Creating universes with thick walls

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We study the dynamics of a spherically symmetric false vacuum bubble embedded in a true vacuum region separated by a "thick wall", which is generated by a scalar field in a quartic potential. We study the "Farhi-Guth-Guven" (FGG) quantum tunneling process by constructing numerical solutions relevant to this process. The ADM mass of the spacetime is calculated, and we show that there is a lower bound that is a significant fraction of the scalar field mass. We argue that the zero mass solutions used to by some to argue against the physicality of the FGG process are artifacts of the thin wall approximation used in earlier work. We argue that the zero mass solutions should not be used to question the viability of the FGG process.

I. INTRODUCTION

While our universe appears to be well described by ΛCDM cosmology and slow-roll inflation, much about the pre-inflationary universe remains speculative. Numerous models rely on quantum tunneling from some previous state to give an inflating universe that eventually leads to the universe we observe today (see for example [1]).

We consider here the Farhi-Guth-Guven" (FGG) process which was originally studied in the "thin wall" limit [2],[3]. In this process, a bubble of false vacuum, known as the seed bubble, is separated by a thin domain wall from a region of true vacuum. Einstein's equation implies two distinct solutions for the motion of the bubble wall; the first eventually collapses while the second expands indefinitely [4]. The possibility of tunneling between these two states is considered. Although FGG consider the case where a seed somehow forms in Minkowski space, other cases were considered (for example in [5]) where the seed forms from Hawking radiation in de Sitter space. Either way, the seed collapses into a black hole but hidden behind the black hole horizon is the expanding solution. The mass of this bubble, M, is the m parameter in the usual Schwarzschild metric, the "ADM" mass.

FGG is known to dominate over Coleman-de Luccia type tunneling[6] and it has been argued that this process can produce inflating universes that do not originate from classical singularities [2, 3, 7]. Despite these features, many calculations that study tunneling in cosmology (for example in the string theory "landscape" [8]) ignore FGG, primarily because of various arguments that this process might not be physical [6, 9]. In this paper we address one of the arguments against the physicality of the FGG process, one that involves taking the bubble mass M to zero[6].

The $M \to 0$ limit of the thin wall formula leads to a prediction of the FGG process that the probability of transitioning from the seed bubble to the inflating bubble remains finite even as the mass of the seed bubble is taken to zero. This is the ultimate free lunch, since it implies our universe was possibly nucleated from zero matter Minkowski space. However, here we argue that this limit is an artifact of the thin wall approximation which assumes that the thickness of the domain wall is small compared to the radius of the bubble. Indeed, as the radius of the bubble is taken to zero (as it is in the $M \to 0$ limit), one should expect the thin wall approximation to breakdown.

In this article we examine bubbles of false vacuum separated by a "thick wall", i.e. scalar field solutions that interpolate between regions of true and false vacuum. We construct numerical solutions for the scalar field coupled to gravity that are relevant to the FGG process. Probably our most important point is an extremely simple one: For a fixed potential the types of possible bubbles are limited and the $M \to 0$ cannot even be taken. So if one has a particular scalar field potential in mind one is
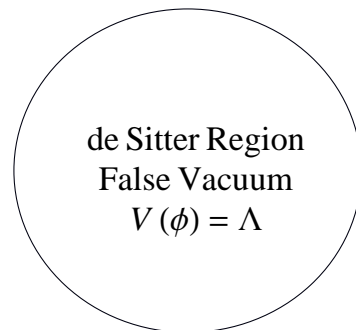
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Schwarschild Region
True Vacuum
$V(\phi) = 0$

de Sitter Region
False Vacuum
$V(\phi) = \Lambda$

FIG. 1. A sketch of a bubble solution.

unlikely to encounter the issues raised in [6] about FGG.

In this work we go beyond this simple point by exploring the parameter space of a general quartic scalar field potential (with an overall scale fixed). We find solutions in such potentials cannot approach the step-function type solutions for ϕ that are assumed in the thin wall case, even when the potential is made as "thin wall" as possible. Instead, the scalar field inevitably "spills over" and zero mass solutions are unattainable. By comparison, the thin wall $M \to 0$ limit relies on exact Schwarzschild space outside the bubble while taking the bubble radius to zero. While it may be possible to find an exotic potential with M arbitrarily small, we show that no quartic potential with a fixed overall scale admits such solutions.

II. THE THIN WALL REVISITED

A. The setup

Imagine embedding a spherically symmetric bubble of false vacuum, the seed bubble, in a region of true vacuum separated by a domain wall of negligible thickness with surface energy density σ as pictured in Fig. 1. The thin wall approximation assumes that the false vacuum is de Sitter space, the true vacuum is Schwarzschild, and the stress energy tensor is discontinuous at the domain wall. Note that we can also allow for a cosmological constant everywhere, and the exterior simply becomes Schwarzschild - de Sitter.

The classical solutions are discussed extensively in [4] and it suffices to repeat a few key results. The mass M of the bubble is the usual Schwarzschild parameter m in the static foliation. This mass can be rewritten as

$$M = \frac{\Lambda^2 r^3}{2G} + 4\pi\sigma r^2 \sqrt{1 + \dot{r}^2 - \Lambda^2 r^2} - 8\pi^2 G \sigma^2 r^3 \quad (1)$$

where Λ is the cosmological constant, G is Newton's constant, σ is the surface energy density of the wall, and
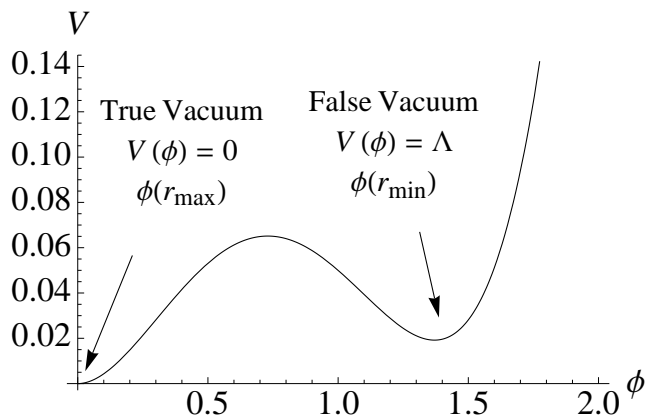

FIG. 2. A particular ϕ^4 potential with true and false vacuum regions labeled.

r is the radial coordinate in the static de Sitter and Schwarzschild foliations. Note that the naive limit $r \to 0$ appears to give zero mass solutions.

One can use the junction formalism developed in [10]. The basic strategy is to place a coordinate system on the wall and demand continuity of the metric tensor. Then utilizing Einstein's equation, the rescaled radial coordinate, z, obeys the following equation

$$\dot{z}^2 + V(z) = E$$

which is identical to that of a particle moving in a one dimensional potential. We know that if $V > E$, then two solutions exist, but classically the particle cannot move across the barrier. We can, however, have quantum tunneling between the two solutions.

B. The two solutions

As previously mentioned, two possible solutions exist for the classical motion provided $M < M_{cr}$ where $M_{cr} \sim \Lambda \hat{r}^3$ is the characteristic mass of the problem [4]. Here $\hat{r}$ is the radius of the bubble wall. Type (a) solutions are bounded solutions that begin at $\hat{r} = 0$, expand to some $\hat{r} = r_{max}$ before collapsing back to zero. These solutions avoid a classical singularity, as discussed in FGG, because the trajectory on the Kruskal diagram crosses to the right of the origin and a closed "anti-trapped" surface no longer exists. This point is further elucidated in [7].

Second, there are bounce solutions in which $\hat{r}$ approaches infinity in the asymptotic past, falls to some minimum value, and expands again to approach infinity in the asymptotic future. The Penrose theorem implies that this space-time must have emerged from an initial singularity, since the bubble radius grows beyond $(\Lambda)^{-\frac{1}{2}}$. The way to avoid this classical singularity yet still produce an inflating universe is to consider tunneling between the two solutions. The two solutions are of identical mass and thus identical energy. This is the FGG process.

The tunneling probability can be calculated using a functional integral [2] or a canonical quantization [3]. In either case, the probability of tunneling between the two solutions remains finite as the mass of the seed bubble, an input parameter, is taken to zero.

III. VISITING THE THICK WALL

Consider a scalar field minimally coupled to gravity in a quartic potential, described by the following action

$$S = \frac{1}{2}m_P^2 \int d^4x \sqrt{-g}(R - \nabla_a\phi\nabla_b\phi g^{ab} - 2V(\phi)) \quad (2)$$

where $V(\phi) = \lambda\phi^4 - \gamma\phi^3 + \frac{m_i^2}{2}\phi^2$, m_i is the inflaton mass, and m_P is the reduced Planck mass. A particular potential is shown in Fig. 2.

We work in a $+2$ metric signature, in reduced Planck units where $\hbar = 1, c = 1$ and $m_P = \sqrt{8\pi G}^{-1}$. One can then nondimensionalize the problem by rescaling the coordinates, for example by using $r^* = rm_P$. This rescales the potential to

$$V(\phi) = \lambda\phi^4 - \gamma\phi^3 + \frac{m_i^2}{2m_P^2}\phi^2$$

Here ϕ, λ and γ are all dimensionless. In what follows, all coordinates and quantities are dimensionless with $m_i = m_P$. Keeping m_i fixed allows us to explore the properties of the bubble solutions without allowing the overall scale of the potential to vanish (in that case one does expect solutions with M approaching zero to be possible). Fixing m_i to the value m_P is convenient for the dynamic range of our numerical work is also a common choice in inflationary models. We use standard spherical, (t, r, θ, ϕ), coordinates. Under the assumption of spherical symmetry, the spacetime line element takes the form

$$ds^2 = -\alpha^2(r,t)dt^2 + a^2(r,t)dr^2 + r^2d\Omega^2 \qquad (3)$$

Note that we have not forced the metric in any region to take the de Sitter or Schwarzschild form, although we do require that the spacetime is asymptotically flat at large r. Here r is both a coordinate and the measure of proper area. The stress-energy tensor of a scalar field in a potential is

$$T_{ab} = \partial_a\phi\partial_b\phi - \frac{1}{2}g_{ab}\left(\partial_c\phi\partial_d\phi g^{cd} + 2V(\phi)\right) \qquad (4)$$

Defining mass in the thick-wall case is more involved, since we no longer have a region of exact Schwarzschild space or a fixed wall position where one can place an observer. Instead, we focus on the ADM mass, which is defined at spatial infinity for asymptotically flat spacetimes. This is the most relevant mass for tunneling calculations [5]. This mass is defined as [11]

$$M = 2\pi \int_0^\infty dr r^2 \left[\left(\frac{\phi'}{a}\right)^2 + \left(\frac{\dot\phi}{\alpha}\right)^2 + 2V(\phi)\right] \qquad (5)$$

where prime denotes differentiation with respect to r while dot denotes differentiation with respect to time.

The tt and rr components of Einstein's equation and the scalar field equation are used to find the turning point. They are listed below. The $\theta\theta$ equation is used as a consistency check.

$$2a^3(r^2V - 1) - 4ra' + a(2 + r^2\phi'^2) = 0$$

$$\frac{2}{r} + a^2(2rV - \frac{2}{r}) + 4\frac{\alpha'}{\alpha} = r\phi'^2$$

$$\frac{a^2}{\alpha^2}(\alpha^2\frac{dV}{d\phi} - \frac{1}{r^2}) + \frac{a'}{a}\phi' - (\frac{2}{r} + \frac{\alpha'}{\alpha})\phi' = \phi''$$

We use a 4th order Runge-Kutta scheme with adaptive mesh refinement. In true S - dS static spherical coordinates, there is a horizon when $1 - 2m/r - \Lambda/r^2 = 0$. We cannot check this condition a priori, because we do not know the mass of the space-time. However, we check it after and make sure that the computational domain does not include the horizon. Note also that, if there was a horizon inside the computational domain, it would be apparent in the metric functions.

We want to find solutions for a bubble of false vacuum embedded in true vacuum, i.e. we want to find the radial profiles and time evolution of ϕ, α and a. This is done by demanding that $\phi_{r=r_{min}}$ take the value of the false minimum of the potential, so that $V(\phi_m)$ acts as a cosmological constant near the origin. We also investigated cases in which the scalar field was not initially at the minimum of the potential, which include the extreme case in which $\phi_{r=r_{min}} = \phi_{r=r_{max}} = 0$. In this case, the solution is not just a constant ϕ profile, and so even this solution has some mass.

A. The turning point

For the purposes of this article it suffices to examine the properties of the classical solutions relevant to the FGG process. There is no need to find the tunneling solutions and the corresponding tunneling actions to make our points. Furthermore, we can understand the relevant properties of these solutions (namely the ADM mass) simply by finding the solution at its turning point, which further simplifies our calculations. At the turning point $\dot\alpha = \dot a = \dot\phi = 0$ but second order time derivatives are nonzero. In this case, equation 5 reduces to
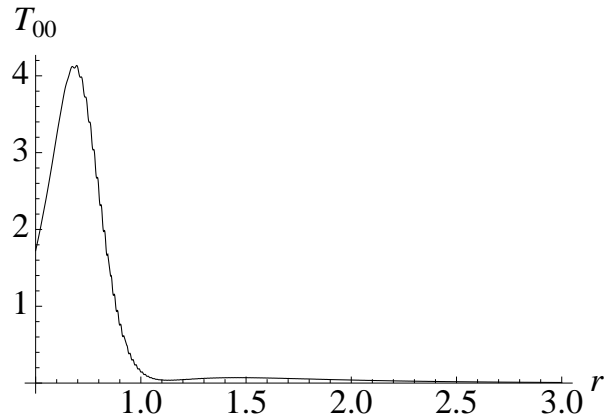

FIG. 3. T_{00}, demonstrating the wall "thickness", at a given time.

$$M = 2\pi \int_0^\infty r^2 \left[\left(\frac{\phi'}{a} \right)^2 + 2V(\phi) \right] dr \qquad (6)$$

Using Einstein's equation, this can be rewritten as (again setting $m_i = m_P$),

$$M = \frac{4\pi}{3} \int_0^\infty \left(\frac{2ra'}{a^3} - \frac{1}{a^2} + 1 \right) dr$$

so we see that $dm/dr = 0$ only for the Schwarzschild metric, as expected. Additionally, substituting the de Sitter metric into 6 gives the energy density times the volume of the bubble, just as one would expect.

Inspection of Eqn. 6 (which is positive definite since $V(\phi)$ is everywhere positive) one can see that we do not expect to find $M \equiv 0$ solutions, but there is no apparent reason why a smooth limit to zero should not exist.

There is freedom to specify the spatial profile of $\ddot\phi$ at the turning point, which we choose to be $\ddot\phi = c/r^2$ for a constant c. This is consistent with spherical symmetry and is sufficiently localized to maintain an asymptotically flat spacetime. Choosing such an ansatz simply enforces locality of the bubble and does not affect the generality of our conclusions.

B. The two solutions

Evolving forward in time from the turning point solution is used to classify the solution character. The energy density of the expanding solution expands into the domain as the metric functions approach de Sitter (see 3). The energy density of the collapsing solution collapses immediately toward the origin while the metric functions approach pure Schwarzschild. A plot of $T_{00}(t = 0)$, showing the "thickness" of the wall, for the expanding solution is given in figure 3. Plots of the field and metric functions for expanding and collapsing solutions are given in figures 4 and 5, respectively.

C. Results of trying to take $M \to 0$

Conceptually, there are two ways in which this can be done. From a cosmological perspective, we can fix the inflaton potential and attempt to take the mass to zero by changing the initial condition on $\ddot\phi$.

On the other hand, we can tune the two constants in the potential while keeping a fixed $\ddot\phi$. (As discussed and motivated above, we are keeping the overall scale m_i fixed for this investigation.) We begin with a parameter scan over three orders of magnitude, i.e. ranging the values of λ and γ from $0.1 \to 10$. Let the value of the field at the false minimum and the maximum be ϕ_{min} and ϕ_{max}, and the potential evaluated at these points be V_{min} and V_{max}, respectively. Figures 6 and 7 show how

the mass of the collapsing turning point solution depends on $\Delta\phi = \phi_{min} - \phi_{max}$ and $\Delta V = V_{max} - V_{min}$.

IV. DISCUSSION

Inspecting Eqn. 5, we see that the integrand, dm/dr, will not be zero unless ϕ is constant and $V(\phi) = 0$, i.e. exactly Schwarzschild space. The crux of our argument is that real potentials and fields do not admit nicely separated solutions; the field spills over into the whole domain and affects the metric functions, preventing the $M \to 0$ limit that appears to exist in the thin-wall formalism.

We still attempted to push the mass smoothly to zero. However, as Figures 6 and 7 show, we are unable to push the mass below about 0.1. This is with a fixed overall scale set by choosing $m_i = m_P$. The point is not the value of m_i (setting $(m_i/m_P)^2 = 10^{-3}$ does not affect our conclusions), but that we have fixed an overall scale.

In any spherically symmetric problem, there is the issue of what happens at $r = 0$. While the numerics cannot evolve such a point, we can make progress analytically by assuming we approach exact de Sitter space, in which the metric functions are regular at the origin. The scalar field potential can then be expanding about the minimum to second order in ϕ. The problem is then analytically tractable and solutions give positive mass contributions. Thus, our calculation of M really is a lower bound.

V. CONCLUSION

We considered classical solutions relevant to the Farhi-Guth-Guven tunneling process. For a generic quartic potential we are unable to take the mass of our turning point solutions smoothly to zero. Other authors have shown using the thin wall approximation that the FGG tunneling amplitude remains finite as $M \to 0$, and this
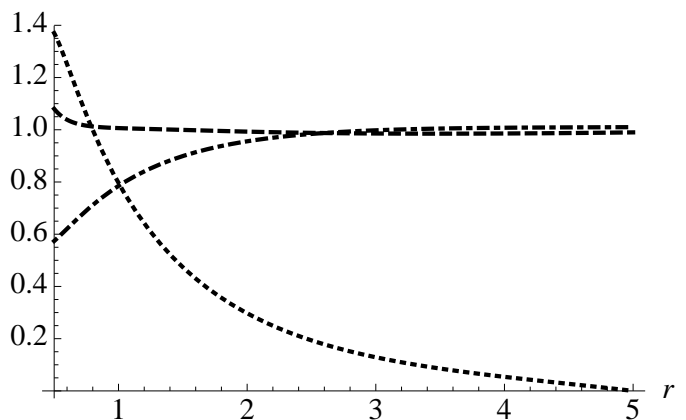

FIG. 4. Turning point slice of geometry and field for the expanding solution. Here ϕ is dotted, a is dash-dotted, and α is dashed.

strange behavior has been used to question the physicality of the FGG process. The absence of $M \to 0$ solutions in our more realistic thick wall calculations suggest that the $M \to 0$ behavior is an artifact of thin wall approximation and should not be used to argue that the FGG process is unphysical.
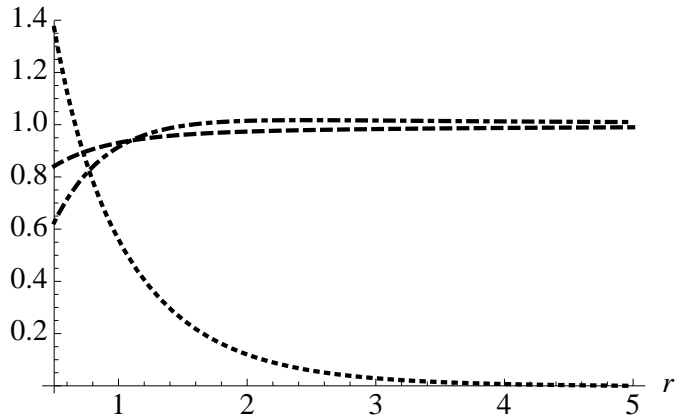
VI. ACKNOWLEDGMENTS

FIG. 5. Turning point slice of the collapsing solution. Here ϕ is dotted, a is dash-dotted, and α is dashed.

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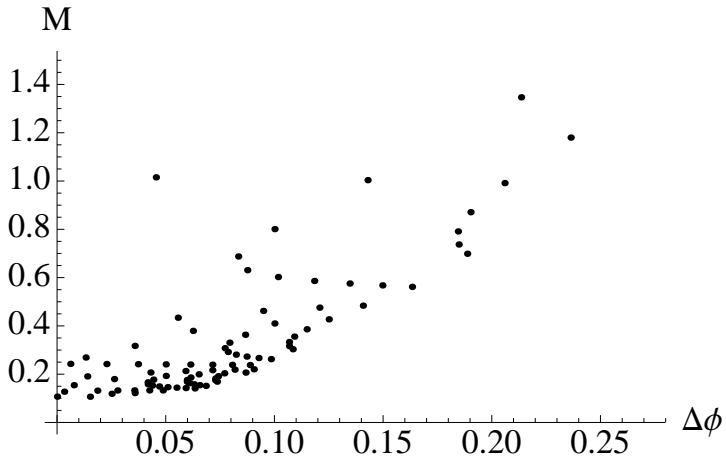[11] M. W. Choptuik, Phys. Rev. Lett., **70**, 9 (1993).

FIG. 6. Mass of the collapsing solution, evaluated at the turning point, as a function of $\Delta\phi$. Points represent a scan of potential parameters λ and γ. Small values of M were not found in the scan.
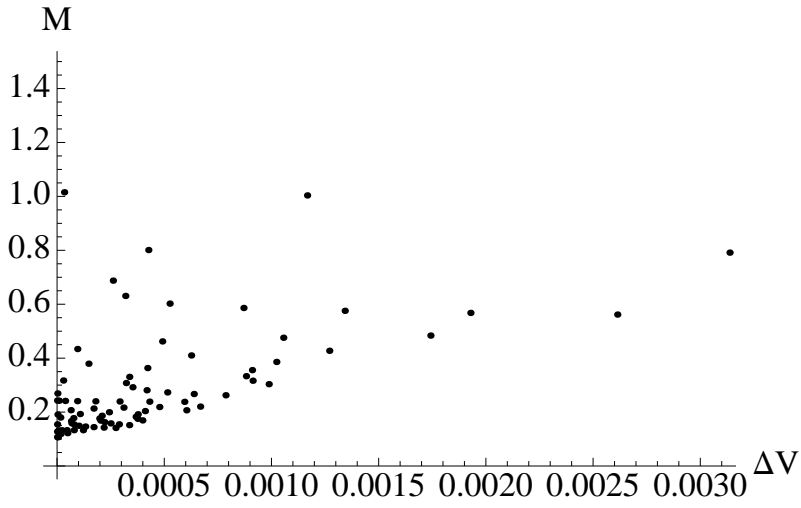

FIG. 7. Mass of the collapsing solution, evaluated at the turning point, as a function of ΔV. Points represent a scan of potential parameters λ and γ. Small values of M were not found in the scan.