Pseudo-rip: Cosmological models intermediate between the cosmological constant and the little rip

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The Pseudo-Rip

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If we assume that the cosmic energy density will remain constant or strictly increase in the future, then the possible fates for the universe can be divided into four categories based on the time asymptotics of the Hubble parameter $H(t)$: the cosmological constant, for which $H(t) = constant$, the big rip, for which $H(t) \to \infty$ at finite time, the little rip, for which $H(t) \to \infty$ as time goes to infinity, and the pseudo-rip, for which $H(t) \to constant$ as time goes to infinity. Here we examine the last of these possibilities in more detail. We provide models that exemplify the pseudo-rip, which is an intermediate case between the cosmological constant and the little rip. Structure disintegration in the pseudo-rip depends on the model parameters. We show that pseudo-rip models for which the density and Hubble parameter increase monotonically can produce an inertial force which does not increase monotonically, but instead peaks at a particular future time and then decreases.

Introduction— Time is an elusive concept of great interest to all physicists since it underlies all dynamical systems. Although we can measure time on everyday scales with exquisite precision, the deeper meaning of time may be addressed only by study of the origin and fate of the universe.

Current observations [1–4] strongly suggest that the universe is dominated by a negative-pressure component, dubbed dark energy. This component can be characterized by an equation of state parameter w, which is simply the ratio of the pressure to the density: $w = p/\rho$; for example, a cosmological constant corresponds to $w = -1$. While it is often assumed that $w \geq -1$ in accordance with the weak energy condition, it has long been known [5] that the observations are consistent with $w < -1$, which corresponds to a dark energy density that increases with time t and scale factor a. If the density increases monotonically in the future, then the universe can undergo a future singularity, called the "big rip," for which $\rho \to \infty$ and $a \to \infty$ at a finite time. Shortly before this singularity is reached, bound structures are disintegrated by the expansion [6].

Note, however, that a dark energy component with a monotonically increasing density that is unbounded from above does not lead inevitably to a future singularity, although it does ultimately lead to the dissolution of all bound structures. Such models, dubbed "little rip" models, were first examined in detail by Frampton et al. [7], who derived the boundary between big rip and little rip models in terms of $\rho(a)$. Properties of little rip models were further investigated in Ref. [8].

Here we investigate a different set of models, in which the density of the dark energy increases monotonically with scale factor but is bounded from above by some limiting density, ρ_∞. Such models can still lead to a dissolution of bound structures for a sufficiently strong inertial force (which we will define), so we dub these models "pseudo-rip" models. This allows us, in the context of monotonically increasing ρ, to distinguish three distinct cosmic futures: the big rip, little rip, and pseudo-rip. (Note that models for which $\rho(a)$ is not monotonic are physically less plausible, and it is more difficult to make any sort of general statement about such models). While the dissolution of bound structures is inevitable in the big rip and little rip scenarios, it may or may not occur in a pseudo-rip, depending on the model parameters.

In the next section, we provide the definition of the pseudo-rip and examine the conditions necessary to dissolve bound structures. We then examine two specific functional forms of the dark energy density and discuss scalar field realizations of pseudo-rip models. We assume a flat FLRW metric and $c = 1$ throughout.

Definition of Pseudo-Rip—Before defining the pseudo-rip, we suggest a compact way to classify all rips. We find it useful to classify ripping behaviour by means of the Hubble parameter $H(t)$. By ripping behavior, we mean any future evolution which can lead to disintegration of structure in the form of bound systems by virtue of the strong inertial force due to dark energy.

Given the Hubble parameter $H(t)$ for $t \geq t_0$, where t_0 is the present time, the density $\rho(t)$ and pressure $p(t)$ are:

$$\rho(t) = \left(\frac{3}{8\pi G}\right) H(t)^2, \qquad (1)$$

$$p(t) = -\left(\frac{1}{8\pi G}\right) \left[2\dot{H}(t) + 3H(t)^2\right]. \qquad (2)$$

The big rip is defined by

$$H(t) \longrightarrow +\infty, \qquad t \longrightarrow t_{rip} < \infty. \qquad (3)$$

In a big rip, all bound structures dissociate in a finite time in the future, and space-time "rips apart" at a finite time in the future, i.e., the scale factor of the FLRW metric goes to infinity at $t = t_{rip}$ [6]. The little rip is defined by

$$H(t) \longrightarrow +\infty, \qquad t \to +\infty. \qquad (4)$$

The little rip dissociates all bound structures, but the strength of the dark energy is not enough to rip apart space-time as there is no finite-time singularity.

An excellent fit to all cosmological data is provided by the ΛCDM model which, in present parlance, is a "no-rip" model defined by

$$H(t) = H(t_0). \tag{5}$$

There remains just one additional possibility for monotonically increasing $H(t)$, namely

$$H(t) \longrightarrow H_\infty < \infty, \qquad t \to +\infty, \tag{6}$$

where H_∞ is a constant. Equation (6) defines the pseudo-rip, the subject of the present article. A pseudo-rip dissociates bound structures that are held together by a binding force at or below a particular threshold that depends on the inertial force in the model. Eqs.(3)-(6) clearly exhaust all possibilities for a monotonically increasing $H(t)$, i.e., a monotonically increasing dark energy density $\rho(a)$.

The equations for monotonically increasing $H(t)$ are the same for $\rho(t)$ *mutatis mutandis*. Pressure $p(t) \longrightarrow -\rho(t)$ as $t \longrightarrow \infty$, provided $\dot{H}(t) \longrightarrow 0$ on the R.H.S. of Eq.(2), which is the case for a pseudo-rip model.

Our division of the future evolution into the categories of big rip, little rip, and pseudo-rip represents a different set of models than those examined in Ref. [9], which provided a classification scheme for future singularities. Our scheme represents a classification of all models with monotonically increasing dark energy density, for which the scale factor a goes smoothly to infinity, at either finite or infinite time, and for which there are no singularities in the derivatives of H unless H itself is singular. In our scheme, the big rip encompasses the type I singularity of Ref. [9], while the type II, III, and IV singularities lie outside the types of models considered in our classification scheme. The little rip and pseudo-rip models are by definition non-singular, so they fall outside of the purview of Ref. [9].

The inertial force F_{inert} on a mass m as seen by a gravitational source separated by a comoving distance l is given by [8]

$$
\begin{aligned}
F_{inert} &= ml(\dot{H}(t) + H(t)^2) \\
&= -ml\frac{4\pi G}{3}(\rho(a) + 3p(a)) \\
&= ml\frac{4\pi G}{3}(2\rho(a) + \rho'(a)a). \tag{7}
\end{aligned}
$$

For simplicity, we set the scale factor at the present time $a_0 = a(t_0) = 1$. A bound structure dissociates when the inertial force, dominated by dark energy, grows in the future to equal the force holding together the bound structure in question. For a pseudo-rip, F_{inert} is asymptotically finite.

However, if the bound structure is massive enough to significantly affect the local space-time metric, it is not accurate to express F_{inert} in terms of the FLRW metric. A more accurate method and local metric is employed in [10], and we use their method to calculate the disintegration times for the Milky Way and the Earth-Sun system.

We analyze two psuedo-rip parameterizations for dark energy density, models 1 and 2, each as a function of the scale factor $a(t)$ with other parameters.

Model One—Model 1 is defined by

$$\rho_1(a, B, f, s) = \rho_0 \frac{\ln[\frac{1}{f+\frac{1}{a}} + \frac{1}{B}]^s}{\ln[\frac{1}{f+1} + \frac{1}{B}]^s}, \tag{8}$$

where ρ_0 is the present value of the dark energy density. Note that ρ_1 is normalized to be ρ_0 at $a = 1$, which we define to represent the present. Then $\rho_1(a, B, f, s)$ is a function of the scale parameter $a(t)$ and of three other parameters B, f, and s. It is most convenient to fix f and s, which mostly control the strength of the rip, and to keep B as a free paramater for fitting the supernova data.

We fix f and s to specify how powerful the psuedo-rip should be. The remaining free parameter is chosen to make a best fit to the latest supernova data from the Supernova Cosmology Project [11] with a reduced χ^2 of $\simeq 0.98$.

As examples of bound states we consider the Milky Way (MW), the Earth-Sun system (ES), the hydrogen atom (H atom), and the proton. The first two, MW and ES, are gravitationally bound, while for the H atom and proton, we must carefully consider the electromagnetic and strong color forces respectively. In all cases the dark energy density increases monotonically from ρ_0 at the present time to an asymptotic value. Depending on the parameters B, f, and s, the inertial force can successively disintegrate the MW, the ES, the H atom and the proton.

In different cases, some of these bounds systems will be disintegrated and not others. If none of them are disintegrated, we shall refer to such a pseudo-rip as a "failed rip."

In Fig. (1) we show five examples of the scaled F_{inert} for $\rho_1(a)$, which include the matter and radiation contributions. Because of these contributions, the curves go to negative infinity as a goes to 0, and the x-intercepts are values of $a < 1$ at which dark energy domination begins. Going from bottom to top, the curves represent respectively a failed rip; a pseudo-rip which disintegrates only the MW; a pseudo-rip which breaks apart the MW and ES; a pseudo-rip which destroys the MW, ES, and H atom; and the highest curve is for a pseudo-rip which succeeds in ripping apart all four of the MW, ES, H atom, and proton. Note that, unlike a little rip, the asymptotic value of $\rho_1(a)$ as $a \longrightarrow \infty$ is finite.

For a given pseudo-rip model with monotonically increasing dark energy density $\rho_{DE}(a)$, the structures with bigger binding forces disintegrate after those with smaller binding forces. But a particular ρ_1 can be constructed such that it leads to, for example, the disintegration of the proton before another ρ_1 disintegrates the Milky Way.

Notice that the more violent the pseudo-rip is required to be, the more extremely small the f parameter is. One may counteract this fine tuning of f by, for example, introducing new factors into the model that help ρ_1 grow
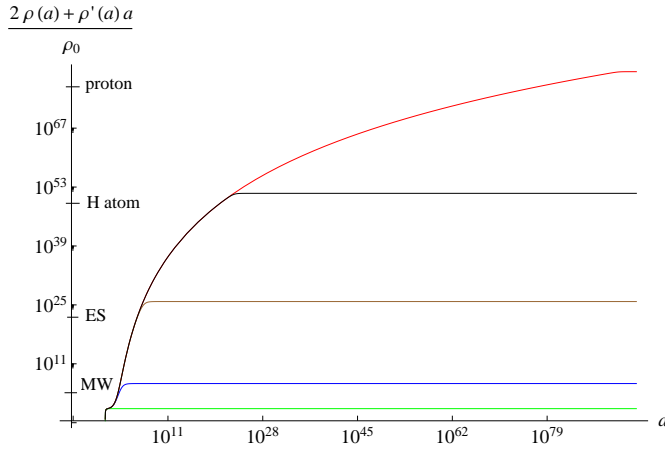
FIG. 1: Plotted from the innermost to the outermost curve is the scaled F_{inert} for $\rho_1(a, 0.29, 10, 48)$, $\rho_1(a, 0.0108, 0.003, 48)$, $\rho_1(a, 0.01078, 2 \times 10^{-7}, 48)$, $\rho_1(a, 0.01078, 10^{-23}, 48)$, and $\rho_1(a, 0.0108, 10^{-92}, 48)$ respectively, for Model 1, given by Eq. (8). Each curve was fitted to supernova data with the extra constraint that $B \geq 0$. The values necessary for structural disintegration are indicated. From the innermost curve to the outermost: failed rip; $t_{MW} - t_0 = 9.2 \times 10^4$ Gyr; $t_{ES} - t_0 = 1.6 \times 10^9$ Gyr; $t_{Hatom} - t_0 = 1.3 \times 10^{33}$ Gyr; $t_{proton} - t_0 = 3.8 \times 10^{117}$ Gyr. (The color plots are in the online version of the paper.)

faster while still leaving it asymptotically finite. One such factor could be $(\frac{a+q}{a+2q}\frac{1+2q}{1+q})^w$. Such a factor is 1 when $a = 1$, so ρ_1 is still ρ_0 at the present time. The new parameters q and w can avoid the fine tuning.

Model 2—Model 2 is defined by

$$\rho_2(a, A, n, m) = \rho_0 \frac{A}{2}(\tan^{-1}(a - n) - \tan^{-1}(1 - n) + 1)^m. \tag{9}$$

Like ρ_1, ρ_2 is normalized to be ρ_0 at $a = 1$. Then $\rho_2(a, A, n, m)$ is a function of the scale factor $a(t)$ and of three other parameters A, n, and m. It is most convenient to fix n and m, which mostly control the strength of the rip, and to keep A as a free paramater for fitting the supernova data. So we fix n and m to specify how powerful the psuedo-rip should be, and the remaining free parameter is chosen to make a best fit to the latest supernova data from the Supernova Cosmology Project [11] with a reduced χ^2 of $\simeq 0.98$.

In Fig. (2), we plot five examples of the scaled F_{inert} for $\rho_2(a)$, which include the contributions from matter and radiation. Just as in Fig. (1), the contributions cause the curves to approach negative infinity as a goes to 0, and the x-intercepts are values of $a < 1$ at which dark energy domination begins. The bottom curve exhibits a failed rip, while the others show pseudo-rips of various strengths. As was mentioned for Model 1, we see from Fig. (2) that the disintegration time for the proton for a particular ρ_2 can be sooner than the disintegration time

for the Milky Way for another ρ_2.

Note in Fig. (2) that each F_{inert} has a local maximum. Because of this bump, $\rho_2(a \to \infty)$ is less than the maximum value of ρ_2. So it is possible, for particular parameterizations, for F_{inert} to reach the level of the binding force of a bound structure and then decrease, allowing the structure to possibly come back together. In principle, a pseudo-rip model can have F_{inert} with an arbitrary number of local maxima that give structures the chance to dissociate and reform multiple times. All this can be achieved using a functional form for dark energy density that is monotonically increasing. However, all the examples shown in Fig. (2) have their asymptotic values higher than the values necessary to rip apart the structures mentioned in the plot.
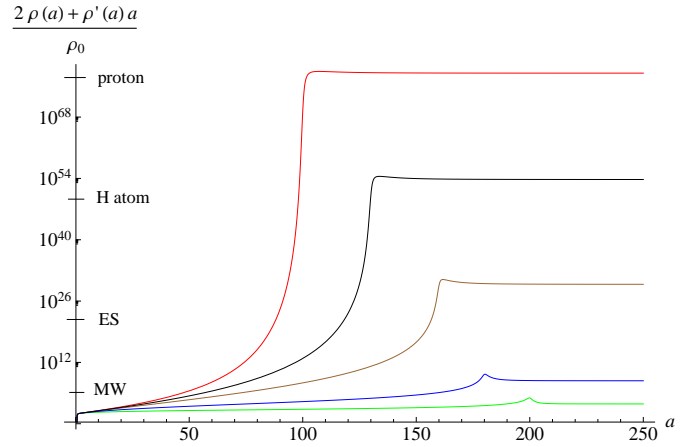

FIG. 2: Plotted from the innermost to the outermost curve is the scaled F_{inert} for $\rho_2(a, 17481.3, 200, 0.5)$, $\rho_2(a, 3601.31, 180, 2)$, $\rho_2(a, 571.1, 160, 10)$, $\rho_2(a, 171.045, 130, 22)$, and $\rho_2(a, 55.45, 100, 40)$ respectively, for Model 2, given by Eq. (9). Each curve was fitted to supernova data. The values necessary for structural disintegration are indicated. As explained in the text, disintegration times from different models should not be directly compared. From the innermost curve to the outermost: failed rip; $t_{MW} - t_0 = 45$ Gyr; $t_{ES} - t_0 = 40$ Gyr; $t_{Hatom} - t_0 = 39$ Gyr; $t_{proton} - t_0 = 38$ Gyr. (The color plots are in the online version of the paper.)

Scalar Field Realizations—One possible realization of pseudo-rip models is a minimally-coupled phantom model, i.e., one that involves a scalar field with a negative kinetic term. The equation of motion for such a field is

$$\ddot{\phi} + 3H\dot{\phi} - V'(\phi) = 0 \,, \tag{10}$$

where the dot is a time derivative, and the prime denotes the derivative with respect to ϕ. A field obeying this equation of motion rolls "uphill" in the potential.

It is clear that a sufficient condition for a pseudo-rip is that $V(\phi) \to V_0$ (where V_0 is a constant) as $\phi \to \infty$, and in Ref. [8] it was shown that this is also a necessary condition for a monotonic potential. In this case,

we simply have $\rho_\infty = V_0$. If the potential is not monotonic, the density of the scalar field can also approach a constant asymptotically if the field gets trapped in a local maximum with $V = V_0$. Phantom fields with bounded potentials have been discussed previously in Ref. [12].

Note, however, that our discussion of pseudo-rips is much more general than the specific example provided by phantom field models. Phantom fields represent only a single possible realization of this much more general class of models for the asymptotic expansion of the universe.

Discussion—We have described merely two illustrative models of the pseudo-rip. Obviously, there is an infinite number of possibilities.

A failed rip will disintegrate nothing because the inertial force will not reach a high enough magnitude. Model 1 exemplifies a pseudo-rip model which can variously disintegrate between one and all four of the chosen systems while ρ_1 asymptotes to a finite density, which implies a finite inertial force, as the scale factor $a(t)$ approaches infinity. As can be seen from Fig. (1), these quantities plateau to a constant value after the last disintegration has taken place.

We included Model 2 as a particularly interesting example in which the inertial force rises relatively abruptly before it plateaus, as shown in Fig. (2). The various disintegrations take place soon after each other in a cosmological sense.

For a function that asymptotically approaches a constant, such as $\rho_1(a)$ or $\rho_2(a)$, a point of inflection allows the function to have an arbitrarily large slope for a portion of the domain and still increase monotonically. A point of inflection is present in all the parameterizations of $\rho_1(a)$ and $\rho_2(a)$ and their derivatives in this letter and it allows them to fit the supernova data (which require the densities to have a very small slope over the relevant portion of a) and still reach a high density in a relatively short time. However, the relevant quantity that determines when a structure dissociates is F_{inert}, which is proportional to $2\rho(a) + \rho'(a)a$. This combination of terms is responsible for any local maxima in F_{inert}, not merely ρ or ρ'. An example of a model that has an inflection in $\rho'_{DE}(a)$ but none in $\rho_{DE}(a)$ is given by $\rho_{DE}(a) = \alpha(a - \ln[1 + e^{a-C}])$, where $\alpha, C > 0$ are constants. Such a model has a local maximum in F_{inert} and if we shift by D, where $D > C$ is a constant, neither the resulting $\rho_{DE}(a + D)$ nor its derivative has an inflection point, but the resulting F_{inert} still has a local maximum.

It is amusing to take examples of Model 2 which are more extreme than the ones illustrated. It is possible to design a pseudo-rip model such that disintegrations happen arbitrarily soon after the present time while still maintaining excellent fits to the supernova data. The Sun may not rise tomorrow.

This is a dramatic illustration of the fact that any amount of observational data, necessarily restricted to the past lightcone and necessarily with non-zero errors, cannot predict anything mathematically about the future even one hour hence without further assumptions. It is also a display of the difference between mathematics and physics: the physicist necessarily employs intuition about the real world.

The earliest support for cosmological futures such as the big rip, little rip, or pseudo-rip might come from the Planck satellite, which is expected to have data around September 2012. If the dark energy equation of state emerges with $w < -1$, it will be a shot in the arm for such exotic ideas.

It will also lend new understanding of the nature of time and perhaps the beginning and possible cyclicity of the universe.

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