A proposal to solve some puzzles in semileptonic B decays

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A proposal to solve some puzzles in semileptonic B decays

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Some long-standing problems in the experimental data for semileptonic $b \to c\ell\bar{\nu}$ decay rates have resisted attempts to resolve them, despite substantial efforts. We summarize the issues, and propose a possible resolution, which may alleviate several of these tensions simultaneously, including the "1/2 vs. 3/2 puzzle" and the composition of the inclusive decay rate in terms of exclusive channels.

I. INTRODUCTION

There are several puzzling features of the semileptonic $b \to c$ decay data, which have existed with varying level of significance for over ten years. While individually these are not many sigma problems, they affect several measurements, and are a source of feeling uneasy about some semileptonic decay results. They relate to tensions between the measurements of inclusive and exclusive decays. The D meson states relevant for our discussion are listed in Table I. We refer to the first two states as $D^{(*)}$, the next four as D^{**}, and the last two as $D'^{(*)}$. The relevant semileptonic decay measurements are [1, 2]:

1. The inclusive rate, $\mathcal{B}(B^+ \to X_c\ell^+\nu) = (10.92 \pm 0.16)\%$, and various inclusive spectra in this decay;
2. The exclusive rates $\mathcal{B}(B^+ \to D\ell^+\nu) = (2.31 \pm 0.09)\,\%$ and $\mathcal{B}(B^+ \to D^*\ell^+\bar{\nu}) = (5.63 \pm 0.18)\,\%$;
3. The sum over the four rates, $\mathcal{B}(B^+ \to D^{**}\ell^+\nu) \times \mathcal{B}(D^{**} \to D^{(*)}\pi) = (1.7 \pm 0.12)\,\%$, composed of roughly equal rates for the sum over the two $s_l^{\pi_l} = \frac{1}{2}^+$ and the two $s_l^{\pi_l} = \frac{3}{2}^+$ states;
4. The semi-inclusive rate $\mathcal{B}(B^+ \to D^{(*)}\pi\ell^+\nu) = (1.53 \pm 0.13)\,\%$, including a $D^{(*)}$ and exactly one π.

The sum of the measured exclusive rates is less than the inclusive one (the value in item 1. is obtained from the more precise average branching ratio for B^0 and $B^\pm$ using equal semileptonic rates), and previous attempts to bring the two into agreement have faced problems. In particular, the inclusive rate (1.) minus those in items 2. and 4. gives $(1.45 \pm 0.67)\%$. Assigning this to semileptonic B decays to other nonresonant channels (i.e., to final states containing more than one hadrons, not included in item 4.), results in a too soft inclusive charged lepton energy spectrum, inconsistent with the data. There has also been a persistent $\sim 2\sigma$ difference between $|V_{cb}|$ extracted from inclusive and exclusive semileptonic B decays.

The charm meson states relevant for our discussion are organized as doublets of heavy quark spin symmetry, and are shown in Table I. The D and D^* states are the $1S$ ground state in the quark model. The next four D^{**} states are the $1P$ orbital excitations (with the spin and parity of the brown muck equal $s_l^{\pi_l} = \frac{1}{2}^+$ and $\frac{3}{2}^+$), and the D' and D'^* states correspond to the first radial excitation in the quark model (the $2S$ states).

Another issue which has received a lot of attention, but concerns a tension not simply between different pieces of data, but the comparison of theory with data, is the "1/2 vs. 3/2 puzzle". Model calculations predict that semileptonic B decays should have a substantially smaller rate to the $s_l^{\pi_l} = \frac{1}{2}^+$ doublet than to the $s_l^{\pi_l} = \frac{3}{2}^+$ doublet [3, 4], contrary to what is observed (item 3. above).

In past experimental analyses there have been various approaches to deal with these issues, typically making cuts with the hope of eliminating the effects of these discrepancies, and/or modifying some of the exclusive rates in the event generators.

Here we propose that these problems could be eased (or maybe even solved) by an unexpectedly large B decay rate to the first radially excited $D'^{(*)}$ states. Recently *BaBar* found evidence for two new states [6], which are most likely these $2S$ states in the quark model picture [7].

II. PROPOSAL AND VIABILITY

We would like to explore the possibility that the sum of the two $D'^{(*)}$ decay rates is substantial,

$$\mathcal{B}\big(B \to D'^{(*)}\ell\bar{\nu}\big) \sim \mathcal{O}(1\,\%)\,, \tag{1}$$

and show that it can help resolve the problems mentioned above, without giving rise to new ones.

1) The rate in Eq. (1) would be a big enough contribution to the sum over exclusive states, so that the

Notation	$s_l^{\pi_l}$	J^P	m (GeV)	Γ (GeV)
D	$\frac{1}{2}^-$	0^-	1.87	
D^*	$\frac{1}{2}^-$	1^-	2.01	
D_0^*	$\frac{1}{2}^+$	0^+	2.40	0.28
D_1^*	$\frac{1}{2}^+$	1^+	2.44	0.38
D_1	$\frac{3}{2}^+$	1^+	2.42	0.03
D_2^*	$\frac{3}{2}^+$	2^+	2.46	0.04
D'	$\frac{1}{2}^-$	0^-	2.54	0.13
D'^*	$\frac{1}{2}^-$	1^-	2.61	0.09

TABLE I. Charm meson states and their isospin averaged masses and widths. $D'^{(*)}$ denote the $2S$ excitation of $D^{(*)}$. The $s_l^{\pi_l}$ is the spin and parity of the light degrees of freedom, which is a good quantum number in the heavy quark limit [5].

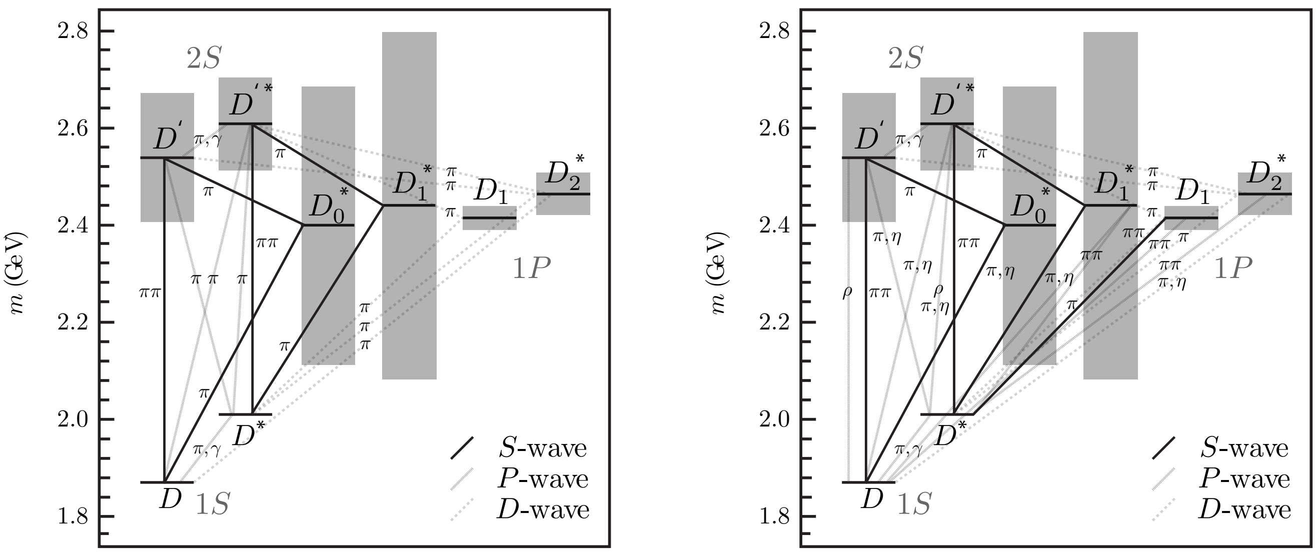

FIG. 1. Strong decays of the D' and D'^* into the $1S$ and $1P$ states involving, one or two pion emissions (left), and all decays including the near off-shell transitions with a ρ and η (right). The style and opacity of the lines connecting the states indicate the orbital angular momentum of the partial wave. The grey bands correspond to the measured widths of the $2S$ and $1P$ states.

nonresonant contribution [8] no longer needs to be large. This would be a problem, because in the soft pion limit a first principles calculation is possible [9], giving a too small rate at this region of phase space. A large nonresonant rate at high $D^{(*)}\pi$ invariant mass would disagree with the inclusive lepton spectrum measurements and the measured semi-exclusive $B \to D^{(*)}\pi\ell\bar\nu$ rate.

2) The $D'^{(*)}$ states decay to one of the $D^{(*)}$ states either with one pion emission in a p-wave, or with two pion emission in an s-wave. However, they can decay with one pion emission in an s-wave to members of the $s_l^{\pi_l} = \frac{1}{2}^+$ states, and could thus enhance the observed decay rate to the $s_l^{\pi_l} = \frac{1}{2}^+$ states, and thus give rise to the "1/2 vs. 3/2 puzzle". The allowed strong decays are illustrated in Figure 1 (including those only allowed by the substantial widths of these particles). It is plausible that the decay modes of the $D'^{(*)}$ to the $1S$ and $1P$ charm meson states may be comparable.

3) With the relatively low mass of the $D'^{(*)}$ states, the inclusive lepton spectrum can stay quite hard, in agreement with the observations.

4) The $\mathcal{B}(B \to D^{(*)}\pi\ell\bar\nu)$ measurement quoted is not in conflict with our hypothesis, since the decay of the $D'^{(*)}$ would yield two or more pions most of the time.

III. THE $B \to D'^{(*)}\ell\bar\nu$ DECAY RATE

Since the quantum numbers of the $D'^{(*)}$ are the same as those of the $D^{(*)}$, the theoretical expressions for the decay rates in terms of the form factors, and the definitions of the form factors themselves, are identical to the well known formulae for $B \to D^{(*)}\ell\bar\nu$ [10]. As for $B \to D^{(*)}\ell\bar\nu$, in the $m_{c,b} \gg \Lambda_{\rm QCD}$ limit, the six form factors are determined by a single universal Isgur-Wise function [11], which we denote by $\xi_2(w)$. Here $w = v \cdot v'$ is the recoil parameter, v is the velocity of the B meson, and v' is that of the $D'^{(*)}$. We define

$$\frac{\mathrm{d}\Gamma_{D'^*}}{\mathrm{d}w} = \frac{G_F^2 |V_{cb}|^2\, m_B^5}{48\pi^3}\, r^3 (1-r)^2 \sqrt{w^2-1}\,(w+1)^2 \times \left[1 + \frac{4w}{w+1}\,\frac{1-2rw+r^2}{(1-r)^2}\right] \big[F(w)\big]^2 , \qquad (2)$$

$$\frac{\mathrm{d}\Gamma_{D'}}{\mathrm{d}w} = \frac{G_F^2 |V_{cb}|^2\, m_B^5}{48\pi^3}\, r^3 (1+r)^2\, (w^2-1)^{3/2} \big[G(w)\big]^2 ,$$

where, in each equation, $r = m_{D'^{(*)}}/m_B$, and in the $m_{c,b} \gg \Lambda_{\rm QCD}$ limit $F(w) = G(w) = \xi_2(w)$.

Heavy quark symmetry implies $\xi_2(1) = 0$, so the rate near zero recoil comes entirely from $\Lambda_{\rm QCD}/m_{c,b}$ corrections. Away from $w = 1$, $\xi_2(w)$ is no longer power suppressed; however, since the kinematic range is only $1 < w < 1.3$, the role of $\Lambda_{\rm QCD}/m_{c,b}$ corrections, which are no longer universal, can be very large [12]. Before turning to model calculations, note that there is a qualitative argument that near $w = 1$ the slope of $\xi_2(w)$, and probably those of $F(w)$ and $G(w)$ as well, should be positive. In $B \to D'^{(*)}$ transition, in the quark model, the main effect of the wave function of the brown muck changing from the $1S$ to the $2S$ state is to increase the expectation value of the distance from the heavy quark of a spherically symmetric wave function. Thus the overlap of the initial and final state wave functions should increase as w increases above 1.

It is not easy to calculate these $B \to D'^{(*)}\ell\bar\nu$ form factors. Below, we use estimates from a quark model prediction [13], hoped to be trustable near $w = 1$, and from

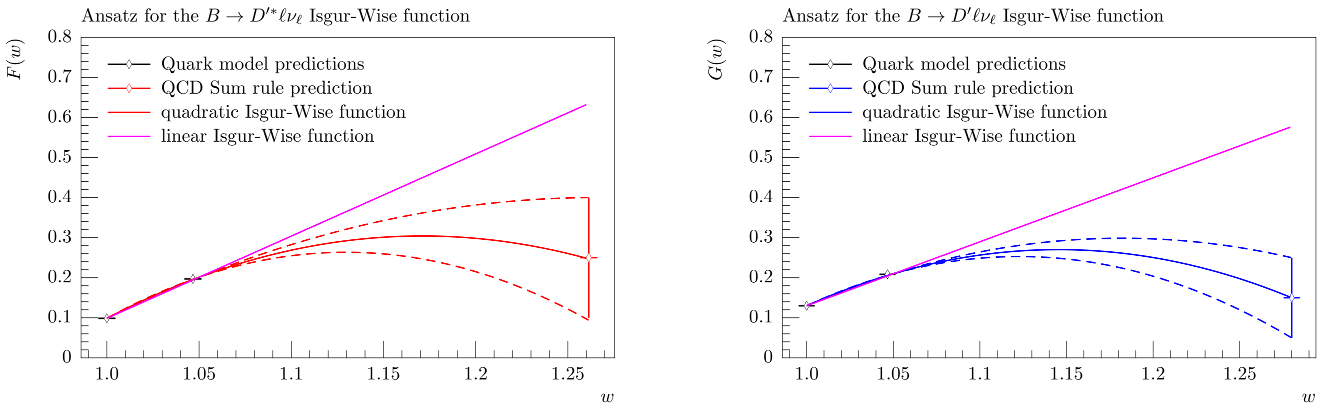

FIG. 2. The function $F(w)$ which determines the $B \to D'^{*}\ell\bar{\nu}$ rate (left) and $G(w)$ which determines $B \to D'\ell\bar{\nu}$ (right). The quark model calculations at $w = 1$ and 1.05 are compared with the sum rule prediction at $w_{\max}$. The solid lines show a quadratic and linear ansatz for the Isgur-Wise function and the dashed lines correspond to the variation of the sum rule parameters.

modifying a QCD light-cone sum rule calculation [14], hoped to be reasonable near maximal recoil. We emphasize that both models were developed, tuned, and tested for states that are the lightest with a given set of quantum numbers. Thus, one should take the following numerical estimates with a truck load of salt, and we present them only to substantiate that the rate in Eq. (1) is plausible and should be searched for experimentally. The same physical problem (and the width of the $D'^{(*)}$) would also provide a formidable challenge to lattice QCD calculations of the $B \to D'^{(*)}\ell\bar{\nu}$ form factors.

A quark model calculation of the values and slopes of the leading and subleading Isgur-Wise functions at zero recoil [13], imply

$$\begin{aligned} F(1.0) &= 0.10\,, \qquad & F(1.05) &= 0.20\,, \\ G(1.0) &= 0.13\,, \qquad & G(1.05) &= 0.21\,, \end{aligned} \tag{3}$$

where the values at $w = 1.05$ are obtained from a linear extrapolation. Perturbative QCD corrections enhance $F'(1)$ compared to $G'(1)$, by about 0.1 [15].

The light-cone sum rule calculation [14] can in principle be adapted to extract the $B \to D'^{(*)}$ form factors, i.e., the first radial excitation, near maximal recoil. We obtain the following estimates (the technical details are described in the Appendix),

$$F(w_{\max}) = 0.25 \pm 0.15\,, \qquad G(w_{\max}) = 0.15 \pm 0.1\,. \tag{4}$$

As one may anticipate, the largest uncertainty originates from the way the suppression of the ground state $D^{(*)}$ contribution is achieved, to project out the first radially excited state from the hadronic dispersion relation.

We parametrize the $F(w)$ and $G(w)$ functions which determine the $D'^{(*)}$ decay rates as quadratic polynomials, which is sufficient for our purposes,

$$\begin{aligned} F(w) &= \beta_0^* + (w-1)\beta_1^* + (w-1)^2\beta_2^*\,, \\ G(w) &= \beta_0 + (w-1)\beta_1 + (w-1)^2\beta_2\,. \end{aligned} \tag{5}$$

To get a rough estimate of the possible $B \to D'^{(*)}\ell\bar{\nu}$ rates, we determine the parameters in Eq. (5) to predict $F(w)$ and $G(w)$, as shown in Figure 2. Using the simple quadratic parametrization in Eq. (5) together with Eqs. (3) and (4) yield for the branching fraction of the sum of the two semileptonic $B \to D'^{(*)}\ell\nu_\ell$ decays

$$\mathcal{B}\big(B \to D'^{(*)}\ell\nu_\ell\big) \sim (0.3 - 0.7)\,\%\,, \tag{6}$$

with the parameters

$$\begin{aligned} \beta_0^* &= 0.10\,, \quad & \beta_1^* &= 2.3 - 2.5\,, \quad & \beta_2^* &= -(4.2 - 9.8)\,, \\ \beta_0 &= 0.13\,, \quad & \beta_1 &= 1.9 - 2.0\,, \quad & \beta_2 &= -(5.1 - 8.2)\,. \end{aligned} \tag{7}$$

Earlier quark model calculations, without accounting for $\Lambda_{\rm QCD}/m_{c,b}$ effects, obtained smaller rates [16, 17], while including $\Lambda_{\rm QCD}/m_{c,b}$ effects, 0.4% was obtained [13]. If, instead, we use a linear parametrization and the quark model results in Eq. (3) only, then we get

$$\mathcal{B}\big(B \to D'^{(*)}\ell\nu_\ell\big) \sim 1.4\%\,, \tag{8}$$

with the parameters

$$\begin{aligned} \beta_0^* &= 0.10\,, \qquad & \beta_1^* &= 2.1\,, \\ \beta_0 &= 0.13\,, \qquad & \beta_1 &= 1.6\,. \end{aligned} \tag{9}$$

We take these as indications that Eq. (1) is plausible, and $B \to D'^{(*)}\ell\bar{\nu}$ may account for a substantial part of the observed "gap" between inclusive and exclusive decays.

Another measurement that can constrain this picture are the nonleptonic rates, $B \to D'^{(*)}\pi$. Factorization, which was proven to leading order in the heavy mass limit in the decays we consider [18], implies that in each channel the nonleptonic rate is related to the semileptonic differential decay rate at maximal recoil,

$$\Gamma(B \to D'^{(*)}\pi) = \frac{3\pi^2 C^2\,|V_{ud}|^2 f_\pi^2}{m_B\, m_{D'^{(*)}}}\, \frac{{\rm d}\Gamma(B \to D'^{(*)}\ell\bar{\nu})}{{\rm d}w}\bigg|_{w_{\max}}. \tag{10}$$

Here C is a combination of Wilson coefficients and numerically $C\,|V_{ud}| \approx 1$, and $w_{\max}$ corresponds to $q^2 = 0 \simeq$

m_π^2. Thus, besides a direct search for $B \to D'^{(*)}\ell\bar{\nu}$ decays, measuring the nonleptonic $B \to D'^{(*)}\pi$ rates would also be very valuable to constrain $F(w)$ and $G(w)$. This type of measurement, including a Dalitz plot analysis of $\overline{B} \to [D^{(*)}\pi^+\pi^-]\pi^-$, would also be valuable in understanding the decay rates of the D'^* states.

IV. CONCLUSIONS

If future measurements find a substantial $B \to D'^{(*)}\ell\bar{\nu}$ decay rate, the precise determination of the branching fraction, the shape of the $F(w)$ and $G(w)$ functions in Eq. (2), and data on the corresponding nonleptonic two-body decays with a pion would be able to test this picture. It may also impact other measurements and the theory of semileptonic decays, e.g., it may yield

- a better understanding of the $b \to c$ background in fully inclusive $b \to u$ measurements, i.e., lead to a more precise determination of $|V_{ub}|$;
- a better understanding of the semileptonic $b \to c$ background in the exclusive $|V_{cb}|$ measurements using $B \to D^{(*)}\ell\bar{\nu}$;
- a better understanding of the missing exclusive contributions to the inclusive $B \to X_c\ell\bar{\nu}$ rate, and the lepton energy and hadronic mass spectrum;
- a better understanding of the measured $B \to D^{(*)}\tau\bar{\nu}$ branching fraction and its tension with respect to the Standard Model expectation [19];
- a more precise determination of the semileptonic branching fractions of the $s_l^{\pi_l} = \frac{1}{2}^+$ and $\frac{3}{2}^+$ states, thus maybe help resolve the "1/2 vs. 3/2 puzzle";
- a stronger sum rule bound [20–22, 12] on the $B \to D^*\ell\bar{\nu}$ form factor, $\mathcal{F}(1)$, relevant for the determination of $|V_{cb}|$ from exclusive decay.

There are a number of measurements that should be possible using the BaBar, Belle, LHCb, and future e^+e^- B factory data samples, which could shed light on whether this possibility is realized in nature.

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Appendix: Details of the Sum Rule Estimate

The crucial ingredient in obtaining Eq. (4) is to modify the light-cone sum rule (LCSR) computation [14] in a manner that the radially excited state can be projected out. We are not aware of similar attempts, so we give some details of our calculation. We write out explicitly the pole of the radial excitation in the hadronic dispersion relation and multiply the formula by the ground state pole, e.g., schematically shown for the decay constant

$$\frac{m_D^4 f_D^2}{m_c^2(m_D^2 - q^2)} + \frac{m_{D'}^4 f_{D'}^2}{m_c^2(m_{D'}^2 - q^2)} + \int_{s_0^{D'}}^{\infty} \mathrm{d}s \frac{\rho(s)}{s - q^2} . \quad \text{(A.1)}$$

Projecting out the radial excited state amounts to modifying the Borel transformation according to

$$B_{q^2} \frac{m_D^2 - q^2}{(s - q^2)^k} = \frac{\left[(k-1)m^2 - (s - m_D^2)\right]}{(k-1)!} \frac{e^{-s/m^2}}{(m^2)^{k-1}} , \quad \text{(A.2)}$$

which leads to a correction term for the sum rule for the form factors of the first radially excited state compared to the expressions in [14]. In order to use the known LCSR up to three particle contributions, one needs to apply a correction term to the Borel transform, $\zeta_k(s, m^2, m_D^2, m_{D'}^2)$ for $k = 1, 2, 3$

$$\zeta_k(s, m^2, m_D^2, m_{D'}^2) = \frac{(k-1)m^2 - (s - m_D^2)}{m_D^2 - m_{D'}^2} . \quad \text{(A.3)}$$

Due to necessary partial integration, it is a nontrivial endeavor to implement this correction term. A more detailed study of the numerical stability is, in principle, possible by multiplying the formulae with higher powers of the ground state pole. This modifies the correction term ζ_k according to

$$(m_D^2 - q^2)^n = \sum_{k=0}^{n} \binom{n}{k} (-q^2)^k (m_D^2)^{n-k} , \quad \text{(A.4)}$$

but does not affect the formal ground state suppression, since $B_{q^2}(q^2)^k = 0$ for $k \geq 0$. However, this is beyond the scope of our estimate in the context of this analysis, so we quote the results for the simplest calculation.

The resulting values for the form factors are sensitive to the numerical input values for the decay constants, the Borel and duality parameters. The latter parameters can be varied to estimate the sensitivity of the final result. The duality parameter, which has to be chosen higher than the corresponding meson mass, approximates the spectral density over the remaining physical resonances.[1] Presumably for higher excited states the ratio s_0^D/m_D^2 of the corresponding state should be

[1] For the $1S$ state this parameter can be estimated by demanding to reproduce the meson mass, which is not possible for the $2S$.

chosen higher than usual, due to the spectral density shape. The Borel parameter m, which suppresses exponentially the higher states, needs to be chosen large enough to obtain a reliable perturbative expansion, but small enough to not loose the sensitivity to the radially excited state (the influence of higher dimensional quark condensates increases with decreasing Borel parameter). Compared to Ref. [14], additional uncertainties emerge from (i) the approximate suppression of the ground state; (ii) the smaller separation to higher excited states; and (iii) larger perturbative and nonperturbative corrections. A further complication arises due to the poor knowledge of the $D'^{(*)}$ decay constants, which are needed as an input to the sum rules. Following a similar approach as in Ref. [23], we estimate the decay constants of the radially excited states, which prove fairly sensitive to the particular choice of Borel and duality parameters. We assume that the ratio of the decay constants for the radial excited states should be similar to that in the ground state, i.e., $f_{D'^*}/f_{D^*} \sim 1.4$, which holds for the parameters we choose,

$$f_{D'^*} \sim 300\,\mathrm{MeV}\,, \qquad f_{D'} \sim 200\,\mathrm{MeV}\,. \tag{A.5}$$

The $D'^{(*)}$ decay constants enter the sum rules, and are an additional source of uncertainty. We find a stable plateau for the various form factors with respect to the Borel and duality parameters, yet at values which should be too high from physical considerations. For the quoted D' and D'^* form factors we choose a duality parameter of $s_0^{D'} = 15\,\mathrm{GeV}^2$ and $s_0^{D'^*} = 17\,\mathrm{GeV}^2$, respectively, and a common Borel parameter of $m^2 = 7\,\mathrm{GeV}^2$, which are smaller than the ones at the plateau, resulting in a smaller form factor. The parameters chosen for computing the ground state yields a value close to zero for both form factors, in agreement with the expected suppression of the ground state contribution.

One may be concerned about the level of heavy quark symmetry violation, such as the deviation of $F(w)/G(w)$ from unity. Deviations are due to $\Lambda_{\mathrm{QCD}}/m_{b,c}$ effects as well as perturbative corrections. Using the sum rule prediction, one obtains

$$F(w_{\max})/G(w_{\max}) = 1.7 \pm 0.6\,. \tag{A.6}$$

where we assumed a 90% correlation between the uncertainties due to the choice of the Borel and duality parameters. For the form factor ratios R_1 and R_2 we obtain at maximal recoil,

$$R_1(w_{\max}) = 2.0 \pm 0.4\,, \qquad R_2(w_{\max}) = 1.1 \pm 0.3\,. \tag{A.7}$$

Interestingly, their ratio, $R_1(w_{\max})/R_2(w_{\max}) = 1.8 \pm 0.2$, is not far from the similar ratio for the D^* case.

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