Neutrinos with velocities greater than c?

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Neutrinos with velocities greater than c ?

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It is shown that possible observation by direct kinematical means of a neutrino moving faster than the conventional velocity of light, c could be an indication of dark matter "bathing" the Earth. In such a situation, the conventional velocity of light would be interpreted as the velocity of light in dark matter. The larger "true velocity" of light in vacuum is designated, c_t.

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I. INTRODUCTION

Conventionally the speed of light in vacuum is taken [1] to be:

$$c = 2.99792458 \times 10^8 m/s. \tag{1}$$

We will be working here in the framework of Special Relativity [2]. In that theory, of course, the energy of a massive particle with velocity greater than c would be pure imaginary and hence unphysical. However if one grants, the widely accepted assumption that the Earth, among other places in our galaxy, is bathed in dark matter [3] there is the likelihood that measurements of the speed of light in the neighborhood of Earth did not correspond to the velocity of light in vacuum but the velocity of light in dark matter. Then the true velocity of light in vacuum could be expected to have a larger value, c_t.

Note that the need for dark matter is well established both in cosmology [4] and for the explanation [5] of the galactic rotation curves. There are many proposed dark matter candidates. Here we consider a possible way to test whether the Earth itself is also "bathed" in dark matter.

It is expected that a low strength interaction of dark matter with photons (and perhaps also neutrinos) exists at some level. If these circumstances are granted, all experimental measurements of the velocity of light on and near Earth have presumably *not* been carried out for the velocity of light in vacuum but for the velocity of light in dark matter. Hence we introduce a dark matter index of refraction n_γ which is greater than unity such that,

$$c = c_t/n_\gamma \tag{2}$$

where c_t is the true velocity of light in vacuum. It is convenient to write,

$$c_t = c + \Delta, \quad \Delta > 0. \tag{3}$$

Clearly c_t should be used instead of Eq. (1) in the formulation of relativistic mechanics.

Of course, in an earth based neutrino experiment, neutrinos typically travel through the earth itself and would be slowed down by ordinary matter. Here we will argue that neutrinos are generally slowed down less by interactions than are photons. Hence it is possible that neutrinos are slowed less by ordinary matter than photons are slowed by dark matter. In that case one could have the neutrino velocity, v greater than the conventional c but less than c_t. Then an earthbound experiment yielding v greater than c would be a possible indication of dark matter surrounding the Earth.

The index of refraction for neutrinos in ordinary matter, n_ν defined by the neutrino velocity v should be written,

$$v = c_t/n_\nu \tag{4}$$

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and, for our argument, should result in v greater than c (or n_ν less than n_γ). Here we are temporarily neglecting, for simplicity, the effect of the small neutrino mass in slowing the neutrino. It is convenient to describe the desired requirement by

$$v = c + \delta, \quad \delta > 0. \tag{5}$$

Evidently we must have,

$$\Delta > \delta. \tag{6}$$

To compare neutrino and photon slowing due to interactions we mention the standard formula [6] for the index of refraction for a wave in a medium:

$$n = 1 + \frac{2\pi N f(0)}{k^2}, \tag{7}$$

where N is the number of scattering centers per unit volume, k is the spatial momentum parameter of the photon or neutrino and $f(0)$ is the forward scattering amplitude for the photon or neutrino scattering off the medium.

The detailed description of either photons or neutrinos interacting with dark matter and/or ordinary matter is rather complicated. However, at least we may say that the photon case involves electromagnetic-strength interaction vertices while the neutrino case involves weak-strength interaction vertices. Hence it is certainly plausible that $f(0)_\gamma$ should be considerably larger than $f(0)_\nu$. Accepting this, we expect n_γ to be considerably larger than n_ν.

As an initial approximation it thus seems reasonable to consider that the neutrino is slowed down just by its nonzero mass while the photon is slowed down by its interaction with dark matter in addition to its interaction with ordinary matter.

II. NEUTRINO ENERGY

Introducing an effective neutrino mass m_ν (to be discussed further in the next section) we write the neutrino energy in the usual way:

$$E_\nu = \frac{m_\nu {c_t}^2}{\sqrt{1 - v_\nu^2/{c_t}^2}}. \tag{8}$$

Using Eqs (3) and (5), while throwing away the very small terms quadratic in Δ and δ, gives for this equation:

$$E_\nu \approx \frac{m_\nu c^2}{\sqrt{2}} \sqrt{\frac{c}{\Delta - \delta}} \tag{9}$$

Note that c in this equation is the conventional one of Eq.(1).

Since this formula may look a little unfamiliar, we compare it with the analogous formula in standard relativity for the energy of a very relativistic particle with velocity $c - \epsilon$ and mass m:

$$E_{standard} \approx \frac{mc^2}{\sqrt{2}} \sqrt{\frac{c}{\epsilon}}. \tag{10}$$

In the present model, the analog of this equation reads:

$$E_{present} \approx \frac{m{c_t}^2}{\sqrt{2}} \sqrt{\frac{c_t}{\epsilon'}}, \tag{11}$$

where $\epsilon' = c_t - v$.

It is clear that rewriting Eq.(9) as,

$$\frac{\Delta - \delta}{c} = \frac{(m_\nu c^2)^2}{2E_\nu^2}, \tag{12}$$

enables us to estimate $(\Delta - \delta)/c$ in terms of the (assumed to be measured) neutrino energy) and plausible values of the effective neutrino mass, m_ν. The result is shown, for a reasonable range of effective neutrino masses and an assumed neutrino energy of 17 GeV) in Fig. 1.. Qualitatively, it seems that Δ is just a tiny bit bigger than δ in this case .

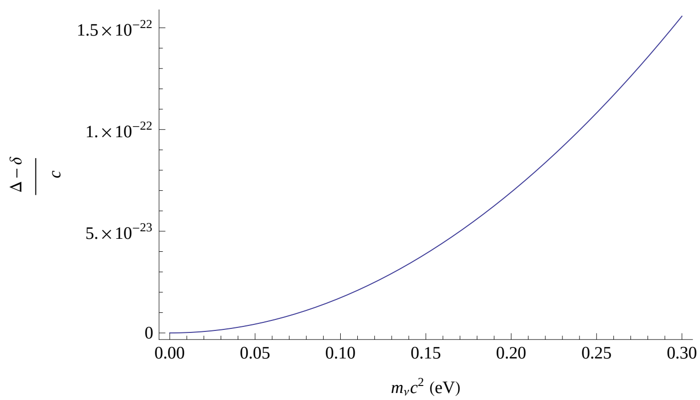

FIG. 1: Plot of $\frac{\Delta-\delta}{c}$ vs the effective neutrino mass

III. EFFECTIVE NEUTRINO MASS

In a typical earthbound experiment of interest to us, one might investigate, for example, the motion of the mu type neutrino ν_μ. This is, of course, not a mass eigenstate, but a linear combination of the three mass eigenstates ν_i :

$$\nu_\mu \approx K_{21}\nu_1 + K_{22}\nu_2 + K_{23}\nu_3, \tag{13}$$

where the K_{2i} are the elements of the leptonic mixing matrix. (For this we made the approximation that the charged lepton mass matrix in the "standard model" is diagonal.) Thus, some prescription is clearly needed to define the "average mass of the muon type neutrino" which was used in the preceding section. In models, say for the case of Majorana neutrinos [7], the mixing results from bringing an "original mass matrix, M to diagonal form, $\hat{M}$,

$$\hat{M} = K^T M K, \tag{14}$$

where K is unitary. It is suggestive to identify the average mass of the muon neutrino as,

$$m_\nu = M_{22} = (K^{-1T})_{2i}\hat{M}_{ii}K^{-1}_{i2}. \tag{15}$$

In the approximation where the matrix K is real and using $m_i = (\hat{M})_{ii}$,

$$m_\nu \approx (K_{2i})^2 m_i \tag{16}$$

This expression clearly has the correct behavior for the formal limit in which $|K_{22}|$ dominates.

The recent experimental situation concerning the $|K_{2i}|$'s is discussed in [8]. For definiteness we will adopt the values obtained in the "tribimaximal" mixing model [9]:

$$K_{21} = 1/\sqrt{6}, \quad K_{22} = 1/\sqrt{3}, \quad K_{23} = 1/\sqrt{2}. \tag{17}$$

Small corrections to these values have been considered by a number of authors.

With the mentioned approximations we get a rough idea of the averaged muon type neutrino mass,

$$m_\nu \approx m_1/6 + m_2/3 + m_3/2 \tag{18}$$

At present, the individual neutrino masses, m_i are not known. Only the differences $m_2^2 - m_1^2$ and $|m_3^2 - m_2^2|$ are available. Thus an absolute mass measurement, like one arising from the study of the tritium decay endpoint spectrum is required to complete the picture. However, we may bound m_ν by using the bound [10] from cosmology,

$$(m_1 + m_2 + m_3)c^2 < 0.3 \quad \text{eV}. \tag{19}$$

Introducing the 3-vectors $\mathbf{A} = (1/6, 1/3, 1/2)$ and $\mathbf{m} = (m_1, m_2, m_3)$ we write,

$$m_\nu = \mathbf{A} \cdot \mathbf{m} < |\mathbf{A}||\mathbf{m}| = \frac{\sqrt{14}}{6}|\mathbf{m}|. \tag{20}$$

Using the positivity of the m_i's we note,

$$|\mathbf{m}| < m_1 + m_2 + m_3. \tag{21}$$

With the help of Eq. (19) we get

$$m_\nu c^2 < 0.3\frac{\sqrt{14}}{6} = 0.187 \quad \text{eV}. \tag{22}$$

From Fig. (1) we finally read off the bound on the extremely small deviation $\frac{\Delta-\delta}{c}$,

$$\frac{\Delta - \delta}{c} < 7 \times 10^{-23}. \tag{23}$$

This very small typical value seems to be compatible with a bound of about 10^{-20} [11] on non trivial contributions to the dispersion relation (Cauchy formula) for free space.

IV. SUMMARY AND DISCUSSION

We discussed the possibility of detecting a cloud of dark matter which may be " bathing" the Earth by using the results of possible future accelerator experiments which measure the time of flight for laboratory produced neutrino beams. If the time of flight corresponds to neutrinos traveling faster than the conventional velocity of light c, it might indicate that the conventional c is not the velocity of light in vacuum but the velocity of light in dark matter. The true velocity of light, designated c_t would then be larger than c and neutrinos would not be violating Relativity provided their speed was less than c_t.

In section I the plausibility of neutrinos being less affected than photons by the effect of matter was argued based on the fact that the photon interactions involve a typical electromagnetic strength vertex while the neutrino interactions contain a typically smaller weak-strength vertex.

In section II the strength of the effect was related the to the energy of the neutrino beam and an effective neutrino mass.

The effective neutrino mass was interpreted in section III as corresponding to a suitable average over the masses of the three neutrino "mass eigenstates", weighted by measured elements of the neutrino mixing matrix. Typically the effect was seen to be numerically small and possibly consistent with the dispersion relation bound for "free space"

It should be mentioned that any future measurement of neutrino velocity greater than the apparent speed of light would have to satisfy, for consistency, the bound on neutrino travel time arising from the observed time difference between neutrino and photon arrivals from the supernova 1987A event (See, for examples ref. [3]).

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