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Galilean currents and charges

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According to Noether’s theorem, each symmetry of the dynamics comes with an associated conservation law. Despite the importance of relativity symmetries—i.e. invariances under Galileo or Lorentz boosts—their associated conservation laws are never emphasized. Here we clarify the physical interpretation of such conservation laws: there exists a collective coordinate—the center-of-mass position—that undergoes uniform motion. This does not happen for theories that do not obey a relativity principle. As an application, we derive the Noether currents and charges associated with an *internal* galilean invariance $\pi(x) \rightarrow \pi(x) + b_\mu x^\mu$ —a symmetry recently postulated in the context of so-called galileon theories.

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Introduction. For any Lagrangian system, Noether’s theorem guarantees the existence of a conserved quantity (or ‘charge’) for each symmetry enjoyed by the dynamics. Standard examples include energy, momentum, and angular momentum, as the conserved charges corresponding to time translations, spacial translations, and rotations, respectively. In field theory we are also accustomed to having internal symmetries acting on the field variables, with associated conserved charges, like e.g. the electric charge. Relativity—be it of the Galileo type or of the Lorentz one—is also a crucial symmetry, in Newtonian theories as well as in ‘relativistic’ ones. Yet its associated conservation law is very rarely mentioned, and somewhat mysterious (see e.g. [11]). It is the purpose of the present letter to clarify the physical meaning of such a conservation law. We will see that it amounts to the statement that the system’s center of mass moves at constant speed. This is simply not true for systems that do not obey a relativity principle. As an application of our results, we will derive the conservation laws associated with the *internal* galilean invariance $\pi(x) \rightarrow \pi(x) + b_\mu x^\mu$ enjoyed by the so-called galileon field theories [1].

Noether theorem for boost invariance. Consider a system whose dynamics are invariant under Galilean boosts,

$$\vec{x} \rightarrow \vec{x} + \vec{v}_0 t . \quad (1)$$

We will discuss the analogous case of Lorentz invariance below. The position \vec{x} can be a dynamical degree of freedom, like for a non-relativistic mechanical system made up of point particles and parameterized by their positions, or an integration variable, that is an argument for fields in a field theory. Typically systems that are galilean invariant are also translationally invariant—for instance this is guaranteed if the system is galilean invariant and *time*-translationally invariant—so that we also

have a symmetry

$$\vec{x} \rightarrow \vec{x} + \vec{x}_0 . \quad (2)$$

Now we apply the standard derivation of the Noether theorem. Let us start with the symmetry (2). Because of it, the variation of the action under an infinitesimal weakly time-dependent translation with parameter $\vec{x}_0(t)$ must start at order $\dot{\vec{x}}_0$:

$$\delta S \simeq \int dt \dot{\vec{x}}_0 \cdot \vec{P} . \quad (3)$$

On a solution to the equations of motion the action is stationary, which implies the conservation of \vec{P} . As is well known, the charge \vec{P} associated with spatial translations is the total momentum of the system.

Now, the fact that the system is also invariant under (1) implies that, in fact, δS in (3) should start at order $\ddot{\vec{x}}_0$,

$$\delta S \simeq - \int dt \ddot{\vec{x}}_0 \cdot \vec{\Xi} \quad (4)$$

(the minus sign upfront is for notational convenience). This is equivalent to saying that \vec{P} in (3) is itself a total time derivative,

$$\vec{P} = \dot{\vec{\Xi}} , \quad (5)$$

which combined with the conservation of \vec{P} , implies that on all solutions $\vec{\Xi}$ is a linear function of time:

$$\vec{\Xi}(t) = \vec{\Xi}_0 + \vec{P} t . \quad (6)$$

$\vec{\Xi}(t)$ is nothing but the center of mass position \vec{X}_{cm} times the total mass of the system. Eq. (6) is the global conservation law associated with Galilean invariance. To explicitly check this, let’s perform a galilean transformation (1) with mildly time-dependent $\vec{v}_0(t)$. We have

$$\delta S \simeq \int dt \dot{\vec{v}}_0 \cdot \vec{Q} , \quad (7)$$

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implying conservation of the quantity \vec{Q} on a solution. We can express \vec{Q} in terms of \vec{P} and $\vec{\Xi}$, by noticing that the time-dependent Galilean transformation we are performing can also be viewed as a time dependent translation with parameter $\vec{x}_0(t) = \vec{v}_0(t)t$. From (4) and (5) we have

$$\delta S \simeq \int dt (\dot{\vec{v}}_0 t + \vec{v}_0) \cdot \dot{\vec{\Xi}} \quad (8)$$

which compared with (7) yields

$$\vec{Q} = \vec{P}t - \vec{\Xi}. \quad (9)$$

The conservation of \vec{Q} is thus equivalent to eq. (6), with $\vec{\Xi}_0 = -\vec{Q}$. Therefore, we see that the conservation law associated with Galilean invariance is more conveniently rephrased as the statement that there is a quantity—the center of mass position—that evolves linearly in time. This is in addition to, and not a consequence of, the conservation of the total momentum. Of course if \vec{P} is conserved, we can always say that $\vec{P}t$ evolves linearly in time. But eq. (6), with $\vec{\Xi}$ defined in (4), is a much stronger statement. It says that there is a quantity $\vec{\Xi}(t)$ that is *a local functional of the dynamical variables, with no explicit time dependence*, that happens to evolve linearly in time. It is as strong as an ordinary conservation law, whereby a local functional of the dynamical variables happens to be constant in time. To be completely clear, by ‘local functional’ we mean, in the point-particle mechanical case, a function of the particles’ coordinates and their time derivatives, all evaluated at the same time, and in the field theory case, the space integral of a local density. Indeed, the standard expressions for $\vec{\Xi}$ for Galilean mechanics is

$$\vec{\Xi}_{\text{mech.}}(t) = \sum_a m_a \vec{x}_a(t) \quad (10)$$

Of course all of the above discussion can be straightforwardly generalized to the relativistic case. Indeed for a Lorentz-invariant field theory the Noether charges associated with the Lorentz boosts are (see e.g. [11])

$$J^{0i} = tP^i - \int d^3x T^{00}x^i, \quad (11)$$

where $T^{\mu\nu}$ is the (symmetric) stress-energy tensor. They are close relatives of the angular momentum, but unlike it, they are usually glossed over. From the above discussion their physical interpretation is clear: their conservation implies the existence of a collective coordinate

$$\vec{\Xi}(t) = \int d^3x T^{00}(x) \vec{x} \equiv E\vec{X}_{\text{cm}}(t) \quad (12)$$

that evolves linearly with time. The point is often made that in special relativity there is a well defined center-of-mass *frame*—that where the total momentum

vanishes—but no well defined center-of-mass *position*—because, given the relativity of simultaneity, it does not make sense to add up the position vectors for different particles at the same time. In (12) we see that there is a well defined quantity that behaves like a center of mass position should: it moves at the same constant speed as the center-of-mass frame, and in the non-relativistic limit it matches the standard Newtonian definition. Only, it does not transform as the spacial part of a position four-vector, but rather as the 0*i* part of a tensor:

$$\Xi^{\mu\nu} \equiv \int d^3x T^{0\mu}(x) x^\nu. \quad (13)$$

It should be emphasized that the Noether charges associated with Galileo or Lorentz boosts do not commute with the Hamiltonian of the system—or in the classical case, they have non-vanishing Poisson brackets with it. The reason is that they are conserved—because of Noether’s theorem—yet they depend *explicitly* on time. As a result, the energy eigenstates of the system do not carry definite charge under boosts. This is partially the reason why we usually ignore these charges.

Counter-examples. To convince ourselves that the linear evolution of $\vec{\Xi}$ and the conservation of \vec{P} are really independent physical laws, we can consider a mechanical system that is invariant under translations but not under boosts, like for instance one made up of many point-particles interacting via a two-body potential and with non-Newtonian (and non-Lorentzian) kinetic energies:

$$S = \int dt \sum_a m_a \left(\frac{1}{2} \dot{\vec{x}}_a^2 + \alpha \dot{\vec{x}}_a^4 \right) - \sum_{a < b} V_{ab}(|\vec{x}_a - \vec{x}_b|). \quad (14)$$

The system is invariant under translations, and thus the total momentum

$$\vec{P} = \sum_a m_a \dot{\vec{x}}_a (1 + 4\alpha \dot{\vec{x}}_a^2) \quad (15)$$

is conserved. Nevertheless, for nonzero α the system is not invariant under galilean transformations, and as a consequence \vec{P} is not associated with the time derivative of a collective coordinate—there is no local combination of \vec{x}_a and $\dot{\vec{x}}_a$ that evolves linearly in time. *The conservation of \vec{P} does not imply that the system as a whole moves at constant speed.* It only does for Galilean (or Lorentz-) invariant theories.

As an other example, consider a free scalar field ϕ living in a background Friedmann-Robertson-Walker (FRW) universe. The (flat) FRW metric,

$$ds^2 = -dt^2 + a^2(t) d\vec{x}^2, \quad (16)$$

is invariant under spacial translations, but does not feature any boost invariance: there is a preferred set of observers—the ‘comoving’ ones—and relative motion with respect to them is physical and measurable. As a consequence, there must exist a global three-dimensional

momentum that is conserved but that cannot be written as the time derivative of a collective center-of-mass coordinate. From the action,

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right], \quad (17)$$

it is straightforward to derive via Noether's theorem the currents associated with spacial translations. They are

$$J^{\mu(i)} \equiv a^5(t) T^{\mu i}, \quad (18)$$

where i labels the translation's direction, and $T^{\mu\nu}$ is ϕ 's stress-energy tensor:

$$T^{\mu\nu} = \partial^\mu \phi \partial^\nu \phi + g^{\mu\nu} \mathcal{L}. \quad (19)$$

These currents are conserved in the *ordinary* sense,

$$\partial_\mu J^{\mu(i)} = 0, \quad (20)$$

as a result of the *covariant* conservation of the stress-energy tensor,

$$\nabla_\mu T^{\mu\nu} = 0. \quad (21)$$

Therefore, there is a conserved total ‘momentum’

$$P^i \equiv \int d^3x J^{0(i)} = - \int d^3x a^3(t) \dot{\phi} \partial_i \phi, \quad \frac{d}{dt} P^i = 0, \quad (22)$$

which takes the same form as it has in Minkowski space, apart from the extra $a^3(t)$ factor, but which now, precisely because of that factor, cannot be rewritten as the time-derivative of a center of mass position. That is, momentum conservation here does not imply uniform motion. The same derivation applied to a free falling point-particle in a flat FRW spacetime yields

$$\vec{p} = a m \frac{a\vec{v}}{\sqrt{1 - a^2 v^2}}, \quad (23)$$

—where $v^i \equiv dx^i/dt$ is the velocity in comoving coordinates—as the conserved momentum. In the non-relativistic limit, the conservation of \vec{p} implies the red-shifting of the physical velocity

$$\vec{v}_{\text{ph}} \equiv a\vec{v} \sim 1/a \quad (24)$$

which is usually attributed to ‘Hubble friction’. Here we see that there is a conserved momentum, yet its conservation looks nothing like uniform motion. As we argued, this is a consequence of the lack of boost invariance.

Application to galileons. As an application of these results we now consider galileon theories [1], which have attracted some interest recently. Such theories have a number of novel field-theoretical properties, at the classical level as well as at the quantum-mechanical one [1–6], which make them potentially relevant for IR-modifications of gravity [1], for consistent violations of

the null energy condition within QFT [7], and for alternatives to slow-roll inflation [8–10]. In their simplest incarnation [1], they correspond to an effective field theory for a Goldstone boson π that is invariant under *internal* Galilean transformations

$$\pi(x) \rightarrow \pi(x) + b_\mu x^\mu. \quad (25)$$

(Several generalizations of the minimal galileon have been proposed, and some of them will be briefly touched upon in the following.) On top of this, the theory also enjoys a more conventional shift invariance

$$\pi(x) \rightarrow \pi(x) + c. \quad (26)$$

The latter yields a standard Noether current j^μ , which we get by performing a weakly x -dependent infinitesimal shift:

$$\delta S \simeq \int d^4x \partial_\mu c j^\mu. \quad (27)$$

However, because of the symmetry (25), this variation should in fact start at second order in derivatives of $c(x)$:

$$\delta S \simeq - \int d^4x \partial_\mu \partial_\nu c \xi^{\mu\nu}, \quad (28)$$

so that the current j^μ is a total divergence:

$$j^\nu = \partial_\mu \xi^{\mu\nu}. \quad (29)$$

(From now on, without loss of generality, we take $\xi^{\mu\nu}$ to be symmetric.) The global charge associated with j^μ is of course

$$Q = \int d^3x j^0. \quad (30)$$

Now, the π equation of motion is equivalent to the conservation of j^μ ,

$$\partial_\mu j^\mu = \partial_\mu \partial_\nu \xi^{\mu\nu} = 0 \quad (31)$$

which implies that the spacial integral of ξ^{00} evolves linearly in time—it is the analogue of our center-of-mass coordinate above:

$$\partial_0^2 \int d^3x \xi^{00} = -2\partial_0 \int d^3x \partial_i \xi^{0i} - \int d^3x \partial_i \partial_j \xi^{ij} = 0 \quad (32)$$

Moreover, its time-derivative is the shift Noether charge Q —the analogue of the total momentum:

$$\partial_0 \int d^3x \xi^{00} = \int d^3x (\partial_\mu \xi^{0\mu} - \partial_i \xi^{0i}) = \int d^3x j^0 \quad (33)$$

In conclusion, there is a local functional of the fields

$$\Xi(t) \equiv \int d^3x \xi^{00}(x) \quad (34)$$

that on all solutions happens to evolve linearly in time,

$$\Xi(t) = \Xi_0 + Q t. \quad (35)$$

Like in our original example, this statement can be seen as a consequence of the internal galilean invariance (25), but there is more. After all, our galilean symmetry has four parameters, and here we discovered just one global conservation law. There should be four locally conserved currents, and four associated global charges. To identify them, we run Noether's theorem with a weakly x -dependent galilean shift (25), with parameter $b_\mu(x)$. The variation of the action starts at order ∂b :

$$\delta S \simeq \int d^4x \partial_\mu b_\alpha \mathcal{J}^{\mu(\alpha)}; \quad (36)$$

α labels the symmetry, and to avoid confusion for the moment we use a different notation than for Lorentz indices. We have four conserved currents $\mathcal{J}^{\mu(\alpha)}$, which we can relate to j^μ and $\xi^{\mu\nu}$ using the same trick as above. We think of our x -dependent galilean transformation as an x -dependent shift (26), with parameter $c(x) = b_\mu(x)x^\mu$. By equating (36) with (28) we thus get

$$\mathcal{J}^{\mu(\alpha)} = \xi^{\mu\alpha} - x^\alpha j^\mu. \quad (37)$$

These are the four Noether currents associated with the symmetry (25). They are conserved on the eom,

$$\partial_\mu \mathcal{J}^{\mu(\alpha)} = \partial_\mu \xi^{\mu\alpha} - \partial_\mu (x^\alpha j^\mu) = -x^\alpha \partial_\mu j^\mu, \quad (38)$$

because j^μ is. The associated global charges are

$$\mathcal{Q}^\alpha = \int d^3x \xi^{0\alpha} - x^\alpha j^0, \quad (39)$$

or more explicitly:

$$\mathcal{Q}^0 = \int d^3x \xi^{00} - t Q = \Xi(t) - t Q \quad (40)$$

$$\mathcal{Q}^i = \int d^3x (\xi^{0i} - x^i j^0) \quad (41)$$

The conservation of \mathcal{Q}^0 is equivalent to the linear evolution of $\Xi(t)$ we discovered above. The conservation of $\vec{\mathcal{Q}}$ is a more traditional conservation law, in that it does not involve an explicit time dependence. Notice that the second piece in $\vec{\mathcal{Q}}$ is the total charge dipole of the system.

In summary, we have five locally conserved currents j^μ and $\mathcal{J}^{\mu(\alpha)}$, and five corresponding global charges Q and \mathcal{Q}^α . As an almost trivial example, we can consider the simplest system with internal galilean invariance—the free massless scalar:

$$S = - \int d^4x \frac{1}{2} (\partial\pi)^2 \quad (42)$$

The various local quantities we have defined above are

$$j^\mu = -\partial^\mu \pi, \quad \xi^{\mu\nu} = -\eta^{\mu\nu} \pi, \quad (43)$$

$$\mathcal{J}^{\mu(\alpha)} = -\eta^{\mu\alpha} \pi + x^\alpha \partial^\mu \pi, \quad (44)$$

which yield the conserved charges

$$Q = \int d^3x \dot{\pi} \quad (45)$$

$$\mathcal{Q}^0 = \int d^3x \pi - t Q \quad (46)$$

$$\vec{\mathcal{Q}} = - \int d^3x \dot{\pi} \vec{x} \quad (47)$$

The first is just the usual charge associated with shift invariance. The third is the total dipole moment of that charge. The fact that it is conserved is here a trivial consequence of the equation of motion (like all conservation laws, to some extent), but it is nonetheless a non-trivial statement (unheard of, at least). And so is the conservation of \mathcal{Q}^0 , which implies that the space integral of π grows linearly in time.

Derivation of the Currents. We can now derive explicit expressions for the currents we have defined, in the case of a generic galileon Lagrangian. The Lagrangian is a function of first and second derivatives of π ,

$$\mathcal{L} = \mathcal{L}(\partial\pi, \partial\partial\pi), \quad (48)$$

with suitable Lorentz contractions to ensure galilean invariance [1]. The shift current j^μ is readily determined, by noticing that under an infinitesimal shift (26) with x -dependent parameter $c(x)$, we have

$$\delta S = \int d^4x \frac{\partial \mathcal{L}}{\partial(\partial_\mu \pi)} \partial_\mu c + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \pi)} \partial_\mu \partial_\nu c \quad (49)$$

which compared with (27) yields

$$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \pi)} - \partial_\nu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \partial_\nu \pi)}. \quad (50)$$

To compute the currents associated with galilean shifts, eq. (37), we need first to determine $\xi^{\mu\nu}$, which is defined simply as a symmetric tensor with divergence j^μ —eq. (29). The second piece in (50) is manifestly the divergence of a symmetric tensor. To rewrite the first piece also as a total divergence requires more work. The reason is that the Lagrangian is *not* invariant under galilean shifts (25)—only the action is. That is, the Lagrangian is invariant only up to a total derivative, which means that the variation of the action under an infinitesimal x -dependent shift is not manifestly of the form (28). To proceed, we need the explicit expression for the galilean invariants. At $(n+1)$ -st order in π we have [1]

$$\mathcal{L}_{n+1} = T^{\mu_1 \nu_1 \mu_2 \nu_2 \dots \mu_n \nu_n} \partial_{\mu_1} \pi \partial_{\nu_1} \pi \partial_{\mu_2} \pi \partial_{\nu_2} \pi \dots \partial_{\mu_n} \pi \partial_{\nu_n} \pi \quad (51)$$

where T is a tensor whose explicit form we will not need. Suffice it to say that it is symmetric under exchanging any two $(\mu\nu)$ pairs, and antisymmetric under exchanging any two like indices (e.g., of the ν type) belonging to different $(\mu\nu)$ pairs. The latter symmetry ensures that the derivative of \mathcal{L}_{n+1} w.r.t. $\partial\pi$ is a total divergence:

$$\frac{\partial \mathcal{L}_{n+1}}{\partial(\partial_\mu \pi)} = \partial_{\nu_2} T^{\mu \nu_1 \mu_2 \nu_2 \dots \mu_n \nu_n} \partial_{\nu_1} \pi \partial_{\mu_2} \pi \dots \partial_{\mu_n} \pi \partial_{\nu_n} \pi +$$

$$\partial_{\mu_2} T^{\nu \mu_2 \nu_2 \dots \mu_n \nu_n} \partial_{\nu} \pi \partial_{\nu_2} \pi \dots \partial_{\mu_n} \pi \partial_{\nu_n} \pi \quad (52)$$

as predicted. We can simplify this expression, by relating it to the derivative of \mathcal{L}_{n+1} w.r.t. $\partial\partial\pi$. By swapping ν with ν_2 in the first term's T and with μ_2 in the second term's T , and by using the symmetry of T under

exchanging whole $(\mu\nu)$ pairs, we get

$$\frac{\partial \mathcal{L}_{n+1}}{\partial(\partial_\mu \pi)} = -\frac{2}{(n-1)} \partial_\nu \frac{\partial \mathcal{L}_{n+1}}{\partial(\partial_\mu \partial_\nu \pi)}. \quad (53)$$

Plugging this into eq (50) we thus get the full n -th order contributions to $\xi^{\mu\nu}$:

$$\xi_{n+1}^{\mu\nu} = -\frac{n+1}{n-1} \frac{\partial \mathcal{L}_{n+1}}{\partial(\partial_\mu \partial_\nu \pi)}. \quad (54)$$

Notice however that this formula only holds when the galilean invariants \mathcal{L}_{n+1} are written as in eq. (51), with T obeying the aforementioned symmetry properties. This is not the case for all the invariants explicitly displayed in ref. [1], where an integration by parts was performed on \mathcal{L}_3 to rewrite it in a more compact form, thus effectively reshuffling its dependence on $\partial\pi$ with that on $\partial\partial\pi$. The ‘canonical’ form for all the invariants relevant in 4D can be found instead in ref. [12] [16]. With this qualification in mind, the currents associated with internal galilean invariance therefore are

$$\mathcal{J}_{n+1}^{\mu(\alpha)} = (x^\alpha \partial_\nu - \frac{n+1}{n-1} \delta_\nu^\alpha) \frac{\partial \mathcal{L}_{n+1}}{\partial(\partial_\mu \partial_\nu \pi)} - x^\alpha \frac{\partial \mathcal{L}_{n+1}}{\partial(\partial_\mu \pi)}. \quad (55)$$

This formula cannot be applied to the lowest-order invariants, $\mathcal{L}_1 = \pi$ and $\mathcal{L}_2 = -\frac{1}{2}(\partial\pi)^2$ —in deriving it we have been assuming that \mathcal{L}_{n+1} depends non-trivially both on $\partial\pi$ and on $\partial\partial\pi$, and that it does not depend on π . For \mathcal{L}_2 , we already gave the relevant expressions in (43, 44). The situation is trickier for \mathcal{L}_1 . By applying Noether’s theorem to it and using the identities $1 = \frac{1}{4}\partial_\mu x^\mu$ and $x^\mu = \frac{1}{2}\partial^\mu x^2$, we discover that its contributions to the shift current and to the galilean ones are

$$j_1^\mu = -\frac{1}{4}x^\mu, \quad \mathcal{J}_1^{\mu(\alpha)} = -\frac{1}{2}\eta^{\mu\alpha}x^2. \quad (56)$$

The global charges (30, 40, 41) thus acquire extra pieces explicitly proportional to t and to t^2 . However, in the presence of the tadpole $\mathcal{L}_1 = \pi$ any solution will have non-trivial boundary conditions at spacial infinity. Like in the case of spontaneous symmetry breaking, this will generically yield divergent global charges, thus making their conservation useless. On the other hand the local current conservation will still be perfectly valid. For certain particularly symmetric solution, like the deSitter ones discussed in [1], the dynamics of perturbations about such asymptotically non-trivial solutions will still be described by a galileon theory, this time without the tadpole of course. In such a case the global charges we derived can be used directly for the perturbations—provided one uses the perturbations’ Lagrangian in our formule.

Generalizations. The generalization of our results to multi-galileon theories [12–14] should be straightforward. Perhaps more interesting is the generalization to the so called IR-completions of the galileon, i.e. to theories that reduce to the galileon in some appropriate

limit—typically at small distances and at small field values—and that away from that limit are invariant under a different symmetry group [1]. So far two possibilities have been proposed—promoting the galileon symmetry group (spacetime Poincaré plus internal shifts plus internal galilean transformations) to the conformal group $SO(4, 2)$ or to the five-dimensional Poincaré group $ISO(4, 1)$ [1, 15]. The generalization of our results to the latter case should be straightforward—as was generalizing the analogous statements we have for *ordinary* Galilean invariance to the Lorentz-invariant case. The reason is that in all these cases the transformation is linear in the relevant coordinates. As clear from our derivation, this is the crucial ingredient for our results. For instance, it never really mattered whether our symmetries acted linearly or non-linearly on the dynamical variables (point particle positions or fields)—the existence of a local functional that grows linearly in time follows purely from the existence of a symmetry that is linear in time. The conformal group case is more complicated. There, the π shift and galilean transformations get promoted to non-linearly realized dilations and special conformal transformations [1] [17]. At the infinitesimal level, the former are still linear in x^μ , thus yielding conservation laws similar to those we discussed here. On the other hand, infinitesimal special conformal transformations are quadratic in x^μ . Among the associated conservation laws, there will be one implying the existence of a local functional that grows *quadratically* with time.

Summary and Acknowledgements. The invariance of a theory under ordinary Galileo or Lorentz boosts implies the existence of a center of mass that moves at constant speed. This generalized conservation law is not a direct consequence of momentum conservation—precisely like invariance under boosts is not a direct consequence of invariance under spacial translations—and is generically violated in theories with no boost-invariance. When generalized to the galileon case, this implies four new conservation laws—which we derived explicitly—one of which is naturally interpreted as the existence of a local functional of the field and its derivatives that evolves linearly in time.

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