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# Graviton Corrections to Maxwell's Equations

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## ABSTRACT

We use dimensional regularization to compute the one loop quantum gravitational contribution to the vacuum polarization on flat space background. Adding the appropriate BPHZ counterterm gives a fully renormalized result which we employ to quantum correct Maxwell's equations. These equations are solved to show that dynamical photons are unchanged, provided the free state wave functional is appropriately corrected. The response to the instantaneous appearance of a point dipole reveals a perturbative version of the long-conjectured, “smearing of the light-cone”. There is no change in the far radiation field produced by an alternating dipole. However, the correction to the static electric field of a point charge shows strengthening at short distances, in contrast to expectations based on the renormalization group. We check for gauge dependence by working out the vacuum polarization in a general 3-parameter family of covariant gauges.

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# 1 Introduction

Electromagnetism provided the first example of a relativistic, unified gauge theory. Later on, it was quantum electrodynamics (QED) which produced the first quantitative successes in the struggle to understand interacting quantum field theories. It is therefore natural to wonder what electromagnetism can tell us about quantum gravity.

Efforts along these lines date back more than half a century, and were at first concerned with a phenomenon termed, “smearing of the light-cone” [1]. The idea is that quantum gravitational effects might soften the divergences of other quantum field theories because those divergences are associated with the singularities all propagators develop for null separations [2],

$$i\Delta[g](x; x') = \frac{1}{2\pi^2} \frac{1}{\sigma[g](x; x')} + \mathcal{O}(\ln(\sigma)) . \quad (1)$$

Here  $i\Delta[g](x; x')$  is the scalar propagator in the presence of a general metric background  $g_{\mu\nu}$ , and  $\sigma[g](x; x')$  is  $\frac{1}{2}$  times the square of the geodesic length from  $x^\mu$  to  $x'^\mu$  in that metric. Although the propagator is a well-defined distribution — its 4-dimensional integral against a test function converges — powers of it are not. That is why there is a quadratic ultraviolet divergence in the two loop “setting sun” contribution to the  $\lambda\phi^4$  self-mass-squared depicted in Fig. 1,

$$-iM_{\text{s.s.}}^2(x; x') = \frac{(-i\lambda)^2}{3!} \sqrt{-g(x)} [i\Delta[g](x; x')]^3 \sqrt{-g(x')} . \quad (2)$$

For fixed  $x^\mu$ , the singularity occurs at different points  $x'^\mu$  as the metric  $g_{\mu\nu}$  is varied. Quantizing gravity entails functionally averaging (2) over metrics, and this might be expected to reduce or eliminate the singularity.

Of course smearing the light-cone would distort the propagation of light, and even a tiny angular deviation might show up over cosmological distances. Metric fluctuations can also induce luminosity and redshift variations, as well as spectral line broadening and angular blurring. These effects have typically been studied indirectly, by working out the scattering of light by a classical metric [3]. A significant example is the computation Sachs and Wolfe made of the anisotropies cosmic microwave photons acquire in propagating through primordial cosmological perturbations [4]. More recent treatments have been given of the effects of various ensembles of gravitons [5]. (It is but a short step to adjust the distribution to

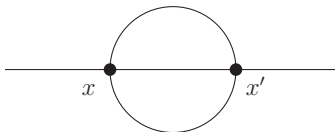


Figure 1: A two loop contribution to the self-mass-squared in  $\lambda\phi^4$  theory.

reproduce quantum 0-point fluctuations, but this does not seem to have been done.) And there has been much recent interest in metric fluctuations engendered by quantum oscillations of matter fields [6], a representative diagram for which is given in Fig. 2.

Quantum gravity also affects electrodynamic forces. Radkowski seems to have performed the first computation of the one loop correction to the Coulomb potential of a point charge [7]. A somewhat different result was inferred from the scattering of charged scalars [8]. Both of those studies found that quantum gravity strengthens the electrostatic force at short distances. Much recent interest attended the proposal of Robinson and Wilczek that the effect goes the other way, based on renormalization group flows [9].

A widely perceived impediment to all of these studies is the fact that general relativity plus electromagnetism is not perturbatively renormalizable, even at one loop order [10]. One consequence is that we cannot compute everything reliably, but that does not mean we are unable to calculate anything. One must simply view quantum gravity as a low energy effective field theory whose divergences are absorbed using BPHZ (Bogoliubov, Parsiuk, Hepp and Zimmermann) counterterms [11]. In this context, loops of massless gravitons and photons engender nonlocal and ultraviolet finite contributions to the effective action which are unique predictions of the theory that cannot be changed by its ultraviolet completion [12]. This is why Bloch and Nordsieck were able to resolve the infrared problem of QED [13], long before that theory's renormalizability was understood. It is also why Weinberg was able to derive a similar resolution for the infrared problem of quantum gravity [14], and why Feinberg and Sucher were able to use Fermi theory to compute the long range force engendered by the exchange of massless neutrinos [15].

One is of course free to criticize the study of quantum gravity plus electromagnetism on the grounds that the predicted effects are too small to be observable. Our own interest in the subject derives from its relevance as the flat space correspondence limit of the regime of primordial inflation, during which quantum gravitational effects are not unobservably small. Indeed, the gravitational response to quantum fluctuations of matter [16] has been resolved [17], and the corresponding fluctuations of gravitational radiation [18] may soon be detected [19].

Whether during primordial inflation or on flat space background, the proper vehicle for studying quantum distortions of electrodynamics is the quantum-corrected Maxwell equation. One gets this by first computing the “vacuum polarization”  $i[\mu\Pi^\nu](x;x')$ , which is the one-particle-irreducible (1PI) 2-point function for the photon. This is then used to quantum-



Figure 2: Lowest order gravitational effect on light due to quantum fluctuations of matter. Photon lines are wavy, graviton lines are winding, and matter lines are solid.

correct Maxwell's equation,

$$\partial_\nu \left[ \sqrt{-g} g^{\nu\rho} g^{\mu\sigma} F_{\rho\sigma}(x) \right] + \int d^4x' \left[ {}^\mu\Pi^\nu \right](x; x') A_\nu(x') = J^\mu(x) . \quad (3)$$

This framework has been employed to infer the effects of inflationary charged scalar production on photons [20], and on electrodynamic forces [21]. The purpose of this paper is to facilitate a similar study of the effects of inflationary graviton production by first working out the flat space correspondence limit. An example of the utility of this exercise is the recent examination of the effects of inflationary scalars on gravitons [22], for which the flat space limit [23] provided crucial guidance in dealing with the vastly more complicated graviton self-energy [24] that pertains during primordial inflation.

The quantum gravitational contribution to the one loop vacuum polarization is derived in section 2. Section 3 solves the quantum-corrected Maxwell equation (3) for photons, for the instantaneous creation of a point dipole, for an alternating point dipole, and for a static point charge. The issue of gauge dependence is discussed in section 4, and our conclusions comprise section 5.

## 2 One Loop Vacuum Polarization

The purpose of this section is to compute the renormalized, one loop contribution to the vacuum polarization from quantum gravity on flat space background. We begin by presenting the necessary Feynman rules. Then we use them to compute the dimensionally regulated result. By a process of successive partial integrations this is expressed as a divergent, local term — which is canceled by a BPHZ counterterm — plus the finite, nonlocal contribution which constitutes the renormalized result.

### 2.1 Feynman Rules

Our total Lagrangian contains three parts,

$$\mathcal{L} = \mathcal{L}_{\text{GR}} + \mathcal{L}_{\text{EM}} + \mathcal{L}_{\text{BPHZ}} . \quad (4)$$

These are, respectively, the Lagrangians of general relativity, electromagnetism and the BPHZ counterterm required for this computation,

$$\mathcal{L}_{\text{GR}} = \frac{1}{16\pi G} R \sqrt{-g} , \quad (5)$$

$$\mathcal{L}_{\text{EM}} = -\frac{1}{4} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma} \sqrt{-g} , \quad (6)$$

$$\mathcal{L}_{\text{BPHZ}} = C_4 D_\alpha F_{\mu\nu} D_\beta F_{\rho\sigma} g^{\alpha\beta} g^{\mu\rho} g^{\nu\sigma} \sqrt{-g} . \quad (7)$$

We employ a  $D$ -dimensional, spacelike metric  $g_{\mu\nu}$ , with inverse  $g^{\mu\nu}$  and determinant  $g = \det(g_{\mu\nu})$ . Our affine connection and Riemann tensor are,

$$\Gamma^\rho_{\mu\nu} \equiv \frac{1}{2}g^{\rho\sigma}[\partial_\nu g_{\sigma\mu} + \partial_\mu g_{\nu\sigma} - \partial_\sigma g_{\mu\nu}] , \quad (8)$$

$$R^\rho_{\sigma\mu\nu} \equiv \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\alpha} \Gamma^\alpha_{\nu\sigma} - \Gamma^\rho_{\nu\alpha} \Gamma^\alpha_{\mu\sigma} . \quad (9)$$

Our Ricci tensor is  $R_{\mu\nu} \equiv R^\rho_{\mu\rho\nu}$  and the associated Ricci scalar is  $R \equiv g^{\mu\nu} R_{\mu\nu}$ . The electromagnetic field strength tensor and its first covariant derivative are,

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu , \quad (10)$$

$$D_\alpha F_{\mu\nu} \equiv \partial_\alpha F_{\mu\nu} - \Gamma^\gamma_{\alpha\mu} F_{\gamma\nu} - \Gamma^\gamma_{\alpha\nu} F_{\mu\gamma} . \quad (11)$$

We define the graviton field  $h_{\mu\nu}(x)$  as the difference between the full metric and its Minkowski background value  $\eta_{\mu\nu}$ ,

$$g_{\mu\nu}(x) \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu}(x) , \quad (12)$$

where  $\kappa^2 \equiv 16\pi G$  is the loop counting parameter of quantum gravity. We follow the usual conventions whereby a comma denotes ordinary differentiation, the trace of the graviton field is  $h \equiv \eta^{\mu\nu} h_{\mu\nu}$ , and graviton indices are raised and lowered using the Minkowski metric,  $h^\mu_\nu \equiv \eta^{\mu\rho} h_{\rho\nu}$  and  $h^{\mu\nu} \equiv \eta^{\mu\rho} \eta^{\nu\sigma} h_{\rho\sigma}$ . After extracting a surface term the gravitational Lagrangian can be written as,

$$\begin{aligned} \mathcal{L}_{\text{GR}} - \text{Surface} &= \sqrt{-g} g^{\alpha\beta} g^{\rho\sigma} g^{\mu\nu} \\ &\times \left\{ \frac{1}{2} h_{\alpha\rho,\mu} h_{\nu\sigma,\beta} - \frac{1}{2} h_{\alpha\beta,\rho} h_{\sigma\mu,\nu} + \frac{1}{4} h_{\alpha\beta,\rho} h_{\mu\nu,\sigma} - \frac{1}{4} h_{\alpha\rho,\mu} h_{\beta\sigma,\nu} \right\} . \end{aligned} \quad (13)$$

The quadratic part of the invariant Lagrangian is,

$$\mathcal{L}_{\text{GR}}^{(2)} = \frac{1}{2} h^{\rho\sigma,\mu} h_{\mu\sigma,\rho} - \frac{1}{2} h^{\mu\nu}_{,\mu} h_{,\nu} + \frac{1}{4} h^{,\mu} h_{,\mu} - \frac{1}{4} h^{\rho\sigma,\mu} h_{\rho\sigma,\mu} . \quad (14)$$

We fix the gauge by adding,

$$\mathcal{L}_{\text{GRfix}} = -\frac{1}{2} \eta^{\mu\nu} F_\mu F_\nu \quad , \quad F_\mu \equiv \eta^{\rho\sigma} \left( h_{\mu\rho,\sigma} - \frac{1}{2} h_{\rho\sigma,\mu} \right) . \quad (15)$$

The resulting graviton propagator can be expressed in terms of the massless scalar propagator  $i\Delta(x; x')$ ,

$$i[\Delta_{\mu\nu} \Delta_{\rho\sigma}](x; x') = \left[ 2\eta_{\mu(\rho} \eta_{\sigma)\nu} - \frac{2}{D-2} \eta_{\mu\nu} \eta_{\rho\sigma} \right] i\Delta(x; x') . \quad (16)$$

The spacetime dependence of the scalar propagator derives from the Lorentz interval  $\Delta x^2(x; x')$ ,

$$\Delta x^2(x; x') \equiv \|\vec{x} - \vec{x}'\|^2 - (|t - t'| - i\varepsilon)^2 \implies i\Delta(x; x') = \frac{\Gamma(\frac{D}{2} - 1)}{4\pi^{\frac{D}{2}}} \left( \frac{1}{\Delta x^2} \right)^{\frac{D}{2} - 1} . \quad (17)$$

The quadratic part of the electromagnetic action is,

$$\mathcal{L}_{\text{EM}} = -\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu + \frac{1}{2}(\partial_\mu A^\mu)^2. \quad (18)$$

We fix the gauge by adding,

$$\mathcal{L}_{\text{EMfix}} = -\frac{1}{2}(\partial_\mu A^\mu)^2. \quad (19)$$

The associated photon propagator is,

$$i[\mu\Delta_\rho](x; x') = \eta_{\mu\rho} i\Delta(x; x'). \quad (20)$$

Electromagnetic interaction vertices descend from the second variational derivative of the action,

$$\frac{\delta^2 S_{\text{EM}}}{\delta A_\mu(x) \delta A_\rho(x')} = \partial_\kappa \left\{ \sqrt{-g(x)} [g^{\kappa\lambda}(x) g^{\mu\rho}(x) - g^{\kappa\rho}(x) g^{\lambda\mu}(x)] \partial_\lambda \delta^D(x - x') \right\}. \quad (21)$$

The necessary vertex functions are obtained by expanding the metric factors,

$$\begin{aligned} \sqrt{-g} (g^{\kappa\lambda} g^{\mu\rho} - g^{\kappa\rho} g^{\lambda\mu}) &\equiv \eta^{\kappa\lambda} \eta^{\mu\rho} - \eta^{\kappa\rho} \eta^{\lambda\mu} \\ &+ \kappa V^{\mu\rho\kappa\lambda\alpha\beta} h_{\alpha\beta} + \kappa^2 U^{\mu\rho\kappa\lambda\alpha\beta\gamma\delta} h_{\alpha\beta} h_{\gamma\delta} + O(\kappa^3). \end{aligned} \quad (22)$$

The 3-point and 4-point vertices are,

$$V^{\mu\rho\kappa\lambda\alpha\beta} = \eta^{\alpha\beta} \eta^{\kappa[\lambda} \eta^{\rho]\mu} + 4\eta^{\alpha[\mu} \eta^{\kappa][\rho} \eta^{\lambda]}(\beta), \quad (23)$$

$$\begin{aligned} U^{\mu\rho\kappa\lambda\alpha\beta\gamma\delta} &= \left[ \frac{1}{4} \eta^{\alpha\beta} \eta^{\gamma\delta} - \frac{1}{2} \eta^{\alpha(\gamma} \eta^{\delta)\beta} \right] \eta^{\kappa[\lambda} \eta^{\rho]\mu} + \eta^{\alpha\beta} \eta^{\gamma(\mu} \eta^{\delta)[\rho} \eta^{\lambda]}(\gamma) \\ &+ \eta^{\gamma\delta} \eta^{\alpha[\mu} \eta^{\kappa][\rho} \eta^{\lambda]}(\beta) + \eta^{\kappa(\alpha} \eta^{\beta)[\lambda} \eta^{\rho](\gamma} \eta^{\delta)\mu} + \eta^{\kappa(\gamma} \eta^{\delta)[\lambda} \eta^{\rho](\alpha} \eta^{\beta)\mu} \\ &+ \eta^{\kappa(\gamma} \eta^{\delta)(\alpha} \eta^{\beta)[\lambda} \eta^{\rho]\mu} + \eta^{\kappa[\lambda} \eta^{\rho](\alpha} \eta^{\beta)(\gamma} \eta^{\delta)\mu} + \eta^{\kappa[\lambda} \eta^{\rho](\gamma} \eta^{\delta)(\alpha} \eta^{\beta)\mu}. \end{aligned} \quad (24)$$

Note that parenthesized indices are symmetrized, whereas indices enclosed in square brackets are antisymmetrized.

## 2.2 Dimensionally Regulated Result

The three one loop diagrams which contribute to the vacuum polarization are depicted in Fig. 3. They can each be expressed using the notation of the previous section. The left hand diagram is,

$$i[\mu\Pi_{3\text{pt}}^\nu](x; x') = (i\kappa)^2 \partial_\kappa \partial'_\theta \left\{ V^{\mu\rho\kappa\lambda\alpha\beta} i[\alpha\beta\Delta_{\gamma\delta}](x; x') V^{\nu\sigma\phi\theta\gamma\delta} \partial_\lambda \partial'_\phi i[\rho\Delta_\sigma](x; x') \right\}. \quad (25)$$

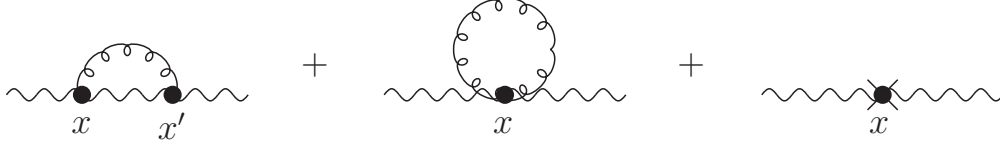


Figure 3: Graviton contributions to the one loop vacuum polarization.

Substituting expressions (16), (20) and (23), acting the inner derivatives and performing the inner contractions gives,

$$i[\mu\Pi_{3\text{pt}}^\nu](x; x') = (i\kappa)^2 \frac{\Gamma^2(\frac{D}{2}-1)}{16\pi^D} \times -(D-3)D\partial_\rho\partial_\sigma \left\{ \frac{2[\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma}]}{\Delta x^{2D-2}} \right. \\ \left. + \frac{D[\Delta x^\mu\Delta x^\rho\eta^{\nu\sigma} - \Delta x^\rho\Delta x^\sigma\eta^{\mu\nu}]}{\Delta x^{2D}} - \frac{D[\Delta x^\mu\Delta x^\nu\eta^{\rho\sigma} - \Delta x^\rho\Delta x^\sigma\eta^{\mu\nu}]}{\Delta x^{2D}} \right\}. \quad (26)$$

The next step is to act the outer derivatives, at which point we can extract a manifestly transverse form,

$$i[\mu\Pi_{3\text{pt}}^\nu](x; x') \\ = -\frac{\kappa^2\Gamma^2(\frac{D}{2}-1)}{16\pi^D} (D-3)(D-2)^2 D \left\{ \frac{(D+1)\eta^{\mu\nu}}{\Delta x^{2D}} - \frac{2D\Delta x^\mu\Delta x^\nu}{\Delta x^{2D+2}} \right\}, \quad (27)$$

$$= -\frac{\kappa^2\Gamma^2(\frac{D}{2}-1)}{16\pi^D} \frac{(D-3)(D-2)^2 D}{2(D-1)} [\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu] \frac{1}{\Delta x^{2D-2}}. \quad (28)$$

The middle diagram of Fig. 1 is,

$$i[\mu\Pi_{4\text{pt}}^\nu](x; x') = i\kappa^2\partial_\kappa \left\{ U^{\mu\nu\kappa\lambda\alpha\beta\gamma\delta} i[\alpha_\beta\Delta_{\gamma\delta}](x; x) \partial_\lambda \delta^D(x-x') \right\}. \quad (29)$$

This diagram vanishes because the coincidence limit of the massless scalar propagator in flat space is zero in dimensional regularization,  $i\Delta(x; x) = 0$ . The diagram on the right of Fig. 1 is,

$$i[\mu\Pi_{\text{ctm}}^\nu](x; x') = i4C_4 (\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu) \partial^2 \delta^D(x-x'). \quad (30)$$

## 2.3 Renormalization

To renormalize (28) we must first localize the ultraviolet divergence so that it can be subtracted by the counterterm (30). This process of localization is accomplished by first partially integrating the factor of  $1/\Delta x^{2D-2}$  in (28) until the remainder is integrable [25]. In dimensional regularization the steps are [26],

$$\frac{1}{\Delta x^{2D-2}} = \frac{\partial^2}{2(D-2)^2} \frac{1}{\Delta x^{2D-4}} = \frac{\partial^4}{4(D-2)^2(D-3)(D-4)} \frac{1}{\Delta x^{2D-6}}. \quad (31)$$



Next we add zero in the form [26],

$$\partial^2 \frac{1}{\Delta x^{D-2}} = \frac{i4\pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2}-1)} \delta^D(x-x') . \quad (32)$$

Adding (32) to the key part of (31) in a dimensionally consistent way gives,

$$\begin{aligned} & \frac{\partial^2}{D-4} \left\{ \frac{1}{\Delta x^{2D-6}} \right\} \\ &= \frac{i4\pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2}-1)} \frac{\mu^{D-4} \delta^D(x-x')}{D-4} + \frac{\partial^2}{D-4} \left\{ \frac{1}{\Delta x^{2D-6}} - \frac{\mu^{D-4}}{\Delta x^{D-2}} \right\} , \end{aligned} \quad (33)$$

$$= \frac{i4\pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2}-1)} \frac{\mu^{D-4} \delta^D(x-x')}{D-4} - \frac{\partial^2}{2} \left\{ \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right\} + O(D-4) . \quad (34)$$

Substituting (31) and (34) into (28) results in the desired localized divergence,

$$\begin{aligned} i[\mu \Pi_{3\text{pt}}^\nu](x; x') &= -\frac{i\kappa^2 \Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}}} \frac{D}{8(D-1)(D-4)} [\eta^{\mu\nu} \partial^2 - \partial^\mu \partial^\nu] \partial^2 \delta^D(x-x') \\ &\quad + \frac{\kappa^2}{192\pi^4} [\eta^{\mu\nu} \partial^2 - \partial^\mu \partial^\nu] \partial^4 \left\{ \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right\} + O(D-4) . \end{aligned} \quad (35)$$

The local divergence of expression (35) will be completely canceled by the counterterm (30) if we make the choice,

$$C_4 = \frac{\kappa^2 \Gamma(\frac{D}{2}-1)}{16\pi^{\frac{D}{2}}} \frac{D}{8(D-1)(D-4)} . \quad (36)$$

We can then take the unregulated limit ( $D \rightarrow 4$ ) to obtain the fully renormalized graviton contribution to the one loop vacuum polarization,

$$[\mu \Pi_{\text{ren}}^\nu](x; x') = -\frac{i\kappa^2}{192\pi^4} [\eta^{\mu\nu} \partial^2 - \partial^\mu \partial^\nu] \partial^4 \left\{ \frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right\} . \quad (37)$$

Note that the ambiguity regarding the finite part of the counterterm is reflected in the dimensional regularization scale  $\mu$ .

### 3 Quantum Corrected Maxwell Equations

The purpose of this section is to use our one result (37) for the one loop vacuum polarization to quantum correct Maxwell's equations, and then infer quantum gravitational corrections to electrodynamics by solving these equations. We begin by deriving the causal effective field equations of the Schwinger-Keldysh formalism. Subsequent subsections solve these equations perturbatively for the special cases of free photons, a point dipole pulse, an alternating point dipole, and a static point charge.

### 3.1 Schwinger-Keldysh Formalism

We come now to the question of what to use for the vacuum polarization  $[\mu\Pi^\nu](x; x')$  in the quantum corrected Maxwell equation (3). It might seem obvious that the in-out result (37) we have just derived should be used, but that would lead to two problems:

- *Causality* — The in-out vacuum polarization (37) is nonzero for points  $x'^\mu$  which lie in the future of  $x^\mu$ , or at spacelike separation from it; and
- *Reality* — The in-out vacuum polarization (37) is not real.

One can get the right result for a static potential by simply ignoring the imaginary part [7, 8], but circumventing the limitations of the in-out formalism becomes more and more difficult as time dependent sources and higher order corrections are included, and these techniques break down entirely for the case of cosmology in which there may not even be asymptotic vacua. Note that there is nothing wrong with the in-out vacuum polarization (37); it is exactly the right thing to correct the photon propagator for asymptotic scattering computations in flat space. The point is rather that employing (37) in equation (3) fails to provide a set of field equations with the same scope and power as the classical Maxwell's equations.

The more appropriate field equations are those of the Schwinger-Keldysh formalism. This technique provides a way of computing true expectation values that is almost as simple as the Feynman diagrams which produce in-out matrix elements [27]. We shall develop the Schwinger-Keldysh rules in the context of a scalar field  $\varphi(x)$  whose Lagrangian (the space integral of its Lagrangian density) at time  $t$  is  $L[\varphi(t)]$ . Suppose we are given a Heisenberg state  $|\Psi\rangle$  whose wave functional in terms of the operator eigenkets at time  $t_0$  is  $\Psi[\varphi(t_0)]$ , and we wish to take the expectation value, in the presence of this state, of a product of two functionals of the field operator:  $A[\varphi]$ , which is anti-time-ordered, and  $B[\varphi]$ , which is time-ordered. The Schwinger-Keldysh functional integral for this is [28],

$$\begin{aligned} \langle \Psi | A[\varphi] B[\varphi] | \Psi \rangle &= \int [d\varphi_+] [d\varphi_-] \delta[\varphi_-(t_1) - \varphi_+(t_1)] \\ &\times A[\varphi_-] B[\varphi_+] \Psi^*[\varphi_-(t_0)] e^{i \int_{t_0}^{t_1} dt \{L[\varphi_+(t)] - L[\varphi_-(t)]\}} \Psi[\varphi_+(t_0)] . \end{aligned} \quad (38)$$

The time  $t_1 > t_0$  is arbitrary as long as no operator in either  $A[\varphi]$  or  $B[\varphi]$  is evaluated at a later time.

The Schwinger-Keldysh rules can be read off from its functional representation (38). Because the same field operator is represented by two different dummy functional variables,  $\varphi_\pm(x)$ , the endpoints of lines carry a  $\pm$  polarity. External lines associated with the anti-time-ordered operator  $A[\varphi]$  have the  $-$  polarity whereas those associated with the time-ordered operator  $B[\varphi]$  have the  $+$  polarity. Interaction vertices are either all  $+$  or all  $-$ . Vertices with  $+$  polarity are the same as in the usual Feynman rules whereas vertices with the  $-$

polarity have an additional minus sign. If the state  $|\Psi\rangle$  is something other than free vacuum then it contributes additional interaction vertices on the initial value surface [29].

Propagators can be  $++$ ,  $+-$ ,  $-+$ , or  $--$ . All four polarity variations can be read off from the fundamental relation (38) when the free Lagrangian is substituted for the full one. It is useful to denote canonical expectation values in the free theory with a subscript 0. With this convention we see that the  $++$  propagator is just the ordinary Feynman propagator,

$$i\Delta_{++}(x; x') = \langle \Omega | T(\varphi(x)\varphi(x')) | \Omega \rangle_0 = i\Delta(x; x'), \quad (39)$$

where  $T$  stands for time-ordering and  $\bar{T}$  denotes anti-time-ordering. The other polarity variations are simple to read off and to relate to the Feynman propagator,

$$i\Delta_{-+}(x; x') = \langle \Omega | \varphi(x)\varphi(x') | \Omega \rangle_0 = \theta(t-t')i\Delta(x; x') + \theta(t'-t)[i\Delta(x; x')]^*, \quad (40)$$

$$i\Delta_{+-}(x; x') = \langle \Omega | \varphi(x')\varphi(x) | \Omega \rangle_0 = \theta(t-t')[i\Delta(x; x')]^* + \theta(t'-t)i\Delta(x; x'), \quad (41)$$

$$i\Delta_{--}(x; x') = \langle \Omega | \bar{T}(\varphi(x)\varphi(x')) | \Omega \rangle_0 = [i\Delta(x; x')]^*. \quad (42)$$

In our case, both the photon and the graviton propagators depend upon the massless scalar propagator (17), which is a function of the Lorentz interval  $\Delta x^2(x; x')$ . It follows from relations (40-42) that the various Schwinger-Keldysh propagators can be obtained by making simple replacements for the Lorentz interval,

$$\Delta x_{++}^2(x; x') \equiv \|\vec{x} - \vec{x}'\|^2 - c^2(|t - t'| - i\epsilon)^2, \quad (43)$$

$$\Delta x_{+-}^2(x; x') \equiv \|\vec{x} - \vec{x}'\|^2 - c^2(t - t' + i\epsilon)^2, \quad (44)$$

$$\Delta x_{-+}^2(x; x') \equiv \|\vec{x} - \vec{x}'\|^2 - c^2(t - t' - i\epsilon)^2, \quad (45)$$

$$\Delta x_{--}^2(x; x') \equiv \|\vec{x} - \vec{x}'\|^2 - c^2(|t - t'| + i\epsilon)^2. \quad (46)$$

Because each external line can be either  $+$  or  $-$  in the Schwinger-Keldysh formalism, every 1PI N-point function of the in-out formalism gives rise to  $2^N$  1PI N-point functions in the Schwinger-Keldysh formalism. For every classical field  $\phi(x)$  of an in-out effective action, the corresponding Schwinger-Keldysh effective action must depend upon two fields — call them  $\phi_+(x)$  and  $\phi_-(x)$  — in order to access the appropriate 1PI function [30]. For the scalar paradigm we have been considering the 1PI 2-point function is the scalar self-mass-squared,  $M_{\pm\pm}^2(x; x')$ , and the effective action takes the form,

$$\begin{aligned} \Gamma[\phi_+, \phi_-] &= S[\phi_+] - S[\phi_-] - \frac{1}{2} \int d^4x \int d^4x' \\ &\times \left\{ \phi_+(x) M_{++}^2(x; x') \phi_+(x') + \phi_+(x) M_{+-}^2(x; x') \phi_-(x') \right. \\ &\left. + \phi_-(x) M_{-+}^2(x; x') \phi_+(x') + \phi_-(x) M_{--}^2(x; x') \phi_-(x') \right\} + O(\phi_{\pm}^3), \end{aligned} \quad (47)$$

where  $S$  is the classical action. The effective field equations are obtained by varying with respect to  $\phi_+$  and then setting both fields equal [30],

$$\left. \frac{\delta \Gamma[\phi_+, \phi_-]}{\delta \phi_+(x)} \right|_{\phi_{\pm}=\phi} = [\partial^2 - m^2]\phi(x) - \int d^4x' [M_{++}^2(x; x') + M_{+-}^2(x; x')]\phi(x') + O(\phi^2). \quad (48)$$

The two 1PI 2-point functions we would need to quantum correct the linearized scalar field equation are  $M_{++}^2(x; x')$  and  $M_{+-}^2(x; x')$ . Their sum in (48) gives effective field equations which are causal in the sense that the two 1PI functions cancel unless  $x'^\mu$  lies on or within the past light-cone of  $x^\mu$ . Their sum is also real, which neither 1PI function is separately.

From the preceding discussion it is apparent that we wish to make the following substitution in equation (3),

$$[\mu\Pi^\nu](x; x') \longrightarrow [\mu\Pi_+^\nu](x; x') + [\mu\Pi_-^\nu](x; x') , \quad (49)$$

where we can read off the appropriate Schwinger-Keldysh vacuum polarization from expression (37),

$$[\mu\Pi_\pm^\nu](x; x') = -\frac{(\pm)(\pm)i\kappa^2}{192\pi^4} [\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu] \partial^4 \left\{ \frac{\ln(\mu^2\Delta x_{\pm\pm}^2)}{\Delta x_{\pm\pm}^2} \right\} , \quad (50)$$

$$= -\frac{(\pm)(\pm)i\kappa^2}{1536\pi^4} [\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu] \partial^6 \left\{ \ln^2(\mu^2\Delta x_{\pm\pm}^2) - 2\ln(\mu^2\Delta x_{\pm\pm}^2) \right\} . \quad (51)$$

Now define the temporal and spatial intervals as,

$$\Delta t \equiv t - t' , \quad \Delta r \equiv \|\vec{x} - \vec{x}'\| . \quad (52)$$

It is apparent from expressions (43-44) that differences of logarithms of the the ++ and +- intervals give,

$$\ln(\mu^2\Delta x_{++}^2) - \ln(\mu^2\Delta x_{+-}^2) = 2\pi i\theta(\Delta t - \Delta r) , \quad (53)$$

$$\ln^2(\mu^2\Delta x_{++}^2) - \ln^2(\mu^2\Delta x_{+-}^2) = 4\pi i\theta(\Delta t - \Delta r) \ln[\mu^2(\Delta t^2 - \Delta r^2)] . \quad (54)$$

Hence the vacuum polarization which belongs in equation (3) is,

$$\begin{aligned} [\mu\Pi^\nu](x; x') &= \frac{\kappa^2}{384\pi^3} [\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu] \\ &\quad \times \partial^6 \left\{ \theta(\Delta t - \Delta r) \left[ \ln[\mu^2\Delta t^2 - \Delta r^2] - 1 \right] \right\} + \mathcal{O}(\kappa^4) . \end{aligned} \quad (55)$$

## 3.2 Photons

Expression (55) gives all the one loop contributions which derive exclusively from interaction vertices, but there are also contributions from perturbative corrections to the initial state wave functionals. In the scalar functional integral (38) these wave functionals are  $\Psi[\varphi_+(t_0)]$  and  $\Psi^*[\varphi_-(t_0)]$ ; for gravity plus electromagnetism they would be functionals of  $A_\mu$  and  $h_{\mu\nu}$ , evaluated at the initial time.

Each state wave functional can be expressed as the wave functional of free vacuum times a series of perturbative corrections,

$$\Psi[A, h] = \Psi_0[A, h] \times \left\{ 1 + \mathcal{O}(\kappa h A^2) \right\} . \quad (56)$$

It is straightforward to show that the free vacuum contribution is what fixes the real part of the propagator in the functional formalism [31]. If there were no perturbative state corrections then merely employing the correct propagators would completely account for the state wave functionals. However, there must be perturbative state corrections because free vacuum cannot be the true vacuum state of an interacting quantum field theory.

Perturbative state corrections manifest as new interactions on the initial value surface [28]. When the initial value surface is in the asymptotic past (or the asymptotic past and future for in-out matrix elements) these interactions have no effect on operators at finite times. However, they can be important when the initial value surface is at a finite time, as it must be in cosmology. The first correction relevant for a massless, minimally coupled  $\lambda\phi^4$  theory has recently been worked out on de Sitter background [29]. In this case the initial state correction is necessary to make the linearized effective field equation well defined at the initial time [32], and to eliminate an infinite series of rapidly redshifting terms from the two loop expectation value of the stress tensor [26].

We shall assume that the missing state corrections exactly cancel the surface terms which arise when (55) is partially integrated. To see what this entails, first note that all orders of the “pure-vertex” part of the vacuum polarization take the manifestly transverse form,

$$\left[{}^\mu\Pi^\nu\right](x;x') = \left(\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu\right)\Pi(x-x') . \quad (57)$$

The partial integration we have in mind concerns the quantum correction to Maxwell’s equation,

$$\begin{aligned} \int d^4x' \left(\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu\right)\Pi(x-x')A_\nu(x') \\ = \int d^4x' \Pi(x-x')\partial'_\nu F^{\nu\mu}(x') + \text{Surface Terms} . \end{aligned} \quad (58)$$

In the Schwinger-Keldysh formalism the  $++$  and  $+-$  contributions exactly cancel on the future temporal surface, as well as on the surface at spatial infinity. Hence the only surface terms come from the initial time. Of course this is also true of perturbative state corrections. We assume that the two contributions exactly cancel, so that the full, quantum-corrected Maxwell equation is,

$$\partial^\nu F_{\nu\mu}(x) + \int d^4x' \Pi(x-x')\partial^\nu F_{\nu\mu}(x') = J_\mu(x) . \quad (59)$$

We are finally ready to consider the case of free photons, which corresponds to  $J_\mu(x) = 0$ . Note from equation (59) that these obey  $\partial^\nu F_{\nu\mu}(x) = 0$ , the same as in the classical theory. One might worry about the potential for solutions of the form  $\partial^\nu F_{\nu\mu}(x) = S_\mu(x)$ , where  $S_\mu(x)$  obeys the integral equation,

$$S_\mu(x) + \int d^4x' \Pi(x-x')S_\mu(x') = 0 . \quad (60)$$

However, an effective field equation such as (59) can only be used to perturbatively correct classical solutions [33], which means we must exclude any such solutions. Hence we conclude that *quantum gravity on flat space background makes no correction to free photons at any order*, except for possible field strength renormalization.

### 3.3 Instantaneously Creating A Point Dipole

The charge density of a static point electric dipole  $\vec{p}$  at the origin is  $\rho = -\vec{p} \cdot \vec{\nabla} \delta^3(\vec{x})$ . We might imagine creating such a dipole at the instant  $t = 0$  by separating the charges in a very small, neutral particle such as a neutron. The conserved 4-current associated with this event is,

$$J^0(t, \vec{x}) = -\theta(t) \vec{p} \cdot \vec{\nabla} \delta^3(\vec{x}) \quad , \quad J^i(t, \vec{x}) = p^i \delta(t) \delta^3(\vec{x}) . \quad (61)$$

The response of the magnetic field provides a good perturbative illustration of the smearing of the light-cone which was conjectured so long ago [1].

Before proceeding it is desirable to reorganize equation (59) in two ways. The first has to do with the limitation inherent in only possessing the first order term in the loop expansion of  $\Pi(x - x')$ ,

$$\Pi(x - x') = \Pi^{(1)}(x - x') + \Pi^{(2)}(x - x') + \mathcal{O}(\kappa^6) . \quad (62)$$

Of course this means we can only infer the one loop correction to the field strength, so we may as well expand it,

$$F_{\mu\nu}(x) = F_{\mu\nu}^{(0)}(x) + F_{\mu\nu}^{(1)}(x) + F_{\mu\nu}^{(2)}(x) + \mathcal{O}(\kappa^6) . \quad (63)$$

Substituting (62) and (63) in the quantum-corrected Maxwell equation (59) and segregating different orders of  $\kappa^2$  produces the hierarchy,

$$\partial^\nu F_{\nu\mu}^{(0)}(x) = J_\mu(x) , \quad (64)$$

$$\partial^\nu F_{\nu\mu}^{(1)}(x) = - \int d^4 x' \Pi(x - x') J_\mu(x') \equiv J_\mu^{(1)}(x) , \quad (65)$$

and so on. Note the classical source  $J_\mu(x)$  is 0th order.

The second reorganization concerns deriving the field strength directly, without constructing the vector potential. Consider taking the curl of the classical Maxwell equation,

$$\epsilon^{\rho\sigma\mu\nu} \partial_\mu \partial^\alpha F_{\alpha\nu} = \partial^2 \epsilon^{\rho\sigma\mu\nu} \partial_\mu A_\nu = \epsilon^{\rho\sigma\mu\nu} \partial_\mu J_\nu \implies \partial^2 F_{\mu\nu} = \partial_\mu J_\nu - \partial_\nu J_\mu . \quad (66)$$

Combining this with (64-65) implies,

$$\partial^2 F_{\mu\nu}^{(0)} = \partial_\mu J_\nu - \partial_\nu J_\mu , \quad (67)$$

$$\partial^2 F_{\mu\nu}^{(1)} = \partial_\mu J_\nu^{(1)} - \partial_\nu J_\mu^{(1)} . \quad (68)$$

Now recall that our one loop current density can be expressed as the d'Alembertian of something,

$$J_\mu^{(1)}(x) \equiv - \int d^4 x' \Pi^{(1)}(x - x') J_\mu(x') , \quad (69)$$

$$= \frac{i\kappa^2 \partial^4}{192\pi^4} \int d^4 x' \left\{ \frac{\ln(\mu^2 \Delta x_{++}^2)}{\Delta x_{++}^2} - \frac{\ln(\mu^2 \Delta x_{+-}^2)}{\Delta x_{+-}^2} \right\} J_\mu(x') , \quad (70)$$

$$= \frac{\kappa^2 \partial^6}{384\pi^3} \int d^4 x' \theta(\Delta t - \Delta r) \left\{ \ln[\mu^2 (\Delta t^2 - \Delta r^2)] - 1 \right\} J_\mu(x') . \quad (71)$$

Comparison of (68) with (70) or (71) implies a result for the one loop field strength, up to possible homogeneous terms,

$$F_{\mu\nu}^{(1)}(x) = \frac{i\kappa^2\partial^2}{192\pi^4} 2\partial_{[\mu} \int d^4x' \left\{ \frac{\ln(\mu^2\Delta x_{++}^2)}{\Delta x_{++}^2} - \frac{\ln(\mu^2\Delta x_{+-}^2)}{\Delta x_{+-}^2} \right\} J_{\nu]}(x') , \quad (72)$$

$$= \frac{\kappa^2\partial^4}{384\pi^3} 2\partial_{[\mu} \int d^4x' \theta(\Delta t - \Delta r) \left\{ \ln[\mu^2(\Delta t^2 - \Delta r^2)] - 1 \right\} J_{\nu]}(x') . \quad (73)$$

We are now ready to specialize to the current density (61) of an instantaneously created dipole. Substituting in (64) and specializing to purely spatial indices gives,

$$\partial^2 F_{ij}^{(0)}(x) = (\partial_i p_j - \partial_j p_i) \delta^4(x) . \quad (74)$$

The solution can be expressed in a convenient form by noting the  $D = 4$  dimensional version of relation (32),

$$\partial^2 \left\{ \frac{1}{\Delta x_{++}^2} \right\} = 4\pi^2 i \delta^4(x - x') \quad , \quad \partial^2 \left\{ \frac{1}{\Delta x_{+-}^2} \right\} = 0 . \quad (75)$$

Hence we have,

$$F_{ij}^{(0)}(x) = -\frac{i}{4\pi^2} (\partial_i p_j - \partial_j p_i) \left\{ \frac{1}{\Delta x_{++}^2} - \frac{1}{\Delta x_{+-}^2} \right\} , \quad (76)$$

where  $x'^\mu = 0$  is understood. Now write out the two intervals,

$$\Delta x_{++}^2 = r^2 - t^2 + \epsilon^2 + 2\epsilon|t|i , \quad (77)$$

$$\Delta x_{+-}^2 = r^2 - t^2 + \epsilon^2 - 2\epsilon ti . \quad (78)$$

Combining these relations with the Dirac identity results in the familiar form for the Liénard-Wiechert potential,

$$F_{ij}^{(0)}(x) = -\frac{1}{2\pi} (\partial_i p_j - \partial_j p_i) \theta(t) \delta(r^2 - t^2) . \quad (79)$$

The most convenient form for the one loop correction is (72),

$$F_{ij}^{(1)}(x) = (\partial_i p_j - \partial_j p_i) \frac{i\kappa^2\partial^2}{192\pi^4} \left\{ \frac{\ln(\mu^2\Delta x_{++}^2)}{\Delta x_{++}^2} - \frac{\ln(\mu^2\Delta x_{+-}^2)}{\Delta x_{+-}^2} \right\} , \quad (80)$$

$$= (\partial_i p_j - \partial_j p_i) \frac{i\kappa^2\partial^2}{48\pi^4} \left\{ \frac{1}{\Delta x_{++}^4} - \frac{1}{\Delta x_{+-}^4} \right\} , \quad (81)$$

$$= (\partial_i p_j - \partial_j p_i) \left( \frac{\kappa^2}{12\pi^2} \frac{\partial}{\partial r^2} \right) \left\{ \frac{\theta(t)\delta(r^2 - t^2)}{2\pi} \right\} . \quad (82)$$

Adding the one loop magnetic field (82) to the tree one (79) leads to an interesting form,

$$F_{ij}(x) = -\frac{1}{2\pi} (\partial_i p_j - \partial_j p_i) \theta(t) \left\{ 1 - \frac{\kappa^2}{12\pi^2} \frac{\partial}{\partial r^2} \right\} \delta(r^2 - t^2) + \mathcal{O}(\kappa^4) , \quad (83)$$

$$= -\frac{1}{2\pi} (\partial_i p_j - \partial_j p_i) \theta(t) \delta\left(r^2 - t^2 - \frac{\kappa^2}{12\pi^2}\right) + \mathcal{O}(\kappa^4) . \quad (84)$$

It would therefore be fair to say that, by time,  $t$  the signal has reached a distance  $r$  slightly outside the classical light-cone,

$$r^2 = t^2 + \frac{\kappa^2}{12\pi^2} + \mathcal{O}(\kappa^4) . \quad (85)$$

Although intriguing, the super-luminality we have just found is unobservably small. In particular, it cannot serve as any sort of explanation for the OPERA result [34]. It also isn't cumulative, so looking at cosmological sources makes the effect no larger. Another thing our effect fails to do is break Lorentz invariance, as could have been predicted from the fact that perturbative quantum gravity provides no mechanism for spontaneously breaking this symmetry. Instead of signals propagating along the classical light-cone  $\eta_{\mu\nu}x^\mu x^\nu = 0$ , they now propagate along  $\eta_{\mu\nu}x^\mu x^\nu = \frac{4G}{3\pi}$ . So it is not that the speed of light or the dispersion relation has been changed.

Of course theories with a nonlinear kinetic operator can show super-luminal propagation even classically [35]. Our effect is different in that it arises from quantum fluctuations of the metric operator which sets the light-cone. One interpretation for the net super-luminal propagation is that there is more volume outside the classical light-cone than inside. This might be checked by extending the graviton expansion of the volume of the past light-cone one order higher than in [36] and then computing its expectation value. If the one loop correction is positive then our conjecture is verified. Note also that this check would be independent of the choice of gauge because the volume of the past light-cone is a gauge invariant operator.

There have been many claims of super-luminal propagation from quantum electrodynamics in nontrivial geometries [37, 38]. Our result is different in that it occurs in flat space, and is due to fluctuations of the metric operator, rather than of some matter field. We also doubt that the earlier claims result from true super-luminal propagation. One cannot compute the vacuum polarization produced by fermions in an arbitrary geometry because the fermion propagator is not known for general metric. What was done instead is a derivative expansion. This should be valid for low energy effective field theory; for example, it should give correct results for the phase velocity of some continuous, low frequency signal. However, demonstrating true super-luminality requires following the propagation of a pulse, and the high frequency modes which are essential for this are not correctly treated by derivative expansions. In fact the Schwinger-Keldysh formalism [27] implies there cannot be super-luminal propagation from the fermionic contribution to vacuum polarization.

### 3.4 An Alternating Point Dipole

The 4-current associated with an alternating point dipole is,

$$J^0(t, \vec{x}) = -\vec{p} \cdot \vec{\nabla} \delta^3(\vec{x}) e^{-i\omega t} \quad , \quad J^i(t, \vec{x}) = -i\omega p^i \delta^3(\vec{x}) e^{-i\omega t} . \quad (86)$$



To find the quantum correction to the current we employ the same expansion technique used in the previous section where the first order correction is defined as,

$$J_{(1)}^\mu(x) = -\frac{G\partial^6}{24\pi^2} \int d^4x' \theta(\Delta t - \Delta x) \left\{ \ln[\mu^2(\Delta t^2 - \Delta x^2)] - 1 \right\} J^\mu(x'). \quad (87)$$

We can evaluate this integral by rewriting  $x$  and the differential operators as  $x = \frac{1}{2}(x+t) + \frac{1}{2}(x-t)$  and  $\partial^2 = \frac{1}{x}(\partial_x - \partial_t)(\partial_x + \partial_t)x$ . Thus we come to the convenient form,

$$\partial^4 = \frac{1}{2x}(\partial_x - \partial_t)^2(\partial_x + \partial_t)^2(x+t) + \frac{1}{2x}(\partial_x - \partial_t)^2(\partial_x + \partial_t)^2(x-t). \quad (88)$$

By substituting (88) for  $\partial^4$  in (87) and applying the zeroth order currents (86) we find the one loop currents to be,

$$J_{(1)}^0(t, \vec{x}) = \partial^2 \left\{ \frac{G\vec{p} \cdot \vec{\nabla}}{6\pi^2} \left[ -\frac{i\omega}{x^2} + \frac{1}{x^3} \right] e^{-i\omega(t-x)} \right\}, \quad (89)$$

$$J_{(1)}^i(t, \vec{x}) = \partial^2 \left\{ \frac{i\omega p^i G}{6\pi^2} \left[ -\frac{i\omega}{x^2} + \frac{1}{x^3} \right] e^{-i\omega(t-x)} \right\}. \quad (90)$$

From (67) we see that the zeroth order field strengths for this source obey,

$$\partial^2 F_{0i}^{(0)}(t, \vec{x}) = -\left[ \omega^2 p_i - \partial_i \vec{p} \cdot \vec{\nabla} \right] \delta^3(\vec{x}) e^{-i\omega t}, \quad (91)$$

$$\partial^2 F_{ij}^{(0)}(t, \vec{x}) = -i\omega \left[ \partial_i p_j - \partial_j p_i \right] \delta^3(\vec{x}) e^{-i\omega t}. \quad (92)$$

Applying the Liénard-Wiechert Green's function we find,

$$F_{0i}^{(0)}(t, \vec{x}) = \frac{1}{4\pi} \left[ \omega^2 p_i - \partial_i \vec{p} \cdot \vec{\nabla} \right] \frac{e^{-i\omega(t-x)}}{x}, \quad (93)$$

$$F_{ij}^{(0)}(t, \vec{x}) = \frac{i\omega}{4\pi} \left[ \partial_i p_j - \partial_j p_i \right] \frac{e^{-i\omega(t-x)}}{x}. \quad (94)$$

From (68) we see that one loop field strengths follow by simply deleting the  $\partial^2$  from (89-90) and acting some derivatives,

$$F_{0i}^{(1)}(t, \vec{x}) = -\frac{i\omega G}{6\pi^2} \left[ \omega^2 p_i - \partial_i \vec{p} \cdot \vec{\nabla} \right] \left[ 1 + \frac{i}{\omega x} \right] \frac{e^{-i\omega(t-x)}}{x^2}, \quad (95)$$

$$F_{ij}^{(1)}(t, \vec{x}) = -\frac{i\omega G}{6\pi^2} (i\omega) \left[ \partial_i p_j - \partial_j p_i \right] \left[ 1 + \frac{i}{\omega x} \right] \frac{e^{-i\omega(t-x)}}{x^2}. \quad (96)$$

Adding the loop correction to the tree results gives,

$$F_{0i}(t, \vec{x}) = \left[ \omega^2 p_i - \partial_i \vec{p} \cdot \vec{\nabla} \right] \times \frac{e^{-i\omega(t-x)}}{4\pi x} \times \left\{ 1 - \frac{2i\omega G}{3\pi x} \left[ 1 + \frac{i}{\omega x} \right] + \mathcal{O}(G^2) \right\}, \quad (97)$$

$$F_{ij}(t, \vec{x}) = i\omega \left[ \partial_i p_j - \partial_j p_i \right] \times \frac{e^{-i\omega(t-x)}}{4\pi x} \times \left\{ 1 - \frac{2i\omega G}{3\pi x} \left[ 1 + \frac{i}{\omega x} \right] + \mathcal{O}(G^2) \right\}. \quad (98)$$

Of course the obvious conclusion is that the one loop corrections have no effect in the far field regime, and the near field regime is unobservably close to the source.

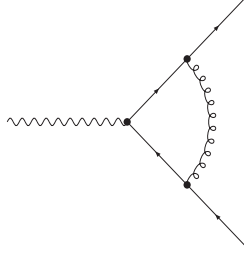


Figure 4: Vertex correction included (along with many other diagrams) in the Bjerrum-Bohr result [8], but not in either our result (101) or that of Radkowski [7]. Charged scalar lines are solid with an arrow, photon lines are wavy and graviton lines are winding.

### 3.5 A Static Point Charge

The charge density of a static point charge  $q$  at the origin is  $\rho = q\delta^3(\vec{x})$ . The conserved 4-current associated with this source is,

$$J^\mu(x) = q\delta^3(\vec{x})\delta_0^\mu. \quad (99)$$

Because the  $\mu = 0$  component differs from the alternating dipole of the previous subsection only by setting  $\omega = 0$  and replacing  $-\vec{p} \cdot \vec{\nabla}$  with  $q$ , we can read off the one loop current density from (89),

$$J_{(1)}^0(t, \vec{x}) = -\frac{Gq}{\pi^2 x^5}. \quad (100)$$

Of course the vector components vanish so we find the correction to the Coulomb potential is,

$$\Phi(r) = \frac{q}{4\pi r} \left\{ 1 + \frac{2G}{3\pi r^2} + \mathcal{O}(G^2) \right\}. \quad (101)$$

Our result (101) agrees with that found in 1970 by Radkowski [7]. The one loop correction that Bjerrum-Bohr inferred from the scattering of charged, gravitating scalars differs from what we got by a factor of nine [8]. Part of this discrepancy may be due to different sources; Bjerrum-Bohr considered a charged scalar whereas we used a point particle with worldline  $\chi^\mu(\tau)$ ,

$$L_{\text{point}} = -m\sqrt{-g_{\mu\nu}(\chi(t))\dot{\chi}^\mu(\tau)\dot{\chi}^\nu(\tau) + q\dot{\chi}^\mu(\tau)A_\mu(\chi(\tau))}. \quad (102)$$

However, we believe the largest part of the discrepancy arises from Bjerrum-Bohr having implicitly included corrections to the current density like the diagram depicted in Fig. 4. We could have and should have done this, but we will see in the next section that it would only have altered all our one loop field strengths by an overall constant.

Radkowski [7], Bjerrum-Bohr [8] and we all agree that quantum gravity strengthens the electromagnetic force at one loop. The opposite conclusion seems to arise from computations of the quantum gravitational contribution to the electromagnetic beta function [9, 39, 40].

These show that quantum gravity decreases the electromagnetic coupling constant at high energy scales. That would normally be assumed to mean that quantum gravity weakens the electromagnetic force at short distances, but it is well to keep in mind that the beta function is not directly observable. The observable thing is the strength scattering between charged particles, and the Bjerrum-Bohr computation shows that one loop quantum gravity effects weaken this, rather than strengthening it.

## 4 Gauge Dependence

The purpose of this section is to examine how the results of the previous section depend upon our choices of the gravitational gauge fixing term (15) and the electromagnetic gauge fixing term (19). We begin with some general considerations which reduce the issue to a single proportionality constant. The graviton and photon propagators are then worked out for a general 3-parameter family of covariant gauges. Although one of these parameters drops out, the other two can change the proportionality constant all the way from minus infinity to plus infinity. We close by exploiting the gauge independent result of Bjerrum-Bohr to argue that this seeming gauge dependence may cancel out if quantum gravitational corrections to the current density are included.

### 4.1 General Considerations

Note from expression (25) that the vacuum polarization is transverse on each of its two indices as a trivial consequence of the antisymmetry of the vertex function on its first and third indices,

$$V^{\mu\rho\kappa\lambda\alpha\beta} = -V^{\kappa\rho\mu\lambda\alpha\beta} . \quad (103)$$

This is completely without regard to the gauges employed to define graviton and photon propagators. Suppose we now restrict attention to gauges which preserve Poincaré invariance. Because the Lagrangians (5-7) and the background are also Poincaré invariant, the vacuum polarization must inherit this symmetry. Then dimensional analysis, transversality and the standard  $\kappa^2$  of a one loop quantum gravity result, together imply a form like that of (28),

$$i\left[{}^{\mu}\Pi_{3\text{pt}}^{\nu}\right](x;x') = -\text{Constant} \times \kappa^2 \left[\eta^{\mu\nu}\partial^2 - \partial^{\mu}\partial^{\nu}\right] \frac{1}{\Delta x^{2D-2}} . \quad (104)$$

However, the constant prefactor might be gauge dependent, and that same gauge dependent constant would multiply all of our one loop corrections.

It is useful to begin at a somewhat earlier point. If different — but Poincaré invariant — graviton and photon propagators had been employed in expression (25), then the combination of Poincaré invariance, dimensional analysis and the algebraic symmetries of the vertex function and the propagators imply that the result can be expressed in terms of two constants

$I$	$\mathcal{C}_I(D, a, b)$	$[\mu\nu\mathcal{T}_{\rho\sigma}^I]$
1	1	$2\eta_{\mu(\rho}\eta_{\sigma)\nu}$
2	$-\frac{2}{D-2}$	$\eta_{\mu\nu}\eta_{\rho\sigma}$
3	$\frac{4(b-1)}{(D-2)(b-2)}$	$\eta_{\mu\nu}\frac{\partial_\rho\partial_\sigma}{\partial^2} + \frac{\partial_\mu\partial_\nu}{\partial^2}\eta_{\rho\sigma}$
4	$a-1$	$4\frac{\partial_{(\mu}\eta_{\nu)(\rho}\partial_{\sigma)}}{\partial^2}$
5	$-\frac{4a(b-1)(b-3)}{(b-2)^2} - \frac{8(b-1)(b+D-3)}{(b-2)^2(D-2)}$	$\frac{\partial_\mu\partial_\nu\partial_\rho\partial_\sigma}{\partial^4}$

Table 1: Coefficient functions  $\mathcal{C}_I(D, a, b)$  and the tensor differential operators  $[\mu\nu\mathcal{T}_{\rho\sigma}^I]$  for the graviton propagator (111) defined with the general gauge fixing functional (110).

$A$  and  $B$ ,

$$\begin{aligned}
& (i\kappa)^2 \partial_\kappa \partial'_\theta \left\{ V^{\mu\rho\kappa\lambda\alpha\beta} i[\alpha\beta\Delta_{\gamma\delta}^{\text{new}}](x; x') V^{\nu\sigma\phi\theta\gamma\delta} \partial_\lambda \partial'_\phi i[\rho\Delta_\sigma^{\text{new}}](x; x') \right\} \\
&= (i\kappa)^2 \frac{\Gamma^2(\frac{D}{2}-1)}{16\pi^D} (D-2) \partial_\kappa \partial'_\theta \left\{ A \times \frac{4\Delta x^{[\mu}\eta^{\kappa][\nu}\Delta x^{\theta]}}{\Delta x^{2D}} + B \times \frac{\eta^{\mu[\nu}\eta^{\theta]\kappa}}{\Delta x^{2D-2}} \right\}, \tag{105}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\kappa^2 \Gamma^2(\frac{D}{2}-1)}{16\pi^D} \times (D-2) [DA - 2(D-1)B] \\
&\quad \times \left\{ \frac{(D+1)\eta^{\mu\nu}}{\Delta x^{2D}} - \frac{2D\Delta x^\mu \Delta x^\nu}{\Delta x^{2D+2}} \right\}, \tag{106}
\end{aligned}$$

$$= -\frac{\kappa^2 \Gamma^2(\frac{D}{2}-1)}{16\pi^D} \times \frac{(D-2)[DA - 2(D-1)B]}{2(D-1)} \times [\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu] \frac{1}{\Delta x^{2D-2}}. \tag{107}$$

We can therefore identify the proportionality constant in (104) as,

$$\text{Constant} = \frac{\Gamma^2(\frac{D}{2}-1)}{16\pi^D} \times \frac{(D-2)[DA - 2(D-1)B]}{2(D-1)}, \tag{108}$$

$$\equiv \frac{\Gamma^2(\frac{D}{2}-1)}{16\pi^D} \times \frac{1}{2} \left( \frac{D-2}{D-1} \right) \times C. \tag{109}$$

## 4.2 General Covariant Gauges

The most general Poincaré invariant extension of the graviton gauge fixing functional (15) depends upon two parameters  $a$  and  $b$ ,

$$\mathcal{L}_{\text{GRnew}} = -\frac{1}{2a} \eta^{\mu\nu} \mathcal{F}_\mu \mathcal{F}_\nu \quad , \quad \mathcal{F}_\mu \equiv \eta^{\rho\sigma} \left( h_{\mu\rho,\sigma} - \frac{b}{2} h_{\rho\sigma,\mu} \right). \tag{110}$$

The associated propagator is [41],

$$i[\alpha\beta\Delta_{\gamma\delta}^{\text{new}}](x; x') = \sum_{I=1}^5 \mathcal{C}_I(D, a, b) \times [\mu\nu\mathcal{T}_{\rho\sigma}^I] \times i\Delta(x; x') , \quad (111)$$

where the coefficient functions  $\mathcal{C}_I(D, a, b, )$  and the tensor differential operators  $[\mu\nu\mathcal{T}_{\rho\sigma}^I]$  are given in Table 1. The propagator can be given a more revealing expression using the transverse projection operator  $\Pi_{\mu\nu} \equiv \eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}$ ,

$$\begin{aligned} i[\alpha\beta\Delta_{\gamma\delta}^{\text{new}}](x; x') = & \left\{ 2\Pi_{\mu(\rho}\Pi_{\sigma)\nu} - \frac{2}{D-1}\Pi_{\mu\nu}\Pi_{\rho\sigma} \right. \\ & - \frac{2}{(D-2)(D-1)} \left[ \eta_{\mu\nu} - \left( \frac{Db-2}{b-2} \right) \frac{\partial_\mu\partial_\nu}{\partial^2} \right] \left[ \eta_{\rho\sigma} - \left( \frac{Db-2}{b-2} \right) \frac{\partial_\rho\partial_\sigma}{\partial^2} \right] \\ & \left. + 4a \times \frac{\partial_{(\mu}\Pi_{\nu)(\rho}\partial_{\sigma)}}{\partial^2} + \frac{4a}{(b-2)^2} \times \frac{\partial_\mu\partial_\nu\partial_\rho\partial_\sigma}{\partial^4} \right\}. \end{aligned} \quad (112)$$

Of course the transverse-traceless term on the first line represents the contribution from dynamical, spin two gravitons. This term looms large in the quantum gravity literature but it is well to recall that it plays no role in the solar system tests of general relativity. The phenomenologically more important parts of the graviton propagator are those on the second and third lines, which mediate the gravitational interaction between sources of stress-energy. Note that the longitudinal terms proportional to the gauge parameter  $a$  would vanish in the exact gauge  $h^\nu_{\mu,\nu} = \frac{b}{2}h_{,\mu}$ .

The most general Poincaré invariant extension of the photon gauge fixing functional (19) depends upon a single parameter  $c$ ,

$$\mathcal{L}_{\text{EMnew}} = -\frac{1}{2c}(\partial^\mu A_\mu)^2 . \quad (113)$$

The associated propagator is,

$$i[\rho\Delta_\sigma^{\text{new}}](x; x') = \left[ \eta_{\rho\sigma} + (c-1)\frac{\partial_\rho\partial_\sigma}{\partial^2} \right] i\Delta(x; x') . \quad (114)$$

The longitudinal term proportional to  $c-1$  can make no contribution to the general gauge vacuum polarization (105) because the vertex function (23) is antisymmetric under interchange of its second and fourth indices,

$$V^{\mu\rho\kappa\lambda\alpha\beta} = -V^{\mu\lambda\kappa\rho\alpha\beta} . \quad (115)$$

It remains to explain how to act the tensor differential operators of Table 1 on the scalar propagator (17). First note that inverse d'Alembertians act on  $1/\Delta x^{D-2}$  to give,

$$\frac{1}{\partial^2} \frac{1}{\Delta x^{D-2}} = -\frac{1}{2(D-4)} \frac{1}{\Delta x^{D-4}} , \quad (116)$$

$$\frac{1}{\partial^4} \frac{1}{\Delta x^{D-2}} = \frac{1}{8(D-4)(D-6)} \frac{1}{\Delta x^{D-6}} . \quad (117)$$

$I$	$A_I$	$B_I$	$C_I$
1	$\frac{1}{2}D(3D-8)$	$3D-8$	$\frac{1}{2}(D-2)^2(3D-8)$
2	$\frac{1}{4}(D-4)^2D$	$\frac{1}{2}(D-4)^2$	$\frac{1}{4}(D-4)^2(D-2)^2$
3	$\frac{1}{2}(D-4)^2(D-1)$	$D-4$	$\frac{1}{2}(D-4)(D-2)^2(D-1)$
4	$(D-1)(3D-8)$	$D^2-2D-2$	$(D-2)^2(D-1)$
5	$\frac{1}{8}(D-2)(D-1)D$	$\frac{1}{4}(D-2)(D-1)$	$\frac{1}{8}(D-2)^3(D-1)$

Table 2: Coefficients  $A_I$ ,  $B_I$  and  $C_I$  defined in relations (105) and (109), under the replacement  $i[\mu\nu\Delta_{\rho\sigma}^{\text{new}}](x; x') \longrightarrow [\mu\nu\mathcal{T}_{\rho\sigma}^I] \times i\Delta(x; x')$  for each of the five tensor differential operators defined in Table 1.

Now just act the derivatives in the numerator to conclude,

$$\frac{\partial_\mu\partial_\nu}{\partial^2} i\Delta(x; x') = \frac{1}{2} \times \left\{ \eta_{\mu\nu} - \frac{(D-2)\Delta x_\mu\Delta x_\nu}{\Delta x^2} \right\} i\Delta(x; x') , \quad (118)$$

$$\frac{\partial_\mu\partial_\nu\partial_\rho\partial_\sigma}{\partial^4} i\Delta(x; x') = \frac{1}{8} \times \left\{ 3\eta_{(\mu\nu}\eta_{\rho\sigma)} - \frac{6(D-2)\eta_{(\mu\nu}\Delta x_\rho\Delta x_\sigma)}{\Delta x^2} \right. \quad (119)$$

$$\left. + \frac{D(D-2)\Delta x_\mu\Delta x_\nu\Delta x_\rho\Delta x_\sigma}{\Delta x^4} \right\} i\Delta(x; x') . \quad (120)$$

### 4.3 Gauge Dependent Proportionality Constant

We are now ready to compute the crucial proportionality constant of relation (104). Because the gauge dependence of the photon propagator drops out, we need only consider the gauge dependence of the graviton propagator. Because the graviton propagator (111) is a sum of gauge-dependent coefficients  $\mathcal{C}_I(D, a, b)$  times tensor operators  $[\mu\nu\mathcal{T}_{\rho\sigma}^I]$ , acting on the scalar propagator, we may as well work out the result for each tensor operator separately. Table 2 presents the coefficients  $A_I(D)$  and  $B_I(D)$  which were defined in relation (105), for each of the five tensor differential operators of Table 1. Also given is the contribution of each tensor differential operator to the coefficient  $C_I(D)$ ,

$$C_I(D) \equiv D \times A_I(D) - 2(D-1) \times B_I(D) . \quad (121)$$

To recover the full result for  $C(D, a, b)$  defined in relation (109) we multiply each  $C_I(D)$  by the appropriate gauge dependent coefficient  $\mathcal{C}(D, a, b)$  from Table 1,

$$C(D, a, b) = \sum_{I=1}^5 \mathcal{C}_I(D, a, b) \times C_I(D) \quad (122)$$

The formula for arbitrary  $D$  is not illuminating, but specializing to  $D = 4$  gives,

$$C(4, a, b) = -12a \times \frac{(3b^2 - 12b + 8)}{(b-2)^2} - \frac{4}{(b-2)^2} . \quad (123)$$

Our original gauge corresponds to  $a = b = 1$ , which gives  $C(4, a, 1) = 8$ . Hence the various one loop corrections computed in section are valid for general  $a$  and  $b$  if we multiply by the proportionality constant,

$$K(a, b) \equiv \frac{C(4, a, b)}{C(4, 1, 1)} = -\frac{3}{2}a \times \frac{(3b^2 - 12b + 8)}{(b-2)^2} - \frac{1}{2(b-2)^2} . \quad (124)$$

It is interesting to note that the gauge independent contribution from dynamical gravitons vanishes in  $D = 4$  dimensions,

$$\begin{aligned} C_1(D) - C_4(D) + 2C_5(D) - \frac{2}{D-1} [C_2(D) - C_3(D) + C_5(D)] \\ = \frac{(D-4)(D-2)^2(D+1)(D+2)}{4(D-1)} . \end{aligned} \quad (125)$$

It is apparent that the gauge dependent proportionality constant (124) can be made to have either sign by varying the gauge parameter  $a$ . Furthermore,  $K(a, b)$  can be made arbitrarily large in magnitude by taking the gauge parameter  $b$  close to 2. Hence it would seem that our results are completely gauge dependent and unphysical. Gauge dependence has also been noted in the renormalization group approach [42, 43].

A moment's thought reveals that all is not lost because *the result of Bjerrum-Bohr [8] for the quantum gravitational correction to the Coulomb potential was derived from the gauge independent S-matrix of scalar QED*. Roughly speaking, this correction derives from the fact that gravity is sourced by the electromagnetic fields of the two charged particles being scattered, and this source changes as the particles move with respect to one another. That is a real effect, not some gauge artifact. And it is crucially important to note that *we agree with Bjerrum-Bohr up to a factor of +9*. We attributed this factor to our having only quantum-corrected the left hand side of the operator Maxwell equation,

$$\partial^\nu [\sqrt{-g} g^{\nu\rho} g^{\mu\sigma} F_{\rho\sigma}(x)] = J^\mu(x) . \quad (126)$$

The right hand side is also an operator and it must also suffer quantum gravitational corrections, one of which is depicted in Fig. 4. By Poincaré invariance, current conservation and dimensional analysis, those corrections must take exactly the same form as we found for the left hand side, up to an overall constant. We conjecture that the gauge dependence  $K(a, b)$  we have just found for corrections to the left hand side is canceled by gauge dependence in corrections to the right hand side. If this is correct, then *the overall, gauge independent correction to the various results derived in section 3 can be inferred by comparing any one of them with its S-matrix analogue*. For scalar QED the correction factor would be 9, and it could be computed for the point particle source we used.

The resolution we have just proposed to the gauge issue recalls some old work by DeWitt [44] about dependence upon the gauge fixing functionals even in the gauge invariant background field effective action  $\Gamma = S + \Sigma$ . DeWitt states [45], “The functional form of  $\Sigma$  is not independent of the choice of these terms. However, the solutions of the *effective*

*field equation* can be shown to be the same for all choices.” At the order we are working there is no distinction between our effective field equations and those of the gauge invariant background field effective action. (One can see this from the transversality of our vacuum polarization.) However, we have just shown that DeWitt’s statement cannot be correct if the source (or the asymptotic field strengths for scattering solutions) is not normalized in some physical way. Our proposal is that including quantum corrections to the right hand side of the equation provides this physical normalization. More work is obviously required, in particular an explicit computation of the quantum gravitational corrections to the source, but it would be wonderful if solutions to the effective field equations could be physically interpreted the same way as classical solutions.

## 5 Discussion

We used dimensional regularization to compute the one loop quantum gravitational contribution (28) to the vacuum polarization on flat space background. A fully renormalized result (37) was obtained by first partially integrating to localize the ultraviolet divergence and then absorbing it into the appropriate BPHZ counterterm (30) with coefficient (36). The Schwinger-Keldysh formalism [27, 30] was then employed to reach the manifestly real and causal form (55).

We used (55) to solve the quantum corrected Maxwell’s equation (3) for various special cases. Provided the appropriate perturbative corrections to the initial state cancel the surface terms involved in reaching the form (59), there is no change in the source-free solutions at any order in the loop expansion. However, sources induce a variety of interesting effects.

Probably the most provocative source is the current density (61) of an instantaneously created, point electric dipole. The pulse (84) which results in the magnetic field propagates slightly outside the classical light-cone. It seems to arise from quantum fluctuations of the metric operator, which are isotropic but favor super-luminal propagation because there is more volume outside the light-cone than inside. That this sort of thing might occur has been realized since the earliest days of quantum gravity [1]. Our super-luminal effect is completely Lorentz invariant, merely changing the characteristic surface from  $\eta_{\mu\nu}x^\mu x^\nu = 0$  to  $\eta_{\mu\nu}x^\mu x^\nu = \frac{4G}{3\pi}$ . Despite many claims to the contrary, this seems to be the first case of super-luminal propagation from a quantum field theory whose classical analogue does not allow super-luminal propagation. All previous claims have been based derivative expansions [37, 38], which are perfectly valid for most applications of low energy effective field theory but which incorrectly treat the high frequency modes needed to resolve the propagation of a pulse.

The other interesting source we studied is the response to a static, point charge. Our result (101) for the quantum corrected Coulomb potential agrees with what Radkowski found more than four decades ago [7]. It does not agree with the Coulomb potential Bjerrum-Bohr inferred from the scattering of charged, gravitating scalars [8], but we all find quantum gravity strengthens the electrostatic force at short distances. We believe that the factor of



nine discrepancy with Bjerrum-Bohr derives from his S-matrix technique implicitly including quantum gravitational corrections to the charge density, like the diagram depicted in Fig. 4. This should have been done for our point particle source, but it would only have changed the one loop field strengths by an overall constant.

Renormalization group analyses [9, 39, 40] seem to provide a more serious discrepancy. These studies find that quantum gravity reduces the electrodynamic coupling constant at large scales. The usual inference would be that quantum gravity weakens the electrostatic force at short distances. However, the beta function is not itself observable; several other effects must be combined to infer the impact of quantum gravity. The S-matrix computation of Bjerrum-Bohr should include all of these effects, and it shows that quantum gravity strengthens the electrostatic force at short distances.

Gauge dependence poses a major obstacle to the physical interpretation of solutions to the effective field equations. If one restricts to Poincaré invariant gauge fixing functionals, the only possible change to our vacuum polarization (55) is rescaling by an overall, gauge-dependent constant. In section 4 we considered the most general 2-parameter family of graviton gauges (110), and the most general 1-parameter family of photon gauges (113). We showed that the vacuum polarization has no dependence upon the electromagnetic gauge fixing parameter  $c$ , but it depends strongly on the two gravitational gauge fixing parameters  $a$  and  $b$ . The effect of being in a general covariant gauge is to rescale (55) by the function  $K(a, b)$  given in equation (124). By varying the constants  $a$  and  $b$ , one can make  $K(a, b)$  assume any values from plus infinity to minus infinity.

Such massive gauge dependence would seem to invalidate any physical inference from the results of section 3, however, the gauge independent result of Bjerrum-Bohr suggests a simple resolution. There is no question that one must include quantum gravitational corrections to the current density operator. This seems to be why Bjerrum-Bohr (who implicitly did this) gets a factor of nine different one loop correction to the Coulomb potential. We conjecture that making such corrections in a general gauge — which seems quite feasible using the techniques of section 4 — would completely cancel the gauge dependence of our result. If this could be demonstrated then it would be possible to realize the old dream [44, 45] of using solutions to the effective field equations as freely as one does classical solutions. *Note also that it would provide an important class of observables in cosmology, for which the S-matrix does not exist.*

The point of this exercise has been to establish the flat space correspondence limit for a planned investigation of the effects of inflationary gravitons on electromagnetism. Our model has been a similar study of the effects of inflationary scalars on gravity [23, 24], the flat space limit of which [22] played a crucial role in guiding the analysis. In retrospect, we can recognize the simplicity of flat space as the ideal venue for sorting out the troublesome issues of dependence upon the choice of field variable and the choice of gauge which are so important to a correct interpretation of the many solutions which now exist to linearized effective field equations on de Sitter background [20, 21, 32, 46, 47, 48, 49, 50].

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