_{\rm final}$. It is useful to express X and $B_{\rm final}$ in terms of $B-L$ so that $X = x(B-L)$ and $B_{\rm final} = b(B-L)$ where b is to be determined later (see Eqs. (28) and (61)). Thus, Eq. (7) can be rewritten as

$$m_{\rm DM} \approx 5 \cdot \frac{b}{x} \cdot 1 \text{ GeV}\,. \tag{8}$$

We will discuss six broad classes of models labeled Models A-F (see Table 1). Models A,B,C are anchored in the standard model while Model D is a two Higgs doublet (2HD) model. For Models A and D, the asymmetry transfer interaction, of the form of Eq. (2), is active only above the electroweak phase transition (EWPT) scale, i.e., $T_{\rm int} > T_{\rm EWPT}$ ($T_{\rm EWPT} \sim 200 - 300$ GeV where the Higgs gets its VEV). For Model B and C, the interaction which transfers the asymmetry could be active *also* below the EWPT scale, i.e., $T_{\rm EWPT} > T_{\rm int}$. More specifically, in Model B we consider the temperature regime $T_{\rm EWPT} > T_{\rm int} > M_t$, and in Model C we discuss $T_t > T_{\rm int} > M_W$ (M_W is the mass of W boson). Similarly, we discuss models based on extensions of the MSSM. Here we will focus on two cases; one of which is when $T_{\rm int} > M_{\rm SUSY}$ (Model E) where $M_{\rm SUSY}$ is the (largest) soft breaking mass. In this case all the sparticles will be in the plasma. The second case (Model F) corresponds to when the first two generations of sparticles (with mass M_1) are heavy and drop out of the plasma (at some temperature $T_1 < M_1$) while the third generation sparticles, the gauginos, the Higgses and the Higgsinos (with mass $M_2 \ll M_1$) remain in the plasma. Thus for this case we

Model A	SM	$T_{\rm int} > T_{\rm EWPT}$
Model B		$T_{\rm EWPT} > T_{\rm int} > M_t$
Model C		$T_t > T_{\rm int} > M_W$
Model D	2HD	$T_{\rm int} > T_{\rm EWPT}$
Model E	MSSM	$T_{\rm int} > M_{\rm SUSY}$
Model F		$T_1 > T_{\rm int} > M_2 > T_{\rm EWPT}$

Table 1: A list of six models which allow for generation of asymmetric dark matter. Models A,B,C are within the framework of the extensions of the standard model (SM) while Model D is an extension of a two Higgs doublet model (2HD). Models E and F are in the framework of an extension of the minimal supersymmetric standard model (MSSM).

have $T_1 > T_{\rm int} > M_2 > T_{\rm EWPT}$.

These six cases are summarized in Table 1. There can be additional subcases for these models corresponding to different choices of the $B-L$ transfer in Eq. (2).

3 Analysis in non-supersymmetric framework

In this section we will determine the dark matter mass in terms of the $B-L$ asymmetry in the non-supersymmetric framework utilizing Eq. (8) for Models A-D. We will discuss three different temperature regimes where the $B-L$ transfer takes place and then deduce a general formula for computing the asymmetric dark matter mass. We note that the dark matter mass depends only on the $(B-L)$-charge of the operator $\mathcal{O}_{\rm asy}$ that enters in Eq. (2) and not on other particulars of the interaction. We will give several examples of the operator $\mathcal{O}_{\rm asy}$ and compute the dark matter mass for them.

3.1 $T > T_{\rm EWPT}$

First we consider the case when the temperature is above the electroweak phase transition scale. In this case the following fields are in the relativistic plasma in the early universe: three generations of left-handed lepton doublets L_i and quark doublets q_i, three generations of right-handed charged leptons e_i and up and down-type quarks u_i and d_i $(i = 1, 2, 3)$, and number λ_H of complex Higgs doublets $H_i = (h_i^+, h_i^0)^T$. Since the Z boson and the photon couple to particle and anti-particle pairs they have a vanishing chemical potential. Further, in this temperature regime, $SU(2)_L$ symmetry is unbroken, the W and Z are part of the same gauge multiplet which requires that the chemical potential of the W vanishes. The chemical potential of the gluon is zero and different color quarks carry the same chemical potential. The flavor (CKM) mixing among quarks ensures that the chemical potential of quarks in different generations are equal. However for the lepton sector, there is no such flavor mixing in the absence of neutrino masses [30]. Thus each of the lepton numbers (L_e, L_μ, L_τ) for the three generations are separately conserved. Our notation is as follows: μ_{L_i}, μ_{e_i} denote the chemical potentials of left-handed and right-handed leptons while $\mu_{q_i}, \mu_{u_i}, \mu_{d_i}$ stand for the chemical potential of left-handed and right-handed quarks. We assume that the chemical potential of all generations is the same and thus drop the subscript i and use μ_H for the chemical potential of the Higgs doublets (we assume all the Higgs doublets have identical chemical potential).

The Yukawa couplings

$$\mathcal{L}_{\text{Yukawa}} = g_{e_i}\bar{L}_i H e_i + g_{u_i}\bar{q}_i H^c u_i + g_{d_i}\bar{q}_i H d_i \tag{9}$$

yield the following relations among the chemical potentials

$$\mu_H = \mu_L - \mu_e = \mu_q - \mu_d = \mu_u - \mu_q\,. \tag{10}$$

Sphaleron processes ($\mathcal{O}_{\text{sph}} \sim \prod_{i=1,2,3} q_i q_i q_i L_i$) give us one additional relation,

$$3\mu_q + \mu_L = 0\,. \tag{11}$$

The temperature at which sphaleron processes decouple is estimated to be $T_{\text{Sph}} \sim [80{+}54(m_h/120\text{ GeV})]$ GeV [31]. It is very likely that T_{Sph} lies below T_{EWPT}, and thus the sphaleron processes are always active at $T > T_{\text{EWPT}}$. Finally, the hypercharge neutrality condition requires the total hypercharge of the Universe to be zero[2]

$$3\mu_q + 6\mu_u - 3\mu_d - 3\mu_L - 3\mu_e + 2\lambda_H\mu_H = 0\,. \tag{12}$$

Solving Eqs. (9)-(12) we can express all the chemical potentials in terms of the chemical potential of one single field, e.g., μ_L. Specifically one finds for Model A with $\lambda_H = 1$ (suppressing a factor of $\beta T^3/6$)

$$B_{\text{A}} = 3 \times [2\mu_q + (\mu_u + \mu_d)] = -4\mu_L\,, \tag{13}$$

$$L_{\text{A}} = 3 \times (2\mu_L + \mu_e) = \frac{51}{7}\mu_L\,, \tag{14}$$

so that $(B-L)_{\text{A}} = -\frac{79}{7}\mu_L$. And for Model D with $\lambda_H = 2$ we have

$$B_{\text{D}} = -4\mu_L\,, \qquad L_{\text{D}} = \frac{15}{2}\mu_L\,, \tag{15}$$

and $(B-L)_{\text{D}} = -\frac{23}{2}\mu_L$.

3.2 $T < T_{\text{EWPT}}$

Now we consider the case when the temperature is below the EWPT scale. After the Higgs gets its VEV, and the $SU(2)_L \times U(1)_Y$ symmetry is broken, one has $W^\pm$, Z, the photon and the Higgs scalar (h) as the physical particles in the thermal bath. Again, since the Z and the photon only couple to two particles with opposite chemical potentials, their chemical potentials are zero. For temperatures above the top quark mass, the relativistic plasma includes three generations of left-handed and right-handed up-type and down-type quarks (u_{iL}, u_{iR}, d_{iL} and d_{iR}), three generations of left-handed leptons (e_{iL} and ν_i) and right-handed charged leptons (e_{iR}), $i = 1, 2, 3$. As in Section 3.1, we will assume that the chemical potentials are generation independent. Thus dropping the generation index we will use $\mu_{u_L}, \mu_{u_R}, \mu_{d_L}, \mu_{d_R}$ to denote the chemical potentials of left-handed and right-handed

[2] The hypercharge of the Universe used in deducing Eq. (12) is computed as follows:

$$Y = 3 \times \left[2 \times 3 \times \tfrac{1}{3}\mu_q + 3 \times \tfrac{4}{3}\mu_u + 3 \times (-\tfrac{2}{3})\mu_d + 2 \times (-1)\mu_L + (-2)\mu_e\right] + 2 \times 2\lambda_H\mu_H\,,$$

where the factor of 3 outside the first brace indicates summation over quark and lepton generations while inside the brace the factor of 3 for quarks indicates summing over colors, the factor of 2 for q, L and H counts two fields inside the doublets, and the additional factor of 2 for the Higgs is due to it being bosonic (see Eq. (5)).

up-type and down-type quarks, μ_{e_L} and μ_ν for left-handed leptons, μ_{e_R} for right-handed charged leptons, μ_W for W^+, and μ_h for h.

In the analysis below we make the following approximations: (1) At $T_{\rm EWPT} > T > M_t$, we still treat the top quark as relativistic gas. (2) At $T_t > T > M_W$, we treat the W boson as relativistic (all other particles, which have non-vanishing chemical potentials, are very light so the limit in Eq. (5) holds for them). (3) We assume $T_t > T_{\rm Sph}$, i.e., the top quark drops out of the thermal bath before the sphaleron processes decouple.

For $T < T_{\rm EWPT}$, the Yukawa couplings have the form

$$\mathcal{L}_{\rm Yukawa} = g_{e_i}\bar{e}_{iL}he_{iR} + g_{u_i}\bar{u}_{iL}hu_{iR} + g_{d_i}\bar{d}_{iL}hd_{iR} + h.c.\,, \tag{16}$$

and since the Higgs boson is a real field and can couple to, for example, both $\bar{e}_{iL}e_{iR}$ and $\bar{e}_{iR}e_{iL}$, we get

$$0 = \mu_h = \mu_{u_L} - \mu_{u_R} = \mu_{d_L} - \mu_{d_R} = \mu_{e_L} - \mu_{e_R}\,. \tag{17}$$

Thus, the chemical potentials of left-handed and right-handed quarks/charged leptons are equal. The gauge interactions involving W bosons ($\mathcal{L} \sim W_\mu \bar{f}\gamma^\mu f$) provide us the following relations,

$$\mu_W = \mu_{u_L} - \mu_{d_L} \qquad (W^+ \leftrightarrow u_L + \bar{d}_L)\,, \tag{18}$$

$$\mu_W = \mu_\nu - \mu_{e_L} \qquad (W^+ \leftrightarrow \nu_i + \bar{e}_{iL})\,. \tag{19}$$

The sphaleron processes give us one additional equation,

$$\mu_{u_L} + 2\mu_{d_L} + \mu_\nu = 0\,. \tag{20}$$

Since $SU(2)_L$ symmetry is broken below the EWPT scale, hypercharge is no longer a good quantum number. Further, the neutrality of the Universe now requires the total electrical charge to be zero[3]

$$2(\mu_{u_L} + \mu_{u_R} + \mu_W) - (\mu_{d_L} + \mu_{d_R} + \mu_{e_L} + \mu_{e_R}) = 0\,. \tag{22}$$

Solving the new set of equations one finds for Model B

$$B_{\rm B} = 3 \times [(\mu_{u_L} + \mu_{u_R}) + (\mu_{d_L} + \mu_{d_R})] = -\frac{36}{7}\mu_e\,, \tag{23}$$

$$L_{\rm B} = 3 \times (\mu_{e_L} + \mu_{e_R} + \mu_\nu) = \frac{75}{7}\mu_e\,, \tag{24}$$

where we have expressed the results in terms of $\mu_e \equiv \mu_{e_L} = \mu_{e_R}$, and $(B-L)_{\rm B} = -\frac{111}{7}\mu_e$.

When the temperature drops below T_t, the top quark drops out from the thermal bath, and we are left with just five flavors of quarks. In this case ($T_t > T > M_W$) one must treat the first

[3] The result of Eq. (22) follows from the computation of the total charge Q which is given by

$$Q = 3 \times \left[3 \times \tfrac{2}{3}(\mu_{u_L} + \mu_{u_R}) + 3 \times (-\tfrac{1}{3})(\mu_{d_L} + \mu_{d_R}) + (-1)(\mu_{e_L} + \mu_{e_R})\right] + 2 \times 3\mu_W\,, \tag{21}$$

where again, the factors of 3 for fermions outside the big brace indicates summing over generations, the other factor of 3 for quarks stands for summing over colors. For the W boson, 2 is the boson factor as given by Eq. (5) and 3 is the degrees of freedom of W.

two generations and the third generations separately. For the first two generations the analysis of Eqs. (17)-(20) still holds. For the remaining third generation leptons, we assume as before that the chemical potentials are identical to those for the first two generation leptons. Further, we note that the charge current process $W^+ \leftrightarrow u_L + \bar{b}_L$ provides us with the relations $\mu_W = \mu_{u_L} - \mu_{b_L}$ and $\mu_{b_L} = \mu_{d_L}$. Thus we can treat Model C similar to Model B with only one modification to the charge neutrality condition, which now becomes

$$4(\mu_{u_L} + \mu_{u_R}) + 6\mu_W - 3(\mu_{d_L} + \mu_{d_R} + \mu_{e_L} + \mu_{e_R}) = 0\,. \tag{25}$$

Solving these equations we obtain for Model C

$$B_\mathrm{C} = 2(\mu_{u_L} + \mu_{u_R}) + 3(\mu_{d_L} + \mu_{d_R}) = -\frac{90}{19}\mu_e\,, \tag{26}$$

$$L_\mathrm{C} = 3 \times (\mu_{e_L} + \mu_{e_R} + \mu_\nu) = \frac{201}{19}\mu_e\,, \tag{27}$$

and $(B-L)_\mathrm{C} = -\frac{291}{19}\mu_e$. We note that the sphaleron processes will decouple below T_Sph as mentioned already. Subsequently the baryon and lepton numbers would be separately conserved. Eqs. (17)-(19), and (25)-(27) would remain valid at $T_\mathrm{Sph} > T > M_W$.

Following our assumptions given earlier, the top quark drops out of the thermal bath before sphaleron processes decouple. After the sphaleron processes decouple, B and L would be separately conserved. In other words, the ratio of $B/(B-L)$ would freeze as soon as the sphaleron processes are no longer active. Thus, we obtain

$$b = \frac{B_\mathrm{final}}{B-L} = \left(\frac{B}{B-L}\right)_\mathrm{C} = \frac{30}{97} \approx 0.31\,. \tag{28}$$

3.3 The AsyDM mass: non-SUSY case

We discuss now in further detail the mechanism by which $B-L$ is transferred from the standard model sector to the dark matter sector and the determination of the dark matter mass. We consider the most general interaction which transfers the $B-L$ asymmetry to dark matter at a high temperature:

$$\mathcal{L}^\mathrm{SM}_\mathrm{asy} = \frac{1}{M^n_\mathrm{asy}} X^k \mathcal{O}^\mathrm{SM}_\mathrm{asy}\,, \tag{29}$$

where the operator $\mathcal{O}^\mathrm{SM}_\mathrm{asy}$ is constructed from the standard model fields, has a $(B-L)$-charge $Q^{\mathcal{O}^\mathrm{SM}}_{B-L}$, and X is the dark particle and has a $(B-L)$-charge $Q^\mathrm{DM}_{B-L} = -Q^{\mathcal{O}^\mathrm{SM}}_{B-L}/k$.[4]

The parameterization of the asymmetric dark matter sector by the charge Q^DM_{B-L} is useful and we will utilize it in our analysis below. Also useful is the parameterization of the interactions in terms of the number of doublets and singlets that enter in $\mathcal{O}^\mathrm{SM}_\mathrm{asy}$, i.e., N_q, N_L, N_H numbers of q, L, H doublets and N_u, N_d, N_e numbers of u_R, d_R, e_R singlets which are all active above the EWPT scale. Eq. (29) leads to the following constraints [13]

$$N_q\mu_q + N_L\mu_L + N_u\mu_u + N_d\mu_d + N_e\mu_e + N_H\mu_H + k\mu_X = 0\,, \tag{30}$$

$$\tfrac{1}{3}N_q + \tfrac{1}{3}N_u + \tfrac{1}{3}N_d - N_L - N_e + kQ^\mathrm{DM}_{B-L} = 0\,, \tag{31}$$

[4]The power of X can only be 2 or greater to ensure the stability of the asymmetric dark matter.

$$\tfrac{1}{3}N_q + \tfrac{4}{3}N_u - \tfrac{2}{3}N_d - N_L - 2N_e + N_H = 0\,. \tag{32}$$

Here Eq. (30) arises from the μ equilibrium of Eq. (29), Eq. (31) arises from the total $(B-L)$-charge conservation of the interaction, and Eq. (32) arises from the hypercharge conservation and the condition that the asymmetric dark matter must have zero hypercharge. Together with Eqs. (10)-(12), for Model A we obtain

$$\mu_X^{\mathrm{A}} = -\frac{11}{7} Q_{B-L}^{\mathrm{DM}} \mu_L\,. \tag{33}$$

If X is fermionic dark matter (FDM), we find,

$$x_{\mathrm{A}} = \frac{X_{\mathrm{A}}}{(B-L)_{\mathrm{A}}} = \frac{k\mu_X^{\mathrm{A}}}{-\frac{79}{7}\mu_L} = -\frac{11}{79} Q_{B-L}^{\mathcal{O}^{\mathrm{SM}}}\,. \tag{34}$$

Using Eqs. (8) and (28), we obtain

$$m_{\mathrm{FDM}}^{\mathrm{A}} \approx -\frac{11.11 \text{ GeV}}{Q_{B-L}^{\mathcal{O}^{\mathrm{SM}}}}\,. \tag{35}$$

Similarly, for Model D with two Higgs doublets, we have

$$\mu_X^{\mathrm{D}} = -\frac{3}{2} Q_{B-L}^{\mathrm{DM}} \mu_L\,, \tag{36}$$

so that

$$m_{\mathrm{FDM}}^{\mathrm{D}} \approx -\frac{11.86 \text{ GeV}}{Q_{B-L}^{\mathcal{O}^{\mathrm{2HD}}}}\,. \tag{37}$$

If the $B-L$ transfer interaction is also active below the EWPT scale, the treatment is similar. Assuming $\mathcal{O}_{\mathrm{asy}}^{\mathrm{SM}}$ has $N_u, N_d, N_e, N_\nu, N_W$ numbers of u, d, e, ν, W^+ fields and recalling that at $T < T_{\mathrm{EWPT}}$, the left-handed and right-handed quarks and charged leptons have the same chemical potentials, one finds the following constraints

$$N_u\mu_u + N_d\mu_d + N_e\mu_e + N_\nu\mu_\nu + N_W\mu_W + k\mu_X = 0\,, \tag{38}$$

$$\tfrac{1}{3}N_u + \tfrac{1}{3}N_d - N_e - N_\nu + kQ_{B-L}^{\mathrm{DM}} = 0\,, \tag{39}$$

$$\tfrac{2}{3}N_u - \tfrac{1}{3}N_d - N_e + N_W = 0\,. \tag{40}$$

We note that the last condition is from the charge neutrality of the operator $\mathcal{O}_{\mathrm{asy}}^{\mathrm{SM}}$. Together with Eqs. (17)-(22), we obtain for Model B,

$$\mu_X^{\mathrm{B}} = -\frac{11}{7} Q_{B-L}^{\mathrm{DM}} \mu_e\,. \tag{41}$$

The fermionic dark matter mass in this model reads

$$m_{\mathrm{FDM}}^{\mathrm{B}} \approx -\frac{15.60 \text{ GeV}}{Q_{B-L}^{\mathcal{O}^{\mathrm{SM}}}}\,. \tag{42}$$

For Model C where the top quark is out of the thermal bath, we find

$$\mu_X^{\mathrm{C}} = -\frac{29}{19} Q_{B-L}^{\mathrm{DM}} \mu_e\,. \tag{43}$$

and

$$m^{\rm C}_{\rm FDM} \approx -\frac{15.52\ {\rm GeV}}{Q^{\mathcal{O}^{\rm SM}}_{B-L}}\,. \tag{44}$$

Now we consider the simplest example of the $B-L$ transfer interaction ($Q^{\mathcal{O}^{\rm SM}}_{B-L} = -1$)

$$\mathcal{L}_{\rm asy} = \frac{1}{M^3_{\rm asy}}\psi^3 LH\,, \tag{45}$$

where ψ is the fermionic dark matter (which carries a lepton number of $-1/3$) and $\psi^3 \equiv \bar{\psi}^c\psi\bar{\psi}^c$. If this interaction is only active above the EWPT scale then the dark matter masses in Models A and D, and more appropriately in Models $\rm A_1$ and $\rm D_1$ since the interaction of Eq. (45) is being used (see Table 2 which also includes a list of additional interactions), are computed to be

$$m_\psi = 11.11\ {\rm GeV} \qquad {\rm Model\ A_1}\,; \qquad m_\psi = 11.86\ {\rm GeV} \qquad {\rm Model\ D_1}\,. \tag{46}$$

If this interaction is also active below the EWPT scale, the dark matter masses in Models B and C are:

$$m_\psi = 15.60\ {\rm GeV} \qquad {\rm Model\ B_1}\,; \qquad m_\psi = 15.52\ {\rm GeV} \qquad {\rm Model\ C_1}\,. \tag{47}$$

Further, applying Eq. (3) and the bounds in Table 1 one can estimate the mass scales for these interactions:

$$M^{\rm A_1/D_1}_{\rm asy} \gtrsim 1.2\times 10^5\ {\rm GeV}\,, \tag{48}$$
$$1.2\times 10^5\ {\rm GeV} \gtrsim M^{\rm B_1}_{\rm asy} \gtrsim 0.9\times 10^5\ {\rm GeV}\,, \tag{49}$$
$$0.9\times 10^5\ {\rm GeV} > M^{\rm C_1}_{\rm asy} \gtrsim 0.4\times 10^5\ {\rm GeV}\,. \tag{50}$$

In the analysis above we focused on asymmetric fermionic dark matter. For bosonic dark matter, the masses would be half the fermionic ones, c.f., Eq. (5). As an example, we consider now an interaction with a higher dimensional operator $\mathcal{O}^{\rm SM}_{\rm asy}$:

$$\mathcal{L}_{\rm asy} = \frac{1}{M^n_{\rm asy}}X^2(LH)^2. \tag{51}$$

In this case, the dark matter could be either a fermion ($X=\psi$, $n=4$) or a boson ($X=\phi$, $n=3$). This interaction gives rise to Models $\rm A_2$-$\rm D_2$ and Models $\rm A_3$-$\rm D_3$. As examples, for Models $\rm A_2$ and $\rm A_3$ where $T_{\rm int} > T_{\rm EWPT}$, applying Eq. (35) we find that the dark matter masses are

$$m_\psi = 5.55\ {\rm GeV} \qquad {\rm Model\ A_2}\,; \qquad m_\phi = 2.78\ {\rm GeV} \qquad {\rm Model\ A_3}\,. \tag{52}$$

We explain now briefly the equality of asymmetric dark mass for the Models $\rm A_1, A_4, A_5, A_6$. From Eq. (2) we can write

$$\mu_{\mathcal{O}_{\rm DM}} + \mu_{\mathcal{O}^{\rm SM}_{\rm asy}} = 0\,. \tag{53}$$

$\frac{1}{M^n}X^k\mathcal{O}_{\rm asy}^{\rm SM}$	Model	DM Mass	Model	DM Mass	Model	DM Mass	Model	DM Mass
$\frac{1}{M^3}\psi^3 LH$	A_1	11.11 GeV	B_1	15.60 GeV	C_1	15.52 GeV	D_1	11.86 GeV
$\frac{1}{M^4}\psi^2(LH)^2$	A_2	5.55 GeV	B_2	7.80 GeV	C_2	7.76 GeV	D_2	5.93 GeV
$\frac{1}{M^3}\phi^2(LH)^2$	A_3	2.78 GeV	B_3	3.90 GeV	C_3	3.88 GeV	D_3	2.96 GeV
$\frac{1}{M^5}\psi^3 LLe^c$	A_4	11.11 GeV	B_4	15.60 GeV	C_4	15.52 GeV	D_4	11.86 GeV
$\frac{1}{M^5}\psi^3 Lqd^c$	A_5	11.11 GeV	B_5	15.60 GeV	C_5	15.52 GeV	D_5	11.86 GeV
$\frac{1}{M^5}\psi^3 u^cd^cd^c$	A_6	11.11 GeV	B_6	15.60 GeV	C_6	15.52 GeV	D_6	11.86 GeV

Table 2: A display of the various interactions that allow a transfer of the $B-L$ asymmetry from the standard model sector to the dark matter sector.

For Models A_1,A_4-A_6 we have

$$LH\ (\mathrm{A}_1): \qquad \mu_{\mathcal{O}^{\rm SM}_{{\rm asy},1}} = \mu_L + \mu_H\,, \tag{54}$$

$$LLe^c\ (\mathrm{A}_4): \qquad \mu_{\mathcal{O}^{\rm SM}_{{\rm asy},4}} = 2\mu_L - \mu_e\,, \tag{55}$$

$$Lqd^c\ (\mathrm{A}_5): \qquad \mu_{\mathcal{O}^{\rm SM}_{{\rm asy},5}} = \mu_L + \mu_q - \mu_d\,, \tag{56}$$

$$u^cd^cd^c\ (\mathrm{A}_6): \qquad \mu_{\mathcal{O}^{\rm SM}_{{\rm asy},6}} = -\mu_u - 2\mu_d\,. \tag{57}$$

From the μ equations Eqs. (10) and (11), it is easy to see that

$$\mu_{\mathcal{O}^{\rm SM}_{{\rm asy},1}} = \mu_{\mathcal{O}^{\rm SM}_{{\rm asy},4}} = \mu_{\mathcal{O}^{\rm SM}_{{\rm asy},5}} = \mu_{\mathcal{O}^{\rm SM}_{{\rm asy},6}}\,. \tag{58}$$

Eq. (58) implies that the dark matter has the same mass for the Models A_1,A_4-A_6. Similar analysis holds for Models B_1,B_4-B_6, C_1,C_4-C_6 and D_1,D_4-D_6.

We summarize all our results in Table 2, where we list the dark matter mass for the various interactions[5] that can transfer the $B-L$ asymmetry from the standard model sector to the dark matter sector. We note that for the first five interactions, the dark matter carries lepton number, while for the last one, it carries a baryon number.

3.4 The AsyDM mass: including the right-handed neutrinos

In the analysis above we used the framework of the standard model where we have no right-handed neutrinos and the neutrinos are assumed massless. The nature of neutrino masses is currently not known, i.e., whether they are Majorana or Dirac, but in the context of a gauged $L_\mu - L_\tau$ symmetry it is more natural for the neutrinos to have Dirac masses which implies that we introduce right-handed neutrinos, one for each generation. We discuss now the effect of this inclusion on the analysis, i.e., on the μ equations, on the $B/(B-L)$ ratio and thus on the DM mass.

Since the right-handed neutrino ν_R has 0 hypercharge and 0 electrical charge, it does not affect the neutrality conditions (such as Eq. (12) or Eq. (22)). ν_R is only involved in one interaction $\mathcal{L} \sim \bar{L}_i H^c \nu_{iR}$ before the electroweak phase transition (or $\mathcal{L} \sim \bar{\nu}_{iL} h \nu_{iR}$ after EWPT), which gives us

[5]In the first column of Table 2, L, H and q stand for $SU(2)_L$ doublets as discussed in $T > T_{\rm EWPT}$ regime (Model A and D). When the temperature drops below EWPT scale (Model B and C), since $SU(2)_L$ symmetry is broken, these interactions should be rewritten in terms of the contents of the original doublets. We omit this step for simplicity.

$\frac{1}{M^n}X^k\mathcal{O}_{\rm asy}^{\rm SM}$	Model	DM Mass	Model	DM Mass	Model	DM Mass	Model	DM Mass
$\frac{1}{M^3}\psi^3 LH$	A$'_1$	12.12 GeV	B$'_1$	15.58 GeV	C$'_1$	15.52 GeV	D$'_1$	12.70 GeV
$\frac{1}{M^4}\psi^2(LH)^2$	A$'_2$	6.06 GeV	B$'_2$	7.79 GeV	C$'_2$	7.76 GeV	D$'_2$	6.35 GeV
$\frac{1}{M^3}\phi^2(LH)^2$	A$'_3$	3.03 GeV	B$'_3$	3.90 GeV	C$'_3$	3.88 GeV	D$'_3$	3.18 GeV
$\frac{1}{M^5}\psi^3 LLe^c$	A$'_4$	12.12 GeV	B$'_4$	15.58 GeV	C$'_4$	15.52 GeV	D$'_4$	12.70 GeV
$\frac{1}{M^5}\psi^3 Lqd^c$	A$'_5$	12.12 GeV	B$'_5$	15.58 GeV	C$'_5$	15.52 GeV	D$'_5$	12.70 GeV
$\frac{1}{M^5}\psi^3 u^cd^cd^c$	A$'_6$	12.12 GeV	B$'_6$	15.58 GeV	C$'_6$	15.52 GeV	D$'_6$	12.70 GeV

Table 3: A display of the various interactions that allow a transfer of the $B-L$ asymmetry from the standard model sector (including the right-handed Dirac neutrinos) to the dark matter sector.

$\mu_H = \mu_{\nu_{iR}} - \mu_{L_i}$ before EWPT (or $\mu_{\nu_{iR}} = \mu_{\nu_{iL}}$ after EWPT). Thus the only change would be the total lepton number since ν_R carries lepton number 1. By including the right-handed neutrinos, a reanalysis gives the following formulas

$$m_{\rm FDM}^{\rm A'} \approx -\frac{12.12\ \text{GeV}}{Q_{B-L}^{\mathcal{O}^{\rm SM}}}\,, \qquad m_{\rm FDM}^{\rm D'} \approx -\frac{12.70\ \text{GeV}}{Q_{B-L}^{\mathcal{O}^{\rm 2HD}}}\,, \tag{59}$$

$$m_{\rm FDM}^{\rm B'} \approx -\frac{15.58\ \text{GeV}}{Q_{B-L}^{\mathcal{O}^{\rm SM}}}\,, \qquad m_{\rm FDM}^{\rm C'} \approx -\frac{15.52\ \text{GeV}}{Q_{B-L}^{\mathcal{O}^{\rm SM}}}\,, \tag{60}$$

where we use a prime to denote all the models with the right-handed neutrinos. With the inclusion of right-handed neutrinos in the thermal bath b' is determined to be (c.f. Eq. (8))

$$b' = \frac{B_{\rm final}}{B-L} = \left(\frac{B}{B-L}\right)_{\rm C'} = \frac{5}{21} \approx 0.24\,. \tag{61}$$

With above formulas, we compute the dark matter masses for the same $B-L$ transfer interactions displayed in Table 2, and they are collected in Table 3. We note that inclusion of right-handed neutrinos generates less than a 10% effect at most and no effect for Model C.[6]

4 Analysis in supersymmetric framework

We now consider the analysis in a supersymmetric framework specifically within an extended MSSM. Since the supersymmetric case can have its own dark matter candidate, i.e., the neutralino, the relic abundance of the neutralino must be depleted. For this reason, we only consider the parameter space where relic density of the neutralino is much smaller than the WMAP value for cold dark matter (CDM) and is thus only a subdominant component. Below we discuss two regimes, one where $T_{\rm int} > M_{\rm SUSY}$ and the other where $T_1 > T_{\rm int} > M_2 > T_{\rm EWPT}$.

[6]The reason for this is simple, i.e., using Eq. (7) and recalling the fact that $B_{\rm final} = B_{\rm C} = B_{\rm C'}$, since the inclusion of right-handed neutrinos does not change the total baryon number, we have

$$m_{\rm DM}^{\rm C'} = 5\cdot\frac{B_{\rm final}}{X}\cdot 1\ \text{GeV} = 5\cdot\frac{B_{\rm C'}}{X}\cdot 1\ \text{GeV} = 5\cdot\frac{B_{\rm C}}{X}\cdot 1\ \text{GeV} = m_{\rm DM}^{\rm C}\,.$$

4.1 $T > M_{\rm SUSY}$

In this regime since the temperature is above the SUSY breaking scale all sparticle masses must be included in the μ equations. This case is very similar to the discussion of $T > T_{\rm EWPT}$ in the standard model framework, except this time our particle spectrum includes all the standard model particles, the extra Higgses as well as the sparticles. For brevity we will use the same symbols for the chemical potentials, though now they stand for not only the standard model fields, but also their super-partners. The chemical potential equations obtained from Yukawa couplings and sphaleron processes remain the same. The only equation modified would be the hypercharge equation, which becomes[7]

$$3\mu_q + 6\mu_u - 3\mu_d - 3\mu_L - 3\mu_e + 2\mu_H = 0\,. \tag{62}$$

Solving the chemical potential equations, we find that for Model E the total baryon and lepton numbers are given by

$$B_{\rm E} = 3\times 3\times[2\mu_q + (\mu_u+\mu_d)] = -12\mu_L\,, \tag{63}$$

$$L_{\rm E} = 3\times 3\times(2\mu_L+\mu_e) = \frac{153}{7}\mu_L\,, \tag{64}$$

so that $(B-L)_{\rm E} = -\frac{237}{7}\mu_L$. Note that in the above equations, the extra factor of $3 = 1+2$ (compare to the standard model case) takes into account the contributions of both fermions and bosons from the superfields, c.f. Eq. (5).

4.2 $T_1 > T > M_2 > T_{\rm EWPT}$

Here we consider two soft breaking mass scales M_1 and M_2 where $M_1 \gg M_2$. When temperature drops below T_1, all the super-particles with masses greater than M_1 would drop out of the thermal bath. We assume that this is the case for the first two generations of squarks and sleptons. Similar to Model C, we simply assume here that these super-particles would drop out of the thermal bath at $M_1 > T_1 > M_2$. Thus the only super-particles remaining in the thermal bath are the third generation sparticles, the gauginos, the Higgses and the Higgsinos. We make the approximation that these particles are relativistic at $T_1 > T > M_2$. This case is labeled Model F. Following the analysis of Eq. (62) we find that the vanishing of the hypercharge for Model F gives

$$5\mu_q + 10\mu_u - 5\mu_d - 5\mu_L - 5\mu_e + 6\mu_H = 0\,. \tag{65}$$

Solving the μ-equations, we obtain

$$B_{\rm F} = (3\times 1+2)\times[2\mu_q + (\mu_u+\mu_d)] = -\frac{20}{3}\mu_L\,, \tag{66}$$

$$L_{\rm F} = (3\times 1+2)\times(2\mu_L+\mu_e) = \frac{485}{39}\mu_L\,, \tag{67}$$

and $(B-L)_{\rm F} = -\frac{745}{39}\mu_L$.

[7]The hypercharge of the Universe for the case when $T > T_{\rm SUSY}$ is given by

$$Y = 3\times\left\{3\times\left[2\times 3\times\tfrac{1}{3}\mu_q + 3\times\tfrac{4}{3}\mu_u + 3\times(-\tfrac{2}{3})\mu_d + 2\times(-1)\mu_L + (-2)\mu_e\right] + 2\times(\mu_{H_u}-\mu_{H_d})\right\},$$

where the counting is similar to discussion in footnote 2. The Higgs mixing term in the superpotential, i.e., $W = \mu H_u H_d$ indicates $\mu_{H_u} + \mu_{H_d} = 0$, and so we define $\mu_H \equiv \mu_{H_u} = -\mu_{H_d}$.

4.3 The AsyDM mass: SUSY case

The supersymmetric interactions which transfer $B-L$ asymmetry typically have a different form than the ones in the non-supersymmetric case. The most general interaction that transfers $B-L$ to the dark sector for the MSSM case is

$$W_{\rm asy} = \frac{1}{M_{\rm asy}^n} X^k \mathcal{O}_{\rm asy}^{\rm MSSM}\,, \tag{68}$$

where the dark matter superfield $X = (\phi_X, \psi_X)$ with ϕ_X as the bosonic and ψ_X as the fermionic component. Now the following possibilities arise in terms of dark matter. First, after soft breaking if ϕ_X and ψ_X have a similar mass, both of them are stable, and could be dark matter candidates. Next, consider the case where one of the components has a much larger mass than the other and would decay into the lighter one. In this case we have two possibilities: either ϕ_X is heavier than ψ_X so that $\phi_X \to \psi_X + \tilde{\chi}^{\rm St}$ (where $\tilde{\chi}^{\rm St}$ is a Stueckelberg neutralino) in which case ψ_X is the dark matter candidate, or ψ_X is heavier than ϕ_X so that $\psi_X \to \phi_X + \tilde{\chi}^{\rm St}$ in which case ϕ_X is the dark matter candidate (The possibility that either $\tilde{\chi}^{\rm St}$ or the MSSM neutralino is a dark matter candidate is discussed in Section 6). For either of these three cases, when computing the total dark particle number from Eq. (5), we need to multiply by an additional factor of 3, since both bosonic and fermionic components of the dark matter superfield would contribute. But for concreteness in our analysis we will assume that ψ_X is lighter than ϕ_X and thus would be the asymmetric dark matter.

Applying the same method we used in Section 3.3, we find

$$m_{\rm DM}^{\rm E} \approx -\frac{11.11 \text{ GeV}}{Q_{B-L}^{\mathcal{O}^{\rm MSSM}}}\,, \qquad m_{\rm DM}^{\rm F} \approx -\frac{6.51 \text{ GeV}}{Q_{B-L}^{\mathcal{O}^{\rm MSSM}}}\,. \tag{69}$$

Thus for the $B-L$ transfer interactions with $Q_{B-L}^{\mathcal{O}^{\rm MSSM}} = -1$, where $\mathcal{O}_{\rm asy}^{\rm MSSM}$ can be LH_u, LLe^c, Lqd^c, or $u^c d^c d^c$, the dark particle masses are

$$m_X = 11.11 \text{ GeV} \qquad \text{Model E}\,; \qquad m_X = 6.51 \text{ GeV} \qquad \text{Model F}\,. \tag{70}$$

For the case $W_{\rm asy} = \frac{1}{M_{\rm asy}^3} X^2 (LH_u)^2$ with $Q_{B-L}^{\mathcal{O}^{\rm MSSM}} = -2$, which we will discuss in Section 6, the dark particle masses are

$$m_X = 5.55 \text{ GeV} \qquad \text{Model E}\,; \qquad m_X = 3.25 \text{ GeV} \qquad \text{Model F}\,, \tag{71}$$

and using Eq. (3) one finds

$$M_{\rm asy}^{\rm E} \gtrsim 3.7 \times 10^5 \text{ GeV}\,. \tag{72}$$

If we include the right-handed neutrinos in the supersymmetric framework, the analysis is similar, and Eq. (69) becomes

$$m_{\rm DM}^{\rm E'} \approx -\frac{12.12 \text{ GeV}}{Q_{B-L}^{\mathcal{O}^{\rm MSSM}}}\,, \qquad m_{\rm DM}^{\rm F'} \approx -\frac{6.99 \text{ GeV}}{Q_{B-L}^{\mathcal{O}^{\rm MSSM}}}\,. \tag{73}$$

5 Asymmetric dark matter in a Stueckelberg extension of the SM

As discussed in the Introduction, one of the major problems for an acceptable AsyDM model is to have an efficient mechanism for the annihilation of dark matter that is produced thermally. In general one has

$$\Omega_{\rm DM} = \Omega^{\rm asy}_{\rm DM} + \Omega^{\rm sym}_{\rm DM}\,, \tag{74}$$

where $\Omega^{\rm asy}_{\rm DM}$ is the relic density of asymmetric dark matter (which carries a nonzero $(B-L)$-charge) and $\Omega^{\rm sym}_{\rm DM}$ is the relic density of dark matter which is produced thermally. For the asymmetric dark matter to be the dominant component, one must significantly deplete the symmetric component of dark matter. Specifically we will use the criteria that $\Omega^{\rm sym}_{\rm DM}\,/\,\Omega_{\rm DM} < 0.1$.[8] Thus we investigate if the symmetric component of dark matter produced by thermal processes can be annihilated efficiently. We accomplish this via the exchange of a gauge field using the Stueckelberg formalism where the gauge field couples to $L_\mu - L_\tau$.

For illustration let us consider Model A$_1$, which is governed by the interaction Eq. (45) operating at $T_{\rm int} > T_{\rm EWPT}$. The corresponding dark matter mass is 11.11 GeV. Further, we require the dark matter particles ψ to have a non-vanishing μ or τ lepton number. The total Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{U(1)} + \mathcal{L}_{\rm St}\ , \tag{75}$$

where $\mathcal{L}_{U(1)}$ is the kinetic energy for the gauge field for the $L_\mu - L_\tau$ symmetry, and for $\mathcal{L}_{\rm St}$ we assume the following form:

$$\mathcal{L}_{\rm St} = -\frac{1}{2}(M_C C_\mu + \partial_\mu \sigma)^2\ . \tag{76}$$

In the unitary gauge the massive vector boson field will be called Z' and its interaction with fermions in the theory is given by

$$\mathcal{L}_{\rm int} = \frac{1}{2} g_C Q^\psi_C \bar\psi\gamma^\mu\psi C_\mu + \frac{1}{2} g_C Q^f_C \bar f\gamma^\mu f C_\mu\ , \tag{77}$$

where f runs over μ and τ families and $Q^\mu_C = -Q^\tau_C$.

5.1 Resonant annihilation of symmetric dark matter

We discuss now the details of the annihilation of the symmetric component of dark matter. We will show that the relic density for such dark matter can be reduced significantly below the WMAP value with resonant annihilation via the Z' pole, i.e., via the process $\psi\bar\psi \to Z' \to f\bar f$.[9] Thus, by using Eq. (77) one can compute the $\psi\bar\psi \to f\bar f$ annihilation cross section and using the Breit-Wigner form for a resonance one has

$$\sigma_{\psi\bar\psi\to f\bar f} \quad = \quad a_\psi \left|\left(s - M^2_{Z'} + i\Gamma_{Z'}M_{Z'}\right)\right|^{-2}\ , \tag{78}$$

[8]The analysis of previous sections was based on the assumption $\Omega^{\rm asy}_{\rm DM}\,/\,\Omega_{\rm matter} \approx 5$. Inclusion of a small contribution (i.e., $\leq 10\%$) of symmetric component to dark matter will proportionately affect the determination of the dark matter mass. It is straightforward to take account of this contribution but we do not carry it out explicitly as it is a relatively small effect.

[9]While the thermal dark matter can annihilate into second and third generation leptons at the tree-level, such an annihilation into the first generation leptons can come about only at the loop level involving the second and third generation leptonic loops. Thus the annihilation of thermal dark matter into first generation leptons is significantly suppressed relative to the annihilation into the second and the third generation leptons.

$$a_\psi = \frac{\beta_f(\frac{1}{2}g_C^2 Q_C^\psi Q_C^f)^2}{64\pi s\beta_\psi}\left[s^2(1+\frac{1}{3}\beta_f^2\beta_\psi^2)+4M_\psi^2(s-2m_f^2)+4m_f^2(s+2M_\psi^2)\right], \tag{79}$$

where $\beta_{f,\psi} = (1-4m_{f,\psi}^2/s)^{1/2}$. The relevant partial Z' decay widths are given by

$$\Gamma(Z'\to f\bar{f}) = \left(\frac{1}{2}g_C Q_C^f\right)^2 r_f\frac{M_{Z'}}{12\pi}, \quad f=\mu,\nu_\mu,\tau,\nu_\tau \tag{80}$$

$$\Gamma(Z'\to\psi\bar{\psi}) = \left(\frac{1}{2}g_C Q_C^\psi\right)^2\frac{M_{Z'}}{12\pi}\left(1+\frac{2M_\psi^2}{M_{Z'}^2}\right)\left(1-\frac{4M_\psi^2}{M_{Z'}^2}\right)^{1/2}\Theta\left(M_{Z'}-2M_\psi\right), \tag{81}$$

where $r_f = 1$ for $f=\mu,\tau$ and $r_f = 1/2$ for $f=\nu_\mu,\nu_\tau$. A constraint on g_C comes from the contribution of the Z' to $g_\mu - 2$ [32,33], which is given by

$$\Delta(g_\mu-2) = \left(\frac{1}{2}g_C Q_C^\mu\right)^2\frac{m_\mu^2}{6\pi^2 M_{Z'}^2}. \tag{82}$$

In the analysis here we impose the constraint that the Z' boson contribution be less than the experimental (4σ) deviation of $\Delta a_\mu \equiv \Delta\big((g_\mu-2)/2\big) = (3.0\pm 0.8)\times 10^{-9}$ [32,33], which is the constraint commonly adopted in analysis of supergravity based models.

The relic densities of ψ and $\bar{\psi}$ are governed by the Boltzmann equations and have been discussed in several works [14,15] for the case of asymmetric dark matter. Typically it is seen that the effect of including the asymmetry in the Boltzmann equations lead to a significant effect on the relic density. In these works the analysis was done in the approximation $\langle\sigma v\rangle = a + bv^2$. Our analysis below differs from these in that for our case annihilation via the Z' pole is the dominant process. Thus in our analysis we need to carry out an explicit thermal averaging over the Breit-Wigner pole. It is convenient to work with the Boltzmann equations for the quantities $f_\psi \equiv n_\psi/(hT^3)$, and $f_{\bar{\psi}} \equiv n_{\bar{\psi}}/(hT^3)$ where n_ψ ($n_{\bar{\psi}}$) is the number density of particle ψ ($\bar{\psi}$) and the combination hT^3 appears in the entropy per unit volume, i.e., $s = (2\pi^2/45)hT^3$ where h is the entropy degrees of freedom. The Boltzmann equations obeyed by f_ψ and $f_{\bar{\psi}}$ take the form

$$\frac{\mathrm{d}f_\psi}{\mathrm{d}x} = \alpha\langle\sigma v\rangle(f_\psi f_{\bar{\psi}} - f_\psi^{\rm eq} f_{\bar{\psi}}^{\rm eq}), \tag{83}$$

$$\frac{\mathrm{d}f_{\bar{\psi}}}{\mathrm{d}x} = \alpha\langle\sigma v\rangle(f_\psi f_{\bar{\psi}} - f_\psi^{\rm eq} f_{\bar{\psi}}^{\rm eq}), \tag{84}$$

where $x = k_B T/m_\psi$ in which k_B is the Boltzmann constant and hereafter we set $k_B = 1$, and α is given by

$$\alpha(T) = \sqrt{90}m_\psi M_{\rm Pl}\frac{h}{\sqrt{g}\pi}\left(1+\frac{1}{4}\frac{T}{g}\frac{\mathrm{d}g}{\mathrm{d}T}\right), \tag{85}$$

where g is the degrees of freedom that enter in the energy per unit volume, i.e., $\rho = \frac{\pi^2}{30}gT^4$, where $T(t) = T_\gamma(t)$ is the photon temperature. Numerically $\alpha(T) = 6.7\times 10^{20}$ GeV^2 for $g = h = 68$ at $T = 0.5$ GeV. $\langle\sigma v\rangle$ is the thermally averaged cross section

$$\langle\sigma v\rangle = \frac{\displaystyle\int_0^\infty \mathrm{d}v\,(\sigma v)v^2 e^{-v^2/4x}}{\displaystyle\int_0^\infty \mathrm{d}v\, v^2 e^{-v^2/4x}}. \tag{86}$$

Further, in Eqs. (83) and (84) $f_\psi^{\rm eq}$ and $f_{\bar\psi}^{\rm eq}$ are values of f_ψ and $f_{\bar\psi}$ at equilibrium. Now one can obtain the result from Eqs. (83) and (84) that the difference of f_ψ and $f_{\bar\psi}$, i.e.,

$$\gamma \equiv f_\psi - f_{\bar\psi}\,, \tag{87}$$

is a constant. Assuming that the asymmetric dark matter currently constitutes a fraction λ of the dark matter relic density one can evaluate γ to be

$$\gamma \simeq \lambda\,\frac{5\rho_c}{6\hbar T^3 m_\psi} \equiv \lambda\gamma_0, \quad \gamma_0 \approx 1.3\times 10^{-10} \quad (m_\psi \sim 10~{\rm GeV})\,, \tag{88}$$

where the 5/6 in γ_0 is due to Eq. (1).

It is now straightforward to obtain the individual relic densities for ψ and $\bar\psi$. Thus one integrates Eqs. (83) and (84) from the freeze-out temperature to the current temperature of $T_\gamma^0 = 2.73$ K. In the integration we will make the following approximation which is conventionally done, i.e., we move α out of the integral and replace it with $\alpha(x_f)$, i.e., by the value of α at the freeze-out temperature. The matter density of ψ at current temperature is given by $\rho_\psi = m_\psi n_\psi(x_0)$ where $x_0 = T_\gamma^0/m_\psi$ and T_γ^0 is the current photon temperature of 2.73 K. The relic density then is

$$\Omega_\psi = m_\psi n_\psi(x_0)/\rho_c\,, \tag{89}$$

where ρ_c is the critical matter density so that $\rho_c = (3\times 10^{-12}~{\rm GeV})^4 h_0^2$ where h_0 is the Hubble parameter. The integration of Eq. (83) straightforwardly gives

$$\Omega_\psi h_0^2 = 2.2\times 10^{-11}\sqrt{g(x_f)}h(x_0,x_f)\left(\frac{T_\gamma}{2.73}\right)^3\left(\frac{1}{\xi} - \frac{f_{\bar\psi}(x_f)}{\xi f_\psi(x_f)}e^{-\xi J(x_f)}\right)^{-1}\,, \tag{90}$$

where

$$J(x_f) \equiv \int_{x_0}^{x_f}\langle\sigma v\rangle\,{\rm d}x\,, \qquad h(x_0,x_f) \equiv \frac{h(x_0)}{h(x_f)}\left[1 + \frac{1}{4}\left(\frac{T}{g}\frac{{\rm d}g}{{\rm d}T}\right)_{x_f}\right]^{-1}\,, \tag{91}$$

and $\xi \equiv \alpha(x_f)\gamma$ where $\alpha(x_f)$ is the value of α evaluated at the freeze-out temperature, and where $g(x_f)$ ($h(x_f)$) are the energy (entropy) degrees of freedom at freeze out and $h(x_0)$ is the entropy degrees of freedom at the current temperature. The derivative term $\frac{1}{4}(\frac{T}{g}\frac{{\rm d}g}{{\rm d}T})_{x_f}$ is small and is often dropped, while $h(x_0) = 3.91$ [30,34] and we estimate $h(x_f) \sim g(x_f)$ given T_f. As discussed below x_f is typically of size $\sim 1/20$ and thus $T_f = m_\psi x_f \sim 0.5$ GeV for $m_\psi \sim 10$ GeV. Now for $T_f \sim 0.5$ GeV, $h(x_f) \sim 68$ which gives $h(x_0,x_f) \sim 1/17.5$. The quantities $f_\psi(x_f)$ ($f_{\bar\psi}(x_f)$) are f_ψ ($f_{\bar\psi}$) evaluated at freeze out. Analogous to the relic density for ψ, we can get the relic density of $\bar\psi$ by integration of $\bar\psi$ and we obtain

$$\Omega_{\bar\psi} h_0^2 = 2.2\times 10^{-11}\sqrt{g(x_f)}h(x_0,x_f)\left(\frac{T_\gamma}{2.73}\right)^3\left(\frac{f_\psi(x_f)}{\xi f_{\bar\psi}(x_f)}e^{\xi J(x_f)} - \frac{1}{\xi}\right)^{-1}\,. \tag{92}$$

The total dark matter relic density is

$$\Omega_{\rm DM} = \Omega_\psi + \Omega_{\bar\psi}\,. \tag{93}$$

From Eqs. (90) and (92) one obtains the ratio of the current relic densities of $\bar{\psi}$ and ψ to be

$$\frac{\Omega_{\bar{\psi}} h_0^2}{\Omega_\psi h_0^2} = \frac{f_{\bar{\psi}}(x_f)}{f_\psi(x_f)} e^{-\xi J(x_f)} . \tag{94}$$

The front factor $f_{\bar{\psi}}(x_f)/f_\psi(x_f)$ in Eq. (94) takes into account the asymmetry that exists at the freeze-out temperature. The size of this effect is estimated at the end of this section and could be as much as 20%, and thus significant. Our result of Eq. (94) is in agreement with the analysis of [14].

We discuss now the evaluation of the freeze-out temperature. The definition of this quantity differs somewhat in various works (see, for example, [35, 36]) but these differences are not very significant. We adopt here the definition of [35] where the freeze-out temperature T_f is defined as the temperature where the annihilation rate per unit volume equals the rate of change of the number density. For our case this implies

$$\frac{\mathrm{d} f_{\bar{\psi}}^{\mathrm{eq}}}{\mathrm{d}x} = \alpha \langle \sigma v \rangle f_\psi^{\mathrm{eq}} f_{\bar{\psi}}^{\mathrm{eq}} , \quad \text{at } x = x_f = T_f/m_\psi , \tag{95}$$

where $f_{\bar{\psi}}^{\mathrm{eq}}$ takes the form

$$f_{\bar{\psi}}^{\mathrm{eq}}(x) = a_{\bar{\psi}}\, x^{-3/2} e^{-1/x} , \tag{96}$$

where $a_{\bar{\psi}} = g_{\bar{\psi}} (2\pi)^{-3/2} h^{-1}(T) \approx 9.3 \times 10^{-4} g_{\bar{\psi}}$ around $T = 0.5$ GeV, and $g_{\bar{\psi}}$ denotes the degrees of freedom of the dark particle ($g_\psi = g_{\bar{\psi}} = 4$ for Dirac spinors). The freeze-out temperature is then determined by the relation

$$(x_f^{-1/2} - \tfrac{3}{2} x_f^{1/2})\, e^{1/x_f} = \alpha \langle \sigma v \rangle (a_{\bar{\psi}} + \gamma x_f^{3/2} e^{1/x_f}) . \tag{97}$$

For the case of no asymmetry, i.e., in the limit $\gamma \to 0$, Eq. (97) reduces down to the well-known result [35]. One may compare the analysis of the freeze-out temperature given by Eq. (97) with the one using the alternate criterion [36]

$$\Delta(x_f) = c f_{\bar{\psi}}^{\mathrm{eq}}(x_f) , \tag{98}$$

where $\Delta(x) \equiv (f_{\bar{\psi}}(x) - f_{\bar{\psi}}^{\mathrm{eq}}(x))$ and c is order unity. Using Eq. (96) in Eq. (98) one gets

$$(x_f^{1/2} - \tfrac{3}{2} x_f^{-1/2} - \alpha \gamma \langle \sigma v \rangle x^{3/2})\, e^{1/x_f} = \alpha a_{\bar{\psi}} c(c+2) \langle \sigma v \rangle . \tag{99}$$

For $\gamma = 0$, Eq. (99) reduces to the result of [36] while $\gamma \neq 0$ gives the correction due to asymmetry. Further, we see that Eq. (99) reduces to Eq. (97) when $c = \sqrt{2} - 1$. To compute the sensitivity of the freeze-out temperature on the asymmetry it is useful to utilize the scale factor λ defined in Eq. (88). On using Eq. (97) we can obtain an approximate expression for $\mathrm{d}x_f/\mathrm{d}\lambda$ so that

$$\mathrm{d}x_f/\mathrm{d}\lambda \simeq -a_{\bar{\psi}}^{-1} \gamma_0 x_f^{7/2} e^{1/x_f} . \tag{100}$$

From above we can compute the first order correction to the freeze-out temperature due to the asymmetry. To the leading order one has

$$x_f \simeq x_f^0 \left[1 - a_{\bar{\psi}}^{-1} \gamma (x_f^0)^{5/2} e^{1/x_f^0} \right] , \tag{101}$$

where x_f^0 is the zeroth order of the x_f, i.e., when $\gamma = 0$. We note that the correction to the freeze-out temperature due to asymmetry is independent of $\langle \sigma v \rangle$ to leading order. Using $a_\psi = 3.7 \times 10^{-3}$, $x_f = 1/17.5$ and $\gamma = \gamma_0 = 1.3 \times 10^{-10}$, one finds that the correction to x_f is around a percent for the choice of the parameters given. Further, as γ (and hence ξ) $\to 0$, one has $\frac{f_\psi(x_f)}{f_{\bar\psi}(x_f)} \to 1$ and in this limit one has

$$\Omega_\psi h_0^2 = \Omega_{\bar\psi} h_0^2 = 2.2 \times 10^{-11} \sqrt{g(x_f)} h(x_0, x_f) \left(\frac{T_\gamma}{2.73} \right)^3 \frac{1}{J(x_f)} \, . \tag{102}$$

Now rapid annihilation of dark matter can occur if the sum of the dark matter masses is close to the Z' pole and there is a Breit-Wigner enhancement [37,38]. Thus for the case we are considering if the mass of the Z' is close to twice the mass of the dark particle, then one can get a large annihilation cross section and correspondingly a small relic density. An analysis of the relic density arising from the annihilation of symmetric dark matter is given in Fig. 1 and the analysis shows that the relic density arising from the symmetric component of dark matter can easily be made negligible, i.e., less than 10% of the cold dark matter density given by WMAP. In Fig. 1 we give the analysis for the case with no asymmetry, i.e., $\gamma = 0$ (left panel) and the case with asymmetry (right panel) where $\gamma = 1.3 \times 10^{-10}$. A comparison of the left and the right panels shows that inclusion of the asymmetry has a substantial effect on the relic density. Specifically it further helps deplete the relic density of $\bar\psi$ (the symmetric component of dark matter). For the case of $g_C = 1$ the allowed upper bound of the Z' mass increases by about ~ 100 GeV in the presence of an asymmetry when $\gamma = 1.3 \times 10^{-10}$. It is also instructive to examine the ratio of the thermal relic density for the cases with and without asymmetry. Here one has

$$\mathrm{R} \equiv \frac{(\Omega_{\bar\psi} h_0^2)_{\gamma=\gamma_0}}{(\Omega_{\bar\psi} h_0^2)_{\gamma=0}} = \frac{\xi J\,(x_f)}{\frac{f_\psi(x_f)}{f_{\bar\psi}(x_f)} e^{\xi J(x_f)} - 1} \, . \tag{103}$$

As $\xi \to 0$, $\frac{f_\psi(x_f)}{f_{\bar\psi}(x_f)} \to 1$ and thus $R \to 1$. However, if we assume that the asymmetric dark matter is responsible for 5/6 of the total relic density, then for $m_\psi \sim 10$ GeV, one has $\gamma = 1.3 \times 10^{-10}$ and $f_{\bar\psi}(x_f) = 6.8 \times 10^{-10}$ which gives $\frac{f_\psi(x_f)}{f_{\bar\psi}(x_f)} = (1 + \gamma_0 / f_{\bar\psi}(x_f)) \sim 1.2$. In this circumstance one finds that R is always less than 1. Thus one finds that the inclusion of asymmetry helps deplete the symmetric component of dark matter.

6 Asymmetric dark matter in a Stueckelberg extension of the MSSM

The analysis of dark matter in the MSSM extension is more complex in that there are now three contributions to the dark matter relic density, i.e., from the asymmetric and symmetric components as in Eq. (74) and from the neutralino. Thus here one has

$$\Omega_{\rm DM} = \Omega_{\rm DM}^{\rm asy} + \Omega_{\rm DM}^{\rm sym} + \Omega_{\tilde\chi^0} \, , \tag{104}$$

where $\Omega_{\tilde\chi^0}$ is the relic density from the neutralino. In this case for the asymmetric dark matter to work, one must significantly deplete not only the symmetric component of dark matter but also the contribution from the neutralino. Thus here we take the criterion that $\Omega_{\rm DM}^{\rm sym} / \Omega_{\rm DM} < 0.1$, and

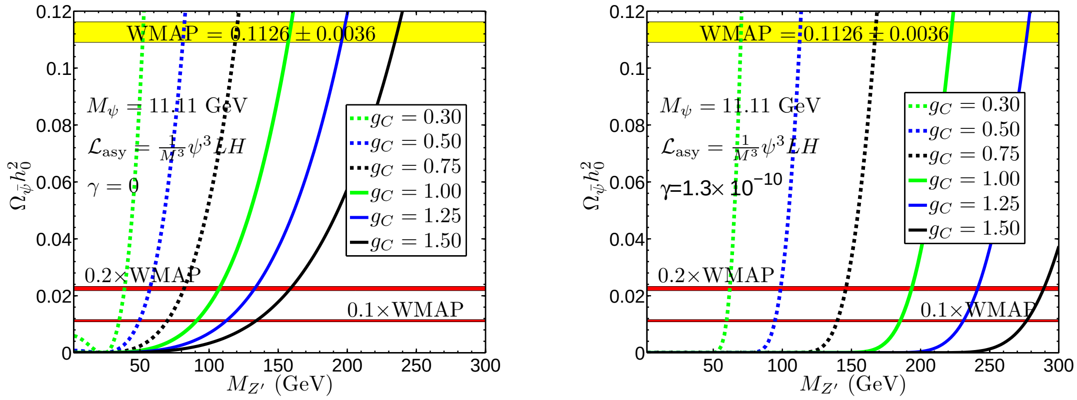

Figure 1: (color online) An exhibition of the thermal relic density of $\bar\psi$ as a function of $M_{Z'}$ in the model with gauged $L_\mu - L_\tau$ for different values of the coupling constant. The left panel shows the case $\gamma = 0$ and the right panel shows $\gamma = 1.3\times 10^{-10}$. In both cases, the analysis shows that the component of dark matter that is thermally produced can be efficiently depleted by resonant annihilation via the Z' pole.

$\Omega_{\tilde\chi^0}\,/\,\Omega_{\rm DM} < 0.1$. For the analysis of AsyDM in extensions of MSSM we consider the interaction

$$W_{\rm asy} = \frac{1}{M_{\rm asy}^3} X^2 (LH_u)^2\,. \tag{105}$$

Here we note that the choice $W_{\rm asy} \sim X^2 LH_u$ would have allowed the decay $\tilde\chi^0 \to XX\nu\cdots$ and would have required the constraint $m_{\tilde\chi^0} < 2m_X$ for the neutralino to be stable. Further, while the choices $W_{\rm asy} \sim X^2 LLe^c, X^2 Lqd^c$ do not allow the neutralino decay at the tree-level, such a decay can occur at the loop level since it is not forbidden by a symmetry. Additionally $W_{\rm asy} \sim X^3 LH_u, X^3 LLe^c, X^3 Lqd^c$ can also preserve the stability of the neutralino. Here and elsewhere we are assuming that the Stueckelberg neutralinos are heavier than the lightest neutralino in the MSSM sector (see the discussion following Eq. (113)). Returning to Eq. (105), the corresponding dark particle masses are computed to be 5.55 GeV (Model E) and 3.25 GeV (Model F). Now the Stueckelberg extension of MSSM, is more complex than the SM extension. We exhibit the relevant parts of this extension below.

For the Stueckelberg Lagrangian of the supersymmetric case we choose [3]

$$\mathcal{L}_{\rm St} = \int d\theta^2 d\bar\theta^2 \, \left[MC + S + \bar S\right]^2\,, \tag{106}$$

where C is the $U(1)_C$ vector multiplet, S and $\bar S$ are chiral multiplets, and M is a mass parameter. We define C in the Wess-Zumino gauge as $C = -\theta\sigma^\mu\bar\theta C_\mu + i\theta\theta\bar\theta\bar\lambda_C - i\bar\theta\bar\theta\theta\lambda_C + \frac{1}{2}\theta\theta\bar\theta\bar\theta D_C$, while $S = \frac{1}{2}(\rho + ia) + \theta\chi + i\theta\sigma^\mu\bar\theta\frac{1}{2}(\partial_\mu\rho + i\partial_\mu a) + \theta\theta F + \frac{i}{2}\theta\theta\bar\theta\bar\sigma^\mu\partial_\mu\chi + \frac{1}{8}\theta\theta\bar\theta\bar\theta(\Box\rho + i\Box a)$. Its complex scalar component contains the axionic pseudo-scalar a, which is the analogue of the real pseudo-scalar that appears in the non-supersymmetric version in [3]. We write $\mathcal{L}_{\rm St}$ in component notation as (see e.g. [3])

$$\mathcal{L}_{\rm St} = -\frac{1}{2}(MC_\mu + \partial_\mu a)^2 - \frac{1}{2}(\partial_\mu\rho)^2 - i\chi\sigma^\mu\partial_\mu\bar\chi + 2|F|^2 + M\rho D_C + M\bar\chi\bar\lambda_C + M\chi\lambda_C\,. \tag{107}$$

For the gauge fields we add the kinetic terms

$$\mathcal{L}_{\rm gkin} = -\frac{1}{4}C_{\mu\nu}C^{\mu\nu} - i\lambda_C\sigma^\mu\partial_\mu\bar{\lambda}_C + \frac{1}{2}D_C^2\,, \tag{108}$$

with $C_{\mu\nu} \equiv \partial_\mu C_\nu - \partial_\nu C_\mu$. For the matter fields (quarks, leptons, Higgs scalars, plus hidden sector matter) chiral superfields with components (f_i, z_i, F_i) are introduced and the matter Lagrangian is given by

$$\mathcal{L}_{\rm matt} = -|D_\mu z_i|^2 - if_i\sigma^\mu D_\mu\bar{f}_i - \Big(i\sqrt{2}g_C Q_C z_i\bar{f}_i\bar{\lambda}_C + \text{h.c.}\Big) + g_C D_C(\bar{z}_i Q_C z_i) + |F_i|^2\ , \tag{109}$$

where (Q_C, g_C) are the charge operator and coupling constant of $U(1)_C$, and $D_\mu = \partial_\mu + ig_C Q_C C_\mu$ is the gauge covariant derivative. It is convenient to replace the two-component Weyl-spinors $(\chi, \bar{\chi}), (\lambda_C, \bar{\lambda}_C)$ by four-component Majorana spinors, which we label as $\psi_S = (\chi_\alpha, \bar{\chi}^{\dot{\alpha}})^T$, and $\lambda_C = (\lambda_{C\alpha}, \bar{\lambda}_C^{\dot{\alpha}})$. The total Lagrangian of the MSSM then takes the form

$$\mathcal{L}_{\rm StMSSM} \;=\; \mathcal{L}_{\rm MSSM} + \mathcal{L}_{U(1)} + \Delta\mathcal{L}_{\rm St}\ , \tag{110}$$

with

$$\begin{aligned}\Delta\mathcal{L}_{\rm St} \;=\; & -\frac{1}{2}(MC_\mu + \partial_\mu a)^2 - \frac{1}{2}(\partial_\mu\rho)^2 - \frac{1}{2}M^2\rho^2 \\ & -\frac{i}{2}\bar{\psi}_S\gamma^\mu\partial_\mu\psi_S - \frac{1}{4}C_{\mu\nu}C^{\mu\nu} - \frac{i}{2}\bar{\lambda}_C\gamma^\mu\partial_\mu\lambda_C + M\bar{\psi}_S\lambda_C \\ & -\sum_i\Big[\,|D_\mu z_i|^2 - |D_\mu z_i|^2_{C_\mu=0} + \rho g_C M(\bar{z}_i Q_C z_i) \\ & +\frac{1}{2}g_C C_\mu\bar{f}_i\gamma^\mu Q_C f_i + \sqrt{2}g_C\Big(iz_i Q_C\bar{f}_i\lambda_C + \text{h.c.}\Big)\Big] - \frac{1}{2}\Big[g_C\sum_i\bar{z}_i Q_C z_i\Big]^2 .\end{aligned} \tag{111}$$

As in the SM case we assume that the $U(1)_C$ is a gauged $L_\mu - L_\tau$. Further, we assume that all hidden sector fields while charged under $U(1)_C$ are neutral under the MSSM gauge group and some of the MSSM particles, i.e., the second and the third generation leptons, are charged under $U(1)_C$. As discussed already an essential ingredient to explain the cosmic coincidence is that the symmetric component of dark matter produced in thermal processes is significantly depleted. For the MSSM Stueckelberg extension the analysis of annihilation is essentially identical to the case of the Stueckelberg extension of the standard model and we do not discuss it further.

We now discuss the fate of the extra particles that arise in the $U(1)_C$ Stueckelberg extension of MSSM. This extension involves the following set of particles: $Z', \rho, \psi, \phi, \psi_S, \lambda_C$. The decay of the Z' has already been discussed. Next we consider the ρ. Eq. (111) gives the interaction of the ρ with the sfermions. Specifically its couplings to the mass diagonal sfermions are given by

$$\mathcal{L}_{\rho\tilde{f}^\dagger\tilde{f}} \;=\; -g_\rho M_\rho\left[\cos(2\theta_{\tilde{f}_i})\left(\tilde{f}_{1i}^\dagger\tilde{f}_{1i} - \tilde{f}_{2i}^\dagger\tilde{f}_{2i}\right) + \sin(2\theta_{\tilde{f}_i})\left(\tilde{f}_{1i}^\dagger\tilde{f}_{2i} + \tilde{f}_{2i}^\dagger\tilde{f}_{1i}\right)\right]\,, \tag{112}$$

where f_i refer to $\mu, \nu_\mu, \tau, \nu_\tau$. Thus the ρ will decay via second and third generation slepton loops into $\mu^+\mu^-, \nu_\mu\bar{\nu}_\mu$, $\tau^+\tau^-$, $\nu_\tau, \bar{\nu}_\tau$ and disappear in the thermal bath quickly (see Appendix B). Next we discuss the neutralino sector. Here in the $U(1)_C$ Stueckelberg extension of MSSM the neutralino sector is enlarged in that one has two more fields, i.e., the gaugino, and the higgsino fields (Ψ_S, Λ_C)

as mentioned earlier. In this case the neutralino mass matrix of the $U(1)_C$ extension of MSSM is given by

$$\mathcal{M}_{\text{neutralino}} = \left(\begin{array}{c|c} \mathcal{M}_{st} & 0_{2\times 4} \\ \hline 0_{4\times 2} & \mathcal{M}_{\text{MSSM}} \end{array}\right) , \qquad \mathcal{M}_{\text{st}} = \begin{pmatrix} 0 & M \\ M & \tilde{M} \end{pmatrix} , \tag{113}$$

where M_{St} is in the basis (Ψ_S, Λ_C), M is the Stueckelberg mass and $\tilde{M}$ is the soft mass. The neutralino mass eigenstates arising from Eq. (113) can be labeled $\tilde{\chi}_1^{\text{St}}, \tilde{\chi}_2^{\text{St}}$. We consider the possibility that the Stueckelberg neutralinos are heavier than the LSP of the MSSM ($\tilde{\chi}_1^0$) and decay into the MSSM neutralino which is assumed to be stable. In this case one will have more than one dark matter particle, i.e., the ψ from the Stueckelberg sector and $\tilde{\chi}_1^0$ from the MSSM sector. Again in the case of AsyDM the relic density of $\tilde{\chi}_1^0$ must be much smaller than the WMAP relic density for CDM. To this end we carry out an explicit analysis of the relic density within supergravity (SUGRA) grand unification [39]. As we show in Fig. 2 the relic density of $\tilde{\chi}_1^0$ can be very small (see the next section for more detail), which allows the dominant component of the dark matter observed today to be the asymmetric dark matter.

7 Detection of dark matter

The AsyDM in the model we consider can interact with the standard model particles only via the Z' and ρ bosons which couple with the second and third generation leptons. Thus the scattering of AsyDM from nuclear targets will not produce any visible signals and the detection of AsyDM in direct detection experiments is difficult. However, it is interesting to investigate if the subdominant component of dark matter could still provide a detectable signature. We discuss this topic in further detail below. First we discuss the depletion of $\tilde{\chi}_1^0$ dark matter to determine the regions of the parameter space where the relic density of $\tilde{\chi}_1^0$ is a negligible fraction of the WMAP relic density for CDM, and is thus indeed a subdominant component of dark matter. Later we will investigate the possibility of detection of this subdominant component in direct detection experiments. Specifically we investigate two classes of models: the supergravity grand unified model (mSUGRA) with universal boundary conditions on soft parameters at the GUT scale, and non-universal SUGRA model with non-universalities in the gaugino sector (see, e.g., [43] and the references there in).

For the mSUGRA case the parameter space investigated was: $m_0 < 10$ TeV, $m_{1/2} < 10$ TeV, $|A_0/m_0| < 10$, $1 < \tan\beta < 60$, and $\mu > 0$. For the case of SUGRA models with non-universalities in the gaugino sector the parameter space investigated was: $M_i = m_{1/2}(1 + \delta_i)$ with the same ranges as in the mSUGRA case with $|\delta_i| < 1$. After radiative breaking of the electroweak symmetry we collected roughly 31.4 million mSUGRA models and 25.6 million non-universal (NU) gaugino models. These models were then subjected to the experimental constraints which included the limits on sparticle masses from LEP [32]: $m_{\tilde{\tau}_1} > 81.9$ GeV, $m_{\tilde{\chi}_1^\pm} > 103.5$ GeV, $m_{\tilde{t}_1} > 95.7$ GeV, $m_{\tilde{b}_1} > 89$ GeV, $m_{\tilde{e}_R} > 107$ GeV, $m_{\tilde{\mu}_R} > 94$ GeV, and $m_{\tilde{g}} > 308$ GeV. as well as the recent bounds on the light CP even, SM-like, Higgs from ATLAS and CMS, i.e 115 GeV $< m_{h^0} <$ 131 GeV [44,45]. More recently, ATLAS has constrained the SM-like Higgs to be between $(117.5 - 118.5)$ GeV and $(122.5 - 129)$ GeV and CMS has constrained the Higgs Mass to be between $(115 - 127)$ GeV [46]. This constraint applied to the mSUGRA parameter space has recently been discussed in [47]. As discussed above if $\tilde{\chi}_1^0 < \tilde{\chi}_1^{\text{St}}$ then the neutralino would contribute to the relic density and for the AsyDM model to work we require that $\Omega_{\tilde{\chi}_1^0} h_0^2$, to be less then 10% of the WMAP-7 limit [1]. Other constraints applied to the parameter points include the $g_\mu - 2$ [33] constraint discussed in

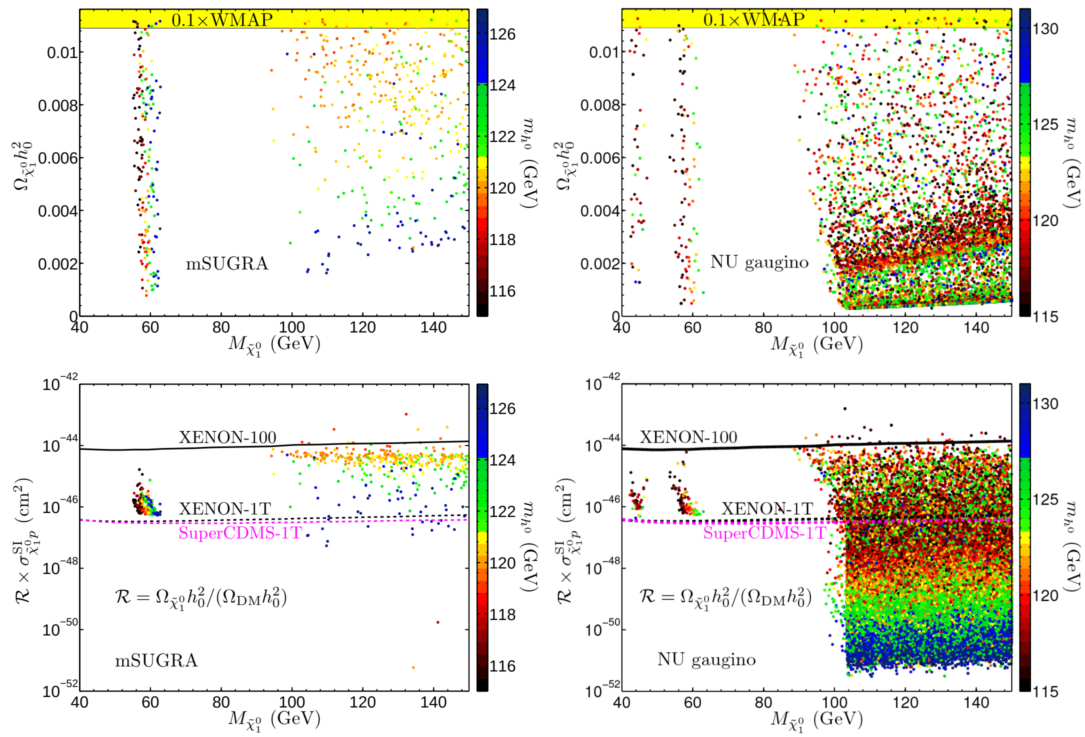

Figure 2: (color online) Top panel: An exhibition of the depletion of the MSSM neutralino dark matter below 10% of the WMAP relic density for cold dark matter. Parameter points are displayed by their light CP even Higgs mass and the yellow band corresponds to 10% of the WMAP-7 observed limit. Bottom panel: An exhibition of the neutralino-proton spin-independent cross section as a function of the neutralino mass. To account for the reduced relic density of the neutralino component of dark matter the spin-independent cross section has been corrected by a factor $\mathcal{R} = \Omega_{\tilde{\chi}_1^0} h_0^2 / \left(\Omega_{\rm DM} h_0^2\right)$. The present experimental limits (solid line) [40] as well as the future projections (dashed lines) are shown [41,42]. The left panel shows the parameter points of mSUGRA and the right panel shows the non-universal gaugino parameter points. All parameter points shown pass the *general constraints*.

Section 5 and constraints from B-physics measurements [48–50] which yield flavor constraints, i.e. $\left(2.77 \times 10^{-4}\right) \leq \mathcal{B}r\left(b \to s\gamma\right) \leq \left(4.37 \times 10^{-4}\right)$ (where this branching ratio has the NNLO correction [51]) and $\mathcal{B}r\left(B_s \to \mu^+\mu^-\right) \leq 4.5 \times 10^{-9}$. As done in [52], we will refer to these constraints as the *general constraints*. These constraints were done by calculating the sparticle mass spectrum with SuSpect [53] and using micrOMEGAs [54] for the relic density as well as for the indirect constraints.

In Fig. 2, we exhibit the mSUGRA (left panels) and the NU gaugino (right panels) parameter points after applying the *general constraints*. In the top two panels we show the thermal relic density of the neutralino. As discussed previously, one finds that there is a significant region of the parameter space with a relic density much less than one tenth of the WMAP relic density. Thus the neutralino is indeed a subdominant component of dark matter. There are many more NU gaugino parameter points that satisfy the relic density compared to mSUGRA parameter points. This comes about because of coannihilation. Thus the non-universal case allows for the light chargino to lie close to

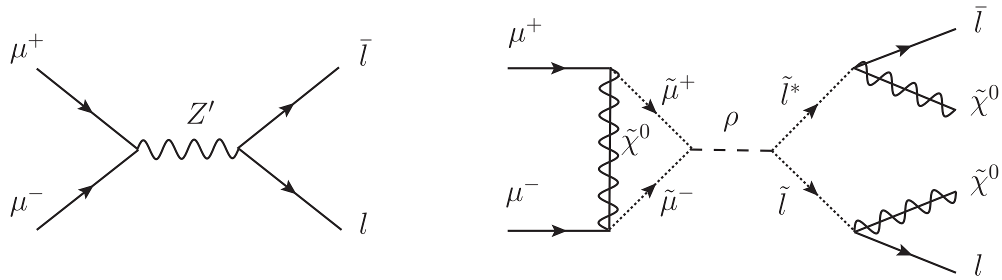

Figure 3: Left: Leptonic final states in a $\mu^+\mu^-$ collider where the $\mu^+\mu^- \to l\bar{l}$, with $l = \mu, \nu_\mu, \tau, \nu_\tau$, final state arising from direct channel poles involving Z'. The Z' pole does not allow for a e^+e^- final state and thus the relative production cross section for $\mu^+\mu^- \to \tau^+\tau^-$ vs $\mu^+\mu^- \to e^+e^-$ can be used to detect the existence of a $L_\mu - L_\tau$ gauged boson. Right: A similar analysis is possible for ρ but its production is suppressed relative to Z' since it must be produced at the loop level.

the LSP, i.e. $(m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0})/m_{\tilde{\chi}_1^0} \ll 1$, allowing for coannihilation to occur. The relevant question then is if such a subdominant component can be detected in dark matter experiments. This is exhibited in the lower two panels of Fig. 2, where the corrected neutralino-proton spin-independent cross section, i.e $\mathcal{R} \times \sigma^{\rm SI}_{\tilde{\chi}_1^0 p}$ where $\mathcal{R} = \Omega_{\tilde{\chi}_1^0} h_0^2 / \left(\Omega_{\rm DM} h_0^2\right)$, is given as a function of the neutralino mass. For comparison we show the current XENON-100 bound [40] and the projected bounds in future experiments [41, 42]. The important observation is that neutralino-proton spin-independent cross section is still detectable even when the neutralino is a subdominant component of dark matter with a relic density less than 10% of the WMAP relic density for CDM.

8 Signatures at colliders

The AsyDM models discussed above can produce a dramatic signature at a muon collider, see Fig. 3, which we now discussed (Signatures of a Z' boson in a gauged $U(1)_{L_\mu - L_\tau}$ model at a muon collider were discussed in [23] but the analysis was only at the tree-level.). This signature arises from a Z' resonance. We note that Z' has no couplings with the first generation leptons and thus a process such as $e^+e^- \to Z' \to \mu^+\mu^-, \tau^+\tau^-$ is absent at the tree-level. This process can only arise at the loop level which, however, is suppressed relative to the tree. This explains why such a resonance has not been observed yet at an e^+e^- collider (see Appendix C). However, dramatic signals will arise at a muon collider where we will have processes of the type

$$\mu^+\mu^- \to Z' \to \mu^+\mu^-, \nu_\mu\bar{\nu}_\mu, \tau^+\tau^-, \nu_\tau\bar{\nu}_\tau \,.$$

Since the final states contain no e^+e^- this would provide a smoking gun signature for the model. In Fig. 4 we exhibit the cross $\sigma(\mu^+\mu^- \to \tau^+\tau^-)$ for various values of g_C when the AsyDM mass is taken to be 11.11 GeV and the Z' mass is 150 GeV. For comparison $\sigma(\mu^+\mu^- \to e^+e^-)$ is also plotted. One finds that the $\sigma(\mu^+\mu^- \to \tau^+\tau^-)$ exhibits a detectable Z' resonance and the cross section varies dramatically as a function of $\sqrt{s}$ relative to $\sigma(\mu^+\mu^- \to e^+e^-)$ which is a rather smoothly falling function beyond the Z boson pole. In Appendix C it is shown that the loop contribution to $\mu^+\mu^- \to e^+e^-$ is suppressed and the Z' resonance is not discernible in this channel at a $\mu^+\mu^-$ collider. We note that there is a second overlapping resonance from a spin 0 ρ state

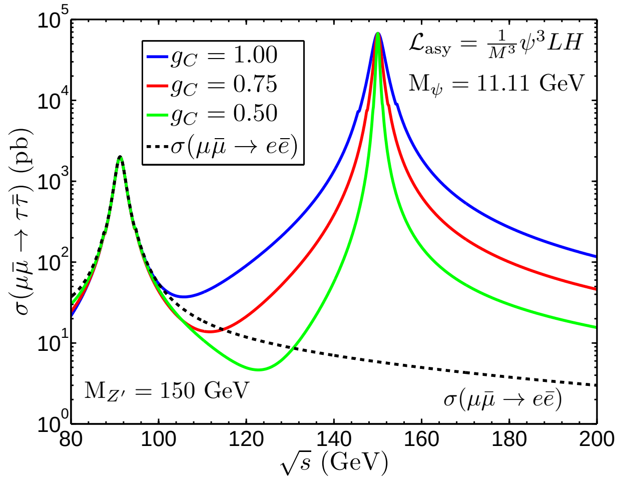

Figure 4: (color online) An exhibition of the relative strength of the $\tau^+\tau^-$ vs e^+e^- signal at a muon collider. The presence of a detectable Z' resonance in the $\mu^+\mu^- \to \tau^+\tau^-$ channel provides a smoking gun signature for the gauged $L_\mu - L_\tau$ AsyDM model. A similar resonance is also present in the $\mu^+\mu^- \to \mu^+\mu^-$ channel while $\mu^+\mu^- \to e^+e^-$ cross section shows no such enhancement in the Z' region.

where $\mu^+\mu^- \to \rho \to \tilde{\mu}^*\tilde{\mu} \to \mu^+\mu^- 2\tilde{\chi}^0$. However, the ρ resonance can only proceed at the loop level and is suppressed relative to the Z' pole.

9 A gauged $B - L$ model

Next we discuss briefly the case where in the Stueckelberg extension we use $U(1)_{B-L}$ rather than $U(1)_{L_\mu - L_\tau}$. Here we consider models *with* the right-handed neutrinos in order to gauge $B - L$ and focus on the model A_2' with the $B - L$ transfer interaction $\mathcal{L}_{\rm asy} = \frac{1}{M^4}\psi^2 (LH)^2$ above the EWPT scale. By including three generations of right-handed neutrinos, the asymmetric dark matter mass is computed to be 6.06 GeV, c.f. Table 3. In this case, there are more experimental constraints to consider which include collider (i.e., LEP, Tevatron, LHC) constraints as well as precision constraints (i.e., the measurements of the ρ parameter, the Υ width). Specifically the LEP constraint gives $M_{Z'}/g_C' \gtrsim 6$ TeV [55] for heavy gauge bosons. A stricter bound within a specific framework is given in [56] where $M_{Z'} \geq 10$ TeV. For lighter gauge bosons, as in the case of [9, 57], the UA2 cross section bound [58] is more stringent. Our analysis here is consistent with these constraints. Now, as in the $L_\mu - L_\tau$ case, the thermal symmetric contribution to the relic density from AsyDM must still be consistent with WMAP, i.e. it must be depleted to below 10% of the WMAP-7 value. An analysis of this is given in Fig. 5 for the model A_2' with $\gamma = 0$ (dashed line) and $\gamma = \gamma_0 = 1.3 \times 10^{-10}$ (solid line). Here one finds that the symmetric component of AsyDM can satisfy our WMAP-7 constraint for a range of Z' masses. If one wishes to keep the Z' mass in the Υ region, i.e. ~ 10 GeV, then a fine-tuned value of g_C' is needed as seen in Fig. 5 to be consistent with the constraints on $M_{Z'}/g_C'$. Heavier Z' masses would have difficulty satisfying both the collider and WMAP-7 constraints discussed above.

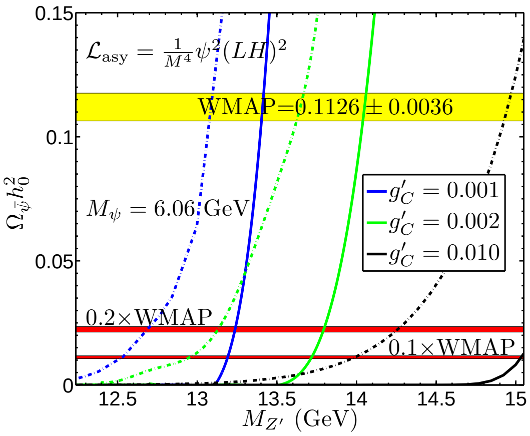

Figure 5: (color online) A display of the thermal relic density of ψ as a function of $M_{Z'}$ for the model with a gauged $B-L$ for different couplings with $\gamma = 0$ (dashed line) and $\gamma = \gamma_0 = 1.3 \times 10^{-10}$ (solid line). It is seen that resonant annihilation of thermal dark matter via the Z' pole allows the relic density of this component to be reduced to below 10% of the WMAP value for values of Z' around twice the mass of the dark particle.

10 Conclusion

In this work we have proposed models of asymmetric dark matter in the framework of Stueckelberg extensions of the SM, the 2HD and the MSSM. Several candidate models for asymmetric dark matter were discussed using a variety of operators constructed out of standard model fields which carry a non-vanishing $B-L$ quantum number which is transferred to the dark matter sector at high temperatures in the early universe consistent with sphaleron interactions which preserve $B-L$. The analysis was done both for models where the interaction temperature at which the $B-L$ transfer takes place above the electroweak phase transition scale as well as below this scale. The details of the $B-L$ transfer determine the mass of the asymmetric dark particle. A master formula was given which generates the asymmetric dark matter mass for a variety of models discussed in the text and allows one to discuss new possibilities. Specific models are discussed, including those anchored in the standard model, the two Higgs doublet model as well as the minimal supersymmetric standard model, with or without the right-handed neutrinos.

A central ingredient in a successful asymmetric dark matter model, and an explanation of cosmic coincidence, is an exhibition of an efficient mechanism for the annihilation of the symmetric dark matter component which is produced by thermal processes. We accomplish this using a Stueckelberg extension of the standard model and of the MSSM. The Stueckelberg extension of the SM is particularly simple and appealing in which, aside from the dark matter field, there is just one more field, a gauge boson (Z') of a $L_\mu - L_\tau$ gauge symmetry, which gains mass via the Stueckelberg mechanism. The symmetric dark matter produced by thermal processes is depleted via resonant annihilation from the exchange of a Z' using a Breit-Wigner pole. This is perhaps the simplest asymmetric dark matter model, in that there are no extra Higgs fields that appear in the model. Moreover, the extra $U(1)$ gauge symmetry in our models forbids the dangerous Majorana mass terms which would generate oscillations of the dark particles and their anti-particles which could washout the asymmetry.

A supersymmetric extension of this model is also given where it is shown that in addition to the Z' boson, there is also a spin 0 boson field along with two additional neutralino states arising from the Stueckelberg gaugino sector. It is shown that the ρ has a rapid decay and does not participate in the dark matter analysis. In the MSSM extension, there is an extra complication, in that, with R parity one can have a stable neutralino which is a possible dark matter candidate, and it must be shown that it is also depleted so does not compete with the asymmetric dark matter candidate. In the analysis presented in this work it was shown that there exists a significant part of the parameter space of mSUGRA where the relic density arising from the neutralino was less than one tenth of the WMAP relic density and thus the neutralino is a subdominant component of dark matter. Interestingly, however, it was shown that the subdominant neutralino is still accessible at future direct detection experiments such as SuperCDMS and XENON-100. It was shown that definitive tests of the model can come from a muon collider where one can produce the Z' which decays only into μ's and τ's (and μ, τ neutrinos).

We also discussed a gauged $B - L$ Stueckelberg model. Again in this model the symmetric dark matter can be efficiently depleted by annihilation near a Breit-Wigner Z' pole. Thus within the Stueckelberg extensions there exist several possibilities for explaining cosmic coincidence. The dominant dark matter in all these models will be light and lies in the range of $1 - 16$ GeV.

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A Master formula for computing the asymmetric dark matter mass

We have discussed various Models A-F and subcases such as A_1-A_6 etc, and also models with right-handed Dirac neutrinos, which will be discussed at the end of this section. We discuss now a master formula which allows one to take some particles in or out of thermal equilibrium. Such a formula would generally be useful before $SU(2)_L$ breaking, i.e., $T > T_{\rm EWPT}$, when we are able to take out some of the super-partners are suppressed in the plasma while others are not. This would allow us to discuss the Models A,D,E,F in a unified way and also allow us to generate new models where some other sets of super-particles are taken out of the relativistic plasma in the early Universe. However, it is not useful to discuss such a formula below the electroweak phase transition scale since the current experimental data indicates the sparticles to be heavy and not below the electroweak phase transition scale.

In obtaining the master formula, we assume: (1) In supersymmetric cases, all particles in a supermultiplet have the same chemical potential; (2) A given particle type in different generations has the same chemical potential, e.g., $\mu_d = \mu_s = \mu_b$; (3) All the additional Higgs doublets have the same chemical potential as the standard model Higgs μ_H. Following the discussion of Section 3.1, for all the fields in the plasma, we have the chemical potential constraints as before, i.e, $\mu_H = \mu_L - \mu_e = \mu_q - \mu_d = \mu_u - \mu_q$ from Yukawa couplings, $3\mu_q + \mu_L = 0$ from sphaleron processes, and $Y = 0$ from the neutrality condition.

It is useful to introduce the temperature-dependent coefficients $c_\alpha^{(i)}$ for the matter fields in the plasma. We define $c_\alpha^{(i)} = c_\alpha^{(i)}(f) + c_\alpha^{(i)}(b)$, where $c_\alpha^{(i)}(f)$ counts the contribution of i^{th} generation particle α (with mass m_α) which is fermionic and $c_\alpha^{(i)}(b)$ counts the contribution of its super-partner $\tilde{\alpha}$ (with mass $m_{\tilde{\alpha}}$) which is bosonic, where $c_\alpha(f)$ and $c_\alpha(b)$ are given by Eq. (4). We note that in the limit when m_α can be neglected, one has a weakly interacting plasma so that $c_\alpha(f) = 1$ and $c_\alpha(b) = 2$. Thus we have, for $T \gg m_{\tilde{\alpha}}$, $c_\alpha^{(i)} = 1 + 2 = 3$; for $m_{\tilde{\alpha}} \gg T \gg m_\alpha$, $c_\alpha^{(i)} = 1 + 0 = 1$; and for $T \ll m_\alpha$, $c_\alpha^{(i)} = 0$. For the Higgs doublets, we have $c_H = c_H(b) = 2$ in the non-supersymmetric case, and $c_H = c_H(b) + c_H(f) = 3$ for the supersymmetric case, and λ_H counts the number of Higgs doublets in the model.

We can then rewrite the hypercharge neutrality condition as

$$\begin{aligned} 2(c_q^{(1)} + c_q^{(2)} + c_q^{(3)})\mu_q + 4(c_u^{(1)} + c_u^{(2)} + c_u^{(3)})\mu_u - 2(c_d^{(1)} + c_d^{(2)} + c_d^{(3)})\mu_d \\ -2(c_L^{(1)} + c_L^{(2)} + c_L^{(3)})\mu_L - 2(c_e^{(1)} + c_e^{(2)} + c_e^{(3)})\mu_e + 2c_H\lambda_H\mu_H = 0 \,. \end{aligned} \tag{114}$$

Further defining $C_\alpha = \sum_i c_\alpha^{(i)} = c_\alpha^{(1)} + c_\alpha^{(2)} + c_\alpha^{(3)}$, and together with Eqs. (30)-(32), we obtain

$$\mu_X = -\frac{C_q + 8C_u + 2C_d + 3C_L + 6C_e + 3c_H\lambda_H}{6C_u + 3C_d + 3C_e + 3c_H\lambda_H} Q_{B-L}^{\text{DM}} \mu_L \,, \tag{115}$$

and

$$\begin{aligned} B - L = & -\Big[C_u\big(3C_q + 6C_d + 9C_L + 2C_e\big) + C_d\big(3C_q + 9C_L + 8C_e\big) + C_e\big(C_q + 3C_L\big) \\ & + c_H\lambda_H\big(2C_q + C_u + C_d + 6C_L + 3C_e\big)\Big]\mu_L \Big/ \big(6C_u + 3C_d + 3C_e + 3c_H\lambda_H\big) \\ \equiv & -\mathcal{N}\,\mu_L \Big/ \big(6C_u + 3C_d + 3C_e + 3c_H\lambda_H\big) \,. \end{aligned} \tag{116}$$

From Eqs. (8) and (28), we find the master formula for computing the dark matter mass

$$m_{\text{DM}} \simeq \frac{\mathcal{N}}{C_q + 8C_u + 2C_d + 3C_L + 6C_e + 3c_H\lambda_H} \cdot \frac{\kappa}{-Q_{B-L}^{\mathcal{O}}} \cdot \frac{150}{97} \text{ GeV} \,, \tag{117}$$

where κ is the parameter indicating the dark matter type: for the non-supersymmetric case, $\kappa = 1$ for fermionic dark matter, and $\kappa = 2$ for bosonic dark matter; for the supersymmetric case, $\kappa = 3$. $Q_{B-L}^{\mathcal{O}}$ is the $(B-L)$-charge of the operator $\mathcal{O}_{\text{asy}}$ in the $B-L$ transfer interaction Eq. (2). The operators that carry $Q_{B-L}^{\mathcal{O}} = -1$ are: $\mathcal{O}_{\text{asy}} = LH, LLe^c, Lqd^c, u^cd^cd^c$. Higher dimensional operators with a larger value of $(B-L)$-charge can be constructed from them.

We can extract the results for Models A,D,E,F from this master formula:

1. Model A: For matter fields we have $c_\alpha^{(i)} = c_\alpha^{(i)}(f) = 1$ (so that $C_\alpha = 3$); $c_H = c_H(b) = 2$ and $\lambda_H = 1$. For fermionic dark matter we take $\kappa = 1$ and we recover Eq. (35).

2. Model D: Here all the parameters are the same as in Model A except that $\lambda_H = 2$. Setting $\kappa = 1$ we recover Eq. (37).

3. Model E: For matter fields, $c_\alpha^{(i)} = c_\alpha^{(i)}(f) + c_\alpha^{(i)}(b) = 3$ (so that $C_\alpha = 9$); $c_H = c_H(b) + c_H(f) = 3$ and $\lambda_H = 2$. Since this is a supersymmetric case, $\kappa = 3$ and we recover Eq. (69).

4. Model F: For matter fields, $c_\alpha^{(1)} = c_\alpha^{(2)} = 1, c_\alpha^{(3)} = 3$ (so that $C_\alpha = 5$); $c_H = 3$, $\lambda_H = 2$. Taking $\kappa = 3$ we recover Eq. (69).

For Models A′, D′, E′, F′ which include three generations of right-handed Dirac neutrinos in the thermal bath, the master formula reads,

$$m_{\rm DM} \simeq \left(\frac{\mathcal{N}}{C_q + 8C_u + 2C_d + 3C_L + 6C_e + 3c_H\lambda_H} + C_{\nu_R} \right) \cdot \frac{\kappa}{-Q^{\mathcal{O}}_{B-L}} \cdot \frac{25}{21} \text{ GeV}, \tag{118}$$

where $C_{\nu_R} = c^{(1)}_{\nu_R} + c^{(2)}_{\nu_R} + c^{(3)}_{\nu_R}$. By taking $C_{\nu_R} = 3, 3, 9, 5$, we can recover the dark matter mass formulas for Models A′, D′, E′, F′ (Eqs. (59) and (73)).

We can also obtain the dark matter mass of other models from the master formula Eq. (117) or Eq. (118) by varying the temperature where certain heavy particles are Boltzmann suppressed in the thermal bath.

B Decay of the ρ

Here we compute the decay of the ρ. From Eq. (112) one finds that ρ couples to smuons, staus, muon sneutrino, and tau sneutrino. This means that the ρ decay has $\mu^+\mu^-, \nu_\mu\bar{\nu}_\mu, \tau^+\tau^-, \nu_\tau\bar{\nu}_\tau$ final states which arise via the exchange of neutralinos and charginos in the loops (a generic diagram is shown in Fig. 6). The amplitude of the generic diagram reads,

$$i\mathcal{M} = -ig_{\rho ij} C_{ki} C^*_{kj} \int \frac{d^4k}{(2\pi)^4} \bar{u}(p') \frac{(\not{k} - \not{p}) + m_{\tilde{\chi}_k}}{(k^2 - m_i^2)(k'^2 - m_j^2)((k-p)^2 - m^2_{\tilde{\chi}_k})} v(p), \tag{119}$$

where $k' = q - k$, m_i, m_j are the masses of the sleptons, and $m_{\tilde{\chi}_k}$ is the mass of the neutralino or of the chargino in the loop, while $g_{\rho ij}, C_{ki}$ are the couplings. Our purpose here is to estimate the size of the lifetime and to that end it is sufficient to estimate the contribution for one set of diagrams. Thus we consider the decay of the ρ to final states $\mu^+\mu^-$ via the exchange of neutralinos. In this case we will have the exchange of smuons and neutralinos in the loop. Further, we will ignore the mixing between the left and the right chiral smuons so that the mixing angle $\theta_{\tilde{f}_i}$ in Eq. (112) can be set to zero. In this circumstance the off-diagonal term involving two smuons in the loop does not contribute and the relevant loop integral takes the form

$$\frac{1}{(k^2 - m_i^2)(k'^2 - m_j^2)((k-p)^2 - m^2_{\tilde{\chi}_k})} = \int_0^1 \mathrm{d}x\mathrm{d}y\mathrm{d}z \delta(x + y + z - 1) \frac{2}{D^3_{ik}}, \tag{120}$$

where $D_{ik} = l^2 - \Delta_{ik} + i\epsilon$ in which $l \equiv k - (yq + zp)$ and

$$\Delta_{ik} = (1 - z)m_i^2 - xym_\rho^2 + zm^2_{\tilde{\chi}_k} + (z^2 - z)m_\mu^2. \tag{121}$$

The masses in the loops are much larger than the muon mass and thus the muon mass can be ignored. The integration on l gives

$$i\mathcal{M} = \frac{-ig_{\rho ij} C_{ki} C^*_{ki}}{(4\pi)^2} \int_0^1 \mathrm{d}x\mathrm{d}y\mathrm{d}z \delta(x + y + z - 1) \frac{\bar{u}(p') m_{\tilde{\chi}_k} v(p)}{\Delta_{ik}}. \tag{122}$$

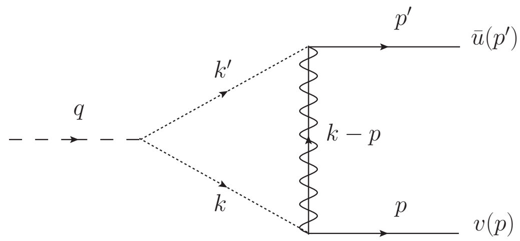

Figure 6: A generic diagram showing the decay of the ρ to one of the final states which could be $\mu^+\mu^-, \nu_\mu\bar{\nu}_\mu$, $\tau^+\tau^-, \nu_\tau\bar{\nu}_\tau$ via exchange of sleptons, charginos and neutralinos at one loop.

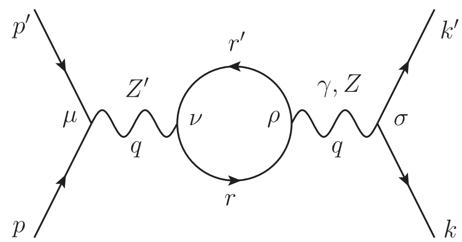

Figure 7: $Z' - \gamma$ and $Z' - Z$ exchange via $\mu^+\mu^-, \nu_\mu\bar{\nu}_\mu$, $\tau^+\tau^-, \nu_\tau\bar{\nu}_\tau$ loops.

Further, an approximate evaluation of integration on the Feynman parameters gives

$$i\mathcal{M} = \frac{-ig_{\rho ii}C_{ki}C^*_{ki}}{(4\pi)^2}\bar{u}(p')\frac{m_{\tilde{\chi}_k}}{m_i^2}v(p)\,, \tag{123}$$

under the assumption $m^2_{\tilde{\chi}_k}/m_i^2 \ll 1$ and $m_\rho^2/m_i^2 \ll 1$. The decay width of $\rho \to \mu^+\mu^-$ is then given by

$$\mathrm{d}\Gamma = \frac{1}{2m_\rho}\int\frac{\mathrm{d}^3\vec{p}}{(2\pi)^3 2E_{\mu^+}}\int\frac{\mathrm{d}^3\vec{p'}}{(2\pi)^3 2E_{\mu^-}}\left|\sum i\mathcal{M}\right|^2(2\pi)^4\delta^{(4)}(q-p-p') = \frac{|\sum i\mathcal{M}|^2}{8\pi m_\rho}\,. \tag{124}$$

Next we note that $g_{\rho 11} = -g_{\rho 22} = g_C Q_C m_\rho$ and thus

$$\left|\sum i\mathcal{M}\right|^2 \simeq \frac{(g_C Q_c)^2 m_\rho^4}{16\pi^4}\left|\sum_{k=1}^{6}\sum_{i=1}^{2}(-1)^{i+1}\frac{C_{ki}C^*_{ki}}{m_i^2}\right|^2\,. \tag{125}$$

A numerical estimate using Eqs. (124) and (125) and the inputs $m_1 = 1$ TeV, $m_2 \gg m_1$, $m_\rho = 100$ GeV, the lightest neutralino mass of 50 GeV gives $\tau_\rho = \hbar/\Gamma \sim 10^{-14\pm1}$ s. Thus the decay of the ρ is very rapid.

C Z' exchange contribution to $\mu^+\mu^- \to e^+e^-$ at loop level

At a muon collider, e^+e^- final states can be created via photon exchange and via a Z boson exchange. Since the Z' has no direct coupling with the first generation leptons, there is no tree-level Z' exchange contribution to e^+e^- final states. However, at the loop level a Z' exchange can make a contribution where the second and third generation leptons are exchanged in the loop as shown in Fig. 7. We now compute this contribution to determine its size. Thus we consider a $\mu^+\mu^- \to e^+e^-$ process with a Z' exchange via the second and third generation leptons loops as shown in Fig. 7. In this case the contribution to the scattering amplitude is

$$
\begin{aligned}
\sum i\mathcal{M} &= i\mathcal{M}_{\gamma Z'} + i\mathcal{M}_{ZZ'} \\
&= \bar{v}(p')(\frac{i}{2}g_C Q_\mu \gamma^\mu)u(p)\frac{-i(g_{\mu\nu} - q_\mu q_\nu/m_{Z'}^2)}{q^2 - m_{Z'}^2}(i\Pi^{\nu\rho}_{\gamma Z'})\frac{-ig_{\rho\sigma}}{q^2}\bar{u}(k)(-ie\gamma^\sigma)v(k') \\
&+ \bar{v}(p')(\frac{i}{2}g_C Q_\mu \gamma^\mu)u(p)\frac{-i(g_{\mu\nu} - q_\mu q_\nu/m_{Z'}^2)}{q^2 - m_{Z'}^2}(i\Pi^{\nu\rho}_{ZZ'}) \\
&\qquad \times \frac{-i(g_{\rho\sigma} - q_\rho q_\sigma/m_Z^2)}{q^2 - m_Z^2}\bar{u}(k)\frac{-ig\gamma^\sigma}{\cos\theta_W}(g_V + g_A\gamma^5)v(k')\,,
\end{aligned}
\tag{126}
$$

where Q_μ is the $U(1)_{L_\mu - L_\tau}$ charge for muon, $g_V = \frac{1}{2}(T_3)_L + \sin^2\theta_W$, $g_A = -\frac{1}{2}(T_3)_L$, and the vacuum polarization tensors $i\Pi^{\nu\rho}_{\gamma Z'}$ and $i\Pi^{\nu\rho}_{ZZ'}$ are the sums of the contributions from $\mu^+\mu^-$, $\nu_\mu\bar{\nu}_\mu$, $\tau^+\tau^-$, $\nu_\tau\bar{\nu}_\tau$ loops

$$
i\Pi^{\nu\rho} = i\Pi^{\nu\rho}_\mu + i\Pi^{\nu\rho}_{\nu_\mu} + i\Pi^{\nu\rho}_\tau + i\Pi^{\nu\rho}_{\nu_\tau}\,. \tag{127}
$$

First we focus on $i\Pi^{\nu\rho}_{\gamma Z',\mu}$ which is the muon loop contribution to the $Z' - \gamma$ exchange. It reads

$$
\begin{aligned}
i\Pi^{\nu\rho}_{\gamma Z',\mu} &= -(\frac{i}{2}g_C Q_\mu)(-ie)\int\frac{\mathrm{d}^4 r}{(2\pi)^4}\mathrm{tr}\Big[\gamma^\nu\frac{i(\not{r} + m_\mu)}{(r^2 - m_\mu^2)}\gamma^\rho\frac{i(\not{r}' + m_\mu)}{(r'^2 - m_\mu^2)}\Big] \\
&= \frac{4ieg_C Q_\mu}{(4\pi)^2}(q^2 g^{\nu\rho} - q^\nu q^\rho)\int_0^1 \mathrm{d}x\, x(1-x)\frac{\Gamma(2 - \frac{d}{2})}{\Delta_\mu^{2-\frac{d}{2}}} \\
&= \frac{4ieg_C Q_\mu}{(4\pi)^2}(q^2 g^{\nu\rho} - q^\nu q^\rho)\int_0^1 \mathrm{d}x\, x(1-x)\big(\frac{2}{\epsilon} - \log\Delta_\mu - \gamma + \log(4\pi) + \mathcal{O}(\epsilon)\big)\,,
\end{aligned}
\tag{128}
$$

where $\Delta_\mu = m_\mu^2 - x(1-x)q^2$, and in the last step we use the dimensional regularization. The expression of $i\Sigma^{\nu\rho}_{\gamma Z',\tau}$ differs from $i\Sigma^{\nu\rho}_{\gamma Z',\mu}$ by only the Q_τ factor, and it takes the form

$$
i\Pi^{\nu\rho}_{\gamma Z',\tau} = \frac{4ieg_C Q_\tau}{(4\pi)^2}(q^2 g^{\nu\rho} - q^\nu q^\rho)\int_0^1 \mathrm{d}x\, x(1-x)\big(\frac{2}{\epsilon} - \log\Delta_\mu - \gamma + \log(4\pi) + \mathcal{O}(\epsilon)\big)\,. \tag{129}
$$

Summing over these two terms, we find a dramatic cancellation of the divergence in the loop due to $Q_\mu = -Q_\tau = 1$, making the loop finite so that

$$
i\Pi^{\nu\rho}_{\gamma Z',\mu} + i\Pi^{\nu\rho}_{\gamma Z',\tau} = \frac{4ieg_C}{(4\pi)^2}(q^2 g^{\nu\rho} - q^\nu q^\rho) \times I\,, \tag{130}
$$

where

$$
I = \int_0^1 \mathrm{d}x\, x(1-x)\log\frac{\Delta_\tau}{\Delta_\mu} = \int_0^1 \mathrm{d}x\, x(1-x)\log\frac{m_\tau^2 - x(1-x)q^2}{m_\mu^2 - x(1-x)q^2}\,. \tag{131}
$$

One can also obtain the neutrino exchange contributions from the above by setting the fermion masses to zero in the equation above (assuming neutrinos to be massless) which gives a vanishing contribution.

Now we want to compare the contribution of the $Z'-\gamma$ exchange loop diagram with the tree-level process $\mu^+\mu^- \to \gamma \to e^+e^-$, whose amplitude reads

$$i\mathcal{M}_\gamma = \bar{v}(p')(-ie\gamma^\mu)u(p)\frac{-ig_{\mu\nu}}{q^2}\bar{u}(k)(-ie\gamma^\nu)v(k')\,. \tag{132}$$

With some manipulation we find

$$i\mathcal{M}_{\gamma Z'} = -\frac{2g_c^2 I}{(4\pi)^2}\cdot\frac{q^2}{q^2-m_{Z'}^2}\times i\mathcal{M}_\gamma \equiv f\times i\mathcal{M}_\gamma\,. \tag{133}$$

Thus, the total squared amplitudes involving a photon can be written as

$$\begin{aligned}|i\mathcal{M}_\gamma + i\mathcal{M}_{\gamma Z'}|^2 &= |1+f|^2\times|i\mathcal{M}_\gamma|^2 \\ &= (1+f+f^*+ff^*)\times|i\mathcal{M}_\gamma|^2\,.\end{aligned} \tag{134}$$

Our numerical analysis shows that $(f+f^*+ff^*)$ is smaller than $\sim 10^{-3}$ and thus the loop makes only a tiny contribution to the total cross section in this case. The analysis of $Z'-Z$ exchange is similar and gives a very small value. Thus we conclude that a Z' peak will not be visible in the $\mu^+\mu^- \to e^+e^-$ process at a muon collider. The above analysis also exhibits why a Z' in this model would not be visible in an e^+e^- machine.

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