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We compare observables to the finite size scaling hypothesis in SU(2) lattice gauge theory with two Dirac fermions in the adjoint representation. The fits that we obtain yield an estimate of the anomalous mass dimension that is consistent with four loop perturbation theory: $\gamma = 0.50 \pm 0.26$, with the error due to systematic uncertainties in the finite size scaling analysis. The result is consistent with Schrödinger functional studies and perturbative results.

I. INTRODUCTION

In technicolor models, the Higgs mechanism occurs through condensation of new fermions that are subject to a gauge interaction that is strong at the TeV scale [1, 2]. Walking technicolor is a version of this theory that can suppress flavor-changing neutral currents by raising the extended technicolor scale, while still having phenomenologically acceptable Standard Model fermion masses, due to condensate enhancement [3–5]. Higher representations of the gauge group are believed to avoid problems with the S-parameter, i.e. electroweak precision constraints [6, 7]. All of this has motivated the study of Minimal Walking Technicolor (MWTC) [8], which is SU(2) gauge theory with two Dirac fermions in the adjoint (triplet) representation.

In order to study technicolor proposals nonperturbatively and from first principles, several groups have been using the techniques of lattice gauge theory; see the review [9] and references therein. One of the key questions is whether the theory "walks" (very slow running of the coupling) or is attracted to an infrared fixed point (IRFP). An important quantity that can be computed in the process of answering this question is the anomalous mass dimension γ, which needs to satify $\gamma \approx 1$ in order for the standard walking technicolor picture to succeed. (Alternatives such as "ideal walking" are now being investigated as improvements over the standard picture [10].) One of the ways in which the lattice community has computed γ is through the Schrödinger functional method. It was employed for SU(3) gauge group with sextet fermions in [11] and for MWTC in [12, 13]. Analysis of the distribution of eigenvalues of the Dirac operator has also been used [14–16].

An alternative approach is to compare observables computed in lattice gauge theory (e.g., meson masses, the "pion" decay constant) to the finite size scaling (FSS) hypothesis. If the theory is indeed driven to an IRFP, then the data on observables should fit the FSS hypothesis. Previous studies of FSS in lattice technicolor include

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[15, 17–19]. Fits to the conformal hypothesis that assume a specific form of the FSS function include [20, 21], where the infinite volume hyperscaling relation is imposed. More general forms of the FSS function have also been considered recently by the authors of [20], with the result that for these forms the resulting conformal hypothesis for SU(3) gauge group and 12 fundamental flavors has a low degree of confidence in fitting the data [22]. By contrast, [19] advocates an approach that does not impose a specific form on the FSS function; this is one of the FSS methods used in the earlier work [15]. In this letter, we apply this method to the case of MWTC in order to extract an estimate of γ under the assumption that an IRFP exists.

II. FINITE SIZE SCALING

In the scaling regime, the correlation length will have an asymptotic behavior dependent on the fermion mass m with exponent y_m:

$$\xi \sim m^{-1/y_m} \qquad (2.1)$$

This exponent is related to the anomalous mass dimension evaluated at the IRFP:

$$y_m = 1 + \gamma(g_*) \qquad (2.2)$$

It is a general consequence of the renormalization group equations that the correlation length ξ_L at finite size L is given by a scaling function of the infinite volume correlation length ξ relative to L:

$$\xi_L/L = F(\xi/L) \qquad (2.3)$$

Thus we obtain the FSS formula in terms of fermion mass:

$$\xi_L/L = f(mL^{y_m}) \qquad (2.4)$$

Corrections to scaling will be an important consideration for us. This translates into a correction that is appreciable for small L, with an exponent ω:

$$\xi_L/L = f(mL^{y_m}) + L^{-\omega}g(mL^{y_m}) \qquad (2.5)$$

This form has also been considered in [22]; there it was pointed out that fitting data to such a hypothesis would require an extensive and highly accurate study. For us the main use of this equation is just that the scaling violations are largest for the smallest values of L. We use this as an interpretation of data on small lattices that does not fall on a scaling curve. Our present study is not extensive enough to fit to this more general form and extract ω. Below, we will consider $\xi_L = 1/M$ or $1/f_\pi$, where M is a meson mass.

We minimize the following function with respect to y_m.

$$P(y_m) = \frac{1}{N_{\text{over}}} \sum_p \sum_{j \neq p} \sum_{i,\text{over}} \left(\frac{\xi_L(m_{i,j})}{L_j} - f_p(L_j^{y_m} m_{i,j}) \right)^2 \tag{3.1}$$

Here i labels the different partially conserved axial current (PCAC) mass values for a given L_j. The effect of this is to find a y_m such that f_p for the other values $L_j, m_{i,j}$ is as close as possible to the curve obtained from fitting $L_p, m_{i,p}$. This is summed over all possibilities p. Also, "over" indicates that only i are used such that $m_{i,j} L_j^{y_m}$ falls within the range of values of $m_{i,p} L_p^{y_m}$, so that the comparison is to an interpolation of the $m_{i,p} L_p^{y_m}$ data, rather than an extrapolation. Unweighted fits were used so that the approximation to the scaling curve would pass through data at small $x = mL^{y_m}$, where absolute (statistical) errors are largest. (Using a weighted fit reduces our conclusion for γ by 4%.)

Type	$f(x)$
Quadratic	$c_0 + c_1 x + c_2 x^2$
Log quadratic	$c_0 + c_1 \ln x + c_2 (\ln x)^2$
Piece-wise log-linear	Straight lines connecting data

TABLE I. Interpolating functions that we use to fit data for a fixed L_p. In the last case, the straight lines interpolating between data are on a semi-log plot.

For the fitting function we have considered the possibilities listed in Table I. In the case of the quadratic we follow one of the methods of [15, 19]. The log quadratic fit was motivated by the behavior of the data when ξ_L/L is plotted versus $\ln(mL^{y_m})$, which is close to a parabola. The piece-wise log-linear form was used as a third choice that trivially passes through the data, giving a reasonable interpolation.

IV. RESULTS

We have used four observables: the "pion" mass m_π, the "rho" mass m_ρ, the "a_1" mass m_{a_1}, and the "pion" decay constant f_π. These are all obtained from standard correlation functions using point sources and sinks. We fit the correlation functions with a single exponential, allowing the first time t_{first} in the fit to be large enough for the excited state contributions to be negligible. This is determined by looking at the mass of the meson as a function of t_{first} and extracting the value on the plateau. Five values of bare masses

III. FITTING METHOD

The method described here seeks to optimize y_m such that all the data falls on a scaling curve. It is due to [23] and was used in [15, 19]. For each L we have a data set p. We use this to obtain a fit f_p. The types of fit functions that we consider will be described below. We then use this fit function on the other values of L, which we label as L_j.
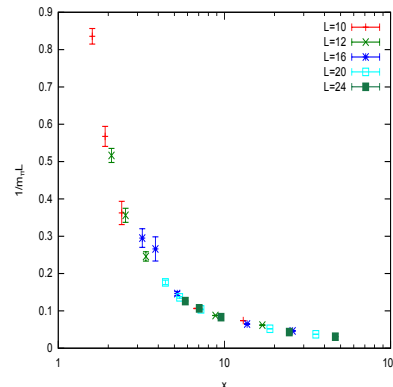

FIG. 1. Collapse of π data for $y_m = 1.46$. Here and in the other figure, $x = mL^{y_m}$.

$m_0 a = -1.0, -1.1, -1.165, -1.175, -1.18$ on lattices of size $L/a = 10, 12, 16, 20, 24$ were simulated, all at $\beta = 2.25$. These are the same configurations as were generated in [24], and the values of the PCAC mass and details on the simulations are given there. Also note that the size of the temporal direction is $T = 2L$.

Using these results, and performing the minimization described in the previous section, we obtain values for y_m. In the case of m_{a_1} and f_π, the quantity ξ_L/L is small, and scaling violations [cf. Eq. (2.5)] can compete with the scaling function for small lattices. For this reason we exclude the small lattices $L/a = 10, 12$ for these channels. The results for y_m are summarized in Table II. It can be seen that each of the channels, and each of the fitting methods are consistent with each other within

Observable	Quadratic	Log Quad	PWL
m_π	1.67(93)	1.26(54)	1.51(33)
m_ρ	1.67(88)	1.37(39)	1.56(31)
m_{a_1}	1.40(52)	1.42(27)	1.41(22)
f_π	1.65(22)	1.49(54)	1.60(29)

TABLE II. The scaling exponent $y_m = 1 + \gamma$ for the various observables and methods of interpolation. In parentheses, jackknife error is shown, obtained from eliminating one $m_{i,j}$ in all possible ways, in the minimization of (3.1). Because we use a large number of configurations, $\mathcal{O}(10^3)$, statistical error is negligible by comparison. Weighted averages and standard deviations are shown in the last column.
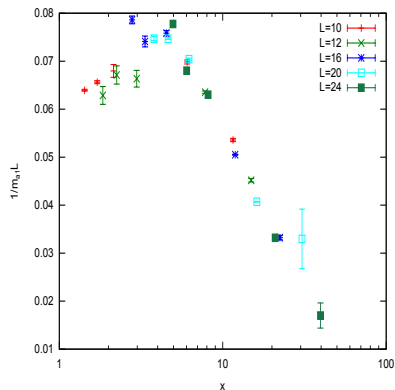

FIG. 2. The a_1 data for $y_m = 1.41$.

errors. The approximate collapse of data in the pion channel is shown in Fig. 1; the rho looks quite similar. In Figs. 2 we show the of scatter that occurs for the a_1; for f_π the spread in data is similar. In both cases it is the small L observables that are pulling away from the curve. We interpret this as being due to scaling violations, though a thorough study extracting ω would be required to demonstrate this. Another interpretation is that the theory does not have an IRFP, and so the FSS fails for some channels. It is also possible that we are seeing the effect of $\beta = 2.25$ not being close enough to the fixed point coupling. However we view the collapse seen in Fig. 1 as favoring our scaling violation interpretation.

We average the twelve values of y_m for the four channels and three fitting methods, weighted by the jackknife errors, to obtain $\overline{\gamma} = 0.50$. The standard deviation of the twelve fits is 0.13. However, the smallest jackknife error from single elimination of data is 0.22. Treated as separate systematic errors, we obtain

$$\gamma = 0.50 \pm 0.26 \qquad (4.1)$$

In Table III, we compare to other results using variety of methods. We are in agreement with all but the FSS studies [17, 18], though only 1.4σ different from their upper limit.

Method	γ
SF [12]	$0.05 < \gamma < 0.56$
SF [13]	0.31 ± 0.06
Perturbative 4-loop [25]	0.500
Schwinger-Dyson [26]	0.653
All-orders hypothesis [27]	0.46
MCRG [28]	$-0.6 < \gamma < 0.6$
FSS [17]	$0.05 < \gamma < 0.20$
FSS [18]	0.22 ± 0.06
FSS (here)	0.50 ± 0.26

TABLE III. Summary of all MWTC results for the anomalous mass dimension. SF is Schrödinger functional and MCRG is Monte Carlo renormalization group. The perturbative result $\gamma = 0.500$ was also given in the later, corrected version of [26], and relied on invariants calculated in [29].

V. CONCLUSIONS

We have applied the FSS approach of [19] (one of the approaches in [15]) to MWTC and find values of the critical exponent that are in agreement with perturbative results and Schrödinger functional results. While there are significant systematic uncertainties, which we interpret as being due to scaling violations on small volumes, the value of γ is too small for phenomenological models of condensate enhancement, which requires $\gamma \approx 1$. The complimentary information obtained by the present method suggests that it be applied in other gauge theories of interest for conformal or near-conformal dynamics. Indeed we expect it to work in any case for which the gauge coupling runs very slowly, so that fixed point behavior is well approximated on the scales probed by the study that is performed. Unfortunately, as explained in [19], a reasonable fit to the FSS assumption does not rule in or out the existence of an IRFP, since all that is required is a very slow running.

We have highlighted some of the systematic uncertainties of the method, and have illustrated how working on small volumes hampers the effort to obtain an accurate y_m. Future work includes simulations on larger volumes so that y_m can be obtained with greater certainty. Also, an improved lattice action should reduce the size of the scaling violations, and we are currently working in that direction for MWTC and other theories. Finally, other values of the bare coupling β should be studied, since there is uncertainty as to the precise location of the putative fixed point.

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