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Baryon Magnetic Moments in the $1/N_c$ Expansion with Flavor Symmetry Breaking

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The magnetic moments and transition magnetic moments of the ground state baryons are analyzed in an expansion in $1/N_c$, $SU(3)$ flavor symmetry breaking and isospin symmetry breaking. There is clear evidence in the experimental data for the hierarchy of magnetic moments of the combined expansion in $1/N_c$ and flavor breaking. $SU(3)$ breaking in the magnetic moments is expected to be enhanced relative to that of other hadronic observables, and significant $SU(3)$ breaking is found.

I. INTRODUCTION

The magnetic moments of the ground state baryons have been of experimental and theoretical interest for over 50 years. The baryon magnetic moments have been studied using a variety of methods, some of which predate QCD and helped lead to the formulation of the theory, including $SU(3)$ flavor symmetry, $SU(3)$ chiral perturbation theory and the $1/N_c$ expansion. This work will consider the baryon magnetic moments in a combined expansion in $1/N_c$ and flavor symmetry breaking, both $SU(3)$ flavor symmetry breaking and isospin breaking. To date, a complete analysis in a combined expansion in $1/N_c$ and $SU(3)$ flavor symmetry breaking has not been done. Analyses in flavor symmetry breaking (i.e. chiral perturbation theory) alone have been performed in works [1–5]. Analyses of the baryon magnetic moments in the $1/N_c$ expansion occur in Refs. [6–13]. The main result of this work is that the combined expansion in $1/N_c$ and $SU(3)$ breaking is needed to understand the hierarchy of baryon magnetic moments found in nature. The derivation given here of the combined expansion to all orders in $SU(3)$ breaking and in $1/N_c$ is analogous to the analysis of the baryon masses of Ref. [14].

The theoretical analysis performed in this work is motivated by new experimental results. Recently, first measurements of two additional decuplet-octet baryon transition magnetic moments have appeared [15, 16]. The new experimental data as well as the promise of future measurements makes it interesting to address the question of the pattern of baryon magnetic moments more completely than in previous work.

The organization of this paper is as follows. Section II analyzes the baryon magnetic moments in the $1/N_c$ expansion in the $SU(3)$ flavor symmetry limit. Four $1/N_c$ operators transforming as spin-1 flavor-octets under the $SU(2) \otimes SU(3)$ spin $\times$ flavor group parametrize the 27 baryon magnetic moments in the $SU(3)$ flavor symmetry limit. Thus, 23 baryon magnetic moment relations are implied by exact $SU(3)$ symmetry. Section III contains the $SU(3)$ flavor symmetry breaking analysis of the baryon magnetic moments to all orders in the $1/N_c$ expansion. The complete set of 27 $1/N_c$ operators transforming according to definite $SU(3)$ representations is determined. In this operator basis, the $SU(3)$ structure of the baryon magnetic moments is manifest. Section IV contains the main results of this work. A hierarchy of magnetic moment relations is obtained in the combined $1/N_c$ and flavor-symmetry breaking expansion. Since many of the ground state baryon magnetic moments are not measured, most of the linear combinations of magnetic moments cannot be evaluated from the experimental data at this time. However, it is possible to perform a variety of fits to the experimental data at successive orders in the combined expansion. Clear evidence in the experimental data for the combined $1/N_c$ and $SU(3)$ symmetry breaking expansion is obtained from these fits. The overall spin-flavor symmetry breaking structure of the baryon magnetic moments is found to be an intricate pattern of $1/N_c$ and $SU(3)$ flavor-symmetry breaking suppressions. It is possible to evaluate all 27 baryon magnetic moments in terms of the leading coefficients extracted from each fit to the experimental data, yielding predictions for the many unmeasured baryon magnetic moments. The complete $SU(3)$ flavor symmetry breaking pattern of the baryon magnetic moments also is explicit in the analysis presented here. Known $SU(3)$-violating relations for the baryon octet magnetic moments [1, 4] are rederived, and corresponding relations for the decuplet magnetic moments and the decuplet-octet transition magnetic moments also are obtained. The $SU(3)$ flavor symmetry breaking of the baryon magnetic moments is predicted on the basis of $SU(3)$ chiral perturbation theory to be dominated by chiral loop corrections which are nonanalytic in the light quark masses. Evidence for the group theory of this flavor symmetry breaking pattern is found. Finally, the $SU(3)$ flavor analysis with perturbative $SU(3)$ breaking is contrasted with the $1/N_c$ analysis for $SU(2) \times U(1)$ flavor symmetry in which $SU(3)$ flavor symmetry is completely broken. Conclusions and final remarks are given in Section V.

II. $SU(3)$ FLAVOR SYMMETRY ANALYSIS

The 27 magnetic moments of the ground state baryons consist of nine magnetic moments of the baryon octet, the eight magnetic moments of the individual octet baryons p, n, etc. and the $\Sigma^0 \to \Lambda$ transition magnetic moment $\Lambda\Sigma^0$, ten magnetic moments of the individual decuplet baryons, and eight transition magnetic moments of a decuplet baryon to an octet baryon. For the flavor-symmetry breaking analysis, it is useful to group the magnetic moments in terms of linear combinations with $I = 0, 1, 2$ and 3. There are ten $I = 0$ combinations,

$$\begin{aligned}
N_0 &\equiv \frac{1}{2}(p+n), \\
\Lambda_0 &\equiv \Lambda, \\
\Sigma_0 &\equiv \frac{1}{3}\left(\Sigma^+ + \Sigma^0 + \Sigma^-\right), \\
\Xi_0 &\equiv \frac{1}{2}\left(\Xi^0 + \Xi^-\right), \\
\Delta_0 &\equiv \frac{1}{4}\left(\Delta^{++} + \Delta^+ + \Delta^0 + \Delta^-\right), \\
\Sigma_0^* &= \frac{1}{3}\left(\Sigma^{*+} + \Sigma^{*0} + \Sigma^{*-}\right), \\
\Xi_0^* &= \frac{1}{2}\left(\Xi^{*0} + \Xi^{*-}\right), \\
\Omega_0 &\equiv \Omega^-, \\
(\Sigma\Sigma^*)_0 &\equiv \frac{1}{3}\left(\Sigma\Sigma^{*+} + \Sigma\Sigma^{*0} + \Sigma\Sigma^{*-}\right), \\
(\Xi\Xi^*)_0 &\equiv \frac{1}{2}\left(\Xi\Xi^{*0} + \Xi\Xi^{*-}\right),
\end{aligned} \tag{1}$$

eleven $I = 1$ combinations,

$$\begin{aligned}
N_1 &\equiv (p-n), \\
(\Lambda\Sigma)_1 &\equiv \Lambda\Sigma^0, \\
\Sigma_1 &\equiv \left(\Sigma^+ - \Sigma^-\right), \\
\Xi_1 &\equiv \left(\Xi^0 - \Xi^-\right), \\
\Delta_1 &\equiv \left(3\Delta^{++} + \Delta^+ - \Delta^0 - 3\Delta^-\right), \\
\Sigma_1^* &\equiv \left(\Sigma^{*+} - \Sigma^{*-}\right), \\
\Xi_1^* &\equiv \left(\Xi^{*0} - \Xi^{*-}\right), \\
(N\Delta)_1 &\equiv \left(p\Delta^+ + n\Delta^0\right), \\
(\Lambda\Sigma^*)_1 &\equiv \Lambda\Sigma^{*0}, \\
(\Sigma\Sigma^*)_1 &\equiv \left(\Sigma\Sigma^{*+} - \Sigma\Sigma^{*-}\right), \\
(\Xi\Xi^*)_1 &\equiv \left(\Xi\Xi^{*0} - \Xi\Xi^{*-}\right),
\end{aligned} \tag{2}$$

five $I = 2$ combinations,

$$\begin{aligned}
\Sigma_2 &\equiv \left(\Sigma^+ - 2\Sigma^0 + \Sigma^-\right), \\
\Delta_2 &\equiv \left(\Delta^{++} - \Delta^+ - \Delta^0 + \Delta^-\right), \\
\Sigma_2^* &\equiv \left(\Sigma^{*+} - 2\Sigma^{*0} + \Sigma^{*-}\right), \\
(N\Delta)_2 &\equiv \left(p\Delta^+ - n\Delta^0\right), \\
(\Sigma\Sigma^*)_2 &\equiv \left(\Sigma\Sigma^{*+} - 2\Sigma\Sigma^{*0} + \Sigma\Sigma^{*-}\right),
\end{aligned} \tag{3}$$

and one $I = 3$ combination,

$$\Delta_3 \equiv \left(\Delta^{++} - 3\Delta^+ + 3\Delta^0 - \Delta^-\right), \tag{4}$$

where the notation used for the transition magnetic moments is the same as Ref. [8]. In the limit of exact isospin symmetry, all magnetic moment operators are linear combinations of isovector $I = 1$ and isoscalar $I = 0$ operators, and so the five $I = 2$ magnetic moment combinations Σ_2, Δ_2, Σ_2^*, $(N\Delta)_2$ and $(\Sigma\Sigma^*)_2$ and the one $I = 3$ magnetic moment combination Δ_3 vanish. The general analysis of this work includes isospin symmetry breaking, allowing these six isospin-violating combinations to be nonzero. However, in much of the analysis, isospin symmetry breaking will be neglected. When isospin breaking is neglected, the six $I = 2$ and $I = 3$ isospin-violating magnetic moment combinations give relations which can be used to determine six magnetic moments in terms of the others. As a consequence, there are only 21 independent magnetic moments in the isospin symmetry limit. Without loss of generality, the six dependent magnetic moments in the isospin symmetry limit are taken to be

$$\begin{aligned}
n\Delta^0 &= p\Delta^+, \\
\Delta^+ &= \frac{1}{3}\left(2\Delta^{++} + \Delta^-\right), \\
\Delta^0 &= \frac{1}{3}\left(\Delta^{++} + 2\Delta^-\right), \\
\Sigma^0 &= \frac{1}{2}\left(\Sigma^+ + \Sigma^-\right), \\
\Sigma^{*0} &= \frac{1}{2}\left(\Sigma^{*+} + \Sigma^{*-}\right), \\
\Sigma\Sigma^{*0} &= \frac{1}{2}\left(\Sigma\Sigma^{*+} + \Sigma\Sigma^{*-}\right).
\end{aligned} \tag{5}$$

Progress in understanding the spin-flavor structure of static baryon properties has resulted from the discovery of a contracted spin-flavor symmetry for baryons in the large N_c limit, where N_c is the number of colors in QCD [6, 17–21]. Expansion in $1/N_c$ about the large-N_c limit [22, 23] has led to a classification of baryon operators in $1/N_c$ and flavor symmetry breaking [6–14, 17–21, 24–27]. A wide variety of baryon properties have been studied. It has been shown that the $1/N_c$ expansion gives a good description of the spin-flavor structure of QCD baryons with $N_c = 3$.

The form of the baryon magnetic moment $1/N_c$ expansion is

$$\sum_{n=1}^{N_c} c_{(n)} \, \frac{1}{N_c^{n-1}} \mathcal{O}_{(n)}, \tag{6}$$

where each independent n-body operator $\mathcal{O}_{(n)}$ in the expansion is a product of n of the baryon spin-flavor generators J^i, T^a and G^{jb} [9, 25]. $1/N_c$ power counting rules dictate the explicit factor of $1/N_c^{n-1}$ accompanying each n-body operator $\mathcal{O}_{(n)}$ in the expansion. The N_c-dependence of each term in the $1/N_c$ expansion is a product of this explicit factor times the implicit N_c-dependence of matrix elements of the operator $\mathcal{O}_{(n)}$. Matrix elements of the spin generators J^i are $O(1)$, so operators with additional powers of the spin operator generally are suppressed in the $1/N_c$ expansion. Matrix elements of T^a and G^{ia} have N_c-dependence which varies in different portions of the flavor weight diagrams of large-N_c baryons and for different values of $a = 1, \cdots, 8$, making the analysis more subtle. Each independent operator in the $1/N_c$ expansion is accompanied by an unknown coefficient $c_{(n)}$ which is order unity at leading order in the

$1/N_c$ expansion. Predictions based on the $1/N_c$ expansion are obtained by neglecting operators which are suppressed by powers of $1/N_c$. For baryons with finite N_c, the baryon $1/N_c$ expansion extends only up to N_c-body operators. Thus, for QCD baryons with $N_c = 3$, the $1/N_c$ expansion of Eq. (6) ends with 3-body operators.

In the $SU(3)$ flavor symmetry limit, the baryon magnetic moment operator transforms as a $(1, \mathbf{8})$ representation under $SU(2) \otimes SU(3)$ spin $\times$ flavor symmetry. For QCD baryons with $N_c = 3$, the $1/N_c$ expansion for the baryon magnetic moments is [9]

$$\begin{aligned} M^{iQ} &= a^{1,\mathbf{8}}_{(1)}\, G^{iQ} + b^{1,\mathbf{8}}_{(2)}\, \frac{1}{N_c} J^i T^Q \\ &\quad + b^{1,\mathbf{8}}_{(3)}\, \frac{1}{N_c^2} \mathcal{D}^{iQ}_{(3)} + c^{1,\mathbf{8}}_{(3)}\, \frac{1}{N_c^2} \mathcal{O}^{iQ}_{(3)}, \end{aligned} \tag{7}$$

where

$$T^Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} = T^3 + \frac{1}{\sqrt{3}} T^8 = Q \tag{8}$$

is the $SU(3)$ flavor generator corresponding to the light quark electromagnetic charge, and

$$G^{iQ} \equiv G^{i3} + \frac{1}{\sqrt{3}} G^{i8}, \tag{9}$$

is the spin-flavor generator in the Q flavor direction. The two 3-body operators in the $1/N_c$ expansion are defined by

$$\begin{aligned} \mathcal{D}^{iQ}_{(3)} &= \{J^i, \{J^j, G^{jQ}\}\}, \\ \mathcal{O}^{iQ}_{(3)} &= \{J^2, G^{iQ}\} - \frac{1}{2}\{J^i, \{J^j, G^{jQ}\}\}. \end{aligned} \tag{10}$$

The 2-body operator $J^i T^Q$ and the 3-body operator $\mathcal{D}^{iQ}_{(3)}$ are purely diagonal operators with no nonvanishing matrix elements between decuplet and octet baryons. The 3-body operator $\mathcal{O}^{iQ}_{(3)}$ is purely off-diagonal; its only nonvanishing matrix elements are decuplet-octet transition matrix elements. In contrast, the 1-body operator G^{iQ} has both diagonal and off-diagonal matrix elements. This 1-body operator is the leading operator of the $1/N_c$ expansion, since the matrix elements of G^{i3} are $O(N_c)$ for large-N_c baryons. The G^{iQ} operator also receives a subleading $O(1)$ contribution from the matrix elements of

$$G^{i8} \equiv \frac{1}{\sqrt{3}} \left(J^i - 3 J^i_s \right). \tag{11}$$

The three subleading operators $J^i T^Q$, $\mathcal{D}^{iQ}_{(3)}$ and $\mathcal{O}^{iQ}_{(3)}$ each contribute at $O(1/N_c)$. This statement requires explanation. The matrix elements of $J^i T^3$ are $O(1)$, so $J^i T^3/N_c$ matrix elements are $O(1/N_c)$. The operator

$$T^8 \equiv \frac{1}{\sqrt{3}} \left(N_c - 3 N_s \right), \tag{12}$$

nominally has a piece of $O(N_c)$, but its contribution to magnetic moment splittings only comes from the term proportional to N_s, so $J^i T^8/N_c$ also contributes to magnetic moment splittings at $O(1/N_c)$. The matrix elements of the 3-body operators $\mathcal{D}^{iQ}_{(3)}/N_c^2$ and $\mathcal{O}^{iQ}_{(3)}/N_c^2$ are suppressed by $1/N_c^2$ relative to the leading operator G^{iQ}, and so are $O(1/N_c)$.

The matrix elements of the four $1/N_c$ operators in Eq. (7) are tabulated in the first four columns of Table I. The matrix elements given in the table are for each operator with $i = 3$. For octet baryons, the matrix elements are taken between octet baryons with $J_z = 1/2$, whereas for decuplet baryons, the matrix elements are given between decuplet baryons with $J_z = 3/2$. For transition magnetic moments between decuplet and octet baryons, the matrix elements are given between $J_z = 1/2$ states for both the decuplet and octet baryons. This same convention for the matrix elements was used in Ref. [8]. Eq. (7) implies that each of the 27 baryon magnetic moments is parametrized by four $1/N_c$ operator coefficients in the $SU(3)$ flavor symmetry limit. For example,

$$\begin{aligned} p &= \frac{1}{2} a^{1,\mathbf{8}}_{(1)} + \frac{1}{2}\frac{1}{N_c} b^{1,\mathbf{8}}_{(2)} + \frac{3}{2}\frac{1}{N_c^2} b^{1,\mathbf{8}}_{(3)}, \\ n &= -\frac{1}{3} a^{1,\mathbf{8}}_{(1)} - \frac{1}{N_c^2} b^{1,\mathbf{8}}_{(3)}, \\ \Delta^+ &= \frac{1}{2} a^{1,\mathbf{8}}_{(1)} + \frac{3}{2}\frac{1}{N_c} b^{1,\mathbf{8}}_{(2)} + \frac{15}{2}\frac{1}{N_c^2} b^{1,\mathbf{8}}_{(3)}, \\ \frac{1}{\sqrt{2}} p\Delta^+ &= \frac{1}{3} a^{1,\mathbf{8}}_{(1)} + \frac{3}{2}\frac{1}{N_c^2} c^{1,\mathbf{8}}_{(3)}, \end{aligned} \tag{13}$$

for the strangeness zero baryons. For QCD baryons, $N_c = 3$, so it is to be understood that $N_c = 3$ in the above equation.

In the $SU(3)$ symmetry limit, the 27 baryon magnetic moments are given in terms of 4 parameters, so there are 23 $SU(3)$ relations for the baryon magnetic moments. The nine magnetic moments of the octet baryons are described in terms of only two linear combinations of the four $1/N_c$ coefficients. Thus, there are seven $SU(3)$ relations for the octet baryon magnetic moments, the Coleman-Glashow relations [28],

$$\begin{aligned} p &= \Sigma^+, \\ \Sigma^- &= \Xi^-, \\ n &= 2\Lambda = \Xi^0 = \frac{2}{\sqrt{3}} \Lambda\Sigma^0 \\ &= -\left(\Sigma^+ + \Sigma^-\right) = -2\Sigma^0. \end{aligned} \tag{14}$$

The ten magnetic moments of the decuplet baryons are given in terms of one linear combination of the four $1/N_c$ coefficients, so there are nine $SU(3)$ relations for the decuplet baryon magnetic moments,

$$\begin{aligned} \frac{1}{2}\Delta^{++} &= \Delta^+ = -\Delta^- = \Sigma^{*+} = -\Sigma^{*-} = -\Xi^{*-} = -\Omega^-, \\ \Delta^0 &= \Sigma^{*0} = \Xi^{*0} = 0. \end{aligned} \tag{15}$$

The eight transition magnetic moments between decuplet and octet baryons also are given in terms of one linear combination of the four $1/N_c$ operator coefficients, yielding seven $SU(3)$ relations for the decuplet-octet transition magnetic moments

$$\begin{aligned} p\Delta^+ &= n\Delta^0 = \Sigma\Sigma^{*+} = \Xi\Xi^{*0} = \frac{2}{\sqrt{3}}\Lambda\Sigma^{*0} = 2\Sigma\Sigma^{*0}, \\ \Sigma\Sigma^{*-} &= \Xi\Xi^{*-} = 0. \end{aligned} \qquad (16)$$

Eqs. (14), (15) and (16) are the 23 $SU(3)$ relations for the magnetic moments. Note that in the $SU(3)$ limit, five baryon magnetic moments vanish. In addition, six of the above 23 $SU(3)$ relations correspond to the six isospin relations Eq. (5).

The level of accuracy at which the $SU(3)$ relations are satisfied gives a quantitative measure of the size of $SU(3)$ breaking for the baryon magnetic moments. Writing each $SU(3)$ relation as LHS = RHS such that all terms on the left- and right-hand sides of the equation contribute with the same sign, the accuracy of an $SU(3)$ relation is defined to be $|\mathrm{LHS} - \mathrm{RHS}|/|\mathrm{LHS} + \mathrm{RHS}|/2$. There are nine $SU(3)$ relations which can be tested by present experimental data. The experimental accuracies of these nine relations are

$$\begin{array}{ll} p = \Sigma^+, & 12.8 \pm 0.4\% \\ \Sigma^- = \Xi^-, & 56.3 \pm 2.9\% \\ n = 2\Lambda, & 43.8 \pm 0.5\% \\ n = \Xi^0, & 41.9 \pm 0.5\% \\ n = \frac{2}{\sqrt{3}}\Lambda\Sigma^0, & 2.8 \pm 4.8\% \\ n = -\left(\Sigma^+ + \Sigma^-\right), & 38.3 \pm 1.7\% \\ \frac{1}{2}\Delta^{++} = -\Omega^-, & 41.4 \pm 10.5\% \\ p\Delta^+ = \Lambda\Sigma^{*0}, & 24.3 \pm 8.6\% \\ p\Delta^+ = \Sigma\Sigma^{*+}, & 8.6 \pm 10.7\% \end{array} \qquad (17)$$

where the experimental data is taken from the Particle Data Group [29] and the two CLAS measurements [15, 16]. The experimental accuracies of Eq. (17) reveal that $SU(3)$ breaking is quite sizeable for the baryon magnetic moments; it is larger than the canonical 30% breaking found for many other hadronic observables. $SU(3)$ chiral perturbation theory predicts a definite pattern of $SU(3)$ breaking, with the leading $SU(3)$ flavor breaking arising from a one-loop chiral correction which is proportional to $m_q^{1/2}$ from the graph in Fig. 1. Thus, one naively expects enhanced $SU(3)$ breaking of order $\sqrt{1/3} \sim 60\%$ for the baryon magnetic moments. $SU(3)$ breaking as large as this is seen in the experimental accuracies of Eq. (17).

One can obtain a quantitative estimate of the four $1/N_c$ coefficients appearing in the $1/N_c$ expansion Eq. (7) for the baryon magnetic moments in the $SU(3)$ symmetry limit. Taking the independent magnetic moments to be the strangeness zero baryon magnetic moments p, n, Δ^{++}, $p\Delta^+$, one finds that the four $1/N_c$ coefficients are given by the linear combinations

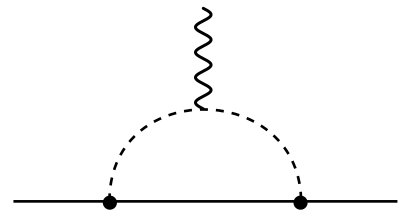

FIG. 1: One loop correction to the baryon magnetic moments of order $m_q^{1/2}$ for $\Delta M = 0$.

$$\begin{aligned} a^{1,\mathbf{8}}_{(1)} &= -p - 4n + \frac{1}{6}\Delta^{++}, \\ \frac{1}{N_c} b^{1,\mathbf{8}}_{(2)} &= 2p + 3n, \\ \frac{1}{N_c^2} b^{1,\mathbf{8}}_{(3)} &= -\frac{1}{2}(p+n) + \frac{1}{12}\Delta^{++}, \\ \frac{1}{N_c^2} c^{1,\mathbf{8}}_{(3)} &= -\frac{1}{3}(p-n) + \frac{1}{18}\Delta^{++} + \frac{2}{3}\frac{1}{\sqrt{2}} p\Delta^+, \end{aligned} \qquad (18)$$

which yields

$$\begin{aligned} a^{1,\mathbf{8}}_{(1)} &= 5.88 \pm 0.09, \\ b^{1,\mathbf{8}}_{(2)} &= -0.459 \pm 0.000, \\ b^{1,\mathbf{8}}_{(3)} &= 0.63 \pm 0.36, \\ c^{1,\mathbf{8}}_{(3)} &= 3.87 \pm 0.45\ . \end{aligned} \qquad (19)$$

These values for the four coefficients parametrizing the $SU(3)$ symmetry limit will be useful for comparison with the values extracted from fits to experimental data given in Section IV.

The analysis of the N_c-dependence of the operator matrix elements implies that the $1/N_c$ expansion of the magnetic moment operator in the $SU(3)$ limit can be truncated after the first operator G^{iQ} up to corrections of order $1/N_c$. The $1/N_c$ expansion,

$$M^{iQ} = a^{1,\mathbf{8}}_{(1)}\, G^{iQ} + O\left(\frac{1}{N_c}\right), \qquad (20)$$

parametrizes the 27 magnetic moments in terms of one $1/N_c$ coefficient, so there are three additional $SU(3)$ relations when the subleading $O(1/N_c)$ terms in the $1/N_c$ expansion are neglected. The three additional relations, holding in the $SU(3)$ limit at $O(1)$ in the $1/N_c$ expansion, are

$$\begin{array}{ll} p = -\frac{3}{2}n, & 2.7 \pm 0.0\%, \\ \frac{1}{2}\Delta^{++} - 3n = 3p, & 5.0 \pm 0.1\%, \\ (p-n) = \frac{1}{6}\Delta^{++} + 2\frac{1}{\sqrt{2}}p\Delta^+, & 24.0 \pm 0.6\% \end{array} \qquad (21)$$

which are obtained from Eq. (18) by setting the coefficients $b^{1,\mathbf{8}}_{(2)}$, $b^{1,\mathbf{8}}_{(3)}$ and $c^{1,\mathbf{8}}_{(3)}$ to zero. These relations for strangeness zero baryons are expected to hold to relative order $1/N_c^2$, or approximately 10% accuracy, in the $SU(3)$ flavor symmetry limit [9, 14].

The additional magnetic moment relations Eq. (21) are predictions of contracted $SU(6)$ spin-flavor symmetry for large-N_c baryons [9, 14]. The relations are identical to the $SU(6)$ magnetic moment relations of Beg, Lee and Pais [30], since only the single operator G^{iQ} is retained in the $1/N_c$ expansion, which is the same operator as the non-relativistic quark model. Contracted spin-flavor symmetry of large-N_c baryons does not predict the absolute normalization of the baryon magnetic moments due to the arbitrary coefficient $a^{1,\mathbf{8}}_{(1)}$ in Eq. (20). The $1/N_c$ expansion gives a quantitative prediction for the accuracy of the spin-flavor relations in terms of factors of $1/N_c$ and $SU(3)$ flavor symmetry breaking. The $1/N_c$ expansion predicts that the $SU(6)$ relations of the strangeness zero baryons will be more accurate than $SU(6)$ relations between the magnetic moments of baryons with different strangeness, which have violations due to $SU(3)$ flavor symmetry breaking. One example of such an $SU(6)$ relation [30] is

$$p = -\Omega^-, \quad 32.1 \pm 2.1\%. \tag{22}$$

III. $SU(3)$ BREAKING ANALYSIS

This section contains the $SU(3)$ symmetry breaking operator analysis for the baryon magnetic moments. The complete operator basis of the $1/N_c$ expansion contains 27 operators which parametrize the 27 baryon magnetic moments. Symmetry-breaking operators are divided into operators which arise at leading order in $SU(3)$ symmetry breaking, operators which occur at subleading orders in $SU(3)$ symmetry breaking, and operators which violate isospin symmetry.

A. Leading Order $SU(3)$ Breaking

The baryon magnetic moment operator is a spin-1 flavor-octet operator in the $SU(3)$ flavor symmetry limit. To leading order in $SU(3)$ breaking, all representations in

$$(1, \mathbf{8} \otimes \mathbf{8}) = (1, \mathbf{1} \oplus \mathbf{8}_S \oplus \mathbf{8}_A \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}) \tag{23}$$

are obtained [9, 10]. Each of these representations is considered separately.

1. $(1, \mathbf{8})$

The four coefficients of the spin-1 flavor-$\mathbf{8}$ operators of the $SU(3)$ symmetry limit each receive $SU(3)$-violating contributions at one-loop in baryon chiral perturbation theory. There are contributions which are order $m_q^{1/2}$ and order $m_q \ln m_q$ in the limit of degenerate decuplet and octet baryons, $\Delta M = 0$.[1] Because each of the four operators of the $1/N_c$ expansion transforms as the $a = Q$ component of a flavor octet, which is traceless, no flavor-singlet subtractions are required for these operators.

Four additional spin-1 flavor-$\mathbf{8}$ operators are obtained at leading order in $SU(3)$ breaking. These operators are

$$d^{Qb8} \left(\bar{a}^{1,\mathbf{8}}_{(1)}\, G^{ib} + \bar{b}^{1,\mathbf{8}}_{(2)}\, \frac{1}{N_c} J^i T^b + \bar{b}^{1,\mathbf{8}}_{(3)}\, \frac{1}{N_c^2} \mathcal{D}^{ib}_{(3)} \right.$$
$$\left. + \bar{c}^{1,\mathbf{8}}_{(3)}\, \frac{1}{N_c^2} \mathcal{O}^{ib}_{(3)} \right), \tag{24}$$

which reduce to

$$\frac{1}{\sqrt{3}} \left(\bar{a}^{1,\mathbf{8}}_{(1)}\, G^{i\overline{Q}} + \bar{b}^{1,\mathbf{8}}_{(2)}\, \frac{1}{N_c} J^i T^{\overline{Q}} + \bar{b}^{1,\mathbf{8}}_{(3)}\, \frac{1}{N_c^2} \mathcal{D}^{i\overline{Q}}_{(3)} \right.$$
$$\left. + \bar{c}^{1,\mathbf{8}}_{(3)}\, \frac{1}{N_c^2} \mathcal{O}^{i\overline{Q}}_{(3)} \right), \tag{25}$$

since

$$\begin{aligned} d^{Qb8} O^{ib} &= \left(d^{3b8} + \frac{1}{\sqrt{3}} d^{8b8} \right) O^{ib} \\ &= \frac{1}{\sqrt{3}} \left(O^{i3} - \frac{1}{\sqrt{3}} O^{i8} \right) \equiv \frac{1}{\sqrt{3}} O^{i\overline{Q}}, \end{aligned} \tag{26}$$

where

$$T^{\overline{Q}} = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & -\frac{2}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix} = T^3 - \frac{1}{\sqrt{3}} T^8 \equiv \overline{Q}. \tag{27}$$

These $a = \overline{Q}$ flavor-octet operators also require no flavor-singlet subtractions since $\overline{Q}$ is traceless. The operators arise in chiral perturbation theory at one loop, with a nonanalytic quark mass dependence of $m_q^{1/2}$ for $\Delta M = 0$.

2. $(1, \mathbf{10} + \overline{\mathbf{10}})$

There are two spin-1 operators in the $\mathbf{10} + \overline{\mathbf{10}}$ flavor representation,

$$c^{1,\mathbf{10}+\overline{\mathbf{10}}}_{(2)}\, \frac{1}{N_c} O^{i[8Q]}_{(2)} + c^{1,\mathbf{10}+\overline{\mathbf{10}}}_{(3)}\, \frac{1}{N_c^2} O^{i[8Q]}_{(3)}, \tag{28}$$

where

$$\begin{aligned} O^{i[8Q]}_{(2)} &\equiv \left(\{T^8, G^{iQ}\} - \{G^{i8}, T^Q\} \right), \\ O^{i[8Q]}_{(3)} &\equiv \left(\{G^{i8}, J^k G^{kQ}\} - \{J^k G^{k8}, G^{iQ}\} \right). \end{aligned} \tag{29}$$

[1] For nonvanishing ΔM, the functional dependence of each of these corrections generalizes to a known function $\mathcal{F}(m_q, \Delta M)$. The chiral corrections are given by these more general functions of m_q and ΔM.

In general, the spin-1 flavor-($\mathbf{10}+\overline{\mathbf{10}}$) operators

$$\begin{aligned} O^{i[ab]}_{(2)} &\equiv \left(\{T^a, G^{ib}\} - \{G^{ia}, T^b\}\right), \\ O^{i[ab]}_{(3)} &\equiv \left(\{G^{ia}, J^k G^{kb}\} - \{J^k G^{ka}, G^{ib}\}\right), \end{aligned} \quad (30)$$

require subtractions of the flavor-octet operators obtained by contracting with f^{abc}. The specific operators Eq. (28), however, do not require flavor-$\mathbf{8}$ subtractions since $f^{8Qc} = 0$. Because of the explicit antisymmetry of the flavor $\mathbf{10}+\overline{\mathbf{10}}$ operators in $[ab]$, only the $Q = 3$ components of the operators in Eq. (28) are nonvanishing. Consequently, no new operators arise when $Q \to \overline{Q}$ at higher order in $SU(3)$ breaking. Each of the operators in Eq. (28) arises in chiral perturbation theory at one loop with a flavor dependence that is $m_q^{1/2}$ for $\Delta M = 0$.

3. $(1, \mathbf{1})$

There are two spin-1 flavor-singlet operators

$$\delta^{Q8}\left(c^{1,\mathbf{1}}_{(1)}\; J^i + c^{1,\mathbf{1}}_{(3)}\; \frac{1}{N_c^2}\{J^2, J^i\}\right). \quad (31)$$

The leading contribution to these operator coefficients is a calculable chiral logarithm of order $m_q \ln m_q$ in the limit that $\Delta M = 0$.

4. $(1, \mathbf{27})$

There are three spin-1 flavor-$\mathbf{27}$ operators generated at leading order in $SU(3)$ breaking,

$$\begin{aligned} &c^{1,\mathbf{27}}_{(2)}\; \frac{1}{N_c} O^{i(8Q)}_{(2)} + b^{1,\mathbf{27}}_{(3)}\; \frac{1}{N_c^2} D^{i(8Q)}_{(3)} \\ &\quad + c^{1,\mathbf{27}}_{(3)}\; \frac{1}{N_c^2} O^{i(8Q)}_{(3)}, \end{aligned} \quad (32)$$

where

$$\begin{aligned} O^{i(ab)}_{(2)} &\equiv \left(\{T^a, G^{ib}\} + \{G^{ia}, T^b\}\right), \\ D^{i(ab)}_{(3)} &\equiv J^i\{T^a, T^b\}, \\ O^{i(ab)}_{(3)} &\equiv \left(\{G^{ia}, J^k G^{kb}\} + \{J^k G^{ka}, G^{ib}\}\right). \end{aligned} \quad (33)$$

These flavor-$\mathbf{27}$ operators require flavor-octet and flavor-singlet subtractions. All three operators arise in chiral perturbation theory at one loop from a calculable chiral logarithm of order $m_q \ln m_q$ in the limit $\Delta M = 0$.

B. Subleading Order $SU(3)$ Breaking

Additional operators are generated at subleading orders in $SU(3)$ breaking. Most of the additional operators occur in the tensor product $(1, \mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8})$. All of the additional operators transform as either $(1, \mathbf{27})$ or $(1, \mathbf{64})$ representations.

1. $(1, \mathbf{27})$

There are three additional $(1, \mathbf{27})$ operators which are of subleading order in $SU(3)$ breaking and which do not break isospin symmetry,

$$\begin{aligned} &d^{Qb8}\left(\bar{c}^{1,\mathbf{27}}_{(2)}\; \frac{1}{N_c} O^{i(8b)}_{(2)} + \bar{b}^{1,\mathbf{27}}_{(3)}\; \frac{1}{N_c^2} D^{i(8b)}_{(3)} \right. \\ &\qquad \left. + \bar{c}^{1,\mathbf{27}}_{(3)}\; \frac{1}{N_c^2} O^{i(8b)}_{(3)}\right) = \\ &\frac{1}{\sqrt{3}}\left(\bar{c}^{1,\mathbf{27}}_{(2)}\; \frac{1}{N_c} O^{i(8\overline{Q})}_{(2)} + \bar{b}^{1,\mathbf{27}}_{(3)}\; \frac{1}{N_c^2} D^{i(8\overline{Q})}_{(3)} \right. \\ &\qquad \left. + \bar{c}^{1,\mathbf{27}}_{(3)}\; \frac{1}{N_c^2} O^{i(8\overline{Q})}_{(3)}\right), \end{aligned} \quad (34)$$

where the operators in this equation are defined in Eq. (33). These operators first arise in chiral perturbation theory at two loops. These flavor-$\mathbf{27}$ operators require flavor-$\mathbf{8}$ and flavor-$\mathbf{1}$ subtractions.

2. $(1, \mathbf{64})$

There are two additional $(1, \mathbf{64})$ operators at second subleading order in $SU(3)$ breaking which do not break isospin symmetry,

$$c^{1,\mathbf{64}}_{(3)}\; \frac{1}{N_c^2} O^{i(88Q)}_{(3)} + d^{1,\mathbf{64}}_{(3)}\; \frac{1}{N_c^2} \tilde{O}^{i(88Q)}_{(3)}, \quad (35)$$

where

$$\begin{aligned} O^{i(88Q)}_{(3)} &\equiv \{G^{i8}, \{T^8, T^Q\}\}, \\ \tilde{O}^{i(88Q)}_{(3)} &\equiv \{T^8, \{T^8, G^{iQ}\}\}. \end{aligned} \quad (36)$$

For flavor $\mathbf{64}$ operators, one needs to perform flavor $\mathbf{1}$, $\mathbf{8}$ and $\mathbf{27}$ subtractions.

Additional spin-1 flavor-$\mathbf{64}$ operators arise at third subleading order via the tensor product $(1, \mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8})$. Naively, there are two additional operators, $d^{Qb8}\{G^{i8}, \{T^8, T^b\}\}$ and $d^{Qb8}\{T^8, \{T^8, G^{ib}\}\}$, which reduce to $1/\sqrt{3}$ times $\{G^{i8}, \{T^8, T^{\overline{Q}}\}\}$ and $\{T^8, \{T^8, G^{i\overline{Q}}\}\}$, respectively. The $(1, \mathbf{64})$ operators are not linearly independent, however, since they satisfy the relation

$$\begin{aligned} &\{G^{i8}, \{T^8, T^Q\}\} - \{G^{i8}, \{T^8, T^{\overline{Q}}\}\} = \\ &\quad \{T^8, \{T^8, G^{iQ}\}\} - \{T^8, \{T^8, G^{i\overline{Q}}\}\}. \end{aligned} \quad (37)$$

Consequently, there are only three independent $(1, \mathbf{64})$ operators. Without loss of generality, only the first of the two $\overline{Q}$ operators

$$\bar{c}^{1,\mathbf{64}}_{(3)}\; \frac{1}{\sqrt{3}}\; \frac{1}{N_c^2} O^{i(88\overline{Q})}_{(3)}, \quad (38)$$

is included in the $1/N_c$ expansion, where

$$O^{i(88\overline{Q})}_{(3)} = \{G^{i8}, \{T^8, T^{\overline{Q}}\}\}. \quad (39)$$

C. Isospin Breaking

Additional operators arise from isospin breaking, which transforms as the $a = 3$ component of a flavor-**8** representation. The only additional operators are either $(1, \mathbf{27})$ or $(1, \mathbf{64})$ operators. There are no new $(1, \mathbf{8})$ operators, since the operators

$$d^{Qb3}\left(G^{ib} + \frac{1}{N_c}J^iT^b + \frac{1}{N_c^2}\mathcal{D}^{ib}_{(3)} + \frac{1}{N_c^2}\mathcal{O}^{ib}_{(3)}\right) \tag{40}$$

reduce to

$$\begin{aligned} d^{Qb3}O^{ib} &= \left(d^{3b3} + \frac{1}{\sqrt{3}}d^{8b3}\right)O^{ib} \\ &= \frac{1}{\sqrt{3}}\left(O^{i8} + \frac{1}{\sqrt{3}}O^{i3}\right), \end{aligned} \tag{41}$$

which are linear combinations of the spin-1 flavor-**8** operators of Eqs. (7) and (25).

1. $(1, \mathbf{27})$

The additional flavor-**27** operators which break isospin symmetry are

$$\begin{aligned} &g^{1,\mathbf{27}}_{(2)}\frac{1}{N_c}O^{i(3Q)}_{(2)} + f^{1,\mathbf{27}}_{(3)}\frac{1}{N_c^2}D^{i(3Q)}_{(3)} \\ &\quad + g^{1,\mathbf{27}}_{(3)}\frac{1}{N_c^2}O^{i(3Q)}_{(3)}, \end{aligned} \tag{42}$$

where

$$\begin{aligned} O^{i(3Q)}_{(2)} &\equiv \left(\{T^3, G^{iQ}\} + \{G^{i3}, T^Q\}\right), \\ D^{i(3Q)}_{(3)} &\equiv J^i\{T^3, T^Q\}, \\ O^{i(3Q)}_{(3)} &\equiv \left(\{G^{i3}, J^kG^{kQ}\} + \{J^kG^{k3}, G^{iQ}\}\right). \end{aligned} \tag{43}$$

These operators contain the $I = 2$ operators $\{T^3, G^{i3}\}$, $J^i\{T^3, T^3\}$, and $\{G^{i3}, J^kG^{k3}\}$.

2. $(1, \mathbf{64})$

The additional flavor-**64** operators which break isospin symmetry are

$$\begin{aligned} &g^{1,\mathbf{64}}_{(3)}\frac{1}{N_c^2}O^{i(33Q)}_{(3)} + f^{1,\mathbf{64}}_{(3)}\frac{1}{N_c^2}\tilde{O}^{i(33Q)}_{(3)} \\ &\quad + \bar{g}^{1,\mathbf{64}}_{(3)}\frac{1}{N_c^2}O^{i(33\overline{Q})}_{(3)}, \end{aligned} \tag{44}$$

where

$$\begin{aligned} O^{i(33Q)}_{(3)} &\equiv \{G^{i3}, \{T^3, T^Q\}\}, \\ \tilde{O}^{i(33Q)}_{(3)} &\equiv \{T^3, \{T^3, G^{iQ}\}\}, \\ O^{i(33\overline{Q})}_{(3)} &\equiv \{G^{i3}, \{T^3, T^{\overline{Q}}\}\}. \end{aligned} \tag{45}$$

These operators contain the $I = 2$ and $I = 3$ operators $\{G^{i3}, \{T^3, T^8\}\}$, $\{T^3, \{T^3, G^{i8}\}\}$, and $\{G^{i3}, \{T^3, T^3\}\}$.

In summary, there are 27 independent operators in the $1/N_c$ expansion when isospin violation is included. Four operators occur in the $SU(3)$ flavor symmetry limit, Eq. (7). The $SU(3)$ flavor-symmetry breaking operators consist of four $(1, \mathbf{8})$ operators, two $(1, \mathbf{10} + \overline{\mathbf{10}})$ operators, two $(1, \mathbf{1})$ operators, six $(1, \mathbf{27})$ operators, and three $(1, \mathbf{64})$ operators. These 17 operators appear in Eqs. (25), (28), (31), (32), (34), (35) and (38). There are six additional operators which break isospin symmetry, Eqs. (42) and (44). If one works in the isospin symmetry limit, there are only 21 independent baryon magnetic moments, and the six isospin-violating operators are not included in the $1/N_c$ expansion.

The matrix elements of the 27 operators of the $1/N_c$ expansion for the baryon magnetic moments are given in Tables I and II. Table I contains the 15 operators which arise at leading order in $SU(3)$ breaking, whereas Table II contains the remaining 12 operators. The first six operators of Table II violate $SU(3)$ symmetry at subleading orders, while the last six operators violate isospin symmetry. The bosonic operator method of Ref. [31] was useful for the calculation of the operator matrix elements.

The implications of this operator basis are studied in the next section.

IV. RESULTS

Each of the 27 operators of the $1/N_c$ expansion contributes to one linear combination of baryon magnetic moments. These combinations are tabulated in Tables III, IV, and V. For convenience, Tables VI and VII give the combinations for the nine $(1, \mathbf{27})$ operators with isospin $I = 0$, 1, and 2, and the six $(1, \mathbf{64})$ operators with isospin $I = 0$, 1, 2 and 3, respectively. Since many of the baryon magnetic moments are not measured, the majority of these linear combinations can not be evaluated at present. Each of the 27 operators of the $1/N_c$ expansion contributes at a specific order in $1/N_c$ given by the explicit factor $1/N_c^{n-1}$ times the N_c-dependence of its operator matrix elements $\langle\mathcal{O}_{(n)}\rangle$. In addition, each operator of the $1/N_c$ expansion transforms as a specific representation of $SU(3)$ flavor symmetry, and so its coefficient $c_{(n)}$ is a function of the $SU(3)$ breaking parameters, the light quark masses m_q. Most of the operators first arise from chiral loop corrections which are nonanalytic in the light quark masses, and their operator coefficients are proportional to these calculable nonanalytic functions.

The leading $1/N_c$ and m_q dependences of each operator and its operator coefficient are summarized in Tables VIII and IX. Table VIII contains the 21 operators which respect isospin symmetry, whereas Table IX contains the six isospin-violating operators. The $SU(3)$ breaking and isospin breaking factors listed in Tables VIII and IX, respectively, give the functional dependences on m_q of the symmetry-breaking factors, which is valid for $\Delta M = 0$. The actual functional dependence is more complicated than this, depending nonanalytically on both the light quark masses m_q through the pion masses M_Π and on the decuplet-octet singlet hyperfine mass splitting ΔM. Closed form expressions for the these generalized flavor symmetry breaking functions for nonvanishing ΔM are known, and can be found in the literature. Since these functions will not be numerically evaluated in this work, they are not given explicitly in the tables. The $1/N_c$ dependences of the operators given in Tables VIII and IX follow from the fact that matrix elements of G^{i3} are order N_c, whereas matrix elements of G^{i8} are order unity. T^3 has matrix elements of order unity, and T^8 has matrix elements of order unity in their contribution to baryon magnetic moment splittings. The identities

$$\begin{aligned}\{J^i, G^{i8}\} &= \frac{1}{2\sqrt{3}}\{J^i, J^i - 3J^i_s\} \\ &= \frac{1}{2\sqrt{3}}\left(-J^2 - 3J_s^2 + 3I^2\right), \\ \{J^i, G^{i3}\} &= \frac{1}{2}\left(V^2 - U^2 + J_u^2 - J_d^2\right), \end{aligned} \tag{46}$$

where V^2 and U^2 refer to V-spin and U-spin, are useful for determining some of the operator matrix elements and their N_c dependences. The matrix elements of $\{J^i, G^{i8}\}$ are order unity, since spin J, strange spin J_s and isospin I are all order unity for the large-N_c baryons. The matrix elements of $\{J^i, G^{i3}\}$ are order N_c at leading order, since for large-N_c baryon flavor representations, both J_u^2 and J_d^2 are order N_c^2, but their difference is order N_c.

From Tables VIII and IX, one is able to determine which operators are the leading operators in the combined expansion in $1/N_c$ and flavor symmetry breaking. There are two operators which contribute at leading order N_c to the baryon magnetic moments, namely G^{iQ} and $G^{i\overline{Q}}$. The G^{iQ} respects $SU(3)$ flavor symmetry, and therefore is order unity in $SU(3)$ breaking. In contrast, the operator $G^{i\overline{Q}}$ breaks $SU(3)$ flavor symmetry. The leading $G^{i\overline{Q}}$ operator contribution arises from a one-loop chiral diagram Fig. 1 which depends nonanalytically on the light quark masses as $m_q^{1/2}$. Thus, the operator $G^{i\overline{Q}}$ is suppressed relative to the operator G^{iQ} by $SU(3)$ breaking, but it is not suppressed in $1/N_c$. It is important to emphasize that $SU(3)$-breaking also contributes to G^{iQ} at one-loop order in chiral perturbation theory, so its coefficient receives subleading contributions proportional to $m_q^{1/2}$ and $m_q \ln m_q$ from one-loop pion graphs in addition to its leading $SU(3)$ symmetric contribution of order unity.

The first 15 operators in Table VIII arise at leading order in $SU(3)$ breaking from one-loop pion corrections in $SU(3)$ chiral perturbation theory. The largest $SU(3)$ breaking contribution is proportional to $m_q^{1/2}$, and the $\overline{Q}$ flavor-octet and the flavor-$\mathbf{10}+\overline{\mathbf{10}}$ operators receive leading contributions proportional to this $SU(3)$ breaking factor. The flavor-singlet and three flavor-27 operators receive chiral logarithmic contributions proportional to $m_q \ln m_q$ at leading order in $SU(3)$ breaking. The 15 operators which occur at leading order in $SU(3)$ breaking occur at various orders in $1/N_c$, as listed in Table VIII. As mentioned already, there are two leading $O(N_c)$ operators, G^{iQ} and $G^{i\overline{Q}}$. The next largest operators in the $1/N_c$ hierarchy are the three $O(1)$ operators, each of which occurs at leading order in $SU(3)$ breaking. These operators are the 2-body $(1, \mathbf{10}+\overline{\mathbf{10}})$ operator, J^i and the 2-body $(1, \mathbf{27})$ operator. The 2-body $(1, \mathbf{10}+\overline{\mathbf{10}})$ operator coefficient is proportional to the $SU(3)$ breaking factor $m_q^{1/2}$; the two other $O(1)$ operators generated at leading order in $SU(3)$ breaking are proportional to $m_q \ln m_q$. At order $1/N_c$ in the $1/N_c$ expansion, there are eight additional operators which are leading order in $SU(3)$ breaking. The two remaining operators which are leading order in $SU(3)$ breaking are $\{J^2, J^i\}$ and $J^i\{T^8, T^Q\}$, which are order $1/N_c^2$ in the $1/N_c$ expansion. All of the other $SU(3)$-violating operators in Table VIII first arise at subleading orders in $SU(3)$ chiral perturbation theory, so the dominant $SU(3)$ breakings proportional to the $m_q^{1/2}$, $m_q \ln m_q$ and m_q from one-loop chiral graphs and from counterterms with one insertion of the quark mass matrix do not generate these subleading operators.

Table IX presents the corresponding $1/N_c$ and isospin-breaking factors for the six isospin-violating operators of the $1/N_c$ expansion. Three of these operators occur at one-loop in chiral perturbation theory, and have coefficients proportional to isospin-breaking with $m_q \ln m_q$ dependence. The remaining three operators arise at two-loop or three-loop order in chiral perturbation theory. Since isospin breaking is much smaller than $SU(3)$ breaking and $1/N_c$, the six isospin-violating operators are much more suppressed than the 21 operators of Table VIII.

A. Fits

The $1/N_c$ and flavor breaking hierarchy given by Tables VIII and IX suggests a number of fits to the experimental data. The first fit, Fit A, is the conventional four parameter $SU(3)$ symmetric fit, which is used for comparison with the subsequent analysis. The $SU(3)$ flavor symmetry breaking parameter $m_q^{1/2}$ is roughly of order $1/\sqrt{N_c}$. This suggest a unified expansion in both flavor symmetry breaking and $1/N_c$, treating $m_q \sim 1/\sqrt{N_c}$ and $m_q \ln m_q \sim 1/N_c$. The successive fits, Fits B, C, D, E and F of Tables X and XI correspond to using this unified power counting. Fit B retains the $O(N_c)$ term; Fit C adds one $O(\sqrt{N_c})$ term; Fit D adds one $O(1/\sqrt{N_c})$ term; Fit E adds five $O(1/N_c)$ terms and Fit F adds four $O(1/N_c^{3/2})$ terms. There are three operators of leading order in $SU(3)$ breaking in Table II that are left out of Fit F, one is $O(1/N_c^2)$ and two are $O(1/N_c^3)$ in the unified expansion, and so are higher order in the unified power counting. Only thirteen magnetic moments have been measured. Once two more magnetic moments are measured, a determination of all of the 15 leading $SU(3)$ breaking operators of Table II becomes possible.

Fits A, B, C, D, E and F are fits to the 13 experimental data points listed in the first column of Table X. Since many of the magnetic moments used in the fits are measured much more accurately than the expected theoretical uncertainties, a theory uncertainty of $\sigma^{\rm theory}$ is added in quadrature to each experimental measurement to avoid biasing the fits. The theory error added for each fit is tuned to yield $\chi^2/{\rm dof} = 1$ for the fit. Consequently, $\sigma^{\rm theory}$ is a measure of how well the data fits the theory formula. The primed fits, Fits A′, B′, C′, D′ and E′, are fits to the 10 most accurately measured magnetic moments. These fits exclude the three primed data points in the first column of Table X, which are the less well-measured Δ^{++} magnetic moment and the CLAS measurements of $\Lambda\Sigma^{*0}$ and $\Sigma\Sigma^{*+}$. Also included in Table X are the numerical values of all 27 magnetic moments evaluated at the central values of the extracted coefficients.

1. Fit A

The first fit, Fit A, is the four-parameter $SU(3)$ symmetric fit using Eq. (7). Fit A yields the coefficients

$$\begin{aligned} a^{1,\mathbf{8}}_{(1)} &= 4.98 \pm 0.49, \\ b^{1,\mathbf{8}}_{(2)} &= 0.66 \pm 1.49, \\ b^{1,\mathbf{8}}_{(3)} &= -0.25 \pm 0.95, \\ c^{1,\mathbf{8}}_{(3)} &= 4.18 \pm 1.71, \end{aligned} \tag{47}$$

with significant error bars, and a theory error of $\sigma^{\rm theory} = 0.35\ \mu_N$. Fit A′ is a fit to only 10 of the 13 measured magnetic moments. Values for the coefficients consistent with those of Fit A are obtained, and $\sigma^{\rm theory} = 0.28\ \mu_N$. The coefficient values obtained for Fits A and A′ also are consistent with the earlier determination using strangeness zero baryon magnetic moments Eq. (19), within their rather large error bars. Again, it is important to note that the Δ^{++} magnetic moment is not included in Fit A′, whereas the determination of Eq. (19) depends upon the Δ^{++} magnetic moment (see Eq. (18)). There is tension between the Δ^{++} and Ω^- measurements and between the $p\Delta^+$ and the transition magnetic moments $\Lambda\Sigma^{*0}$ and $\Sigma\Sigma^{*+}$ in an $SU(3)$ symmetry fit, since $SU(3)$ relations imply that these magnetic moments are simply related. The Fit A′, which does not contain this additional data, therefore finds smaller $SU(3)$ breaking, but this result is not physically significant. Significant $SU(3)$ breaking in the decuplet and the transition magnetic moments is allowed.

2. Fit B

Fit B is a one-parameter fit of the data to the single operator G^{iQ}, which is the leading operator of the $SU(3)$ symmetric limit and of the combined $1/N_c$ and flavor symmetry breaking expansion. From Fit B, one extracts the leading operator coefficient

$$a^{1,\mathbf{8}}_{(1)} = 5.45 \pm 0.31, \tag{48}$$

with $\sigma^{\rm theory} = 0.41\ \mu_N$. For the one-parameter fit, again there is tension between obtaining the experimental values of both Δ^{++} and Ω^- magnetic moments in an $SU(3)$ symmetric fit, and there is tension between obtaining $p\Delta^+$ and the new two CLAS measurements. The extracted value of $a^{1,\mathbf{8}}_{(1)}$ of Fit B is consistent with the experimental determination of (5.88 ± 0.09) from the strangeness zero baryon magnetic moments of Eq. (18) in Section I. Fit B′ including only 10 of the 13 magnetic moments yields the value

$$a^{1,\mathbf{8}}_{(1)} = 5.07 \pm 0.35, \tag{49}$$

with $\sigma^{\rm theory} = 0.39\ \mu_N$. The greater discrepancy in this extracted value with the determination of Eq. (18) occurs because the latter determination depends upon the Δ^{++} magnetic moment, which is not one of the 10 magnetic moments included in Fit B′.

3. Fit C

Fit C is a two-parameter fit to the leading operator G^{iQ} and the first subleading operator $G^{i\overline{Q}}$. Both of these operators are $O(N_c)$. The G^{iQ} operator respects $SU(3)$ flavor symmetry. The $G^{i\overline{Q}}$ operator violates $SU(3)$ symmetry at leading order $m_q^{1/2}$. A theory error of $0.27\ \mu_N$ is found for Fit C, so Fit C is a better fit to the data than the four-parameter $SU(3)$ symmetry fit, Fit A. The extracted coefficients are

$$\begin{aligned} a^{1,\mathbf{8}}_{(1)} &= 5.07 \pm 0.23, \\ \bar{a}^{1,\mathbf{8}}_{(1)} &= 1.60 \pm 0.42\ . \end{aligned} \tag{50}$$

The suppression of the $\bar{a}^{1,\mathbf{8}}_{(1)}$ coefficient relative to the $a^{1,\mathbf{8}}_{(1)}$ coefficient by a factor of 0.3 is a reflection of the $SU(3)$-breaking suppression factor $m_q^{1/2}$, since both operators are $O(N_c)$. Fit C′ to only 10 of the 13 magnetic moments yields similar values for the two coefficients,

$$\begin{aligned} a^{1,\mathbf{8}}_{(1)} &= 4.85 \pm 0.21, \\ \bar{a}^{1,\mathbf{8}}_{(1)} &= 1.54 \pm 0.37, \end{aligned} \tag{51}$$

and for the theory error, $\sigma^{\rm theory} = 0.23\ \mu_N$. These 2-parameter fits provide clear evidence for the $1/N_c$ and $SU(3)$ flavor breaking hierarchy of the combined $1/N_c$

and $SU(3)$ breaking expansion. The unified expansion predicts that the dominant $SU(3)$ breaking in the magnetic moments is due to the $G^{i\overline{Q}}$ operator, and Fits C and C$'$ support this prediction. For this 2-parameter fit, all baryon magnetic moments except Σ^{*0} are nonzero.

4. Fit D

Fit D adds one additional operator to the fit, namely the $(1, \mathbf{10}+\overline{\mathbf{10}})$ 2-body operator, which is $O(1)$ in $1/N_c$ and proportional to $m_q^{1/2}$. All of the other operators proportional to $m_q^{1/2}$ flavor symmetry breaking which are not included in the fit are $O(1/N_c)$, and therefore suppressed by one factor of $1/N_c$ relative to the operators included in Fit D. Fit D has a theory error of 0.275 μ_N, and so it also is a better fit to the data than Fit A, even though it has fewer parameters. Fit D yields coefficients $a_{(1)}^{1,\mathbf{8}}$ and $\bar{a}_{(1)}^{1,\mathbf{8}}$ essentially identical to those of Fit C, with identical error bars. The third coefficient $c_{(2)}^{1,\mathbf{10}+\overline{\mathbf{10}}}$ is determined to be 0.37 ± 0.52 in Fit D and 0.71 ± 0.40 in Fit D$'$. The three operators of Fit D do not contribute to the Σ^{*0} magnetic moment, which remains zero.

5. Fit E

Fit E is an eight-parameter fit which adds to the three operators of Fit D, the three $1/N_c$ suppressed operators of the $SU(3)$ symmetric fit Eq. (7), J^i and the leading 2-body $(1, \mathbf{27})$ operator. This eight-operator fit includes all of the $SU(3)$ breaking contributions occurring at one loop in chiral perturbation theory which are leading order in $1/N_c$. Relative to Fit D, the five added operators are the three $O(1/N_c)$ $SU(3)$-symmetric $(1, \mathbf{8})$ operators Eq. (7) which are $O(1)$ in $SU(3)$ flavor symmetry breaking, the $(1, \mathbf{1})$ operator J^i which is $O(1)$ in $1/N_c$ and proportional to $m_q \ln m_q$, and the leading 2-body $(1, \mathbf{27})$ operator which is $O(1)$ in $1/N_c$ and proportional to $m_q \ln m_q$. The theory error σ^{theory} drops dramatically for Fit E to 0.024 μ_N.

For Fit E$'$, the theory error is 0.035 μ_N, so Fit E is a better than Fit E$'$. One can view this result as evidence that $SU(3)$ breaking in the decuplet and decuplet-octet transition magnetic moments is substantial, since the primed fit leaves out the data which indicates significant $SU(3)$ breaking for the decuplet and transition magnetic moments. For Fits E and E$'$, the values for the coefficients already obtained in the earlier fits are mainly stable, and the experimental data is reproduced at the central values of the coefficients to within the theory errors for the included data points. The Σ^{*0} is nonvanishing for Fit E, due to the inclusion of the J^i operator. From Table II matrix elements of the 15 leading operators in $SU(3)$ breaking, one sees that only the J^i operator of these 15 leading operators contributes to the Σ^{*0} magnetic moment. Thus, this magnetic moment is a sensitive probe of the J^i operator.

6. Fit F

Fit F corresponds to a 12 operator fit. Relative to Fit E, it adds four operators, namely the three $O(1/N_c)$ flavor-$\mathbf{8}$ operators in the $\overline{Q}$ flavor direction, proportional to $m_q^{1/2}$, as well as the 3-body $(1, \mathbf{10}+\overline{\mathbf{10}})$ operator which also is $O(1/N_c)$ and proportional to $m_q^{1/2}$. Fit F necessarily includes all 13 measured magnetic moments. A theory error of 0.06 μ_N is found for the fit, so Fit F is not a better fit to the data than Fit E. The coefficients of the operators which are present in Fit E are stable, with the exception of $c_{(3)}^{1,\mathbf{8}}$. The coefficients of the four additional operators in Fit F, however, have very significant error bars, and are not well determined. The large errors on $c_{(3)}^{1,\mathbf{8}}$, $\bar{c}_{(3)}^{1,\mathbf{8}}$, and $c_{(3)}^{1,\mathbf{10}+\overline{\mathbf{10}}}$ reflects degeneracies in fitting the decuplet-octet transition magnetic moments. Only three of these transition magnetic moments are measured, and the values of these three ill-determined coefficients of Fit F can compensate for one another to return the three experimental values. Measurement of additional transition magnetic moments in the future are necessary to break these degeneracies.

One clear result of the fits is that the experimental data is reproduced by the operators of leading order in the combined expansion in $1/N_c$ and $SU(3)$ breaking. There is evidence for both the $1/N_c$ and $SU(3)$ breaking pattern in the experimental data. Fits to the unified expansion with fewer parameters, Fits C and D, are better fits to the data than the $SU(3)$ symmetry fit, Fit A. The extracted values of the coefficients of the combined expansion are stable when more operators are added, with the exception of Fit F, which suffers from degeneracies. The eight operator fit, Fit E, is able to reproduce the experimental data within a theory error of 0.025 μ_N.

B. $SU(3)$ Hierarchy

Finally, it is worth returning to the magnetic moment linear combinations of Tables III, IV and V. It is not possible to evaluate most of the linear combinations because of their dependence on unmeasured magnetic moments. However, it is possible to consider the various linear combinations of the octet baryon magnetic moments in an expansion in $SU(3)$ breaking alone. These combinations yield a hierarchy of relations in $SU(3)$ breaking for the octet baryon magnetic moments. The $SU(3)$ breaking analysis of the decuplet magnetic moment combinations and of the decuplet-octet transition magnetic moment combinations gives analogous relations for these magnetic moments.

First, consider the baryon octet magnetic moment combinations in an expansion in $SU(3)$ breaking. At order $m_q^{1/2}$ in the $SU(3)$ chiral expansion, three $SU(3)$ combinations vanish, namely

$$\begin{aligned} &2N_0 + \Lambda_0 + 3\Sigma_0 + 2\Xi_0, \\ &2N_0 - 3\Lambda_0 - \Sigma_0 + 2\Xi_0, \\ &N_1 + 2\sqrt{3}\,(\Lambda\Sigma)_1 - \Xi_1. \end{aligned} \tag{52}$$

These three magnetic moment relations were first derived by Caldi and Pagels [1] using $SU(3)$ chiral perturbation theory for the octet baryons. In Ref. [4], it was shown that these relations continue to hold when the decuplet baryons are included in heavy baryon chiral perturbation theory [32, 33]. Including order $m_q \ln m_q$ and order m_q counterterm effects, a single combination of the octet baryon magnetic moments continues to vanish. This combination,

$$\left[N_1 + 2\sqrt{3}\,(\Lambda\Sigma)_1 - \Xi_1\right] - \left[2N_0 - 3\Lambda_0 - \Sigma_0 + 2\Xi_0\right], \tag{53}$$

is equal to

$$-2n + 3\Lambda + 2\sqrt{3}\Lambda\Sigma^0 + \frac{1}{2}\left(\Sigma^+ + \Sigma^-\right) - 2\Xi^0, \tag{54}$$

which is the same magnetic moment combination first found in Ref. [4].

Repeating this same $SU(3)$ symmetry breaking analysis for the baryon decuplet magnetic moments at order $m_q^{1/2}$ in the $SU(3)$ chiral expansion yields five linear combinations which vanish,

$$\begin{aligned} &4\Delta_0 + 3\Sigma_0^* + 2\Xi_0^* + \Omega_0, \\ &\Delta_1 - 3\Sigma_1^* - 4\Xi_1^*, \\ &4\Delta_0 - 5\Sigma_0^* - 2\Xi_0^* + 3\Omega_0, \\ &\Delta_0 - 3\Sigma_0^* + 3\Xi_0^* - \Omega_0, \\ &\Delta_1 - 10\Sigma_1^* + 10\Xi_1^*. \end{aligned} \tag{55}$$

Including order $m_q \ln m_q$ and order m_q effects, three of the combinations

$$\begin{aligned} &\left[\Delta_1 - 3\Sigma_1^* - 4\Xi_1^*\right] - 2\left[4\Delta_0 - 5\Sigma_0^* - 2\Xi_0^* + 3\Omega_0\right], \\ &\Delta_0 - 3\Sigma_0^* + 3\Xi_0^* - \Omega_0, \\ &\Delta_1 - 10\Sigma_1^* + 10\Xi_1^*, \end{aligned} \tag{56}$$

continue to vanish.

A similar $SU(3)$ analysis applies to the decuplet-octet transition magnetic moments. At order $m_q^{1/2}$ in the $SU(3)$ chiral expansion, the three linear combinations

$$\begin{aligned} &(\Sigma\Sigma^*)_0 - (\Xi\Xi^*)_0\,, \\ &(N\Delta)_1 - 2\sqrt{3}\,(\Lambda\Sigma^*)_1 + 3\,(\Sigma\Sigma^*)_1 - 2\,(\Xi\Xi^*)_1\,, \\ &-(N\Delta)_1 + 2\sqrt{3}\,(\Lambda\Sigma^*)_1 + (\Sigma\Sigma^*)_1 - 2\,(\Xi\Xi^*)_1\,, \end{aligned} \tag{57}$$

vanish. Including order $m_q \ln m_q$ and order m_q effects, two of these combinations,

$$\begin{aligned} &\left[(N\Delta)_1 - 2\sqrt{3}\,(\Lambda\Sigma^*)_1 + 3\,(\Sigma\Sigma^*)_1 - 2\,(\Xi\Xi^*)_1\right] \\ &\quad -8\left[(\Sigma\Sigma^*)_0 - (\Xi\Xi^*)_0\right], \\ &-(N\Delta)_1 + 2\sqrt{3}\,(\Lambda\Sigma^*)_1 + (\Sigma\Sigma^*)_1 - 2\,(\Xi\Xi^*)_1\,, \end{aligned} \tag{58}$$

continue to vanish.

This $SU(3)$ hierarchy of the baryon magnetic moments represents a subhierarchy of the combined $1/N_c$ and $SU(3)$ breaking expansion. Numerical evaluation of the baryon octet magnetic moment combinations and their experimental accuracies supports the conclusion of this work that the $SU(3)$ hierarchy alone is not sufficient to understand the spin-flavor structure of the baryon magnetic moments. For example, the isovector combination

$$N_1 - \Sigma_1 + \Xi_1 \tag{59}$$

in the $(1, \mathbf{10} + \overline{\mathbf{10}})$ representation has an experimental accuracy of $11.0 \pm 0.7\%$, which is to be compared with $1/N_c$ times the $SU(3)$ suppression factor corresponding to $m_q^{1/2}$ breaking. Another example is the isovector combination in the $(1, \mathbf{8})$ representation,

$$-N_1 + 2\sqrt{3}\,(\Lambda\Sigma)_1 + 4\Sigma_1 + 5\Xi_1, \tag{60}$$

which has an experimental accuracy of $8.6 \pm 2.2\%$. This combination is order unity in $SU(3)$ breaking, but is suppressed by $1/N_c^2$ relative to the leading $O(N_c)$ contribution to the isovector magnetic moments. For the isoscalar combinations, the combination

$$7N_0 + 2\Lambda_0 - 6\Sigma_0 - 3\Xi_0, \tag{61}$$

is order unity in $SU(3)$ breaking, but suppressed by $1/N_c$ relative to the leading $O(1)$ contribution to the isoscalar magnetic moments. This combination has an experimental accuracy of $14.7 \pm 1.5\%$. In each of these cases, the experimental accuracy of the relation is not explained by $SU(3)$ breaking alone.

C. $1/N_c$ Hierarchy for $SU(2) \times U(1)$ Flavor Symmetry

The $1/N_c$ expansion of the baryon magnetic moments for approximate $SU(3)$ flavor symmetry with perturbative $SU(3)$ flavor symmetry breaking can be contrasted with the analysis for $SU(2) \times U(1)$ flavor symmetry in which $SU(3)$ flavor symmetry is completely broken. Most of the analysis for $SU(2) \times U(1)$ flavor symmetry was performed in Ref. [8]. The analysis is completed in this subsection. All 27 operators of the $1/N_c$ expansion for $SU(2) \times U(1)$ flavor symmetry are determined, and the magnetic moment combinations corresponding to each of the isoscalar and isovector operators are obtained. One important new result of this analysis is that there are two isovector magnetic moment combinations which are suppressed by $1/N_c^2$, or by three powers of $1/N_c$ relative to the leading isovector contribution which is $O(N_c)$.

For $SU(2) \times U(1)$ flavor symmetry, the spin-flavor symmetry of the baryon $1/N_c$ expansion is $SU(4) \times SU(2)$, where the $SU(4)$ is the spin-flavor symmetry for the u and d quarks and $SU(2)$ is strange quark spin symmetry. The generators of the $SU(4)$ spin-flavor symmetry are the isospin generators I^a, the spin of the u and d quarks J^i_{ud}, and G^{ia} for $i = 1, 2, 3$ and $a = 1, 2, 3$. The $SU(2)$ spin symmetry is generated by the strange quark spin generators J^i_s. In addition, the strange quark number operator is N_s is required to distinguish between baryons with different numbers of strange quarks. For a baryon containing N_s strange quarks, the strange quark spin J_s is equal to $N_s/2$, so that

$$J_s^2 \ = \ \frac{N_s}{2}\left(\frac{N_s}{2}+1\right) \ . \tag{62}$$

The total spin J^i of the baryon is equal to

$$J^i \ = \ J^i_u + J^i_d + J^i_s = J^i_{ud} + J^i_s \ . \tag{63}$$

Subsequently, it follows that

$$\begin{aligned} J_{ud}\cdot J_s \ &= \ \frac{1}{2}\left(J^2 - J^2_{ud} - J^2_s\right) \\ &= \ \frac{1}{2}\left(J^2 - I^2 - J^2_s\right), \end{aligned} \tag{64}$$

since $J^2_{ud} = I^2$ is an $SU(4)$ spin-flavor operator identity.

The 27 baryon magnetic moments are parametrized by 27 $1/N_c$ operators. These operators and their operator matrix elements are given in Tables XII, XIII and XIV. In the strict isospin symmetry limit, the only nonvanishing baryon magnetic moments are the ten $I = 0$ and the eleven $I = 1$ magnetic moments of Eqs. (1) and (2), respectively. The linear combinations of baryon magnetic moments corresponding to the isoscalar and isotensor magnetic moment operators are tabulated in Tables XV and XVI, respectively, together with their orders in $1/N_c$.

At leading order in the $1/N_c$ expansion, the isoscalar magnetic moments are given by the two operators J^i_{ud} and J^i_s, with order unity matrix elements [6, 8, 9]. At order $1/N_c$, there are two additional operators, $J^i_{ud}N_s$ and $J^i_sN_s$. The six remaining isoscalar operators first appear at order $1/N_c^2$. The linear combinations corresponding to these operators are linear combinations of the six $1/N_c^2$ isoscalar combinations $S1 - S6$ of Ref. [8]. Table XVII gives these six isoscalar combinations in the quark representation.

At leading order in the $1/N_c$ expansion, the isovector magnetic moments are given by a single operator G^{i3} with $O(N_c)$ matrix elements [17, 19]. At order unity, there is one additional operators $G^{i3}N_s$ [6]. At order $1/N_c$, there are seven additional operators. Finally, at order $1/N_c^2$, there are two additional operators $J^i_{ud}I^3N_s$ and $J^i_sI^3N_s$. Ref. [8] obtained nine $1/N_c$ isovector combinations $V1 - V9$, given in Table XVIII for the quark representation. It is a new result of this work that there are two combinations of these nine $1/N_c$ relations which are suppressed by $1/N_c^2$.

D. Comparison with Literature

It is worthwhile to compare and contrast the analysis performed in this work to that of prior treatments of the ground state baryon magnetic moments in the $1/N_c$ expansion.

The first ground-breaking papers [17, 19] by Dashen and Manohar showed that the baryon isovector magnetic moments μ_V^{ia}, $a = 1, 2, 3$, were given by a single operator $gN_cX_0^{ia}$ with arbitrary normalization g at leading order and first subleading order in the $1/N_c$ expansion. Dashen, Jenkins and Manohar [6] extended these results to $N_f = 3$ flavors, determining the isovector and isoscalar couplings and the F/D ratio for the magnetic moments at leading and first subleading orders in $1/N_c$.

Luty, March-Russell and White [7] considered the baryon magnetic moments in an expansion in $1/N_c$ and m_s. They obtained the leading operator G^{iQ}, and they studied the one-loop corrections [4] to the baryon magnetic moments in heavy baryon $SU(3)$ chiral perturbation theory [32, 33] in the $1/N_c$ expansion for $m_s \neq 0$. They recognized that the leading chiral correction was $O(N_c m_s^{1/2})$, and therefore unsuppressed in $1/N_c$ relative to the leading contribution. Luty and March-Russell also give the order in $1/N_c$ of the chiral logarithmic corrections and the one-loop counterterms.

Jenkins and Manohar [8] considered the isovector and isoscalar magnetic moment expansions in $1/N_c$, obtaining relations suppressed by a relative factor of $1/N_c^2$ for the $I = 0$ and $I = 1$ magnetic moments. Additional relations were obtained in the $SU(3)$ flavor symmetry limit by combining isovector and isoscalar magnetic moments. The predictions of the $1/N_c$ expansion were contrasted with those of nonrelativistic quark model.

Dashen, Jenkins and Manohar [9] found the complete set of operator identities in the quark representation, enabling the construction of a basis of $1/N_c$ operators for any QCD operator transforming according a given spin × flavor representation. The four magnetic moment operators of the $1/N_c$ expansion in the $SU(3)$ flavor symmetry limit, Eq. (7), were obtained. The leading and first subleading $1/N_c$ expansions for the isovector and isoscalar magnetic moment operators also were obtained.

Dai, Dashen, Jenkins and Manohar [10] studied $SU(3)$ flavor symmetry breaking of the magnetic moments in the $1/N_c$ expansion, including the leading $SU(3)$-breaking operators transforming as flavor $\mathbf{8}$, $\mathbf{10} + \overline{\mathbf{10}}$, $\mathbf{27}$ and $\mathbf{1}$ representations. It was shown that the magnetic moments have a dominant pattern of $SU(3)$ breaking with the group theoretic structure of the one-loop diagram of Fig. 1.

Lebed and Martin [11] obtained 27 linearly independent magnetic moment $1/N_c$ operators, in terms of products of the 1-body operators G^{i3}, G^{i8}, J^i, I^a and N_s. Fits in a combined expansion in $1/N_c$ and $SU(3)$ breaking were performed by assigning flavor-symmetry breaking suppression factors to the different operators. The operators chosen did not transform as definite $SU(3)$

representations, as they do in this work. In principle, the operator basis of Lebed and Martin can be written as linear combinations of the operator basis of the current basis, leading to identical fits. However, the fits performed by Lebed and Martin do not correspond to the fits of the current paper. In detail, Lebed and Martin perform a seven-operator fit and a twelve-operator fit. The seven-operator fit includes the four $1/N_c$ operators of the $SU(3)$ symmetry limit, Eq. (7), plus the additional three operators J_s^i, $N_s G^{iQ}$ and $J_s^i T^Q$. This set of seven operators is equivalent to the four operators of the $SU(3)$ limit, namely G^{iQ}, $J^i T^Q$, $\mathcal{D}_{(3)}^{iQ}$ and $\mathcal{O}_{(3)}^{iQ}$, and the three additional operators J_s^i, $\{T^8, G^{iQ}\}$ and $\{G^{i8}, T^Q\}$. The latter two operators are equivalent to the leading $\mathbf{10}+\overline{\mathbf{10}}$ operator $O_{(2)}^{i[8Q]}$ and the leading $\mathbf{27}$ operator $O_{(2)}^{i(8Q)}$. This set of operators includes only seven of the eight operators of Fit E. The difference is that Fit E contains two independent operators $G^{i\overline{Q}}$ and J^i, whereas the Lebed and Martin fit only contains a single operator J_s^i. Thus, the seven-operator fit of Lebed and Martin is not equivalent to Fit E. The twelve-operator fit of Lebed and Martin adds the five additional operators $N_s J^i$, $N_s J^i T^Q$, $\{J \cdot J_s, G^{iQ}\}$, $\{J_s^j G^{jQ}, J^i\}$ and $\{J^j G^{jQ}, J_s^i\}$. An equivalent set of five operators is $J^i T^8$, $J^i\{T^8, T^Q\}$, $\{J^j G^{j8}, G^{iQ}\}$, $\{J_s^j G^{jQ}, J^i\}$, and $\{J^j G^{jQ}, G^{i8}\}$. This set of twelve operators is not equivalent to the twelve operators of Fit F. Relative to Fit E, Fit F adds the four operators $J^i T^{\overline{Q}}$, $\{J^2, G^{i\overline{Q}}\}$, $\{J^i, J^j G^{j\overline{Q}}\}$ and the subleading three-body $\mathbf{10}+\overline{\mathbf{10}}$ operator $O_{(3)}^{i[8Q]}$ which is equal to $\{G^{i8}, J^k G^{kQ}\} - \{J^k G^{k8}, G^{iQ}\}$. Fit F includes only 12 of the 15 operators of Table II, leaving out $\{J^2, J^i\}$, and the subleading 3-body flavor $\mathbf{27}$ operators $D_{(3)}^{i(8Q)}$ and $O_{(3)}^{i(8Q)}$. These latter two operators are equal to $J^i\{T^8, T^Q\}$ and $\{G^{i8}, J^k G^{kQ}\} + \{J^k G^{k8}, G^{iQ}\}$, both of which are included in the Lebed-Martin twelve-operator fit. Thus, the Lebed-Martin twelve operator fit is not equivalent to Fit F.

Refs. [12, 13] by Flores-Mendieta and collaborators compute the one-loop corrections to the baryon magnetic moments in chiral perturbation theory explicitly in the $1/N_c$ expansion for vanishing and nonvanishing baryon mass differences. The explicit expressions for the nonanalytic corrections are obtained in terms of the four pion coupling coefficients of the $1/N_c$ expansion and the leading order tree-level magnetic moment operators of the $1/N_c$ expansion.

The $1/N_c$ operators which transform according to definite $SU(3)$ representations are the operators which appear with definite flavor suppressions in the $SU(3)$-breaking expansion. For this operator basis, the $1/N_c$ and $SU(3)$-breaking flavor suppressions are manifest. The hierarchy of fits at different orders in the unified expansion performed here is evident in this operator basis. Obtaining the correct hierarchy is not easy in another operator basis. Fit B, the one-operator fit using G^{iQ}, corresponds to the $SU(6)$ predictions of Beg, Lee and Pais [30]. The successive fits, Fit C and D, which are two- and three-operator fits in the unified expansion, have not been performed previously in the literature. These fits are better fits than the $SU(3)$ symmetry fit, Fit A, which contains four operators. Finally, the dramatic reduction of σ^{theory} in the eight-operator fit, Fit E, shows that the leading $SU(3)$ breaking operators of the $1/N_c$ expansion reproduce the experimental data well.

V. CONCLUSIONS

There are several clear results of the analysis of the baryon magnetic moments in a combined expansion in $1/N_c$ and $SU(3)$ flavor symmetry breaking of this work. First, there is clear evidence in the experimental data for the combined hierarchy from the fits of Tables X and XI. The successive fits of the combined expansion are Fits B, C, D, and E. There is not enough experimental data to resolve the rest of the hierarchy at present. Fit E is the most reliable fit obtained in this work. Second, the dominant $SU(3)$ flavor symmetry breaking for the baryon magnetic moments is due to the leading $O(N_c m_q^{1/2})$ contribution of $SU(3)$ chiral perturbation theory. This $SU(3)$ breaking is described by the operator $G^{i\overline{Q}}$. The next largest $SU(3)$ breaking is a $m_q^{1/2}$ breaking at $O(1)$ in the $1/N_c$ expansion from the the leading 2-body $(1, \mathbf{10}+\overline{\mathbf{10}})$ operator $O_{(2)}^{i[8Q]}$.

The nonanalytic $m_q^{1/2}$ chiral correction to the baryon magnetic moments is similar to the $O(m_q^{3/2})$ correction to the baryon masses, in that it is not suppressed in $1/N_c$. In the case of the magnetic moments, however, the $m_q^{1/2}$ contribution is the leading order term in $SU(3)$ breaking, whereas the $m_q^{3/2}$ correction for the baryon masses is subdominant relative to the analytic $O(m_q)$ contribution. This fact makes the baryon magnetic moments a particularly useful observable for studying $SU(3)$ flavor symmetry breaking, since $SU(3)$ breaking is enhanced.

The analysis of this work is a group theoretic one. A hierarchy of baryon magnetic moments is obtained in terms of linear combinations of the 27 baryon magnetic moments. The complete set of magnetic moment combinations to all orders in $1/N_c$ and $SU(3)$ flavor symmetry breaking is obtained. Further progress now requires additional experimental measurements. The very precisely measured baryon octet magnetic moments contain substantial $SU(3)$ flavor symmetry breaking. The measurements of the decuplet baryon magnetic moments Ω^- and Δ^{++} also indicate significant $SU(3)$ breaking, although the error bar on the Δ^{++} magnetic moment is still quite large. The recent CLAS measurements of the $\Lambda\Sigma^{*0}$ and $\Sigma\Sigma^{*+}$ transition magnetic moments reveal that there is substantial $SU(3)$ breaking in the transition magnetic moments as well. Fit E supports the conclusion that $SU(3)$ breaking of the decuplet and the decuplet-octet transition magnetic moments is substantial. The $1/N_c$ expansion implies that certain linear combinations of the octet, decuplet and transition magnetic moments will be highly suppressed in $1/N_c$ and flavor symmetry breaking. Further data is needed to verify these predictions.

The eight operator fit, Fit E, of Table XI is able to reproduce the experimental data to within a theory error of 0.025 μ_N. Estimates for all of the unmeasured baryon magnetic moments are obtained from this fit. A number of features can be seen. There is an equal space rule for the four Δ baryon magnetic moments implied by $SU(3)$ symmetry. This equal spacing approximately persists even in the presence of $SU(3)$ breaking. The Σ^{*0} magnetic moment is particularly small. It vanishes in the $SU(3)$ limit, and only receives a nonvanishing contribution from one of the fifteen operators of Table I which is leading order in $SU(3)$ breaking. The $\Sigma\Sigma^{*-}$ and the $\Xi\Xi^{*-}$ transition magnetic moments, which also vanish in the $SU(3)$ symmetry limit, also remain quite small even in the presence of $SU(3)$ breaking.

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TABLE I: Matrix Elements of Magnetic Moment Operators for Exact $SU(3)$ Flavor Symmetry and to Leading Order in $SU(3)$ Flavor Symmetry Breaking. In the $SU(3)$ symmetry limit, there are four operators G^{iQ}, J^iT^Q, $\mathcal{D}^{iQ}_{(3)}$ and $\mathcal{O}^{iQ}_{(3)}$. Leading order $SU(3)$ symmetry breaking contributes to all fifteen operators.

	G^{iQ}	J^iT^Q	$\mathcal{D}^{iQ}_{(3)}$	$\mathcal{O}^{iQ}_{(3)}$	J^i	$\{J^2,J^i\}$	$G^{i\overline{Q}}$	$J^iT^{\overline{Q}}$	$\mathcal{D}^{i\overline{Q}}_{(3)}$	$\mathcal{O}^{i\overline{Q}}_{(3)}$	$\frac{1}{\sqrt{3}}O^{i[8Q]}_{(2)}$	$\frac{1}{\sqrt{3}}O^{i[8Q]}_{(3)}$	$\frac{1}{\sqrt{3}}O^{i(8Q)}_{(2)}$	$\frac{1}{\sqrt{3}}D^{i(8Q)}_{(3)}$	$\frac{1}{\sqrt{3}}O^{i(8Q)}_{(3)}$
	$(1,\mathbf{8})$	$(1,\mathbf{8})$	$(1,\mathbf{8})$	$(1,\mathbf{8})$	$(1,\mathbf{1})$	$(1,\mathbf{1})$	$(1,\mathbf{8})$	$(1,\mathbf{8})$	$(1,\mathbf{8})$	$(1,\mathbf{8})$	$(1,\mathbf{10}+\overline{\mathbf{10}})$	$(1,\mathbf{10}+\overline{\mathbf{10}})$	$(1,\mathbf{27})$	$(1,\mathbf{27})$	$(1,\mathbf{27})$
p	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	0	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{3}$	0	1	0	$\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{4}$
n	$-\frac{1}{3}$	0	-1	0	$\frac{1}{2}$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	0	$-\frac{1}{3}$	0	$-\frac{1}{3}$	0	$-\frac{1}{6}$
Λ	$-\frac{1}{6}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{6}$	0	$\frac{1}{2}$	0	0	0	0	0	$\frac{1}{6}$
$\Lambda\Sigma^0$	$-\frac{1}{2\sqrt{3}}$	0	$-\frac{\sqrt{3}}{2}$	0	0	0	$-\frac{1}{2\sqrt{3}}$	0	$-\frac{\sqrt{3}}{2}$	0	0	0	0	0	0
Σ^+	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	0	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{3}$	0	$\frac{1}{3}$	0	$\frac{1}{2}$
Σ^0	$\frac{1}{6}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{3}{4}$	$-\frac{1}{6}$	0	$-\frac{1}{2}$	0	0	0	0	0	$\frac{1}{6}$
Σ^-	$-\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	0	$\frac{1}{3}$	0	$-\frac{1}{3}$	0	$-\frac{1}{6}$
Ξ^0	$-\frac{1}{3}$	0	-1	0	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{3}$	0	$\frac{1}{3}$	0	$\frac{1}{2}$
Ξ^-	$-\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{3}$	0	1	0	$-\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{4}$
Δ^{++}	1	3	15	0	$\frac{3}{2}$	$\frac{45}{4}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{15}{2}$	0	0	0	2	3	$\frac{5}{2}$
Δ^+	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{15}{2}$	0	$\frac{3}{2}$	$\frac{45}{4}$	0	0	0	0	0	0	1	$\frac{3}{2}$	$\frac{5}{4}$
Δ^0	0	0	0	0	$\frac{3}{2}$	$\frac{45}{4}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{15}{2}$	0	0	0	0	0	0
Δ^-	$-\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{15}{2}$	0	$\frac{3}{2}$	$\frac{45}{4}$	-1	-3	-15	0	0	0	-1	$-\frac{3}{2}$	$-\frac{5}{4}$
Σ^{*+}	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{15}{2}$	0	$\frac{3}{2}$	$\frac{45}{4}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{15}{2}$	0	0	0	0	0	0
Σ^{*0}	0	0	0	0	$\frac{3}{2}$	$\frac{45}{4}$	0	0	0	0	0	0	0	0	0
Σ^{*-}	$-\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{15}{2}$	0	$\frac{3}{2}$	$\frac{45}{4}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{15}{2}$	0	0	0	0	0	0
Ξ^{*0}	0	0	0	0	$\frac{3}{2}$	$\frac{45}{4}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{15}{2}$	0	0	0	0	0	0
Ξ^{*-}	$-\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{15}{2}$	0	$\frac{3}{2}$	$\frac{45}{4}$	0	0	0	0	0	0	1	$\frac{3}{2}$	$\frac{5}{4}$
Ω^-	$-\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{15}{2}$	0	$\frac{3}{2}$	$\frac{45}{4}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{15}{2}$	0	0	0	2	3	$\frac{5}{2}$
$\frac{1}{\sqrt{2}}p\Delta^+$	$\frac{1}{3}$	0	0	$\frac{3}{2}$	0	0	$\frac{1}{3}$	0	0	$\frac{3}{2}$	$\frac{1}{3}$	$-\frac{1}{4}$	$\frac{1}{3}$	0	$\frac{1}{4}$
$\frac{1}{\sqrt{2}}n\Delta^0$	$\frac{1}{3}$	0	0	$\frac{3}{2}$	0	0	$\frac{1}{3}$	0	0	$\frac{3}{2}$	$\frac{1}{3}$	$-\frac{1}{4}$	$\frac{1}{3}$	0	$\frac{1}{4}$
$\frac{1}{\sqrt{2}}\Lambda\Sigma^{*0}$	$\frac{1}{2\sqrt{3}}$	0	0	$\frac{3\sqrt{3}}{4}$	0	0	$\frac{1}{2\sqrt{3}}$	0	0	$\frac{3\sqrt{3}}{4}$	0	0	0	0	$-\frac{1}{4\sqrt{3}}$
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*+}$	$\frac{1}{3}$	0	0	$\frac{3}{2}$	0	0	0	0	0	0	$-\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{3}$	0	$\frac{5}{12}$
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*0}$	$\frac{1}{6}$	0	0	$\frac{3}{4}$	0	0	$-\frac{1}{6}$	0	0	$-\frac{3}{4}$	0	0	0	0	$\frac{1}{12}$
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*-}$	0	0	0	0	0	0	$-\frac{1}{3}$	0	0	$-\frac{3}{2}$	$\frac{1}{3}$	$-\frac{1}{4}$	$-\frac{1}{3}$	0	$-\frac{1}{4}$
$\frac{1}{\sqrt{2}}\Xi\Xi^{*0}$	$\frac{1}{3}$	0	0	$\frac{3}{2}$	0	0	0	0	0	0	$-\frac{1}{3}$	$\frac{1}{4}$	$-\frac{1}{3}$	0	$-\frac{5}{12}$
$\frac{1}{\sqrt{2}}\Xi\Xi^{*-}$	0	0	0	0	0	0	$-\frac{1}{3}$	0	0	$-\frac{3}{2}$	$\frac{1}{3}$	$-\frac{1}{4}$	$-\frac{1}{3}$	0	$-\frac{1}{4}$

TABLE II: Matrix Elements of Subleading $SU(3)$-Violating and Isospin-Violating Magnetic Moment Operators. The subleading $SU(3)$-violating operators are the first six operators, and the isospin-violating operators are the second six operators.

	$\frac{1}{\sqrt{3}}O^{i(8\overline{Q})}_{(2)}$	$\frac{1}{\sqrt{3}}D^{i(8\overline{Q})}_{(3)}$	$\frac{1}{\sqrt{3}}O^{i(8\overline{Q})}_{(3)}$	$O^{i(88Q)}_{(3)}$	$\tilde{O}^{i(88Q)}_{(3)}$	$O^{i(88\overline{Q})}_{(3)}$	$O^{i(3Q)}_{(2)}$	$D^{i(3Q)}_{(3)}$	$O^{i(3Q)}_{(3)}$	$O^{i(33Q)}_{(3)}$	$\tilde{O}^{i(33Q)}_{(3)}$	$O^{i(33\overline{Q})}_{(3)}$
	$(1,\mathbf{27})$	$(1,\mathbf{27})$	$(1,\mathbf{27})$	$(1,\mathbf{64})$	$(1,\mathbf{64})$	$(1,\mathbf{64})$	$(1,\mathbf{27})$	$(1,\mathbf{27})$	$(1,\mathbf{27})$	$(1,\mathbf{64})$	$(1,\mathbf{64})$	$(1,\mathbf{64})$
p	$\frac{1}{3}$	0	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{2}$	0	$\frac{4}{3}$	$\frac{1}{2}$	$\frac{5}{4}$	$\frac{5}{6}$	$\frac{1}{2}$	0
n	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{1}{4}$	0	-1	$-\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{5}{6}$	0	$-\frac{1}{3}$	$-\frac{5}{6}$
Λ	0	0	$-\frac{1}{6}$	0	0	0	0	0	$\frac{1}{2}$	0	0	0
$\Lambda\Sigma^0$	0	0	0	0	0	0	0	0	0	0	0	0
Σ^+	$\frac{1}{3}$	0	$\frac{1}{6}$	0	0	0	$\frac{5}{3}$	1	1	$\frac{4}{3}$	2	$\frac{4}{3}$
Σ^0	0	0	$-\frac{1}{6}$	0	0	0	0	0	$\frac{1}{2}$	0	0	0
Σ^-	$-\frac{1}{3}$	0	$-\frac{1}{2}$	0	0	0	1	1	$\frac{1}{3}$	$-\frac{4}{3}$	$-\frac{2}{3}$	$-\frac{4}{3}$
Ξ^0	$-\frac{2}{3}$	$-\frac{1}{2}$	$-\frac{1}{4}$	0	-1	$\frac{3}{2}$	$-\frac{1}{3}$	0	$\frac{1}{6}$	0	$-\frac{1}{3}$	$-\frac{1}{6}$
Ξ^-	$-\frac{1}{3}$	0	$-\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{12}$	$\frac{1}{6}$	$-\frac{1}{6}$	0
Δ^{++}	1	$\frac{3}{2}$	$\frac{5}{4}$	3	3	$\frac{3}{2}$	6	9	$\frac{15}{2}$	9	9	$\frac{9}{2}$
Δ^+	0	0	0	$\frac{3}{2}$	$\frac{3}{2}$	0	1	$\frac{3}{2}$	$\frac{5}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	0
Δ^0	-1	$-\frac{3}{2}$	$-\frac{5}{4}$	0	0	$-\frac{3}{2}$	0	0	0	0	0	$-\frac{1}{2}$
Δ^-	-2	-3	$-\frac{5}{2}$	$-\frac{3}{2}$	$-\frac{3}{2}$	-3	3	$\frac{9}{2}$	$\frac{15}{4}$	$-\frac{9}{2}$	$-\frac{9}{2}$	-9
Σ^{*+}	0	0	0	0	0	0	2	3	$\frac{5}{2}$	2	2	2
Σ^{*0}	0	0	0	0	0	0	0	0	0	0	0	0
Σ^{*-}	0	0	0	0	0	0	2	3	$\frac{5}{2}$	-2	-2	-2
Ξ^{*0}	-1	$-\frac{3}{2}$	$-\frac{5}{4}$	0	0	$\frac{3}{2}$	0	0	0	0	0	$\frac{1}{2}$
Ξ^{*-}	0	0	0	$-\frac{3}{2}$	$-\frac{3}{2}$	0	1	$\frac{3}{2}$	$\frac{5}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0
Ω^-	-2	-3	$-\frac{5}{2}$	-6	-6	6	0	0	0	0	0	0
$\frac{1}{\sqrt{2}}p\Delta^+$	$\frac{1}{3}$	0	$\frac{1}{4}$	0	1	0	1	0	$\frac{13}{12}$	$\frac{2}{3}$	$\frac{1}{3}$	0
$\frac{1}{\sqrt{2}}n\Delta^0$	$\frac{1}{3}$	0	$\frac{1}{4}$	0	1	0	$-\frac{1}{3}$	0	$-\frac{7}{12}$	0	$\frac{1}{3}$	$\frac{2}{3}$
$\frac{1}{\sqrt{2}}\Lambda\Sigma^{*0}$	0	0	$-\frac{1}{4\sqrt{3}}$	0	0	0	0	0	$-\frac{1}{4\sqrt{3}}$	0	0	0
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*+}$	$\frac{1}{3}$	0	$\frac{1}{4}$	0	0	0	1	0	$\frac{11}{12}$	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{2}{3}$
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*0}$	0	0	$-\frac{1}{12}$	0	0	0	0	0	$-\frac{1}{4}$	0	0	0
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*-}$	$-\frac{1}{3}$	0	$-\frac{5}{12}$	0	0	0	$\frac{1}{3}$	0	$\frac{1}{4}$	$-\frac{2}{3}$	0	$-\frac{2}{3}$
$\frac{1}{\sqrt{2}}\Xi\Xi^{*0}$	$\frac{1}{3}$	0	$\frac{1}{4}$	0	1	-1	$\frac{1}{3}$	0	$\frac{1}{12}$	0	$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{\sqrt{2}}\Xi\Xi^{*-}$	$\frac{1}{3}$	0	$\frac{5}{12}$	1	0	0	$\frac{1}{3}$	0	$\frac{1}{4}$	$-\frac{1}{3}$	0	0

TABLE III: Coefficients of Magnetic Moment Operators Including $SU(3)$ Flavor Symmetry Violation at Leading Order

Coefficient	Operator	Magnetic Moment Combination
$a_{(1)}^{1,\mathbf{8}}$	G^{iQ}	$\frac{1}{32}\left[13N_1-6\sqrt{3}\left(\Lambda\Sigma\right)_1+8\Sigma_1-5\Xi_1\right]-\frac{1}{60}\left[\Delta_1+2\Sigma_1^*+\Xi_1^*\right]$
		$+\frac{3}{20}\left[2N_0-3\Lambda_0+9\Sigma_0-8\Xi_0\right]-\frac{1}{10}\left[2\Delta_0-\Xi_0^*-\Omega_0\right]$
$b_{(2)}^{1,\mathbf{8}}$	J^iT^Q	$\frac{1}{12}\left[-N_1+2\sqrt{3}\left(\Lambda\Sigma\right)_1+4\Sigma_1+5\Xi_1\right]+\frac{1}{5}\left[7N_0+2\Lambda_0-6\Sigma_0-3\Xi_0\right]$
$b_{(3)}^{1,\mathbf{8}}$	$\mathcal{D}_{(3)}^{iQ}$	$-\frac{1}{96}\left[N_1+2\sqrt{3}\left(\Lambda\Sigma\right)_1+8\Sigma_1+7\Xi_1\right]+\frac{1}{180}\left[\Delta_1+2\Sigma_1^*+\Xi_1^*\right]$
		$-\frac{1}{20}\left[6N_0+\Lambda_0-3\Sigma_0-4\Xi_0\right]+\frac{1}{30}\left[2\Delta_0-\Xi_0^*-\Omega_0\right]$
$c_{(3)}^{1,\mathbf{8}}$	$\mathcal{O}_{(3)}^{iQ}$	$-\frac{1}{144}\left[13N_1-6\sqrt{3}\left(\Lambda\Sigma\right)_1+8\Sigma_1-5\Xi_1\right]+\frac{1}{270}\left[\Delta_1+2\Sigma_1^*+\Xi_1^*\right]$
		$+\frac{2}{45}\frac{1}{\sqrt{2}}\left[2\left(N\Delta\right)_1+\sqrt{3}\left(\Lambda\Sigma^*\right)_1+\left(\Sigma\Sigma^*\right)_1+\left(\Xi\Xi^*\right)_1\right]$
		$-\frac{1}{30}\left[2N_0-3\Lambda_0+9\Sigma_0-8\Xi_0\right]+\frac{1}{45}\left[2\Delta_0-\Xi_0^*-\Omega_0\right]+\frac{2}{15}\frac{1}{\sqrt{2}}\left[3\left(\Sigma\Sigma^*\right)_0+2\left(\Xi\Xi^*\right)_0\right]$
$c_{(1)}^{1,\mathbf{1}}$	J^i	$\frac{5}{16}\left[2N_0+\Lambda_0+3\Sigma_0+2\Xi_0\right]-\frac{1}{60}\left[4\Delta_0+3\Sigma_0^*+2\Xi_0^*+\Omega_0\right]$
$c_{(3)}^{1,\mathbf{1}}$	$\{J^2,J^i\}$	$-\frac{1}{24}\left[2N_0+\Lambda_0+3\Sigma_0+2\Xi_0\right]+\frac{1}{90}\left[4\Delta_0+3\Sigma_0^*+2\Xi_0^*+\Omega_0\right]$
$\frac{1}{\sqrt{3}}\overline{a}_{(1)}^{1,\mathbf{8}}$	$G^{i\overline{Q}}$	$\frac{1}{32}\left[13N_1-6\sqrt{3}\left(\Lambda\Sigma\right)_1+8\Sigma_1-5\Xi_1\right]-\frac{1}{60}\left[\Delta_1+2\Sigma_1^*+\Xi_1^*\right]$
		$-\frac{3}{20}\left[2N_0-3\Lambda_0+9\Sigma_0-8\Xi_0\right]+\frac{1}{10}\left[2\Delta_0-\Xi_0^*-\Omega_0\right]$
$\frac{1}{\sqrt{3}}\overline{b}_{(2)}^{1,\mathbf{8}}$	$J^iT^{\overline{Q}}$	$\frac{1}{12}\left[-N_1+2\sqrt{3}\left(\Lambda\Sigma\right)_1+4\Sigma_1+5\Xi_1\right]-\frac{1}{5}\left[7N_0+2\Lambda_0-6\Sigma_0-3\Xi_0\right]$
$\frac{1}{\sqrt{3}}\overline{b}_{(3)}^{1,\mathbf{8}}$	$\mathcal{D}_{(3)}^{i\overline{Q}}$	$-\frac{1}{96}\left[N_1+2\sqrt{3}\left(\Lambda\Sigma\right)_1+8\Sigma_1+7\Xi_1\right]+\frac{1}{180}\left[\Delta_1+2\Sigma_1^*+\Xi_1^*\right]$
		$+\frac{1}{20}\left[6N_0+\Lambda_0-3\Sigma_0-4\Xi_0\right]-\frac{1}{30}\left[2\Delta_0-\Xi_0^*-\Omega_0\right]$
$\frac{1}{\sqrt{3}}\overline{c}_{(3)}^{1,\mathbf{8}}$	$\mathcal{O}_{(3)}^{i\overline{Q}}$	$-\frac{1}{144}\left[13N_1-6\sqrt{3}\left(\Lambda\Sigma\right)_1+8\Sigma_1-5\Xi_1\right]+\frac{1}{270}\left[\Delta_1+2\Sigma_1^*+\Xi_1^*\right]$
		$+\frac{2}{45}\frac{1}{\sqrt{2}}\left[2\left(N\Delta\right)_1+\sqrt{3}\Lambda\Sigma^{*0}+\left(\Sigma\Sigma^*\right)_1+\left(\Xi\Xi^*\right)_1\right]$
		$+\frac{1}{30}\left[2N_0-3\Lambda+9\Sigma_0-8\Xi_0\right]-\frac{1}{45}\left[2\Delta_0-\Xi_0^*-\Omega\right]-\frac{2}{15}\frac{1}{\sqrt{2}}\left[3\left(\Sigma\Sigma^*\right)_0+2\left(\Xi\Xi^*\right)_0\right]$
$\frac{1}{\sqrt{3}}c_{(2)}^{1,\mathbf{10}+\overline{\mathbf{10}}}$	$O_{(2)}^{i[8Q]}$	$\frac{1}{2}\left[N_1-\Sigma_1+\Xi_1\right]$
$\frac{1}{\sqrt{3}}c_{(3)}^{1,\mathbf{10}+\overline{\mathbf{10}}}$	$O_{(3)}^{i[8Q]}$	$\frac{2}{3}\left[N_1-\Sigma_1+\Xi_1\right]+\frac{2}{3}\left[-\frac{1}{\sqrt{2}}\left(N\Delta\right)_1+\frac{1}{\sqrt{2}}\left(\Sigma\Sigma^*\right)_1+\frac{1}{\sqrt{2}}\left(\Xi\Xi^*\right)_1\right]$
$\frac{1}{\sqrt{3}}c_{(2)}^{1,\mathbf{27}}$	$O_{(2)}^{i(8Q)}$	$\frac{5}{6}\left[N_1+2\sqrt{3}\left(\Lambda\Sigma\right)_1-\Xi_1\right]-\frac{5}{126}\left[\Delta_1-3\Sigma_1^*-4\Xi_1^*\right]$
		$+\frac{1}{6}\frac{1}{\sqrt{2}}\left[\left(N\Delta\right)_1-2\sqrt{3}\left(\Lambda\Sigma^*\right)_1+3\left(\Sigma\Sigma^*\right)_1-2\left(\Xi\Xi^*\right)_1\right]$
		$+\frac{5}{6}\left[2N_0-3\Lambda_0-\Sigma_0+2\Xi_0\right]-\frac{5}{63}\left[4\Delta_0-5\Sigma_0^*-2\Xi_0^*+3\Omega_0\right]$
		$+\frac{4}{3}\frac{1}{\sqrt{2}}\left[\left(\Sigma\Sigma^*\right)_0-\left(\Xi\Xi^*\right)_0\right]$
$\frac{1}{\sqrt{3}}b_{(3)}^{1,\mathbf{27}}$	$D_{(3)}^{i(8Q)}$	$\frac{1}{21}\left[\Delta_1-3\Sigma_1^*-4\Xi_1^*\right]-\frac{1}{4}\frac{1}{\sqrt{2}}\left[\left(N\Delta\right)_1-2\sqrt{3}\left(\Lambda\Sigma^*\right)_1+3\left(\Sigma\Sigma^*\right)_1-2\left(\Xi\Xi^*\right)_1\right]$
		$+\frac{2}{21}\left[4\Delta_0-5\Sigma_0^*-2\Xi_0^*+3\Omega_0\right]-2\frac{1}{\sqrt{2}}\left[\left(\Sigma\Sigma^*\right)_0-\left(\Xi\Xi^*\right)_0\right]$
$\frac{1}{\sqrt{3}}c_{(3)}^{1,\mathbf{27}}$	$O_{(3)}^{i(8Q)}$	$-\frac{2}{3}\left[N_1+2\sqrt{3}\left(\Lambda\Sigma\right)_1-\Xi_1\right]+\frac{2}{63}\left[\Delta_1-3\Sigma_1^*-4\Xi_1^*\right]$
		$+\frac{1}{6}\frac{1}{\sqrt{2}}\left[\left(N\Delta\right)_1-2\sqrt{3}\left(\Lambda\Sigma^*\right)_1+3\left(\Sigma\Sigma^*\right)_1-2\left(\Xi\Xi^*\right)_1\right]$
		$-\frac{2}{3}\left[2N_0-3\Lambda_0-\Sigma_0+2\Xi_0\right]+\frac{4}{63}\left[4\Delta_0-5\Sigma_0^*-2\Xi_0^*+3\Omega_0\right]$
		$+\frac{4}{3}\frac{1}{\sqrt{2}}\left[\left(\Sigma\Sigma^*\right)_0-\left(\Xi\Xi^*\right)_0\right]$

TABLE IV: Coefficients of Magnetic Moment Operators Breaking $SU(3)$ Flavor Symmetry at Subleading Order

Coefficient	Operator	Magnetic Moment Combination
$\frac{1}{3}\overline{c}^{1,\mathbf{27}}_{(2)}$	$O^{i(8\overline{Q})}_{(2)}$	$\frac{5}{6}\left[N_1+2\sqrt{3}\left(\Lambda\Sigma\right)_1-\Xi_1\right]-\frac{5}{126}\left[\Delta_1-3\Sigma_1^*-4\Xi_1^*\right]$
		$+\frac{1}{6}\frac{1}{\sqrt{2}}\left[\left(N\Delta\right)_1-2\sqrt{3}\left(\Lambda\Sigma^*\right)_1+3\left(\Sigma\Sigma^*\right)_1-2\left(\Xi\Xi^*\right)_1\right]$
		$-\frac{5}{6}\left[2N_0-3\Lambda_0-\Sigma_0+2\Xi_0\right]+\frac{5}{63}\left[4\Delta_0-5\Sigma_0^*-2\Xi_0^*+3\Omega_0\right]-\frac{4}{3}\frac{1}{\sqrt{2}}\left[\left(\Sigma\Sigma^*\right)_0-\left(\Xi\Xi^*\right)_0\right]$
$\frac{1}{3}\overline{b}^{1,\mathbf{27}}_{(3)}$	$D^{i(8\overline{Q})}_{(3)}$	$\frac{1}{21}\left[\Delta_1-3\Sigma_1^*-4\Xi_1^*\right]-\frac{1}{4}\frac{1}{\sqrt{2}}\left[\left(N\Delta\right)_1-2\sqrt{3}\left(\Lambda\Sigma^*\right)_1+3\left(\Sigma\Sigma^*\right)_1-2\left(\Xi\Xi^*\right)_1\right]$
		$-\frac{2}{21}\left[4\Delta_0-5\Sigma_0^*-2\Xi_0^*+3\Omega_0\right]+2\frac{1}{\sqrt{2}}\left[\left(\Sigma\Sigma^*\right)_0-\left(\Xi\Xi^*\right)_0\right]$
$\frac{1}{3}\overline{c}^{1,\mathbf{27}}_{(3)}$	$O^{i(8\overline{Q})}_{(3)}$	$-\frac{2}{3}\left[N_1+2\sqrt{3}\left(\Lambda\Sigma\right)_1-\Xi_1\right]+\frac{2}{63}\left[\Delta_1-3\Sigma_1^*-4\Xi_1^*\right]$
		$+\frac{1}{6}\frac{1}{\sqrt{2}}\left[\left(N\Delta\right)_1-2\sqrt{3}\left(\Lambda\Sigma^*\right)_1+3\left(\Sigma\Sigma^*\right)_1-2\left(\Xi\Xi^*\right)_1\right]$
		$+\frac{2}{3}\left[2N_0-3\Lambda_0-\Sigma_0+2\Xi_0\right]-\frac{4}{63}\left[4\Delta_0-5\Sigma_0^*-2\Xi_0^*+3\Omega_0\right]$
		$-\frac{4}{3}\frac{1}{\sqrt{2}}\left[\left(\Sigma\Sigma^*\right)_0-\left(\Xi\Xi^*\right)_0\right]$
$c^{1,\mathbf{64}}_{(3)}$	$O^{i(88Q)}_{(3)}$	$\frac{1}{45}\left[\Delta_1-10\Sigma_1^*+10\Xi_1^*\right]+\frac{1}{6}\frac{1}{\sqrt{2}}\left[-\left(N\Delta\right)_1+2\sqrt{3}\left(\Lambda\Sigma^*\right)_1+\left(\Sigma\Sigma^*\right)_1-2\left(\Xi\Xi^*\right)_1\right]$
		$+\frac{2}{9}\left[\Delta_0-3\Sigma_0^*+3\Xi_0^*-\Omega_0\right]$
$d^{1,\mathbf{64}}_{(3)}$	$\tilde{O}^{i(88Q)}_{(3)}$	$\frac{1}{90}\left[\Delta_1-10\Sigma_1^*+10\Xi_1^*\right]-\frac{1}{6}\frac{1}{\sqrt{2}}\left[-\left(N\Delta\right)_1+2\sqrt{3}\left(\Lambda\Sigma^*\right)_1+\left(\Sigma\Sigma^*\right)_1-2\left(\Xi\Xi^*\right)_1\right]$
$\frac{1}{\sqrt{3}}\overline{c}^{1,\mathbf{64}}_{(3)}$	$O^{i(88\overline{Q})}_{(3)}$	$\frac{1}{45}\left[\Delta_1-10\Sigma_1^*+10\Xi_1^*\right]+\frac{1}{6}\frac{1}{\sqrt{2}}\left[-\left(N\Delta\right)_1+2\sqrt{3}\left(\Lambda\Sigma^*\right)_1+\left(\Sigma\Sigma^*\right)_1-2\left(\Xi\Xi^*\right)_1\right]$
		$-\frac{2}{9}\left[\Delta_0-3\Sigma_0^*+3\Xi_0^*-\Omega_0\right]$

TABLE V: Coefficients of Magnetic Moment Operators Violating Isospin Symmetry

Coefficient	Operator	Magnetic Moment Combination
$g^{1,\mathbf{27}}_{(2)}$	$O^{i(3Q)}_{(2)}$	$\frac{5}{12}\Sigma_2-\frac{5}{252}\left[3\Delta_2+\Sigma_2^*\right]+\frac{1}{12}\frac{1}{\sqrt{3}}\left[3\left(N\Delta\right)_2+\left(\Sigma\Sigma^*\right)_2\right]$
$f^{1,\mathbf{27}}_{(3)}$	$D^{i(3Q)}_{(3)}$	$\frac{1}{42}\left[3\Delta_2+\Sigma_2^*\right]-\frac{1}{8}\frac{1}{\sqrt{2}}\left[3\left(N\Delta\right)_2+\left(\Sigma\Sigma^*\right)_2\right]$
$g^{1,\mathbf{27}}_{(3)}$	$O^{i(3Q)}_{(3)}$	$-\frac{1}{3}\Sigma_2+\frac{1}{63}\left[3\Delta_2+\Sigma_2^*\right]+\frac{1}{12}\frac{1}{\sqrt{2}}\left[3\left(N\Delta\right)_2+\left(\Sigma\Sigma^*\right)_2\right]$
$g^{1,\mathbf{64}}_{(3)}$	$O^{i(33Q)}_{(3)}$	$\frac{1}{12}\Delta_3+\frac{1}{6}\left[\Delta_2-2\Sigma_2^*\right]+\frac{1}{2}\frac{1}{\sqrt{2}}\left[\left(N\Delta\right)_2-\left(\Sigma\Sigma^*\right)_2\right]$
$f^{1,\mathbf{64}}_{(3)}$	$\tilde{O}^{i(33Q)}_{(3)}$	$\frac{1}{12}\left[\Delta_2-2\Sigma_2^*\right]-\frac{1}{2}\frac{1}{\sqrt{2}}\left[\left(N\Delta\right)_2-\left(\Sigma\Sigma^*\right)_2\right]$
$\overline{g}^{1,\mathbf{64}}_{(3)}$	$O^{i(33\overline{Q})}_{(3)}$	$\frac{1}{12}\Delta_3-\frac{1}{6}\left[\Delta_2-2\Sigma_2^*\right]-\frac{1}{2}\frac{1}{\sqrt{2}}\left[\left(N\Delta\right)_2-\left(\Sigma\Sigma^*\right)_2\right]$

TABLE VI: Coefficients of $(1, \mathbf{27})$ Magnetic Moment Operators with Isospin $I = 0$, 1, and 2.

Coefficient	Operator	Magnetic Moment Combination
$\left(c^{1,\mathbf{27}}_{(2)}\right)_{I=0}$	$\{G^{i8}, T^8\}$	$\frac{5}{6}\left[2N_0 - 3\Lambda_0 - \Sigma_0 + 2\Xi_0\right] - \frac{5}{63}\left[4\Delta_0 - 5\Sigma_0^* - 2\Xi_0^* + 3\Omega_0\right]$
		$+\frac{4}{3}\frac{1}{\sqrt{2}}\left[(\Sigma\Sigma^*)_0 - (\Xi\Xi^*)_0\right]$
$\left(c^{1,\mathbf{27}}_{(3)}\right)_{I=0}$	$\{G^{i8}, J^k G^{k8}\}$	$-\frac{2}{3}\left[2N_0 - 3\Lambda_0 - \Sigma_0 + 2\Xi_0\right] + \frac{4}{63}\left[4\Delta_0 - 5\Sigma_0^* - 2\Xi_0^* + 3\Omega_0\right]$
		$+\frac{4}{3}\frac{1}{\sqrt{2}}\left[(\Sigma\Sigma^*)_0 - (\Xi\Xi^*)_0\right]$
$\left(b^{1,\mathbf{27}}_{(3)}\right)_{I=0}$	$J^i\{T^8, T^8\}$	$\frac{2}{21}\left[4\Delta_0 - 5\Sigma_0^* - 2\Xi_0^* + 3\Omega_0\right] - 2\frac{1}{\sqrt{2}}\left[(\Sigma\Sigma^*)_0 - (\Xi\Xi^*)_0\right]$
$\left(c^{1,\mathbf{27}}_{(2)}\right)_{I=1}$	$\{G^{i3}, T^8\} + \{G^{i8}, T^3\}$	$\frac{5}{6}\left[N_1 + 2\sqrt{3}\,(\Lambda\Sigma)_1 - \Xi_1\right] - \frac{5}{126}\left[\Delta_1 - 3\Sigma_1^* - 4\Xi_1^*\right]$
		$+\frac{1}{6}\frac{1}{\sqrt{2}}\left[(N\Delta)_1 - 2\sqrt{3}\,(\Lambda\Sigma^*)_1 + 3\,(\Sigma\Sigma^*)_1 - 2\,(\Xi\Xi^*)_1\right]$
$\left(c^{1,\mathbf{27}}_{(3)}\right)_{I=1}$	$\{G^{i3}, J^k G^{k8}\} + \{G^{i8}, J^k G^{k3}\}$	$-\frac{2}{3}\left[N_1 + 2\sqrt{3}\,(\Lambda\Sigma)_1 - \Xi_1\right] + \frac{2}{63}\left[\Delta_1 - 3\Sigma_1^* - 4\Xi_1^*\right]$
		$+\frac{1}{6}\frac{1}{\sqrt{2}}\left[(N\Delta)_1 - 2\sqrt{3}\,(\Lambda\Sigma^*)_1 + 3\,(\Sigma\Sigma^*)_1 - 2\,(\Xi\Xi^*)_1\right]$
$\left(b^{1,\mathbf{27}}_{(3)}\right)_{I=1}$	$J^i\{T^8, T^3\}$	$\frac{1}{21}\left[\Delta_1 - 3\Sigma_1^* - 4\Xi_1^*\right] - \frac{1}{4}\frac{1}{\sqrt{2}}\left[(N\Delta)_1 - 2\sqrt{3}\,(\Lambda\Sigma^*)_1 + 3\,(\Sigma\Sigma^*)_1 - 2\,(\Xi\Xi^*)_1\right]$
$\left(c^{1,\mathbf{27}}_{(2)}\right)_{I=2}$	$\{G^{i3}, T^3\}$	$\frac{5}{12}\Sigma_2 - \frac{5}{252}\left[3\Delta_2 + \Sigma_2^*\right] + \frac{1}{12}\frac{1}{\sqrt{2}}\left[3\,(N\Delta)_2 + (\Sigma\Sigma^*)_2\right]$
$\left(c^{1,\mathbf{27}}_{(3)}\right)_{I=2}$	$\{G^{i3}, J^k G^{k3}\}$	$-\frac{1}{3}\Sigma_2 + \frac{1}{63}\left[3\Delta_2 + \Sigma_2^*\right] + \frac{1}{12}\frac{1}{\sqrt{2}}\left[3\,(N\Delta)_2 + (\Sigma\Sigma^*)_2\right]$
$\left(b^{1,\mathbf{27}}_{(3)}\right)_{I=2}$	$J^i\{T^3, T^3\}$	$\frac{1}{42}\left[3\Delta_2 + \Sigma_2^*\right] - \frac{1}{8}\frac{1}{\sqrt{2}}\left[3\,(N\Delta)_2 + (\Sigma\Sigma^*)_2\right]$

TABLE VII: Coefficients of $(1, \mathbf{64})$ Magnetic Moment Operators with Isospin $I = 0$, 1, 2, and 3.

Coefficient	Operator	Magnetic Moment Combination
$\left(c^{1,\mathbf{64}}_{(3)}\right)_{I=0}$	$\{G^{i8}, \{T^8, T^8\}\}$	$\frac{2}{9}\left[\Delta_0 - 3\Sigma_0^* + 3\Xi_0^* - \Omega_0\right]$
$\left(c^{1,\mathbf{64}}_{(3)}\right)_{I=1}$	$\{G^{i3}, \{T^8, T^8\}\}$	$\frac{1}{90}\left[\Delta_1 - 10\Sigma_1^* + 10\Xi_1^*\right] - \frac{1}{6}\frac{1}{\sqrt{2}}\left[-(N\Delta)_1 + 2\sqrt{3}\,(\Lambda\Sigma^*)_1 + (\Sigma\Sigma^*)_1 - 2\,(\Xi\Xi^*)_1\right]$
$\left(c^{1,\mathbf{64}}_{(3)}\right)_{I=1}$	$\{G^{i8}, \{T^8, T^3\}\}$	$\frac{1}{45}\left[\Delta_1 - 10\Sigma_1^* + 10\Xi_1^*\right] + \frac{1}{6}\frac{1}{\sqrt{2}}\left[-(N\Delta)_1 + 2\sqrt{3}\,(\Lambda\Sigma^*)_1 + (\Sigma\Sigma^*)_1 - 2\,(\Xi\Xi^*)_1\right]$
$\left(c^{1,\mathbf{64}}_{(3)}\right)_{I=2}$	$\{G^{i3}, \{T^8, T^3\}\}$	$\frac{1}{6}\left[\Delta_2 - 2\Sigma_2^*\right] + \frac{1}{2}\frac{1}{\sqrt{2}}\left[(N\Delta)_2 - (\Sigma\Sigma^*)_2\right]$
$\left(c^{1,\mathbf{64}}_{(3)}\right)_{I=2}$	$\{G^{i8}, \{T^3, T^3\}\}$	$\frac{1}{12}\left[\Delta_2 - 2\Sigma_2^*\right] - \frac{1}{2}\frac{1}{\sqrt{2}}\left[(N\Delta)_2 - (\Sigma\Sigma^*)_2\right]$
$\left(c^{1,\mathbf{64}}_{(3)}\right)_{I=3}$	$\{G^{i3}, \{T^3, T^3\}\}$	$\frac{1}{12}\Delta_3$

TABLE VIII: Coefficients of Magnetic Moment Operators to all orders in $SU(3)$ Flavor Symmetry Breaking. The first four operators contribute in the $SU(3)$ flavor symmetry limit. There are 21 operators.

Coefficient	Operator	N_c	$SU(3)$ Breaking
$a_{(1)}^{1,\mathbf{8}}$	G^{iQ}	N_c	$1,\ m_q^{1/2},\ m_q \ln m_q$
$b_{(2)}^{1,\mathbf{8}}$	$\frac{1}{N_c} J^i T^Q$	$\frac{1}{N_c}$	$1,\ m_q^{1/2},\ m_q \ln m_q$
$b_{(3)}^{1,\mathbf{8}}$	$\frac{1}{N_c^2} \mathcal{D}_{(3)}^{iQ}$	$\frac{1}{N_c}$	$1,\ m_q^{1/2},\ m_q \ln m_q$
$c_{(3)}^{1,\mathbf{8}}$	$\frac{1}{N_c^2} \mathcal{O}_{(3)}^{iQ}$	$\frac{1}{N_c}$	$1,\ m_q^{1/2},\ m_q \ln m_q$
$c_{(1)}^{1,\mathbf{1}}$	J^i	1	$m_q \ln m_q$
$c_{(3)}^{1,\mathbf{1}}$	$\frac{1}{N_c^2} \{J^2, J^i\}$	$\frac{1}{N_c^2}$	$m_q \ln m_q$
$\overline{a}_{(1)}^{1,\mathbf{8}}$	$G^{i\overline{Q}}$	N_c	$m_q^{1/2}$
$\overline{b}_{(2)}^{1,\mathbf{8}}$	$\frac{1}{N_c} J^i T^{\overline{Q}}$	$\frac{1}{N_c}$	$m_q^{1/2}$
$\overline{b}_{(3)}^{1,\mathbf{8}}$	$\frac{1}{N_c^2} \mathcal{D}_{(3)}^{i\overline{Q}}$	$\frac{1}{N_c}$	$m_q^{1/2}$
$\overline{c}_{(3)}^{1,\mathbf{8}}$	$\frac{1}{N_c^2} \mathcal{O}_{(3)}^{i\overline{Q}}$	$\frac{1}{N_c}$	$m_q^{1/2}$
$c_{(2)}^{1,\mathbf{10}+\overline{\mathbf{10}}}$	$\frac{1}{N_c} \left(\{T^8, G^{iQ}\} - \{G^{i8}, T^Q\} \right)$	1	$m_q^{1/2}$
$c_{(3)}^{1,\mathbf{10}+\overline{\mathbf{10}}}$	$\frac{1}{N_c^2} \left(\{G^{i8}, J^k G^{kQ}\} - \{J^k G^{k8}, G^{iQ}\} \right)$	$\frac{1}{N_c}$	$m_q^{1/2}$
$c_{(2)}^{1,\mathbf{27}}$	$\frac{1}{N_c} \left(\{T^8, G^{iQ}\} + \{G^{i8}, T^Q\} \right)$	1	$m_q \ln m_q$
$b_{(3)}^{1,\mathbf{27}}$	$\frac{1}{N_c^2} J^i \{T^8, T^Q\}$	$\frac{1}{N_c^2}$	$m_q \ln m_q$
$c_{(3)}^{1,\mathbf{27}}$	$\frac{1}{N_c^2} \left(\{G^{i8}, J^k G^{kQ}\} + \{J^k G^{k8}, G^{iQ}\} \right)$	$\frac{1}{N_c}$	$m_q \ln m_q$
$\overline{c}_{(2)}^{1,\mathbf{27}}$	$\frac{1}{N_c} \left(\{T^8, G^{i\overline{Q}}\} + \{G^{i8}, T^{\overline{Q}}\} \right)$	1	2 loop χPT
$\overline{b}_{(3)}^{1,\mathbf{27}}$	$\frac{1}{N_c^2} J^i \{T^8, T^{\overline{Q}}\}$	$\frac{1}{N_c^2}$	2 loop χPT
$\overline{c}_{(3)}^{1,\mathbf{27}}$	$\frac{1}{N_c^2} \left(\{G^{i8}, J^k G^{k\overline{Q}}\} + \{J^k G^{k8}, G^{i\overline{Q}}\} \right)$	$\frac{1}{N_c}$	2 loop χPT
$c_{(3)}^{1,\mathbf{64}}$	$\frac{1}{N_c^2} \{G^{i8}, \{T^8, T^Q\}\}$	$\frac{1}{N_c^2}$	2 loop χPT
$d_{(3)}^{1,\mathbf{64}}$	$\frac{1}{N_c^2} \{T^8, \{T^8, G^{iQ}\}\}$	$\frac{1}{N_c}$	2 loop χPT
$\overline{c}_{(3)}^{1,\mathbf{64}}$	$\frac{1}{N_c^2} \{G^{i8}, \{T^8, T^{\overline{Q}}\}\}$	$\frac{1}{N_c^2}$	3 loop χPT

TABLE IX: Coefficients of Magnetic Moment Operators Violating Isospin Symmetry. There are six operators.

Coefficient	Operator	N_c	Isospin Breaking
$g_{(2)}^{1,\mathbf{27}}$	$\frac{1}{N_c^2} \left(\{T^3, G^{iQ}\} + \{G^{i3}, T^Q\} \right)$	$\frac{1}{N_c}$	$m_q \ln m_q$
$f_{(3)}^{1,\mathbf{27}}$	$\frac{1}{N_c^2} J^i \{T^3, T^Q\}$	$\frac{1}{N_c^2}$	$m_q \ln m_q$
$g_{(3)}^{1,\mathbf{27}}$	$\frac{1}{N_c^2} \left(\{G^{i3}, J^k G^{kQ}\} + \{J^k G^{k3}, G^{iQ}\} \right)$	1	$m_q \ln m_q$
$g_{(3)}^{1,\mathbf{64}}$	$\frac{1}{N_c^2} \{G^{i3}, \{T^3, T^Q\}\}$	$\frac{1}{N_c}$	2 loop χPT
$f_{(3)}^{1,\mathbf{64}}$	$\frac{1}{N_c^2} \{T^3, \{T^3, G^{iQ}\}\}$	$\frac{1}{N_c}$	2 loop χPT
$\overline{g}_{(3)}^{1,\mathbf{64}}$	$\frac{1}{N_c^2} \{G^{i3}, \{T^3, T^{\overline{Q}}\}\}$	$\frac{1}{N_c}$	3 loop χPT

TABLE X: Fits to Experimental Data.

	Experiment	Fit A	Fit A$'$	Fit B	Fit B$'$	Fit C	Fit C$'$	Fit D	Fit D$'$
p	2.79 ± 0.00	2.56	2.56	2.72	2.53	2.84	2.72	2.91	2.83
n	-1.91 ± 0.00	-1.63	-1.63	-1.82	-1.69	-2.15	-2.06	-2.22	-2.17
Λ	-0.613 ± 0.004	-0.82	-0.82	-0.91	-0.84	-0.69	-0.66	-0.69	-0.66
$\Lambda\Sigma^0$	-1.61 ± 0.08	-1.41	-1.41	-1.57	-1.46	-1.73	-1.66	-1.73	-1.64
Σ^+	2.458 ± 0.010	2.56	2.56	2.72	2.53	2.69	2.57	2.62	2.42
Σ^0		0.82	0.82	0.91	0.84	0.69	0.66	0.69	0.66
Σ^-	-1.160 ± 0.025	-0.93	-0.93	-0.91	-0.84	-1.31	-1.25	-1.23	-1.10
Ξ^0	-1.250 ± 0.014	-1.63	-1.63	-1.82	-1.69	-1.53	-1.47	-1.47	-1.33
Ξ^-	-0.6507 ± 0.0025	-0.93	-0.93	-0.91	-0.84	-0.54	-0.51	-0.61	-0.65
Δ^{++}	$6.14 \pm 0.51'$	5.23	4.04	5.45	5.07	5.53	5.29	5.53	5.25
Δ^+		2.61	2.02	2.72	2.53	2.53	2.42	2.54	2.41
Δ^0		0	0	0	0	-0.46	-0.44	-0.45	-0.43
Δ^-		-2.61	-2.02	-2.72	-2.53	-3.46	-3.31	-3.44	-3.27
Σ^{*+}		2.61	2.02	2.72	2.53	3.00	2.87	2.99	2.84
Σ^{*0}		0	0	0	0	0	0	0	0
Σ^{*-}		-2.61	-2.02	-2.72	-2.53	-3.00	-2.87	-2.99	-2.84
Ξ^{*0}		0	0	0	0	0.46	0.44	0.45	0.43
Ξ^{*-}		-2.61	-2.02	-2.72	-2.53	-2.53	-2.42	-2.54	-2.41
Ω^-	-2.02 ± 0.05	-2.61	-2.02	-2.72	-2.53	-2.07	-1.98	-2.08	-1.98
$\frac{1}{\sqrt{2}} p\Delta^+$	2.48 ± 0.06	2.36	2.48	1.82	1.69	2.00	1.91	2.07	2.03
$\frac{1}{\sqrt{2}} n\Delta^0$		2.36	2.48	1.82	1.69	2.00	1.91	2.07	2.03
$\frac{1}{\sqrt{2}} \Lambda\Sigma^{*0}$	$1.94 \pm 0.18'$	2.04	2.15	1.57	1.46	1.73	1.66	1.73	1.64
$\frac{1}{\sqrt{2}} \Sigma\Sigma^{*+}$	$2.28 \pm 0.25'$	2.36	2.48	1.82	1.69	1.69	1.62	1.62	1.47
$\frac{1}{\sqrt{2}} \Sigma\Sigma^{*0}$		1.18	1.24	0.91	0.84	0.69	0.66	0.69	0.66
$\frac{1}{\sqrt{2}} \Sigma\Sigma^{*-}$		0	0	0	0	-0.31	-0.30	-0.23	-0.15
$\frac{1}{\sqrt{2}} \Xi\Xi^{*0}$		2.36	2.48	1.82	1.69	1.69	1.62	1.62	1.47
$\frac{1}{\sqrt{2}} \Xi\Xi^{*-}$		0	0	0	0	-0.31	-0.30	-0.23	-0.15
σ^{theory}		0.35	0.28	0.41	0.39	0.27	0.23	0.275	0.20
dof		9	6	12	9	11	8	10	7
$a_{(1)}^{1,\mathbf{8}}$		4.98 ± 0.49	5.28 ± 0.40	5.45 ± 0.31	5.07 ± 0.35	5.07 ± 0.23	4.85 ± 0.21	5.07 ± 0.23	4.82 ± 0.19
$b_{(2)}^{1,\mathbf{8}}$		0.66 ± 1.49	0.66 ± 1.18						
$b_{(3)}^{1,\mathbf{8}}$		-0.25 ± 0.95	-1.14 ± 0.82						
$c_{(3)}^{1,\mathbf{8}}$		4.18 ± 1.71	4.33 ± 1.89						
$\bar{a}_{(1)}^{1,\mathbf{8}}$						1.60 ± 0.42	1.54 ± 0.37	1.57 ± 0.44	1.49 ± 0.33
$\bar{b}_{(2)}^{1,\mathbf{8}}$									
$\bar{b}_{(3)}^{1,\mathbf{8}}$									
$\bar{c}_{(3)}^{1,\mathbf{8}}$									
$c_{(2)}^{1,\mathbf{10}+\overline{\mathbf{10}}}$								0.37 ± 0.52	0.71 ± 0.40
$c_{(3)}^{1,\mathbf{10}+\overline{\mathbf{10}}}$									
$c_{(1)}^{1,\mathbf{1}}$									
$c_{(2)}^{1,\mathbf{27}}$									

TABLE XI: Fits to Experimental Data.

	Fit E	Fit E$'$	Fit F
p	2.72	2.71	2.70
n	-1.91	-1.92	-1.93
Λ	-0.61	-0.59	-0.59
$\Lambda\Sigma^0$	-1.49	-1.51	-1.53
Σ^+	2.45	2.45	2.46
Σ^0	0.64	0.64	0.65
Σ^-	-1.16	-1.16	-1.15
Ξ^0	-1.26	-1.27	-1.27
Ξ^-	-0.64	-0.65	-0.65
Δ^{++}	5.64	5.47	6.14
Δ^+	2.67	2.58	2.79
Δ^0	-0.30	-0.30	-0.56
Δ^-	-3.28	-3.19	-3.91
Σ^{*+}	2.97	2.95	3.49
Σ^{*0}	0.05	0.08	0.10
Σ^{*-}	-2.86	-2.80	-3.28
Ξ^{*0}	0.41	0.46	0.77
Ξ^{*-}	-2.45	-2.41	-2.65
Ω^-	-2.03	-2.02	-2.02
$\frac{1}{\sqrt{2}}p\Delta^+$	2.48	2.48	2.48
$\frac{1}{\sqrt{2}}n\Delta^0$	2.48	2.48	2.48
$\frac{1}{\sqrt{2}}\Lambda\Sigma^{*0}$	2.07	2.09	1.94
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*+}$	2.11	2.09	2.28
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*0}$	0.96	0.95	1.42
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*-}$	-0.19	-0.19	0.56
$\frac{1}{\sqrt{2}}\Xi\Xi^{*0}$	2.07	2.09	2.30
$\frac{1}{\sqrt{2}}\Xi\Xi^{*-}$	-0.19	-0.19	0.56
σ^{theory}	0.024	0.035	0.06
dof	5	2	1
$a_{(1)}^{1,\mathbf{8}}$	4.48 ± 0.07	4.53 ± 0.10	4.44 ± 0.14
$b_{(2)}^{1,\mathbf{8}}$	0.78 ± 0.13	0.73 ± 0.18	0.65 ± 0.33
$b_{(3)}^{1,\mathbf{8}}$	-0.09 ± 0.14	-0.16 ± 0.19	0.21 ± 0.32
$c_{(3)}^{1,\mathbf{8}}$	3.96 ± 0.38	3.88 ± 0.46	6.38 ± 2.18
$\bar{a}_{(1)}^{1,\mathbf{8}}$	1.23 ± 0.12	1.32 ± 0.17	1.16 ± 0.27
$\bar{b}_{(2)}^{1,\mathbf{8}}$			0.13 ± 0.55
$\bar{b}_{(3)}^{1,\mathbf{8}}$			0.60 ± 0.71
$\bar{c}_{(3)}^{1,\mathbf{8}}$			-5.46 ± 5.55
$c_{(2)}^{1,\mathbf{10}+\overline{\mathbf{10}}}$	0.34 ± 0.06	0.34 ± 0.08	0.34 ± 0.14
$c_{(3)}^{1,\mathbf{10}+\overline{\mathbf{10}}}$			-3.83 ± 5.05
$c_{(1)}^{1,\mathbf{1}}$	0.03 ± 0.03	0.05 ± 0.04	0.07 ± 0.07
$c_{(2)}^{1,\mathbf{27}}$	0.10 ± 0.10	0.02 ± 0.16	-0.06 ± 0.25

TABLE XII: Matrix Elements of $I=0$ Isoscalar Magnetic Moment Operators for $SU(2)\times U(1)$ Flavor Symmetry.

	J^i_{ud}	J^i_s	$J^i_{ud}N_s$	$J^i_sN_s$	$J^i_{ud}N_s^2$	$J^i_sN_s^2$	$\{I^2,J^i_{ud}\}$	$\{I^2,J^i_s\}$	$\{J_{ud}\cdot J_s,J^i_{ud}\}$	$\{J_{ud}\cdot J_s,J^i_s\}$
p	$\frac{1}{2}$	0	0	0	0	0	$\frac{3}{4}$	0	0	0
n	$\frac{1}{2}$	0	0	0	0	0	$\frac{3}{4}$	0	0	0
Λ	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	0	0
$\Lambda\Sigma^0$	0	0	0	0	0	0	0	0	0	0
Σ^+	$\frac{2}{3}$	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{6}$	$\frac{8}{3}$	$-\frac{2}{3}$	$-\frac{4}{3}$	$\frac{1}{3}$
Σ^0	$\frac{2}{3}$	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{6}$	$\frac{8}{3}$	$-\frac{2}{3}$	$-\frac{4}{3}$	$\frac{1}{3}$
Σ^-	$\frac{2}{3}$	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{6}$	$\frac{8}{3}$	$-\frac{2}{3}$	$-\frac{4}{3}$	$\frac{1}{3}$
Ξ^0	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$	$\frac{8}{3}$	$-\frac{1}{4}$	1	$\frac{1}{3}$	$-\frac{4}{3}$
Ξ^-	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$	$\frac{8}{3}$	$-\frac{1}{4}$	1	$\frac{1}{3}$	$-\frac{4}{3}$
Δ^{++}	$\frac{3}{2}$	0	0	0	0	0	$\frac{45}{4}$	0	0	0
Δ^+	$\frac{3}{2}$	0	0	0	0	0	$\frac{45}{4}$	0	0	0
Δ^0	$\frac{3}{2}$	0	0	0	0	0	$\frac{45}{4}$	0	0	0
Δ^-	$\frac{3}{2}$	0	0	0	0	0	$\frac{45}{4}$	0	0	0
Σ^{*+}	1	$\frac{1}{2}$	1	$\frac{1}{2}$	1	$\frac{1}{2}$	4	2	1	$\frac{1}{2}$
Σ^{*0}	1	$\frac{1}{2}$	1	$\frac{1}{2}$	1	$\frac{1}{2}$	4	2	1	$\frac{1}{2}$
Σ^{*-}	1	$\frac{1}{2}$	1	$\frac{1}{2}$	1	$\frac{1}{2}$	4	2	1	$\frac{1}{2}$
Ξ^{*0}	$\frac{1}{2}$	1	1	2	2	4	$\frac{3}{4}$	$\frac{3}{2}$	$\frac{1}{2}$	1
Ξ^{*-}	$\frac{1}{2}$	1	1	2	2	4	$\frac{3}{4}$	$\frac{3}{2}$	$\frac{1}{2}$	1
Ω^-	0	$\frac{3}{2}$	0	$\frac{9}{2}$	0	$\frac{27}{2}$	0	0	0	0
$\frac{1}{\sqrt{2}}p\Delta^+$	0	0	0	0	0	0	0	0	0	0
$\frac{1}{\sqrt{2}}n\Delta^0$	0	0	0	0	0	0	0	0	0	0
$\frac{1}{\sqrt{2}}\Lambda\Sigma^{*0}$	0	0	0	0	0	0	0	0	0	0
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*+}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{4}{3}$	$-\frac{4}{3}$	$-\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*0}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{4}{3}$	$-\frac{4}{3}$	$-\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*-}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{4}{3}$	$-\frac{4}{3}$	$-\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{\sqrt{2}}\Xi\Xi^{*0}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	$\frac{4}{3}$	$-\frac{4}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{\sqrt{2}}\Xi\Xi^{*-}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	$\frac{4}{3}$	$-\frac{4}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{6}$	$\frac{1}{6}$

TABLE XIII: Matrix Elements of $I = 1$ Isovector Magnetic Moment Operators for $SU(2)\times U(1)$ Flavor Symmetry.

	G^{i3}	$G^{i3}N_s$	$G^{i3}N_s^2$	$J^i_{ud}I^3$	$J^i_sI^3$	$\{I^2,G^{i3}\}$	$\{J_{ud}\cdot J_s,G^{i3}\}$	$\{J^i_{ud},J^k_sG^{k3}\}$	$\{J^i_s,J^k_sG^{k3}\}$	$J^i_{ud}I^3N_s$	$J^i_sI^3N_s$
p	$\frac{5}{12}$	0	0	$\frac{1}{4}$	0	$\frac{5}{8}$	0	0	0	0	0
n	$-\frac{5}{12}$	0	0	$-\frac{1}{4}$	0	$-\frac{5}{8}$	0	0	0	0	0
Λ	0	0	0	0	0	0	0	0	0	0	0
$\Lambda\Sigma^0$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{2\sqrt{3}}$	0	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{4\sqrt{3}}$	0	0
Σ^+	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{6}$	$\frac{4}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{6}$
Σ^0	0	0	0	0	0	0	0	0	0	0	0
Σ^-	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{6}$	$-\frac{4}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$\frac{1}{6}$
Ξ^0	$-\frac{1}{12}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{12}$	$\frac{1}{3}$	$-\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{1}{6}$	$\frac{2}{3}$
Ξ^-	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{12}$	$-\frac{1}{3}$	$\frac{1}{8}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$	$-\frac{2}{3}$
Δ^{++}	$\frac{3}{4}$	0	0	$\frac{9}{4}$	0	$\frac{45}{8}$	0	0	0	0	0
Δ^+	$\frac{1}{4}$	0	0	$\frac{3}{4}$	0	$\frac{15}{8}$	0	0	0	0	0
Δ^0	$-\frac{1}{4}$	0	0	$-\frac{3}{4}$	0	$-\frac{15}{8}$	0	0	0	0	0
Δ^-	$-\frac{3}{4}$	0	0	$-\frac{9}{4}$	0	$-\frac{45}{8}$	0	0	0	0	0
Σ^{*+}	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	2	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	1	$\frac{1}{2}$
Σ^{*0}	0	0	0	0	0	0	0	0	0	0	0
Σ^{*-}	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	-2	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{4}$	-1	$-\frac{1}{2}$
Ξ^{*0}	$\frac{1}{4}$	$\frac{1}{2}$	1	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	1
Ξ^{*-}	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{3}{8}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
Ω^-	0	0	0	0	0	0	0	0	0	0	0
$\frac{1}{\sqrt{2}}p\Delta^+$	$\frac{1}{3}$	0	0	0	0	$\frac{3}{2}$	0	0	0	0	0
$\frac{1}{\sqrt{2}}n\Delta^0$	$\frac{1}{3}$	0	0	0	0	$\frac{3}{2}$	0	0	0	0	0
$\frac{1}{\sqrt{2}}\Lambda\Sigma^{*0}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$	0	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{4\sqrt{3}}$	0	0	0	0
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*+}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{12}$	$-\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{3}$	$-\frac{1}{3}$
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*0}$	0	0	0	0	0	0	0	0	0	0	0
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*-}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{12}$	$\frac{1}{12}$	$-\frac{1}{12}$	$-\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{\sqrt{2}}\Xi\Xi^{*0}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{4}$	$-\frac{1}{12}$	$-\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{3}$	$-\frac{1}{3}$
$\frac{1}{\sqrt{2}}\Xi\Xi^{*-}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{12}$	$-\frac{1}{12}$	$-\frac{1}{3}$	$\frac{1}{3}$

TABLE XIV: Matrix Elements of $I=2$ and $I=3$ Isotensor Magnetic Moment Operators for $SU(2)\times U(1)$ Flavor Symmetry.

	$\{G^{i3},I^3\}$	$\{G^{i3},I^3\}N_s$	$J^i_{ud}\{I^3,I^3\}$	$J^i_s\{I^3,I^3\}$	$\{G^{i3},J^k_sG^{k3}\}$	$\{G^{i3},\{I^3,I^3\}\}$
p	$\frac{5}{12}$	0	$\frac{1}{4}$	0	0	$\frac{5}{12}$
n	$\frac{5}{12}$	0	$\frac{1}{4}$	0	0	$-\frac{5}{12}$
Λ	0	0	0	0	0	0
$\Lambda\Sigma^0$	0	0	0	0	0	0
Σ^+	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{4}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{4}{3}$
Σ^0	0	0	0	0	0	0
Σ^-	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{4}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{4}{3}$
Ξ^0	$-\frac{1}{12}$	$-\frac{1}{6}$	$-\frac{1}{12}$	$\frac{1}{3}$	$\frac{1}{12}$	$-\frac{1}{12}$
Ξ^-	$-\frac{1}{12}$	$-\frac{1}{6}$	$-\frac{1}{12}$	$\frac{1}{3}$	$\frac{1}{12}$	$\frac{1}{12}$
Δ^{++}	$\frac{9}{4}$	0	$\frac{27}{4}$	0	0	$\frac{27}{4}$
Δ^+	$\frac{1}{4}$	0	$\frac{3}{4}$	0	0	$\frac{1}{4}$
Δ^0	$\frac{1}{4}$	0	$\frac{3}{4}$	0	0	$-\frac{1}{4}$
Δ^-	$\frac{9}{4}$	0	$\frac{27}{4}$	0	0	$-\frac{27}{4}$
Σ^{*+}	1	1	2	1	$\frac{1}{4}$	2
Σ^{*0}	0	0	0	0	0	0
Σ^{*-}	1	1	2	1	$\frac{1}{4}$	-2
Ξ^{*0}	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{4}$
Ξ^{*-}	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$	$-\frac{1}{4}$
Ω^-	0	0	0	0	0	0
$\frac{1}{\sqrt{2}}p\Delta^+$	$\frac{1}{3}$	0	0	0	0	$\frac{1}{3}$
$\frac{1}{\sqrt{2}}n\Delta^0$	$-\frac{1}{3}$	0	0	0	0	$\frac{1}{3}$
$\frac{1}{\sqrt{2}}\Lambda\Sigma^{*0}$	0	0	0	0	0	0
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*+}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{24}$	$\frac{2}{3}$
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*0}$	0	0	0	0	0	0
$\frac{1}{\sqrt{2}}\Sigma\Sigma^{*-}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{24}$	$-\frac{2}{3}$
$\frac{1}{\sqrt{2}}\Xi\Xi^{*0}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{24}$	$\frac{1}{6}$
$\frac{1}{\sqrt{2}}\Xi\Xi^{*-}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{24}$	$-\frac{1}{6}$

TABLE XV: $I = 0$ Magnetic Moment Combinations

Operator	Magnetic Moment Combination	$\frac{1}{N_c}$
J^i_{ud}	$\frac{5}{2}N_0 - \frac{1}{6}\Delta_0$	1
J^i_s	$\frac{15}{4}\Lambda_0 - \frac{1}{4}\Sigma_0 - \Xi_0 + \frac{5}{6}\Sigma^*_0 - \frac{5}{3}\Xi^*_0 + \frac{2}{3}\Omega_0 - 2\frac{1}{\sqrt{2}}\left(\Sigma\Sigma^*\right)_0 + 2\frac{1}{\sqrt{2}}\left(\Xi\Xi^*\right)_0$	1
$J^i_{ud}N_s$	$-\frac{4}{3}N_0 + \frac{2}{3}\Sigma_0 + \frac{1}{3}\Xi_0 - \frac{5}{9}\Delta_0 + \frac{10}{9}\Sigma^*_0 - \frac{4}{9}\Xi^*_0 + \frac{4}{3}\frac{1}{\sqrt{2}}\left(\Sigma\Sigma^*\right)_0 - \frac{2}{3}\frac{1}{\sqrt{2}}\left(\Xi\Xi^*\right)_0$	$\frac{1}{N_c}$
$J^i_sN_s$	$-2\Lambda_0 + \frac{1}{3}\Sigma_0 + \frac{4}{3}\Xi_0 - \frac{10}{9}\Sigma^*_0 + \frac{20}{9}\Xi^*_0 - \Omega_0 + \frac{8}{3}\frac{1}{\sqrt{2}}\left(\Sigma\Sigma^*\right)_0 - \frac{8}{3}\frac{1}{\sqrt{2}}\left(\Xi\Xi^*\right)_0$	$\frac{1}{N_c}$
$J^i_{ud}N^2_s$	$\frac{1}{6}N_0 - \frac{1}{3}\Sigma_0 + \frac{5}{18}\Delta_0 - \frac{5}{9}\Sigma^*_0 + \frac{1}{3}\Xi^*_0 - \frac{2}{3}\frac{1}{\sqrt{2}}\left(\Sigma\Sigma^*\right)_0 + \frac{1}{\sqrt{2}}\left(\Xi\Xi^*\right)_0$	$\frac{1}{N^2_c}$
$J^i_sN^2_s$	$\frac{1}{4}\Lambda_0 - \frac{1}{12}\Sigma_0 - \frac{1}{3}\Xi_0 + \frac{5}{18}\Sigma^*_0 - \frac{5}{9}\Xi^*_0 + \frac{1}{3}\Omega_0 - \frac{2}{3}\frac{1}{\sqrt{2}}\left(\Sigma\Sigma^*\right)_0 + \frac{2}{3}\frac{1}{\sqrt{2}}\left(\Xi\Xi^*\right)_0$	$\frac{1}{N^2_c}$
$\{I^2, J^i_{ud}\}$	$-\frac{1}{3}N_0 + \frac{1}{9}\Delta_0$	$\frac{1}{N^2_c}$
$\{I^2, J^i_s\}$	$-\frac{1}{2}\Lambda_0 + \frac{1}{6}\Sigma_0 + \frac{1}{9}\Sigma^*_0 - \frac{2}{3}\frac{1}{\sqrt{2}}\left(\Sigma\Sigma^*\right)_0$	$\frac{1}{N^2_c}$
$\{J_{ud} \cdot J_s, J^i_{ud}\}$	$-\frac{2}{3}\Sigma_0 + \frac{2}{9}\Sigma^*_0 + \frac{2}{3}\frac{1}{\sqrt{2}}\left(\Sigma\Sigma^*\right)_0$	$\frac{1}{N^2_c}$
$\{J_{ud} \cdot J_s, J^i_s\}$	$-\frac{2}{3}\Xi_0 + \frac{2}{9}\Xi^*_0 - \frac{2}{3}\frac{1}{\sqrt{2}}\left(\Xi\Xi^*\right)_0$	$\frac{1}{N^2_c}$

TABLE XVI: $I = 1$ Magnetic Moment Combinations

Operator	Magnetic Moment Combination	$\frac{1}{N_c}$
G^{i3}	$\frac{3}{2}N_1 - \frac{1}{20}\Delta_1$	N_c
$G^{i3}N_s$	$-N_1 - \frac{3\sqrt{3}}{5}\left(\Lambda\Sigma\right)_1 + \frac{11}{30}\Sigma_1 - \frac{1}{15}\Xi_1 + \frac{19}{90}\Sigma^*_1 - \frac{14}{45}\Xi^*_1 - \frac{1}{\sqrt{2}}\left(N\Delta\right)_1 + \frac{12\sqrt{3}}{5}\frac{1}{\sqrt{2}}\left(\Lambda\Sigma^*\right)_1 + \frac{2}{15}\frac{1}{\sqrt{2}}\left(\Sigma\Sigma^*\right)_1 - \frac{16}{15}\frac{1}{\sqrt{2}}\left(\Xi\Xi^*\right)_1$	1
$G^{i3}N^2_s$	$\frac{1}{6}N_1 + \frac{\sqrt{3}}{5}\left(\Lambda\Sigma\right)_1 - \frac{7}{30}\Sigma_1 + \frac{2}{15}\Xi_1 + \frac{1}{36}\Delta_1 - \frac{23}{90}\Sigma^*_1 + \frac{13}{45}\Xi^*_1$	$\frac{1}{N_c}$
	$+\frac{1}{3}\frac{1}{\sqrt{2}}\left(N\Delta\right)_1 - \frac{4\sqrt{3}}{5}\frac{1}{\sqrt{2}}\left(\Lambda\Sigma^*\right)_1 - \frac{4}{15}\frac{1}{\sqrt{2}}\left(\Sigma\Sigma^*\right)_1 + \frac{17}{15}\frac{1}{\sqrt{2}}\left(\Xi\Xi^*\right)_1$	
$J^i_{ud}I^3$	$\frac{1}{3}N_1 + \frac{1}{18}\Delta_1 - \frac{5}{6}\frac{1}{\sqrt{2}}\left(N\Delta\right)_1$	$\frac{1}{N_c}$
$J^i_sI^3$	$\frac{1}{3}\Sigma_1 - \frac{1}{3}\Xi_1 + \frac{5}{9}\Sigma^*_1 - \frac{5}{9}\Xi^*_1 - \frac{7}{3}\frac{1}{\sqrt{2}}\left(\Sigma\Sigma^*\right)_1 + \frac{2}{3}\frac{1}{\sqrt{2}}\left(\Xi\Xi^*\right)_1$	$\frac{1}{N_c}$
$\{I^2, G^{i3}\}$	$-\frac{1}{3}N_1 + \frac{1}{90}\Delta_1 + \frac{1}{3}\frac{1}{\sqrt{2}}\left(N\Delta\right)_1$	$\frac{1}{N_c}$
$\{J_{ud} \cdot J_s, G^{i3}\}$	$\frac{4\sqrt{3}}{5}\left(\Lambda\Sigma\right)_1 - \frac{4}{15}\Sigma_1 - \frac{2}{15}\Xi_1 + \frac{4}{45}\Sigma^*_1 + \frac{2}{45}\Xi^*_1 + \frac{4\sqrt{3}}{5}\frac{1}{\sqrt{2}}\left(\Lambda\Sigma^*\right)_1 + \frac{4}{15}\frac{1}{\sqrt{2}}\left(\Sigma\Sigma^*\right)_1 - \frac{2}{15}\frac{1}{\sqrt{2}}\left(\Xi\Xi^*\right)_1$	$\frac{1}{N_c}$
$\{J^i_{ud}, J^k_sG^{k3}\}$	$-\frac{4\sqrt{3}}{5}\left(\Lambda\Sigma\right)_1 - \frac{2}{5}\Sigma_1 + \frac{2}{15}\Xi_1 + \frac{2}{15}\Sigma^*_1 - \frac{2}{45}\Xi^*_1 - \frac{4\sqrt{3}}{5}\frac{1}{\sqrt{2}}\left(\Lambda\Sigma^*\right)_1 + \frac{2}{5}\frac{1}{\sqrt{2}}\left(\Sigma\Sigma^*\right)_1 + \frac{2}{15}\frac{1}{\sqrt{2}}\left(\Xi\Xi^*\right)_1$	$\frac{1}{N_c}$
$\{J^i_s, J^k_sG^{k3}\}$	$-\frac{2}{3}\Xi_1 + \frac{2}{9}\Xi^*_1 - \frac{2}{3}\frac{1}{\sqrt{2}}\left(\Xi\Xi^*\right)_1$	$\frac{1}{N_c}$
$J^i_{ud}I^3N_s$	$\frac{\sqrt{3}}{5}\left(\Lambda\Sigma\right)_1 + \frac{1}{10}\Sigma_1 + \frac{2}{15}\Xi_1 - \frac{1}{15}\Delta_1 + \frac{3}{10}\Sigma^*_1 - \frac{2}{45}\Xi^*_1 + \frac{1}{2}\frac{1}{\sqrt{2}}\left(N\Delta\right)_1 - \frac{4\sqrt{3}}{5}\frac{1}{\sqrt{2}}\left(\Lambda\Sigma^*\right)_1 + \frac{2}{5}\frac{1}{\sqrt{2}}\left(\Sigma\Sigma^*\right)_1 + \frac{2}{15}\frac{1}{\sqrt{2}}\left(\Xi\Xi^*\right)_1$	$\frac{1}{N^2_c}$
$J^i_sI^3N_s$	$\frac{1}{3}\Xi_1 - \frac{1}{3}\Sigma^*_1 + \frac{5}{9}\Xi^*_1 + \frac{1}{\sqrt{2}}\left(\Sigma\Sigma^*\right)_1 - \frac{2}{3}\frac{1}{\sqrt{2}}\left(\Xi\Xi^*\right)_1$	$\frac{1}{N^2_c}$

TABLE XVII: $\frac{1}{N^2_c}$ Isoscalar Magnetic Moment Combinations

$S1$	$2N_0 + 3\Lambda_0 - 3\Sigma_0 - 6\Xi_0 + \frac{4}{3}\Omega_0$
$S2$	$-3N_0 + \Delta_0$
$S3$	$-2N_0 + \Lambda_0 + 3\Sigma_0 + 2\Xi_0 - \frac{4}{3}\Xi^*_0$
$S4$	$\Lambda_0 + \Sigma_0 - \frac{2}{3}\Sigma^*_0$
$S5$	$\Sigma_0 - \frac{1}{3}\Sigma^*_0 - \frac{1}{\sqrt{2}}\left(\Sigma\Sigma^*\right)_0$
$S6$	$-\Xi_0 + \frac{1}{3}\Xi^*_0 - \frac{1}{\sqrt{2}}\left(\Xi\Xi^*\right)_0$

TABLE XVIII: $\frac{1}{N_c}$ Isovector Magnetic Moment Combinations

$V1$	$\frac{1}{5}N_1 - \frac{1}{2}\Sigma_1 - \Xi_1$
$V2$	$-\frac{9}{5}N_1 + \frac{3}{10}\Delta_1$
$V3$	$-\frac{\sqrt{3}}{4}\Sigma_1 + \frac{1}{\sqrt{2}}\left(\Lambda\Sigma^*\right)_1$
$V4$	$-\frac{3}{2}\Sigma_1 + \Sigma_1^*$
$V5$	$3\Xi_1 + \Xi_1^*$
$V6$	$-\frac{1}{2}\Sigma_1 + \frac{1}{\sqrt{2}}\left(\Sigma\Sigma^*\right)_1$
$V7$	$2\Xi_1 + \frac{1}{\sqrt{2}}\left(\Xi\Xi^*\right)_1$
$V8$	$\frac{\sqrt{3}}{4}\Sigma_1 + \left(\Lambda\Sigma\right)_1$
$V9$	$-\frac{2}{5}N_1 + \frac{1}{2}\frac{1}{\sqrt{2}}\left(N\Delta\right)_1$