Energy loss calculations of moving defects for general holographic metrics

John F. Fuini, III and Andreas Karch

Phys. Rev. D **85**, 066006 — Published 27 March 2012

DOI: 10.1103/PhysRevD.85.066006

Energy Loss Calculations of Moving Defects for General Holographic Metrics

John F. Fuini III[1,*] and Andreas Karch[1,†]

[1] *Physics Department, University of Washington, Seattle, WA 98195-1560, USA*

We extend the ideas of using AdS/CFT to calculate energy loss of extended defects in strongly coupled systems to general holographic metrics. We find the equations of motion governing uniformly moving defects of various dimension and determine the corresponding energy loss rates in terms of the metric coefficients. We apply our formulae to the specific examples of both bulk geometries created by general Dp-branes, as well as to holographic superfluids. For the Dp-branes, we find that the energy loss of our defect, in addition to the expected quadratic dependence on velocity, depends on velocity only via an effective blueshifted temperature - despite the existence of a microscopic length scale in the theory. We also find, for a certain value of p and dimension of the defect, that the energy loss has no dependence on temperature or velocity other than the aforementioned quadratic dependence on velocity. For the superfluid example, we find agreement with previous results on the existence of a cutoff velocity, below which the probe experiences no drag force. For both examples we can easily extend the equations of motion and energy loss to defects of larger dimension.

PACS numbers:
Keywords: AdS/CFT

I. INTRODUCTION

The duality between Type IIB string theory on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ SU(N) supersymmetric Yang-Mills (SYM) [1–3] has been studied extensively as a means to provide insight to the inner workings of strongly coupled systems where perturbation theory is not valid, and where lattice gauge theory and Monte Carlo techniques are available but struggle with real time physics. Motivated by the QGP created at RHIC, holography has helped guide our understanding of shear viscosity, drag, screening length and jet quenching to name a few; see [4] for a recent perspective on these developments by one of us. The dual description of a classical string ending on a probe brane has given information on the characteristics of a point particle, possibly a quark, traveling through the Yang-Mills plasma, which is thought to be a good approximation for the strongly coupled QCD. Recently it has been seen that one can also study extended defects with dimensions larger than a point, living in AdS_d spaces, the idea being that one can shed new light on the dynamics of energy loss at strong coupling [5]. We extend the ideas of [5–7] to a general metric with a compact internal space. We produce general solutions for the equation of motion and the energy loss of an extended defect moving uniformly through the bulk whose geometry is described by a generic brane-like metric. We study two examples in detail, general Dp-brane metrics and holographic superfluids. After working out the energy loss for Dp-branes, we find that the energy loss of these extended probes is given by simple power laws in velocity and temperature, revealing that the energy loss depends only on an effective temperature multiplied by the velocity squared - the moving probe being affected only by a blue-shifted energy density - despite the existence of an intrinsic scale in the underlying theory. We then apply our results to dragging string- and sheet-like defects through holographic superfluids.

We organize these ideas as follows: In section II, we will calculate a general equation of motion and solution to a uniformly moving defect in a general bulk theory. We then find a general energy loss formula for said defect. In section III, we apply these results to metrics created by general Dp-branes. In section IV, we apply our results to a holographic superfluid.

II. CALCULATIONS

We want to study a general holographic brane-like metric. This metric will preserve the symmetries of the dual field theory. Therefore, it should be rotationally invariant and translationally invariant in both the spatial and time dimensions. In addition it should preserve the isometries of the internal space. To these ends, we introduce the following diagonal metric

$$\begin{aligned} ds^2 &= G_{\mu\nu} dx^\mu dx^\nu \\ &= G_{tt} dt^2 + G_{xx} \sum_{i=1} dx_i^2 + G_{uu} du^2 + G_{\theta\theta} d\Omega^2, \end{aligned} \qquad (1)$$

with $N+2$ total space-time dimensions. At this point, the $N+1$ spatial coordinates are arbitrarily separated into M Cartesian coordinates and $N-M$ spherical angles of the internal space. The additional radial coordinate is denoted by u. It is understood that $G_{\theta\theta}$ is only a function of u and since none of our results depend on the details of the internal space we will take $d\Omega^2$ be a unit sphere - i.e. $d\Omega^2$ contains the appropriate terms for the angular variables of a unit sphere of arbitrary

*Email: fuini@u.washington.edu
†Email: akarch@uw.edu

dimension. The background will continue to solve the same equations of motion should the sphere be replaced with any other compact Einstein manifold. We are only considering metrics whose G_{ii} depend only on u, and whose G_{tt} and G_{xx} grow with u at the same rate, both of which grow faster with u than $G_{\theta\theta}$. The rationale behind this is that the gravity theory in the bulk has a dual interpretation in terms of a field theory at the $u=\infty$ boundary which lives in $M+1$ space-time dimensions.

In a manner similar to [5], we introduce a defect of spatial dimension $n+1$ in the bulk. The defect has n spatial dimensions orthogonal to u which are divided into m infinitely extended Cartesian directions on the boundary, denoted $\vec{y}$, and $n-m$ angular directions, denoted $\vec{\theta}$. It will then move in an additional transverse direction, x. Since the dimensionality of the bulk puts constraints on the size of the defect we find $m+1 \leq M$ and $n+1 \leq N$.

In the static gauge, the world-volume map is of the form $X = (t, u, \vec{y}, \vec{\theta}, x(t, u, \vec{y}, \vec{\theta}), \vec{z} = const)$ where $\vec{z}$ are any additional unused orthogonal coordinates in the bulk. The induced metric on the world volume is defined as $g_{\alpha\beta} = G_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta}$. We find $g = \det(g_{\alpha\beta})$ to be

$$\begin{aligned} g = G_{xx}^m G_{\theta\theta}^{n-m} [&G_{tt} G_{uu} + G_{tt} G_{xx} (x_{,u})^2 \\ &+ G_{uu} G_{xx} (x_{,t})^2 + G_{tt} G_{uu} \sum_i^m (x_{,y_i})^2 \\ &+ \frac{G_{tt} G_{uu} G_{xx}}{G_{\theta\theta}} \sum_j^{n-m} (x_{,\theta_j})^2] \end{aligned} \tag{2}$$

where $(x_{,\sigma})$ denotes the partial derivative of x with respect to the coordinate σ.

Our general metric will have some intrinsic scale governed by its radii of curvature, all of which we take parametrically to be of the same order R. The bulk theory will be governed by classical gravity if R abides by $M_{pl} R \gg 1$, where $M_{pl}^N = 1/16\pi G$, (ensuring our curvature is not too large in Planck units). We want to describe the defect by the following Nambu-Goto-like action

$$S = -T_0 \int e^{-\Phi} \sqrt{-g} \prod_i d\sigma_i \tag{3}$$

where g is given by (2), T_0 is the tension and the integral is over all world-volume coordinates. Φ is a function of the background scalar fields, the dilaton field, ϕ, for example. Unlike the coupling to the background metric, the functional form of the coupling to the background scalars is not fixed by diffeomorphism invariance and in principle $e^{-\Phi}$ can be a complicated function of the background scalars. As long as the background scalars respect the symmetries of the metric, they can only depend on the radial coordinate u and we can treat Φ as a function of u. Two important probes we are going to consider in examples for $\Phi(u)$ are a fundamental string, whose action has the form (3) with $\Phi = 0$, and probe D-branes, whose action is of the form (3) with $\Phi = \phi$. We can trust the classical treatment of this action so long as $T_0 R^{n+2} \gg 1$. Additionally, demanding that $(M_{pl} R)^N \gg T_0 R^{n+2}$ will render gravitational back-reaction negligible.

With $\mathcal{L} = -T_0 e^{-\Phi} \sqrt{-g}$ we find the following canonical momentum densities:

$$\begin{aligned} \Pi_x^t &= \frac{\partial \mathcal{L}}{\partial (x_{,t})} = -T_0 \frac{G_{uu} G_{xx}^{m+1} G_{\theta\theta}^{n-m}}{\sqrt{-g}} (x_{,t}) e^{-\Phi} \\ \Pi_x^u &= \frac{\partial \mathcal{L}}{\partial (x_{,u})} = -T_0 \frac{G_{tt} G_{xx}^{m+1} G_{\theta\theta}^{n-m}}{\sqrt{-g}} (x_{,u}) e^{-\Phi} \\ \Pi_x^{y_i} &= \frac{\partial \mathcal{L}}{\partial (x_{,y_i})} = -T_0 \frac{G_{tt} G_{uu} G_{xx}^m G_{\theta\theta}^{n-m}}{\sqrt{-g}} (x_{,y_i}) e^{-\Phi} \\ \Pi_x^{\theta_i} &= \frac{\partial \mathcal{L}}{\partial (x_{,\theta_i})} = -T_0 \frac{G_{tt} G_{uu} G_{xx}^{m+1} G_{\theta\theta}^{n-m-1}}{\sqrt{-g}} (x_{,\theta_i}) e^{-\Phi} \end{aligned} \tag{4}$$

Requiring a vanishing variation in our action yields

$$\sum_i^n \partial_{\sigma_i} (\Pi_\mu^{\sigma_i}) = 0 \tag{5}$$

Setting μ to x gives us our equation of motion.

We are interested in a solution of a uniformly moving object. The defect should move in a direction transverse to its spatial extent and travel with a constant velocity in the x direction. Thus, our ansatz is

$$x(t, u, \vec{y}) = vt + x(u).$$

With this form, g becomes independent of time and our equation of motion, $\sum_i^n \partial_{\sigma_i} \Pi_{mu}^{\sigma_i} = 0$, will then reduce to

$$\partial_u \left(\frac{G_{tt} G_{xx}^{m+1} G_{\theta\theta}^{n-m}}{\sqrt{-g}} (x_{,u}) e^{-\Phi} \right) = 0,$$

which gives us

$$(x_{,u})^2 = \frac{C^2 e^{2\Phi} (-g)}{G_{tt}^2 G_{xx}^{2(m+1)} G_{\theta\theta}^{2(n-m)}}, \tag{6}$$

and plugging in for g

$$(x_{,u})^2 = -(C e^{\Phi})^2 \frac{G_{uu}}{G_{tt} G_{xx} e^{2\Phi}} \left\{ \frac{G_{tt} + G_{xx} v^2}{G_{tt} G_{xx}^{m+1} G_{\theta\theta}^{n-m} e^{-2\Phi} + C^2} \right\} \tag{7}$$

We expect on physical grounds, that $(x_{,u})$ should be real, and thus $(x_{,u})^2$ and $(-g)$ should be positive. From (7), we see that this will be true if $G_{tt} + G_{xx} v^2$ and $G_{tt} G_{xx}^{m+1} G_{\theta\theta}^{n-m} e^{-2\Phi} + C^2$ both switch sign at the same value of u. Assuming that there is at most one root, call it u_c, we can solve for it from

$$G_{tt}(u_c) + G_{xx}(u_c) v^2 = 0 \tag{8}$$

and if this root exists, we have

$$C = \pm e^{-\Phi} \sqrt{-G_{tt} G_{xx}^{m+1} G_{\theta\theta}^{n-m}} |_{u=u_c}.$$

The case where u_c does not exist occurs in our example of a holographic superfluid and will be discussed in section

IV. The induced metric can be diagonalized and this diagonal form has a time component which vanishes when $G_{tt}(u_c) + G_{xx}(u_c)v^2 = 0$. This tells us that u_c denotes the worldvolume horizon.

Using (8) to plug in for G_{tt} and defining $\widetilde{C} = C/v$ we have

$$\widetilde{C} = \pm e^{-\Phi}\sqrt{G_{xx}^{m+2}G_{\theta\theta}}|_{u=u_c} \qquad (9)$$

The momentum loss rate, due to momentum flowing along the defect and towards the horizon, is given by -Π_x^u which is seen from (4) and (6) to be

$$-\Pi_x^u = T_0 C. \qquad (10)$$

The momentum loss rate directly gives us the drag force density. The energy loss rate, $-\Pi_t^u$, is simply v times the momentum loss rate. Physically, we expect that we have energy flowing towards the horizon of the black brane, which requires our loss rate to be positive. We thus pick the positive sign for $\widetilde{C}$.

$$\widetilde{C} = e^{-\Phi}\sqrt{G_{xx}^{m+2}G_{\theta\theta}^{n-m}}|_{u=u_c}. \qquad (11)$$

While we have established that this stationary solution is a consistent solution to the equations of motion, what is less obvious is that it is stable. Small fluctuations around dragging sheets in AdS$_7$ have recently been studied in [8] and it has been found that these fluctuations do not exhibit any instabilities, that is any modes that grow exponentially in time. A new potential instability in our case is the slipping mode on the internal space, that is fluctuations in the θ directions that take our defect off the equator of the internal sphere. As such fluctuations reduce the volume of the defect (and hence its potential energy), they clearly correspond to negative mass squared modes and are hence potentially problematic. It is well known from the case of static defects, starting with the work on flavor probe branes [9], that these negative mass squared slipping modes often are actually stable. For a background AdS space the basic physics behind this is the BF bound [10]. The potential energy gain of the fluctuation is offset by the kinetic energy cost of any fluctuation in a spacetime geometry that effectively corresponds to a finite size box. A similar effect also occurs in the more general holographic metrics. In particular, it has been shown in [11] that supersymmetric Dq-brane defects in black Dp-brane backgrounds (which will be the first example we apply our results to) have stable slipping modes. For non-supersymmetric defects stability of the slipping mode will have to be checked on a case by case basis.

III. P-BRANES

We now turn to the example of general Dp-branes which create geometries dual to SYM in $p+1$ dimensions on the boundary, at finite temperature. From [12] we see that in the limit that

$$g_{YM}^2 = (2\pi)^{p-2} g_s \alpha'^{(p-3)/2} = \mathsf{fixed},$$

as

$$\alpha' \to 0$$

where $g_s = e^{\phi_\infty}$, and g_{YM} is the Yang-Mills coupling constant, the Dp brane metric becomes,

$$ds^2 = \alpha'\{\frac{u^{(7-p)/2}}{g_{YM}\sqrt{d_p N'}}\left[-(1-\frac{u_0^{7-p}}{u^{7-p}})dt^2 + dy_{\|}^2\right] + \frac{g_{YM}\sqrt{d_p N'}}{u^{(7-p)/2}(1-\frac{u_0^{7-p}}{u^{7-p}})}du^2 + g_{YM}\sqrt{d_p N'}u^{(p-3)/2}d\Omega_{8-p}^2\} \qquad (12)$$

which indeed is of the general form (1). Here, N' is the number of branes and $d_p = 2^{7-2p}\pi^{\frac{9-3p}{2}}\Gamma(\frac{7-p}{2})$.

The dilaton is

$$e^{\phi} = (2\pi)^{2-p} g_{YM}^2 (\frac{g_{YM}^2 d_p N'}{u^{7-p}})^{\frac{3-p}{4}}, \qquad (13)$$

and

$$u_0^{7-p} = \frac{\Gamma(\frac{9-p}{2})2^{11-2p}\pi^{\frac{13-3p}{2}}}{(9-p)} g_{YM}^4 \epsilon, \qquad (14)$$

where ϵ corresponds to the energy density of the Yang-Mills theory.

Following the outline laid out in the previous section, we first need to find u_c from (8). Extracting G_{tt} and G_{xx} from (12), and plugging into (8) we find

$$u_c = \frac{u_0}{(1-v^2)^{1/(7-p)}} \qquad (15)$$

At this stage we need to commit to the nature of the probe, that is we need to chose a particular function Φ. Let us first focus on the case where the dragging object is a D$(n+1)$-brane itself, in which case it couples to the string theory dilaton ϕ with an overall prefactor of $e^{-\phi}$ in the action, that is $\Phi = \phi$. From (10) we find we can isolate the dependence of $\widetilde{C}$ on v and T

$$\widetilde{C} = P(1-v^2)^A T^B \qquad (16)$$

Where the prefactor is,

$$P = \left(\frac{(\alpha')^{n+2}(2\pi)^{2(p-2)}}{g_{YM}^4 (g_{YM}\sqrt{d_p N})^{2m-n-p+5}}\right)^{\frac{1}{2}} \left(\frac{4\pi}{7-p}\right)^B, \qquad (17)$$

$$A = -\frac{1}{4}\left(5 + m - p + \frac{(p-3)(n-m)}{(7-p)}\right) \qquad (18)$$

and

$$B = -2\frac{(7-p)}{(5-p)}A. \qquad (19)$$

$p\backslash n$	0	1	2	3	4
1	$\{-1,3\}$	$\{-\frac{5}{4},\frac{15}{4}\}$	N/A	N/A	N/A
2	$\{-\frac{3}{4},\frac{5}{2}\}$	$\{-1,\frac{10}{3}\}$	$\{-\frac{5}{4},\frac{25}{6}\}$	N/A	N/A
3	$\{-\frac{1}{2},2\}$	$\{-\frac{3}{4},3\}$	$\{-1,4\}$	$\{-\frac{5}{4},5\}$	N/A
4	$\{-\frac{1}{4},\frac{3}{2}\}$	$\{-\frac{1}{2},3\}$	$\{-\frac{3}{4},\frac{9}{2}\}$	$\{-1,6\}$	$\{-\frac{5}{4},\frac{15}{2}\}$

TABLE I: $\{A,B\}$ displayed for the case where the defect does not extend into the internal space, $m = n$.

$p\backslash n$	1	2	3	4
1	$\{-\frac{11}{12},\frac{11}{4}\}$	N/A	N/A	N/A
2	$\{-\frac{7}{10},\frac{7}{3}\}$	$\{-\frac{19}{20},\frac{19}{6}\}$	N/A	N/A
3	$\{-\frac{1}{2},2\}$	$\{-\frac{3}{4},3\}$	$\{-1,4\}$	N/A
4	$\{-\frac{1}{3},2\}$	$\{-\frac{7}{12},\frac{7}{2}\}$	$\{-\frac{5}{6},5\}$	$\{-\frac{13}{12},\frac{13}{2}\}$

TABLE II: $\{A,B\}$ displayed for the case where the defect extends into one dimension in the internal space, $m = n - 1$.

Here we have made use of the relation

$$u_0 = \left(\frac{4\pi T}{7-p}\right)^{\frac{2}{5-p}}. \tag{20}$$

Various dependencies of $\widetilde{C}$ on velocity and temperature are shown in the following tables for different values of p (rows) and n (columns). Each table has a specific value of m, and the entries in the table are of the form $\{A,B\}$, where A and B are defined as in (16).

Table I shows the dependencies for allowing our defect to have a point on the internal sphere, $n = m$. For table II we allow one spatial dimension of the defect to go to the internal space (our defect must have a minimum dimension of $n = 1$, as we are insisting that one of our spatial dimensions lives in the internal sphere). For table III we allow two spatial dimensions of our defect to go to the internal space (now our defect must have at least 2 spatial dimensions, $n = 2$, and p must contain n, meaning $p \geq 2$).

The Dp-brane energy density obeys the following relation, $\epsilon \sim T^{2\frac{(7-p)}{5-p}}$. Under a boost, we expect $\epsilon \to \gamma^2\epsilon$ and thus $T^{2\frac{(7-p)}{(5-p)}} \to \gamma^2 T^{2\frac{(7-p)}{(5-p)}}$. This motivates the definition of

$$T_{\mathsf{eff}} = \gamma^{\frac{(5-p)}{(7-p)}} T = \gamma^{-2A/B} T. \tag{21}$$

With this definition of the effective temperature we see that, analogous to relation (20), we have an identical relation between the worldvolume horizon and the effective temperature,

$$u_c = \left(\frac{4\pi T_{\mathsf{eff}}}{7-p}\right)^{\frac{2}{5-p}}. \tag{22}$$

Consequently we see that $T^B = (1-v^2)^{-A} T^B_{\mathsf{eff}}$ and so we can rewrite $\widetilde{C}$ as

$$\widetilde{C} = P T^B_{\mathsf{eff}}. \tag{23}$$

This tells us that the loss rate of the moving defect is only dependent on velocity in a trivial way - the defect only sees a blueshifted energy density - and that there are no sensitivities to the microscopic details of the plasma despite the fact that g_{YM} is a dimensionfull quantity and hence defines a microscopic scale in the system. Presumably this is a consequence of the hidden conformal invariance that is present in the Dp brane systems as first exhibited in [13].

There is an area of overlap between this work and [5], where in the latter, various dimensional defects are studied is AdS spaces of variable dimension. Our results for a Dp-brane with $p = 3$ reproduce the equation of motions and loss rates found in [5] for the case AdS_5, as it should. For the case of a pointlike defect, $m = n = 0$, our results can be compared to the formulas quoted for the dragging string in [14] following the analysis of dragging strings in general holographic metrics performed in [7]. Our $m = n = 0$ results are for a dragging D-string, as we included an overall $e^{-\phi}$ coupling in the action. To compare with the results for the dragging fundamental string we have to set $\Phi = 0$ in our analysis (that is, the worldvolume action is independent of the dilaton). It is easy to see that in this case our general expression (10) indeed nicely reduces to the result of [14]. Other than the trivial velocity dependence, the energy loss rate still only depends on velocity and temperature via a power of u_c, and hence, due to (22), via the effective temperature.

Last but not least, it is interesting to note that for the case $m = n = 1$ and $p = 6$, our defect's energy loss rate is independent of both velocity and temperature, other than the expected velocity squared dependence. In $p = 6$ case, there is no good decoupling limit [12] and it is not clear what significance should be attached to this result.

$p\backslash n$	2	3	4
2	$\{-\frac{13}{20},\frac{13}{6}\}$	N/A	N/A
3	$\{-\frac{1}{2},2\}$	$\{-\frac{3}{4},3\}$	N/A
4	$\{-\frac{5}{12},\frac{5}{2}\}$	$\{-\frac{2}{3},4\}$	$\{-\frac{11}{12},\frac{11}{2}\}$

TABLE III: $\{A,B\}$ displayed for the case where the defect extends into two dimensions of the internal space, $m = n - 2$.

IV. SUPERFLUIDITY

Pointlike probes have been used to study superfluids that have a gravity dual [15, 16]. This is an area in which strongly interacting extended defects exist in nature (vortices in Liquid Helium) and thus suggests a possible analysis using a gauge-string duality. Following the layout of [17], we are interested in using a superconducting black hole in AdS_5.

The bulk theory has metric, gauge field, and a complex scalar field (magnitude η and phase θ) degrees of freedom, and is governed by

$$\mathcal{L}_{bulk} = R - \frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}\left[(\partial_\mu \eta)^2 + \Sigma(\eta)(\partial_\mu \theta - qA_\mu)^2\right] - V(\eta).$$

This Lagrangian density allows a charged black brane solution to the metric of the form

$$ds^2 = e^{2A(u)}(-h(u)dt^2 + d\vec{x}^2) + \frac{du^2}{h(u)}, \tag{24}$$

where $A_\mu dx^\mu = A_0(u)dt$, $\eta = \eta(u)$, $\theta = 0$, $A(u)$ is the warp factor, $h(u)$ is the blackening function, and u is the "radial coordinate" that is defined between $-\infty$ and ∞. V and Σ are in principle free functions of η which in reference [17] are taken to be $V(\eta) = -\frac{3}{L^2}\cosh^2(\frac{\eta}{2})(5 - \cosh(\eta))$ and $\Sigma(\eta) = \sinh^2(\eta)$. These particular forms are required for a consistent truncation of Type II B supergravity on a Sasaki-Einstein manifold [18]. The blackening function smoothly interpolates from 1 at large u, to its asymptotically value v_{IR}^2 as $u \to -\infty$.

We can now apply our general formulae to the gravity dual metric for this holographic superfluid. We first reproduce some results of [17]. A string in the bulk has the following action,

$$S = -\int d\sigma d\tau \frac{1}{2\pi\alpha'} Q(\eta)\sqrt{-g}, \tag{25}$$

where $Q = \cosh(\frac{\eta}{2})$, and α' is the square of the string length scale and goes to zero in the limit of infinite string tension. We compare their action for the string (20) to our general formula (3) setting $m = n$ and $n = 0$ so that we are discussing the same defect. We find that we should make the associations $T_0 \to \frac{1}{2\pi\alpha'}$, and $e^{-\Phi(u)} \to Q(\eta(u))$.

Following our prescription for finding the solution to a uniformly moving defect, we first find the root of equation (8) where we are now using the metric appropriate for our bulk theory in AdS_5 (24). We see that $G_{tt} = -e^{2A}h$, $G_{xx} = e^{2A}$ and $G_{uu} = h^{-1}$.

From (8), u_c should be given by $h(u_c) = v^2$. It is clear from the form of $h(u)$ that if $v^2 < v_{IR}^2$ there is no solution, and thus the value $h(u)$ approaches as $u \to -\infty$ defines a cutoff velocity, v_{IR} [17]. Defects whose velocities are below this cutoff experience zero drag force.

For velocities above the cutoff, we find the non-zero drag force density from the momentum density of our uniformly moving defect, $\Pi_x^u = -T_0\widetilde{C}v$, which comes from (4), with $(x_{,u})$ given by (6) and $\widetilde{C}$ defined in (11). We correctly reproduce the following,

$$\widetilde{C} = \pm e^{2A(u_c)}Q(u_c) \tag{26}$$

and again choosing the positive sign we have,

$$\Pi_x^u = -\frac{e^{2A(u_c)}}{2\pi\alpha'}Q(u_c)v = f_{\mathsf{drag}}. \tag{27}$$

We can now easily extend these arguments to a sheet-like defect. This comes down to setting $n = 1$ and continuing to demand that $m = n$. Since our general solution to (8) does not depend on the dimensionality of the defect, we will again find the same cutoff velocity for the sheet. This supports the interpretation of [17] as this cutoff velocity is a property of the system and not of the defect. The drag force will be modified as it is proportional to $\widetilde{C}$, which depends on n through (11). We find,

$$\widetilde{C}_{\mathsf{sheet}} = \pm e^{3A(u_c)}Q(u_c) \tag{28}$$

and

$$\Pi_x^u = -\frac{e^{3A(u_c)}}{2\pi\alpha'}Q(u_c)v = f_{\mathsf{sheet,drag}} \tag{29}$$

Like in the case of a dragging string in this holographic superfluid background, our analysis has been performed entirely in the effective four dimensional language. While the background itself is a consistent truncation of a full ten dimensional solution, it is not entirely obvious what sort of object is described by the defect action eq. (25) with the specific form fo $Q(\eta)$ from the ten 10 dimensional point of view. For the case of dragging strings in the background of five-dimensional charged black holes that correspond to spinning black branes in ten dimension this question has been carefully addressed in [19] and indeed the use of the analog of eq. (25) turned out to be questionable in that case. Here we take the point of view of simply being interested in an effective four dimensional description and take the action of the form eq. (25) as it is the most general two derivative action consistent with symmetries.

V. DISCUSSION

We gave a systematic study of dragging sheets in arbitrary holographic metrics. Our results reconfirm in this most general setting the general structure that was found for pointlike defects in general holographic metrics as well as for the study of dragging sheets in anti-de Sitter spaces: the energy loss is completely insensitive to microscopic details of the system and only depends on the velocity via an overall blueshifted energy density. This seems to be the most general characteristic of energy loss at "strong coupling", where a particle interpretation of the medium is not possible.

An example that may have a real world counterpart is the study of string like defects (corresponding to dragging membranes) in holographic superconductors. Vortices in superfluid Helium and their energy loss can be studied experimentally. To the extent that holographic superfluids and superconductors are candidates for real world systems, the loss rate experienced by a vortex in such a medium could be a physical observable.

Acknowledgments

We'd like to thank Christopher Herzog and Stefan Janiszewski for useful discussions. This work was funded in part by the U.S. DOE under grant No. DE-FG02- 96ER40956.

---

[1] J. M. Maldacena, Adv.Theor.Math.Phys. **2**, 231 (1998), hep-th/9711200.
[2] S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys.Lett. **B428**, 105 (1998), hep-th/9802109.
[3] E. Witten, Adv.Theor.Math.Phys. **2**, 253 (1998), hep-th/9802150.
[4] A. Karch (2011), 1108.4014.
[5] S. Janiszewski and A. Karch, JHEP **1111**, 044 (2011), 1106.4010.
[6] C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, and L. G. Yaffe, JHEP **07**, 013 (2006), hep-th/0605158.
[7] C. P. Herzog, JHEP **09**, 032 (2006), hep-th/0605191.
[8] S. Janiszewski (2011), * Temporary entry *, 1112.0085.
[9] A. Karch and E. Katz, JHEP **0206**, 043 (2002), hep-th/0205236.
[10] P. Breitenlohner and D. Z. Freedman, Phys.Lett. **B115**, 197 (1982).
[11] J. Camino, A. Paredes, and A. Ramallo, JHEP **0105**, 011 (2001), hep-th/0104082.
[12] N. Itzhaki, J. M. Maldacena, J. Sonnenschein, and S. Yankielowicz, Phys.Rev. **D58**, 046004 (1998), hep-th/9802042.
[13] I. Kanitscheider and K. Skenderis, JHEP **0904**, 062 (2009), 0901.1487.
[14] A. Karch, M. Kulaxizi, and A. Parnachev, JHEP **0911**, 017 (2009), 0908.3493.
[15] S. S. Gubser, Phys.Rev. **D78**, 065034 (2008), 0801.2977.
[16] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, Phys.Rev.Lett. **101**, 031601 (2008), 0803.3295.
[17] S. S. Gubser and A. Yarom, JHEP **1003**, 041 (2010), 0908.1392.
[18] S. S. Gubser, C. P. Herzog, S. S. Pufu, and T. Tesileanu, Phys. Rev. Lett. **103**, 141601 (2009), 0907.3510.
[19] C. Herzog and A. Vuorinen, JHEP **0710**, 087 (2007), 0708.0609.