Testing for new physics in singly Cabibbo suppressed D decays

Yuval Grossman, Alexander L. Kagan, and Jure Zupan

Testing for new physics in singly Cabibbo suppressed D decays

Yuval Grossman,[1,*] Alexander L. Kagan,[2,†] and Jure Zupan[2,‡]

[1]*Institute for High Energy Phenomenology, Newman Laboratory of Elementary Particle Physics, Cornell University, Ithaca, NY14853, USA*

[2]*Department of Physics, University of Cincinnati, Cincinnati, Ohio 45221,USA*

We devise tests for a new physics origin of the recently measured direct CP violation in singly Cabibbo suppressed D decays. The tests take the form of sum rules for the CP asymmetries in various D decays. They are based on the fact that within the standard model CP violation arises from interference of the dominant tree amplitudes with the $\Delta I = 1/2$ penguin amplitudes. The sum rules would be violated if the observed CP violation is due to new physics contributions to the effective weak Hamiltonian that change isospin by $\Delta I = 3/2$.

I. INTRODUCTION

Recently, the LHCb collaboration announced a measurement of the difference between the time-integrated CP asymmetries in two singly Cabibbo supressed (SCS) D meson decay modes [1],

$$\Delta \mathcal{A}_{CP} \equiv \mathcal{A}_{CP}(D \to K^+ K^-) - \mathcal{A}_{CP}(D \to \pi^+ \pi^-)$$
$$= (-0.82 \pm 0.21 \pm 0.11)\%, \tag{1}$$

which has been confirmed by the CDF measurement [2]

$$\Delta \mathcal{A}_{CP} = (-0.62 \pm 0.21 \pm 0.10)\%. \tag{2}$$

The updated world average for the difference of the direct CP asymmetries is then $\Delta \mathcal{A}_{CP}^{\rm dir} = (-0.67 \pm 0.16)\%$ [2]. In the SM, CP violation (CPV) in SCS D decays comes from the interference of the tree and penguin amplitudes and is parametrically suppressed by $\mathcal{O}(V_{cb}V_{ub}/V_{cs}V_{us}) \sim 10^{-3}$. However, the uncertainties on these order of magnitude estimates are large [3]. In particular, the penguin contraction power corrections could be significantly enhanced, possibly leading to the observed CP asymmetry.

The predictions for direct CPV in charm decays are notoriously difficult to make, even if one is aiming at order of magnitude estimates. For instance, in [4] and recently in [3, 5–8] it was argued that large CP asymmetries can be expected in SCS D decays. In [9–11], small CP asymmetries were obtained, while marginal consistency with measurements (1), (2) was found in [12]. At the same time, there are viable new physics models (NP) that can enhance $\Delta \mathcal{A}_{CP}$ and simultaneously avoid constraints from other flavor violation searches, such as $D - \bar{D}$ mixing, as shown in [7, 13–21].

The aim of this work is to provide experimental tests that can distinguish between a SM and NP origin for $\Delta \mathcal{A}_{CP}$. The basic idea it to take advantage of the fact that the penguin amplitudes are $\Delta I = 1/2$ transitions. A prediction of the SM is that CPV effects are confined to the $\Delta I = 1/2$ amplitude, to very good approximation (to be quantified below). Any observation of CPV effects due to the other possible isospin reduced matrix element, namely the $\Delta I = 3/2$ amplitude, would be a clear signal of new physics.

This insight can be used to experimentally search for NP in charm decays without relying on theoretical calculations. To do so we construct a set of CP asymmetry sum rules that will be obeyed if the observed CP asymmetries are due to the SM, but violated if they are due to NP. It is important to note that we can search in this way only for a subset of NP models – the ones that generate CPV in the $\Delta I = 3/2$ reduced matrix elements. An example is a model with a single new scalar field with nontrivial flavor couplings, as recently proposed in [16] to explain the observed $\Delta \mathcal{A}_{CP}$. On the other hand, NP models that only contribute through penguin $\Delta I = 1/2$ operators would not violate the derived sum rules and are thus much harder to distinguish from the SM contributions to $\Delta \mathcal{A}_{CP}$. Examples are provided by flavor violating supersymmetric squark-gluino loops that mediate the $c \to ug$ transition [13, 18, 21].

In the derivation of the sum rules we use the isospin flavor symmetry of QCD. Isospin symmetry is broken at $\mathcal{O}(1\%)$, which is also the size of the CP asymmetries we are interested in. Thus special care is needed in order to avoid the introduction of large errors in the sum rules. There are several sources of isospin breaking that could modify the sum rules. The u and d quark masses and electromagnetic interactions break isospin in a CP conserving manner. Their effects are explicitly included in the sum rules so that they cancel up to quadratic order in isospin breaking. The electroweak penguin operators provide CP violating sources for isospin breaking. However, their effects are suppressed by $\alpha/\alpha_S \sim \mathcal{O}(10^{-2})$ compared to the leading CP violating but isospin conserving penguin contractions of the $Q_{1,2}$ operators, and can thus be safely neglected.

The paper is organized as follows. Our notation is introduced in Section II, the sum rules are derived in Section III, and we conclude in Section IV.

---

*Electronic address:yg73@cornell.edu

†Electronic address:kaganalexander@gmail.com

‡Electronic address:zupanje@ucmail.uc.edu; On leave of absence from Josef Stefan Institute and U. of Ljubljana, Ljubljana, Slovenia.

II. PRELIMINARIES

The CP-conjugate decay amplitudes for SCS decays can be written as

$$A_f(D \to f) = T_f + P_f e^{i(\delta_f^P - \gamma)} + A_f^{NP} e^{i(\delta^{NP} - \phi)},$$
$$\overline{A}_{\bar{f}}(\bar{D} \to \bar{f}) = T_f + P_f e^{i(\delta_f^P + \gamma)} + A_f^{NP} e^{i(\delta^{NP} + \phi)}, \quad (3)$$

where T_f is the dominant SM "tree" amplitude. It is proportional to $V_{ud}V_{cd}^*$ and is taken to be real by convention. The SM "penguin" amplitude has magnitude P_f. It is CKM suppressed by $\mathcal{O}(V_{cb}V_{ub}/V_{cs}V_{us})$ compared to the tree amplitude. It carries the CKM weak phase $\gamma = (67.3^{+4.2}_{-3.5})°$ [22] and the relative strong phase δ_f. The NP amplitude has magnitude A_f^{NP}, and carries a strong phase δ^{NP} and a weak phase ϕ relative to the tree amplitude.

The direct CP asymmetry is given by

$$\mathcal{A}_f^{\mathrm{dir}} \equiv \frac{|A_f|^2 - |\bar{A}_{\bar{f}}|^2}{|A_f|^2 + |\bar{A}_{\bar{f}}|^2} = $$
$$= 2r_f^P \sin\gamma \sin\delta_f^P + 2r_f^{NP} \sin\phi \sin\delta_f^{NP}, \quad (4)$$

where $r_f^P \equiv P_f/T_f$, $r_f^{NP} \equiv A_f^{NP}/T_f$, and we have neglected higher orders in r_f^P and r_f^{NP}. The question we are interested in is how one can distinguish between the SM contributions to the direct CP asymmetries, proportional to r_f^P, and the NP contributions to the direct CP asymmetries, proportional to r_f^{NP}. To do so, we will utilize the transformation properties of the SM and NP contributions under isospin.

We first review the transformation properties of the SM contributions under isospin. The effective $\Delta C = 1$ weak Hamiltonian, H_{eff}, is [23]

$$H_{\mathrm{eff}}^{\Delta C=1} = \frac{G_F}{\sqrt{2}} \Big[\sum_{p=d,s} V_{cp}^* V_{up} (C_1 Q_1^p + C_2 Q_2^p) $$
$$- V_{cb}^* V_{ub} \sum_{i=3}^{6} C_i Q_i + C_{8g} Q_{8g} \Big] + \mathrm{h.c.}. \quad (5)$$

The "tree" operators are

$$Q_1^p = (\bar{p}c)_{V-A}(\bar{u}p)_{V-A},$$
$$Q_2^p = (\bar{p}_\alpha c_\beta)_{V-A}(\bar{u}_\beta p_\alpha)_{V-A} \quad (6)$$

with summation over color indices α, β understood. The QCD penguin operators are

$$Q_{3,5} = (\bar{u}c)_{V-A} \sum_{q=u,d,s} (\bar{q}q)_{V\mp A},$$
$$Q_{4,6} = (\bar{u}_\alpha c_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V\mp A}, \quad (7)$$
$$Q_{8g} = -\frac{g_s}{8\pi^2} m_c \bar{u} \sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu}c,$$

and we do not display the numerically further suppressed electroweak penguin operators.

The flavor structure of the tree operators for $D \to \pi\pi$ and $D_s \to K\pi$ decays is $(\bar{d}c)(\bar{u}d)$. Thus they have both $\Delta I = 3/2$ and $\Delta I = 1/2$ components. The remaining operators, i.e., the $(\bar{s}c)(\bar{u}s)$ tree operators for $D \to KK$ decays and the penguin operators, are purely $\Delta I = 1/2$. Note that the penguin contraction contributions of the $(\bar{d}c)(\bar{u}d)$ tree operators to $D \to KK$ are $\Delta I = 1/2$.

The NP models that contribute to the CP asymmetries in SCS D decays can be grouped in to two sets, (i) those in which the NP operators are purely $\Delta I = 1/2$, and (ii) those in which the NP operators also have $\Delta I = 3/2$ components. As we show below, one can use the isospin decomposition and the resulting sum rules to search for the presence of $\Delta I = 3/2$ NP just using experimental information.

III. SEARCHING FOR NEW PHYSICS VIA ISOSPIN

We now derive CP asymmetry sum rules that can be used to probe for the presence of $\Delta I = 3/2$ NP contributions. Among the SCS decays, the $D \to \pi\pi$, $D \to \rho\pi$, $D \to \rho\rho$, $D \to K\bar{K}\pi$ and $D_s \to K^*\pi$ modes carry enough information for such tests. We discuss each of them in turn.

A. $D \to \pi\pi$ and $D \to \rho\rho$ decays

The isospin decomposition of the $D^0 \to \pi\pi$ decays is

$$A_{\pi^+\pi^-} = \sqrt{2}\mathcal{A}_3 + \sqrt{2}\mathcal{A}_1, \quad (8a)$$
$$A_{\pi^0\pi^0} = 2\mathcal{A}_3 - \mathcal{A}_1, \quad (8b)$$
$$A_{\pi^+\pi^0} = 3\mathcal{A}_3, \quad (8c)$$

where $\mathcal{A}_3$ and $\mathcal{A}_1$ are the reduced matrix elements for the $\Delta I = 3/2$ and $\Delta I = 1/2$ Hamiltonians. The phase convention used is such that (u,d), $(\bar{d}, -\bar{u})$, (D^+, D^0), (K^+, K^0) and $(\bar{K}^0, K^-)$ form isospin doublets, while (π^+, π^0, π^-) form a triplet. For the $\bar{D}^0 \to \pi\pi$ system the isospin decomposition is similar to (8), with $\mathcal{A}_3$, $\mathcal{A}_1$ replaced by the CP conjugate matrix elements $\bar{\mathcal{A}}_3$, $\bar{\mathcal{A}}_1$.

We decompose the reduced matrix elements into SM and NP contributions, with magnitudes A_k and a_k, respectively,

$$\mathcal{A}_k = A_k e^{i(\delta_k^A - \phi_k^A)} + a_k e^{i(\delta_k^a - \phi_k^a)}, \quad k = 1, 3. \quad (9)$$

By convention we can set the strong phase $\delta_3^A = 0$. In the SM the weak phase of the $\Delta I = 3/2$ amplitude is also zero to excellent approximation, so that we can set $\phi_3^A = 0$. Thus, in the SM the purely $\Delta I = 3/2$ decay $D^+ \to \pi^+\pi^0$ has $\mathcal{A}^{\mathrm{dir}}(D^+ \to \pi^+\pi^0) = 0$. However, the rate difference can be nonzero in the presence of NP, being given by

$$|A_{\pi^+\pi^0}|^2 - |\bar{A}_{\pi^-\pi^0}|^2 = 36 a_3 A_3 \sin\phi_3^a \sin\delta_3^a. \quad (10)$$

Note that the CP asymmetry is proportional to the $\Delta I = 3/2$ NP coefficient a_3.

Let us comment on the isospin breaking effects that have been ignored in the decomposition of (8). The isospin breaking due to the u, d quark masses and due to the electromagnetic interactions can be safely neglected since they are CP conserving. Thus, they only modify $\mathcal{A}_{CP}(D^+ \to \pi^+\pi^0)$ at second order in small parameters. While $\mathcal{A}_{CP}(D^+ \to \pi^+\pi^0) \sim \mathcal{O}(r_f^{NP})$, the effect of isospin breaking is $\mathcal{O}(\epsilon_I r_f^{NP,EWP})$, where ϵ_I is the typical size of isospin breaking. It is of order 1% and may be enhanced by at most a factor of a few. Similarly the electroweak penguins can be neglected due to the small sizes of their Wilson coefficients. Thus, we conclude that a measured nonzero CP asymmetry in $D^+ \to \pi^+\pi^0$ would be a signal for $\Delta I = 3/2$ NP.

Note that if a direct CP asymmetry is not found in $D^+ \to \pi^+\pi^0$, this does not mean that $\Delta\mathcal{A}_{CP}$ cannot be due to a new $\Delta I = 3/2$ amplitude. It is possible, for instance, that the strong phase difference δ_3^a between the NP and SM $\Delta I = 3/2$ amplitudes is simply smaller than the strong phase difference between the $\Delta I = 3/2$ and $\Delta I = 1/2$ amplitudes.

We therefore devise two more tests for the presence of new CP violating phases in the $\Delta I = 3/2$ operators. The first involves the sum of rate differences

$$|A_{\pi^+\pi^-}|^2 - |\bar{A}_{\pi^-\pi^+}|^2 + |A_{\pi^0\pi^0}|^2 - |\bar{A}_{\pi^0\pi^0}|^2$$
$$- \frac{2}{3}\left(|A_{\pi^+\pi^0}|^2 - |\bar{A}_{\pi^-\pi^0}|^2\right) = 3\left(|\mathcal{A}_1|^2 - |\bar{\mathcal{A}}_1|^2\right). \quad (11)$$

The important point is that this sum only depends on the $\Delta I = 1/2$ amplitudes. Thus, if the sum is found to be nonzero this means that there are $\Delta I = 1/2$ contributions to the CP asymmetries. They could be due to NP or they could be due to the SM. However, if the sum (11) is found to be zero, while the individual rate differences are nonzero, this would indicate that the CP asymmetries are likely dominated by $\Delta I = 3/2$ NP contributions. This statement does come with a caveat. It would still be possible that, whereas the CPV weak phases are only present in the $\Delta I = 1/2$ amplitude, the strong phases between terms in $\mathcal{A}_1$ with different weak phases are small. In this case, $\mathcal{A}_{CP}(\pi^+\pi^-)$, and $\mathcal{A}_{CP}(\pi^0\pi^0)$ would be nonzero due to interference of the $\Delta I = 1/2$ and $\Delta I = 3/2$ amplitudes.

This possibility can be checked with more data if time dependent $D(t) \to \pi^+\pi^-$ and $D(t) \to \pi^0\pi^0$ measurements become available, or if there is additional information on relative phases from a charm factory running on the $\Psi(3770)$ (for feasibility see, e.g. [24]). It amounts to measuring the weak phase of the $\Delta I = 3/2$ amplitude $\mathcal{A}_3$ via generalized triangle constructions that also take isospin breaking into account. From the isospin decomposition we have an isospin sum rule

$$\frac{1}{\sqrt{2}}A_{\pi^+\pi^-} + A_{\pi^0\pi^0} - A_{\pi^+\pi^0} = A_{\text{break}}, \quad (12)$$

and a similar sum rule for the CP-conjugate decays. The amplitude A_{break} is due to isospin breaking and is of order $\mathcal{O}(\epsilon_I A_i)$. It is equal in $D \to \pi\pi$ and $\bar{D} \to \pi\pi$ decays, i.e., $A_{\text{break}} = \bar{A}_{\text{break}}$, up to very small CP violating corrections which are down by an extra factor of $r_f \lesssim \mathcal{O}(0.01)$. One therefore has the following sum rule, valid even in the presence of isospin breaking,

$$\frac{1}{\sqrt{2}}A_{\pi^+\pi^-} + A_{\pi^0\pi^0} - \frac{1}{\sqrt{2}}\bar{A}_{\pi^-\pi^+} - \bar{A}_{\pi^0\pi^0} =$$
$$3\left(\mathcal{A}_3 - \bar{\mathcal{A}}_3\right) = -6ia_3 e^{i\delta_3^a}\sin\phi_3^a, \quad (13)$$

where in the last stage we use the fact that A_3 carries a negligible CP violating phase in the SM. Note that isospin breaking in this relation has canceled (up to corrections quadratic in small parameters). Therefore, if

$$\frac{1}{\sqrt{2}}\left(A_{\pi^+\pi^-} - \bar{A}_{\pi^-\pi^+}\right) \neq -\left(A_{\pi^0\pi^0} - \bar{A}_{\pi^0\pi^0}\right), \quad (14)$$

is found, this would mean there is CPV NP in the $\Delta I = 3/2$ amplitude. The relative phases between the $A_{\pi^+\pi^-}$ and $\bar{A}_{\pi^-\pi^+}$ amplitudes and between the $A_{\pi^0\pi^0}$ and $\bar{A}_{\pi^0\pi^0}$ amplitudes can be measured in entangled $\psi(3770) \to D\bar{D}$ decays. In addition, the phase between the $A_{\pi^+\pi^-}$ and $\bar{A}_{\pi^-\pi^+}$ amplitudes can be obtained from the time dependent $D(t) \to \pi^+\pi^-$ decay. Similarly, the phase between the $A_{\pi^0\pi^0}$ and $\bar{A}_{\pi^0\pi^0}$ amplitudes can be obtained from the time dependent $D(t) \to \pi^0\pi^0$ decay. The magnitudes of the amplitudes can be measured in their respective time integrated decays. We can thus form an experimental test. If

$$\frac{1}{\sqrt{2}}\left|A_{\pi^+\pi^-} - \bar{A}_{\pi^-\pi^+}\right| \neq \left|A_{\pi^0\pi^0} - \bar{A}_{\pi^0\pi^0}\right|, \quad (15)$$

then a $\Delta I = 3/2$ NP amplitude has been discovered.

While the above formalism has been written down for $D \to \pi\pi$ decays, it applies without changes to $D \to \rho\rho$ decays, but for each polarization amplitude separately. The $I = 1$ $\rho\rho$ combination is absent in the limit of narrow ρ resonances due to Bose symmetry. Since $J = 0$ then $S = L$, while the symmetry of the $\rho\rho$ system under particle exchange is $(-1)^{L+S+I}$ [25]. The corrections due to finite ρ width can be controlled experimentally in the same way as in $B \to \rho\rho$ decays [26]. As long as the polarizations of the ρ resonances are measured (or if the longitudinal decay modes dominate, as is the case in $B \to \rho\rho$ decays), the search for $\Delta I = 3/2$ NP could be easier experimentally in $D \to \rho\rho$ decays since there are more charged tracks in the final state. The most promising observable where polarization measurement is not needed, is $A_{CP}(\rho^+\rho^0)$, which if found nonzero (after the correction for the effect of finite ρ decay widths) would signal $\Delta I = 3/2$ NP.

B. $D \to \rho\pi$ decays

Another experimentally favorable probe is the isospin analysis of the $D \to \pi^+\pi^-\pi^0$ Dalitz plot in terms of

the $D \to \rho\pi$ decays. The isospin decomposition for D^0 decays is

$$A_{\rho^+\pi^-} = \mathcal{A}_3 + \mathcal{B}_3 + \frac{1}{\sqrt{2}}\mathcal{A}_1 + \mathcal{B}_1, \qquad (16)$$

$$A_{\rho^0\pi^0} = 2\mathcal{A}_3 - \mathcal{B}_1, \qquad (17)$$

$$A_{\rho^-\pi^+} = \mathcal{A}_3 - \mathcal{B}_3 - \frac{1}{\sqrt{2}}\mathcal{A}_1 + \mathcal{B}_1, \qquad (18)$$

and for D^+ decays it is

$$A_{\rho^+\pi^0} = \frac{3}{\sqrt{2}}\mathcal{A}_3 - \frac{1}{\sqrt{2}}\mathcal{B}_3 + \mathcal{A}_1, \qquad (19)$$

$$A_{\rho^0\pi^+} = \frac{3}{\sqrt{2}}\mathcal{A}_3 + \frac{1}{\sqrt{2}}\mathcal{B}_3 - \mathcal{A}_1, \qquad (20)$$

where $\mathcal{A}_3, \mathcal{B}_3$ are the $\Delta I = 3/2$ amplitudes for $I = 2, 1$ final states, while $\mathcal{A}_1, \mathcal{B}_1$ are $\Delta I = 1/2$ amplitudes for $I = 1, 0$ final states, and we have assumed that the $\Delta I = 5/2$ amplitude is negligibly small (since these interactions are small and CP conserving they would introduce corrections to our result that are only quadratic in small parameters). The $D \to \pi^+\pi^-\pi^0$ decay is dominated by the isospin 0 final state [27], which means that the reduced amplitude $\mathcal{B}_1$ is expected to be the largest.

From the Dalitz plot for $D^0 \to \pi^+\pi^-\pi^0$ one can measure the relative phases of $A_{\rho^+\pi^-}, A_{\rho^0\pi^0}$ and $A_{\rho^-\pi^+}$, as well as their magnitudes. The sensitivity to phases comes from the overlaps of the ρ resonances in the Dalitz plot. This means that the magnitudes and phases (up to an overall phase) of the reduced matrix elements $\mathcal{B}_1$, $\mathcal{A}_3$, and $\mathcal{B}_3 + \mathcal{A}_1/\sqrt{2}$ are measurable. If the time dependent Dalitz plot is measured then the relative phases between the $\mathcal{A}_i$ and CP conjugate $\bar{\mathcal{A}}_i$ could be measured.

We first discuss CP asymmetry sum rules that can be obtained from time integrated Dalitz plot measurements. We again employ a notation in which the strong and weak phases of the SM and NP contributions appear explicitly, as in (9). The notation we use is the straightforward generalization of (9), with $\delta_{1,3}^A$, $\delta_{1,3}^B$ the SM strong phases, $\phi_{1,3}^A$, $\phi_{1,3}^B$ the SM weak phases, $A_{1,3}$, $B_{1,3}$ the magnitudes of the SM reduced amplitudes, while NP contributions are denoted by small letters, $A \to a$, $B \to b$. By convention, the strong phase of the SM amplitude A_3 is taken to be zero, $\delta_3^A = 0$. The weak phases of the SM tree amplitudes A_3 and B_3 are also zero, $\phi_3^A = \phi_3^B = 0$. There are two combinations of measured amplitudes that are proportional to $\Delta I = 3/2$ amplitudes

$$A_{\rho^+\pi^0} + A_{\rho^0\pi^+} = 3\sqrt{2}\mathcal{A}_3,$$
$$A_{\rho^+\pi^-} + 2A_{\rho^0\pi^0} + A_{\rho^-\pi^+} = 6\mathcal{A}_3. \qquad (21)$$

A measurement of the second sum can be obtained from the $D^0 \to \pi^+\pi^-\pi^0$ Dalitz plot. If the related CP asymmetry

$$|A_{\rho^+\pi^-} + 2A_{\rho^0\pi^0} + A_{\rho^-\pi^+}|^2 - |\bar{A}_{\rho^-\pi^+} + 2\bar{A}_{\rho^0\pi^0} + \bar{A}_{\rho^+\pi^-}|^2$$
$$= 36(|\mathcal{A}_3|^2 - |\bar{\mathcal{A}}_3|^2) = 144 A_3 a_3 \sin\phi_3^a \sin\delta_3^a, \qquad (22)$$

is found to be nonzero, this would mean that the NP contribution a_3 is nonzero. If it is found to vanish, it could still be that this is due to the strong phase difference δ_3^a being vanishingly small.

Assuming that this is the case, i.e. that $\delta_3^a = 0$, one can still test for the presence of $\Delta I = 3/2$ CP violating NP. The weighted sum

$$2(|A_{\rho^0\pi^0}|^2 - |\bar{A}_{\rho^0\pi^0}|^2) +$$
$$|A_{\rho^+\pi^-} + A_{\rho^-\pi^+}|^2 - |\bar{A}_{\rho^+\pi^-} + \bar{A}_{\rho^-\pi^+}|^2 \qquad (23)$$
$$= 12(|\mathcal{A}_3|^2 - |\bar{\mathcal{A}}_3|^2) + 6(|\mathcal{B}_1|^2 - |\bar{\mathcal{B}}_1|^2),$$

measures whether there is direct CP violation in the $\mathcal{A}_3$ or $\mathcal{B}_1$ reduced amplitudes. Let us assume that (22) is found to be vanishingly small, so that $|\mathcal{A}_3| = |\bar{\mathcal{A}}_3|$. If the sum (23) is found to be zero as well, while the individual CP asymmetries are nonzero, this would be a strong indication for $\Delta I = 3/2$ NP. Again, as in the case of $\pi\pi$, there is a caveat, namely that it is possible that there is no direct CPV in $\mathcal{B}_1$ even though there are weak phases in $\mathcal{B}_1$. For instance, this would be the case if the strong phases for terms with different weak phases in $\mathcal{B}_1$ would be the same. The individual CP asymmetries would then be nonzero due to interference of $\mathcal{B}_1$ with the other amplitudes, rather than $\Delta I = 3/2$ NP.

A definitive answer can be provided by another test that is directly sensitive to the weak phase of $\mathcal{A}_3$. This test is possible if the time dependent $D(t) \to \pi^+\pi^-\pi^0$ Dalitz plot is measured. In this case the relative phases between the $D^0 \to \rho\pi$ and $\bar{D}^0 \to \rho\pi$ amplitudes can be obtained (alternatively one could use time integrated entangled decays of $\psi(3770)$ at the charm factory). The presence of a weak phase in $\mathcal{A}_3$ can then be determined from the following sum-rule

$$(A_{\rho^+\pi^-} + A_{\rho^-\pi^+} + 2A_{\rho^0\pi^0}) -$$
$$(\bar{A}_{\rho^-\pi^+} + \bar{A}_{\rho^+\pi^-} + 2\bar{A}_{\rho^0\pi^0}) = \qquad (24)$$
$$6(\mathcal{A}_3 - \bar{\mathcal{A}}_3) = -12 i a_3 e^{i\delta_3^a} \sin\phi_3^a,$$

where in the last stage we use the fact that the SM amplitude A_3 does not carry a weak phase. Thus, a nonvanishing result for (24) would provide definitive proof for $\Delta I = 3/2$ NP. A similar sum rule for the CP asymmetries rather than the amplitudes was given in (22). In that case the time integrated Dalitz plot suffices to determine the sum rule inputs.

C. $D \to K\bar{K}\pi$ decays

The isospin decomposition for the D^0 decays is

$$A_{K^+\bar{K}^0\pi^-} = \mathcal{B}_1 - \mathcal{A}_1 + \mathcal{C}_3 + \mathcal{B}_3, \qquad (25)$$

$$A_{K^+K^-\pi^0} = \mathcal{B}_1' + \frac{1}{\sqrt{2}}\mathcal{A}_1 + \sqrt{2}\mathcal{C}_3 + \mathcal{B}_3', \qquad (26)$$

$$A_{K^0\bar{K}^0\pi^0} = -\mathcal{B}_1' + \frac{1}{\sqrt{2}}\mathcal{A}_1 + \sqrt{2}\mathcal{C}_3 - \mathcal{B}_3', \qquad (27)$$

$$A_{K^0K^-\pi^+} = -\mathcal{B}_1 - \mathcal{A}_1 + \mathcal{C}_3 - \mathcal{B}_3, \qquad (28)$$

and for the D^+ decays it is

$$A_{K^+\bar{K}^0\pi^0} = \sqrt{2}\mathcal{B}_1 + \frac{3}{\sqrt{2}}\mathcal{C}_3 - \frac{1}{\sqrt{2}}\mathcal{B}_3, \qquad (29)$$

$$A_{K^+K^-\pi^+} = -\mathcal{B}_1 + \sqrt{2}\mathcal{B}_1' + \frac{3}{2}\mathcal{C}_3 + \frac{1}{2}\mathcal{B}_3 - \frac{1}{\sqrt{2}}\mathcal{B}_3', \quad (30)$$

$$A_{K^0\bar{K}^0\pi^+} = -\mathcal{B}_1 - \sqrt{2}\mathcal{B}_1' + \frac{3}{2}\mathcal{C}_3 + \frac{1}{2}\mathcal{B}_3 + \frac{1}{\sqrt{2}}\mathcal{B}_3', \quad (31)$$

where $\mathcal{B}_3, \mathcal{B}_3', \mathcal{C}_3$ are $\Delta I = 3/2$ amplitudes for $I = 1, 1, 2$ final states with the two kaons in the $I = 1, 0, 1$ isospin state, while $\mathcal{A}_1, \mathcal{B}_1, \mathcal{B}_1'$ are $\Delta I = 1/2$ amplitudes for $I = 0, 1, 1$ final states with the two kaons in $I = 1, 1, 0$ isospin state. The same results apply for $D \to K^*\bar{K}\pi$, $D \to K\bar{K}^*\pi$ and $D \to K^*\bar{K}^*\pi$ decays (and decays with a ρ instead of a π in the final state) with obvious replacements.

In the case of D^+ decays it is possible to construct a purely $\Delta I = 3/2$ matrix element by summing only three decay amplitudes, while in the case of D^0 decays four amplitudes are needed. For this reason we only consider the D^+ decays. For instance, for D^+ decays to $K^*\bar{K}^*$ resonances we have for each polarization (to shorten the notation we do not show the polarizations explicitly)

$$\sqrt{2}A_{K^{*+}\bar{K}^{*0}\pi^0} + A_{K^{*+}K^{*-}\pi^+} + A_{K^{*0}\bar{K}^{*0}\pi^+} = 6\,\mathcal{C}_3. \tag{32}$$

Thus, if the CP violating difference

$$\begin{aligned}&|\sqrt{2}A_{K^{*+}\bar{K}^{*0}\pi^0} + A_{K^{*+}K^{*-}\pi^+} + A_{K^{*0}\bar{K}^{*0}\pi^+}|^2\\&- |\sqrt{2}\bar{A}_{K^{*-}K^{*0}\pi^0} + \bar{A}_{K^{*-}K^{*+}\pi^-} + \bar{A}_{\bar{K}^{*0}K^{*0}\pi^-}|^2,\end{aligned} \tag{33}$$

is found to be nonzero, this would mean that there is $\Delta I = 3/2$ NP. The relative phases of the three amplitudes can be measured in the five-body decay $D^+ \to K^0K^-\pi^0\pi^+\pi^+$ and its CP conjugate. All three resonant decays, $D^+ \to K^{*+}\bar{K}^{*0}\pi^0, D^+ \to K^{*+}K^{*-}\pi^+$ and $D^+ \to K^{*0}\bar{K}^{*0}\pi^+$ are part of this final state. The relative phases between the amplitudes can then be obtained from the overlaps of the resonances in the five body final state phase space.

A somewhat more complicated possibility is represented by the $D \to K\bar{K}^*\pi$ and $D \to K^*\bar{K}\pi$ decays. A test that is similar to (33) can be devised for each of the two sets of decays. If either one of the CP violating differences

$$\begin{aligned}&|\sqrt{2}A_{K^+\bar{K}^{*0}\pi^0} + A_{K^+K^{*-}\pi^+} + A_{K^0\bar{K}^{*0}\pi^+}|^2\\&- |\sqrt{2}\bar{A}_{K^-K^{*0}\pi^0} + \bar{A}_{K^-K^{*+}\pi^-} + \bar{A}_{\bar{K}^0K^{*0}\pi^-}|^2,\end{aligned} \tag{34}$$

and

$$\begin{aligned}&|\sqrt{2}A_{K^{*+}\bar{K}^0\pi^0} + A_{K^{*+}K^-\pi^+} + A_{K^{*0}\bar{K}^0\pi^+}|^2\\&- |\sqrt{2}\bar{A}_{K^{*-}K^0\pi^0} + \bar{A}_{K^{*-}K^+\pi^-} + \bar{A}_{K^{*-}K^+\pi^-}|^2,\end{aligned} \tag{35}$$

is found to be nonzero, this would mean that there is $\Delta I = 3/2$ NP.

In order to experimentally construct (34) or (35), the magnitudes of the amplitudes and their relative phases need to be measured. To determine the relative phase differences a number of four body decays and their CP conjugates need to be measured. The phase difference between $A_{K^{*+}K^-\pi^+}$ and $A_{K^0\bar{K}^{0*}\pi^+}$ can be measured from the decay $D^+ \to K^0K^-\pi^+\pi^+$ (the two amplitudes appear in (35) and (34), respectively). The phase difference between $A_{K^+K^{*-}\pi^+}$ and $A_{K^{0*}\bar{K}^0\pi^+}$ can be measured from the decay $D^+ \to K^+\bar{K}^0\pi^-\pi^+$ (they appear in (34) and (35), respectively). In order to completely fix all of the required phase differences, the decay $D^+ \to K^0\bar{K}^0\pi^0\pi^+$ or the decay $D^+ \to K^+K^-\pi^0\pi^+$ also needs to be measured (as well as the CP conjugated decays of all the above mentioned modes). From the resonance overlaps in the decay $D^+ \to K^0\bar{K}^0\pi^0\pi^+$, the relative phases of $A_{K^{*0}\bar{K}^0\pi^+}$, $A_{K^{*+}\bar{K}^0\pi^0}$ and $A_{K^0\bar{K}^{*0}\pi^+}$ can be obtained, so that (35) is fully determined. Similarly, from the decay $D^+ \to K^+K^-\pi^0\pi^+$ the relative phases of $A_{K^+\bar{K}^{*0}\pi^0}$, $A_{K^{*+}K^-\pi^+}$ and $A_{K^+K^{*-}\pi^+}$ can be obtained so that, (34) is fully determined.

D. D_s decays

It is also possible to search for CP violation in $\Delta I = 3/2$ amplitudes using $D_s^+ \to K^*\pi$ decays. The isospin decomposition is

$$\begin{aligned}A(D_s^+ \to \pi^0 K^{*+}) &= \sqrt{2}\mathcal{A}_3 - \mathcal{A}_1,\\A(D_s^+ \to \pi^+ K^{*0}) &= \mathcal{A}_3 + \sqrt{2}\mathcal{A}_1.\end{aligned} \tag{36}$$

The two decays can be measured from the common Dalitz plot for $D_s^+ \to K_S\pi^+\pi^0$, which has K^{*+} and K^{*0} bands that overlap with the ρ^+ band, while the two K^* bands do no overlap directly. From the Dalitz plot analysis one can deduce the phase difference between the two amplitudes and construct the quantity

$$\sqrt{2}A(D_s^+ \to \pi^0 K^{*+}) + A(D_s^+ \to \pi^+ K^{*0}) = 3\mathcal{A}_3. \tag{37}$$

Direct CP violation in this sum, i.e.,

$$\begin{aligned}&|\sqrt{2}A(D_s^+ \to \pi^0 K^{*+}) + A(D_s^+ \to \pi^+ K^{*0})|^2-\\&|\sqrt{2}A(D_s^- \to \pi^0 K^{*-}) + A(D_s^- \to \pi^- \bar{K}^{*0})|^2 \neq 0,\end{aligned} \tag{38}$$

would necessarily be due to $\Delta I = 3/2$ NP contributions. Additional information on the absolute value of $|A(D_s^+ \to \pi^+ K^{*0})|$ can be obtained from the $D_s^+ \to \pi^+K^+\pi^-$ three body decay.

An analogous test using $D_s \to \rho K^*$ decays also exists, with expressions obtained from the above via the replacement $\pi \to \rho$ and valid for each polarization separately. The relative phase between $A(D_s^+ \to \rho^0 K^{*+})$ and $A(D_s^+ \to \rho^+ K^{*0})$ can be measured from the four body decay $D_s^+ \to \pi^+\pi^-K^+\pi^0$. The absolute magnitude $|A(D_s^+ \to \rho^0 K^{*+})|$ can be obtained from the more easily measured decay $D_s^+ \to \pi^+\pi^-K_S\pi^+$, and can be used as a further constraint.

In most of the manuscript we kept the final states K^0 and $\bar{K}^0$ mesons explicit in the notation. When measurements are performed they will be part of the K_S meson. In checking for the presence of $\Delta I = 3/2$ NP one thus needs to keep track of the CP violation in the neutral kaon system. This effect cannot be neglected as it generates CP asymmetries of order ϵ_K. However, this effect can be taken into account explicitly by appropriately modifying the above sum rule equations and also by correcting for the time dependence efficiency for detecting the K_S [28].

IV. CONCLUSIONS

We have presented a set of isospin sum rules for CP asymmetries in singly Cabibbo suppressed D decays that can be used to test for NP explanations of the measured $\Delta\mathcal{A}_{CP} = \mathcal{A}_{CP}(D \to K^+K^-) - \mathcal{A}_{CP}(D \to \pi^+\pi^-)$ that originate from a $\Delta I = 3/2$ matrix element. The simplest test only requires the measurement of $\mathcal{A}_{CP}(D^+ \to \pi^+\pi^0)$. If this is found to be nonzero then one has discovered NP in the $\Delta I = 3/2$ transition. The same is true if $\mathcal{A}_{CP}(D^+ \to \rho^+\rho^0) \neq 0$ is found. Similar sum rules

involving several $D \to \pi\pi, \rho\rho, \rho\pi, K^{(*)}\bar{K}^{(*)}\pi, K^{(*)}\bar{K}^{(*)}\rho$ or $D_s \to K^*\pi, K^*\rho$ decay amplitudes were also derived. The isospin sum rules (22), (34), (35), (38) only require time integrated measurements, while the isospin sum rules (15), (24) need time dependent measurements. Generically, if this type of NP is responsible for the bulk of the measured $\Delta\mathcal{A}_{CP}$, then violations of the isospin sum rules at the order of $\sim \mathcal{O}(0.5\%)$ can be expected, while the sum rules would be zero in the SM, up to corrections that are second order in isospin breaking.

[1] *et al.* [LHCb Collaboration], arXiv:1112.0938.

[2] A. Di Canto, talk at XXVI Rencontres de Physique de la Vallee d'Aoste Feb 26th-Mar 3rd 2012, La Thuile, Italy; CDF Note 10784.

[3] J. Brod, A. L. Kagan and J. Zupan, arXiv:1111.5000 [hep-ph].

[4] M. Golden, B. Grinstein, Phys. Lett. **B222**, 501 (1989).

[5] B. Bhattacharya, M. Gronau and J. L. Rosner, arXiv:1201.2351 [hep-ph].

[6] D. Pirtskhalava and P. Uttayarat, arXiv:1112.5451 [hep-ph].

[7] T. Feldmann, S. Nandi and A. Soni, arXiv:1202.3795 [hep-ph].

[8] J. Brod, Y. Grossman, A. L. Kagan and J. Zupan, arXiv:1203.6659 [hep-ph].

[9] F. Buccella, M. Lusignoli, G. Miele, A. Pugliese and P. Santorelli, Phys. Rev. D **51** (1995) 3478 [arXiv:hep-ph/9411286].

[10] H. Y. Cheng and C. W. Chiang, arXiv:1201.0785 [hep-ph].

[11] H. n. Li, C. D. Lu and F. S. Yu, arXiv:1203.3120 [hep-ph].

[12] E. Franco, S. Mishima and L. Silvestrini, arXiv:1203.3131 [hep-ph].

[13] Y. Grossman, A. L. Kagan, Y. Nir, Phys. Rev. **D75**, 036008 (2007). [hep-ph/0609178].

[14] G. Isidori, J. F. Kamenik, Z. Ligeti and G. Perez, arXiv:1111.4987 [hep-ph].

[15] K. Wang and G. Zhu, arXiv:1111.5196 [hep-ph].

[16] Y. Hochberg and Y. Nir, arXiv:1112.5268 [hep-ph].

[17] X. Chang, M. -K. Du, C. Liu, J. -S. Lu and S. Yang, arXiv:1201.2565 [hep-ph].

[18] G. F. Giudice, G. Isidori and P. Paradisi, arXiv:1201.6204 [hep-ph].

[19] W. Altmannshofer, R. Primulando, C. -T. Yu and F. Yu, arXiv:1202.2866 [hep-ph].

[20] C. -H. Chen, C. -Q. Geng and W. Wang, arXiv:1202.3300 [hep-ph].

[21] G. Hiller, Y. Hochberg and Y. Nir, arXiv:1204.1046 [hep-ph].

[22] J. Charles, O. Deschamps, S. Descotes-Genon, R. Itoh, H. Lacker, A. Menzel, S. Monteil, V. Niess *et al.*, Phys. Rev. **D84**, 033005 (2011). [arXiv:1106.4041 [hep-ph]]; updates available at `http://ckmfitter.in2p3.fr`

[23] G. Buchalla, A. J. Buras, M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125-1144 (1996). [hep-ph/9512380].

[24] A. J. Bevan, G. Inguglia and B. Meadows, Phys. Rev. D **84**, 114009 (2011) [arXiv:1106.5075 [hep-ph]].

[25] H. J. Lipkin, Y. Nir, H. R. Quinn and A. Snyder, Phys. Rev. D **44** (1991) 1454.

[26] A. F. Falk, Z. Ligeti, Y. Nir and H. Quinn, Phys. Rev. D **69**, 011502 (2004) [hep-ph/0310242].

[27] M. Gaspero, B. Meadows, K. Mishra and A. Soffer, Phys. Rev. D **78**, 014015 (2008) [arXiv:0805.4050 [hep-ph]].

[28] Y. Grossman and Y. Nir, arXiv:1110.3790 [hep-ph].