Non-Abelian Higgs model of magnetic field generation during a cosmological first-order electroweak phase transition

Trevor Stevens, Mikkel B. Johnson, Leonard S. Kisslinger, and Ernest M. Henley

Phys. Rev. D **85**, 063003 — Published 23 March 2012

DOI: 10.1103/PhysRevD.85.063003

Non-Abelian Higgs Model of Magnetic Field Generation during a Cosmological First-Order Electroweak Phase Transition

Trevor Stevens
Department of Physics, New Mexico State University, Las Cruces, NM 88003
Mikkel B. Johnson
Los Alamos National Laboratory, Los Alamos, NM 87545
Leonard S. Kisslinger
Department of Physics, Carnegie-Mellon University, Pittsburgh, PA 15213
Ernest M. Henley
Department of Physics, University of Washington, Seattle, WA 98195

PACS Indices98.62.En,98.80.Cq,12.60.-i

Abstract

We present a theory of the generation of magnetic fields in bubble collisions during the electroweak phase transition (EWPT), which is first order in the minimal supersymmetric Standard Model (MSSM). Using the equations of motion (EOM) derived from this theory we derive the magnetic field using a model of gentle collisions of the EWPT bubbles, discussed in our earlier work. Solutions of the relevant EOM for the magnetic field are examined in O(1,2) space-time symmetry with boundary conditions applied at the time of collision. These solutions indicate that the magnetic fields based on the MSSM are somewhat larger in magnitude and extend more uniformly through the available volume of the bubble than those found in the Abelian Higgs model. The magnetic fields so produced might possibly seed galactic and extra-galactic magnetic fields observed today.

1 Introduction

Identifying the source of the observed large-scale galactic and extra-galactic magnetic fields remains an unresolved problem of astrophysics [1]. One of the interesting possible sources is cosmological magnetogenesis, where the seed fields would have arisen during one of the early-universe phase transitions. Our first work on magnetic field creation was the study of bubble collisions during the Quantum Chromodynamic phase transition (QCDPT) at about 10^{-5} seconds[2, 3], in which it was shown that the magnetic fields could lead to detectable correlations in the CMBR.

In our present work our interest is in magnetic field production during the electroweak phase transition (EWPT) at about 10^{-11} seconds, during which the Gauge fields and the other particles acquired their masses. Studies of the evolution of primary magnetic fields from the QCDPT and EWPT[4, 5, 6] showed that it is very unlikely that the EWPT produces seeds of galactic and extra-galactic magnetic fields, while it is possible that the QCDPT, with a much larger scale, could produce the seeds. These calculations, however, used random magnetic seed fields, while in our MSSM EW theory, discussed below, the electromagnetic fields produced are coherent, and therefore have a much larger scale.

If magnetogenesis occurred during the EWPT, it most likely required a first-order phase transition, in which bubbles of matter in the broken phase nucleate within the unbroken phase. Although it was shown that there is no first-order EWPT in the Standard Model, with the known minimum Higgs mass [7], there has been a great deal of activity in minimal supersymmetric extensions (MSSM) of standard EW theory [8, 9, 10]. In models with the Stop, the SS partner of the top quark, with certain Stop masses there can be a first-order phase transition and baryogenesis [11, 12, 13, 14]. Also, limits on the parameter-space of (MSSM) are given by electric dipole moment measurements and dark matter [14].

Interest in these issues has led to quantitative studies of EWPT magnetogenesis based on the solution of equations of motion (EOM) derived from specific models. Nucleation was studied in a model using a Higgs potential [15]. In the Abelian Higgs model (AHM) [16, 17, 18], the first-order phase transition developed as the Universe condensed into bubbles consisting of localized regions of space filled by the Higgs field in a broken phase. These models, based on earlier work of Coleman [19], were some of the earliest attempts to derive magnetic field production during the EWPT. The EOM related the magnetic fields to gradients in the phase of the Higgs field produced when bubbles merged following nucleation. Simple and transparent solutions to the EOM evolved from specific field configurations applied at the point of collision in a relativistic $O(1,2)$ symmetric model. In Refs [16, 17, 18], the importance of the bubble wall velocity v_{wall} and conductivity σ [20], and how they affect seed field formation, have been discussed.

We carried out the first work that uses the basic EW theory with MSSM for the production of electromagnetic (*em*) fields via bubble nucleation in a first order phase transition without[21] and with charged lepton currents [22, 23]. Due to the spherical symmetry, however, magnetic fields are not produced via nucleation. This approach was extended to EWPT bubble collisions, during which magnetic fields are produced, with an $O(1,2)$ symmetric EOM approach [24, 25], similar to the isospin *ansatz* in our nucleation work [21, 23]. Since lepton currents were neglected, only the charged gauge fields are the source of the *em* currents. Numerical studies [25] found magnetic fields similar to those of the Abelian Higgs model even though the source of the current in the two approaches was quite different. In our more recent studies [26, 27] bubble surface dynamics was taken into account in the same model, and the results presented therein showed that the magnetic fields produced could be possibly even larger than those calculated in its absence.

It was shown [5, 6] that helicity plays a major role for magnetic fields produced in cosmological phase transitions to be the seeds for galactic and extra-galactic magnetic fields, or that give measureble effects in CMBR. For the QCDPT helicity was determined [2], and measurable efects in CMBR polarization were found. For our theory of the EWPT, including the present work, we do not derive helicity. This will be done in the future. In our study of gravity wave production from the QCDPT and EWPT [28] MHD turbulence caused by the magnetic fields created during the phase tranitions produce the helicity needed to produce gravity waves.

In Sect. 2, we review the derivation of the exact EOM from the MSSM Lagrangian. We also identify constraints on the initial conditions required for solving the EOM for bubble collisions.

In Sect. 3 we review the general framework within which our theory is applied[25], discussing thermal erasure and gentle collisions. Restricting the application to the regime of gentle collisions has the advantages that the EOM linearize an the Higgs field decouples from the other fields, similar to the models of Ref. [16, 17, 18]. In this section we also show explicitly how thermal erasure of the gauge fields makes it possible for the phase of the Higgs field to become the source of the *em* current in the MSSM. We find EOM for the Higgs phase, obtain a corresponding Maxwell equation for the magnetic field, and identify specific requirements on the Z field necessary for internal consistency of the theory. For our study based on MSSM for gentle collision we refer to our present work as the Non-abelian Higgs model (NAHM), in contrast to the Abelian Higgs Models (AHM).

Then, in Sect. 4 we express our EOM in $O(1,2)$ symmetric form and give explicitly the initial conditions imposed by thermal erasure. Additionally, it is shown that the Z field satisfies the requirement for internal consistency identified in Sect. 3.4, and we discuss issues for determining the other fields in the NAHM. A closed-form expression for the magnetic fields is also given here.

Numerical results for the NAHM are then presented in the following section along with a comparison of these results of the Abelian Higgs model obtained in Ref. [17]. We identify significant qualitative differences arising from the Non-abelian character of the underlying Lagrangian. Comparing to the numerical results of Ref. [17] we find that our magnetic fields are larger in both scale and magnitude, which could be important for them to be seeds of galactic and extra-galactic magnetic fields.

2 Equation of motion for a MSSM EW theory

In this sction we review the EOM derived from a MSSM extension of the standard EW theory, with parameters giving a first order phase transition [21, 23]

2.1 Equations of Motion

The Lagrangian which we use[23] is

$$\mathcal{L}^{MSSM} = \mathcal{L}^1 + \mathcal{L}^2 + \Delta\mathcal{L} \ , \tag{1}$$

where $\mathcal{L}^1$ and $\mathcal{L}^2$ are contributions to $\mathcal{L}^{MSSM}$ from the gauge and the Higgs fields Φ of the Standard Model, and where $\Delta\mathcal{L}$ accounts for leptonic, quark, and supersymmetric partner interactions. Thus,

$$\begin{aligned} \mathcal{L}^1 &= -\frac{1}{4}W^i_{\mu\nu}W^{i\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \ , \\ W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon_{ijk}W^j_\mu W^k_\nu \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \ , \end{aligned} \tag{2}$$

with W^i for i = (1,2) the W^+, W^- fields, and

$$\begin{aligned} \mathcal{L}^2 &= |(i\partial_\mu - \frac{g}{2}\tau \cdot W_\mu - \frac{g'}{2}B_\mu)\Phi|^2 \\ &- V(\Phi) \ , \end{aligned} \tag{3}$$

with τ^i the SU(2) generator and $V(\Phi)$ the Higgs potential. The various parameters are discussed in many publications [11]. For our calculations we use the values consistent with experiments,

$$\begin{aligned} g &= e/\sin\theta_W = 0.646 \ , \\ g' &= g\tan\theta_W = 0.343 \ , \\ m_W &= 80.4 \text{ GeV} \\ m_Z &= 91.2 \text{ GeV} \ , \end{aligned} \tag{4}$$

where m_W is the mass of the charged gauge bosons, M_Z the mass of the Z, and

$$G \equiv gg'/\sqrt{g^2 + g'^2} = 0.303 \ . \tag{5}$$

In this section and throughout the paper units are such that $\hbar = c = 1$, with distance and time expressed in units of m_W.

In the picture we are developing, the Higgs field plays a central dynamical role in EW bubble nucleation and collisions. In AHM models the Higgs potential $V(\Phi, T)$ at temperature T is an essential element in the theory, while it is not relevant for the purposes of this paper. We require only that the EWPT is first-order, consistent with certain MSSM extensions, including, for example, those with a right-handed Stop [10, 23].

As in Ref. [25], we derive"exact" EOM by minimizing the action using an effective SU(2)×U(1) invariant Lagrangian at the classical level from which the supersymmetric partners have been projected out as explicit degrees of freedom, but whose effect is retained by a renormalization of the Higgs potential to maintain the properties of the first-order phase transition. Fermions are not explicitly considered since our study EWPT nucleation [23] showed charged lepton current have a small effect, and earlier work to which we want to compare also ignored them. The EOM obtained are complicated nonlinear partial differential equations (PDE) coupling the W, B, and Φ fields.

From the solution of the EOM one may obtain the physical Z and A^{em} fields,

$$\begin{aligned} A^{em}_\mu &= \frac{1}{\sqrt{g^2+g'^2}}(g'W^3_\mu + gB_\mu) \\ Z_\mu &= \frac{1}{\sqrt{g^2+g'^2}}(gW^3_\mu - g'B_\mu) \ . \end{aligned} \tag{6}$$

In order that the photon be massless, the SU(2)×U(1) symmetry of the theory is spontaneously broken (by the form of the Higgs, Eq. (7) below), except in the sector containing the electromagnetic field A^{em}_μ. By the nature of spontaneous symmetry breaking, physical observables continue to be invariant under corresponding local S(2)×U(1) gauge transformations, although the fields themselves are not.

The fields in the plasma outside the bubbles are assumed to occupy thermal modes described by a partition function at a local temperature T. We further assume that once the phase transition begins and bubbles form, the fields within the bubbles must be treated as evolving through non-equilibrium dynamics described by the EOM obtained by minimizing the action corresponding to the Lagrangian in Eq. (1). We next present these EOM [24, 25] in a particular unitary gauge in which the Higgs field doublet has the form

$$\Phi(x) = \begin{pmatrix} 0 \\ \rho(x) \exp i\Theta(x) \end{pmatrix} , \tag{7}$$

where $\Theta(x)$ is the phase of the Higgs field and $\rho(x)$ its magnitude.

2.1.1 EOM for Higgs, B, and W Fields

For our choice of gauge, the modulus ρ of the Higgs field satisfies the "ρ-equation",

$$\begin{aligned} 0 &= \partial^2\rho(x) - \frac{g^2}{4}\rho(x)[W^1 \cdot W^1 \\ &\quad + W^2 \cdot W^2] - \rho(x)\psi_\nu\psi^\nu + \rho(x)\frac{\partial V}{\partial \rho^2} \ , \end{aligned} \tag{8}$$

and the gauge field B field satisfies the "B-equation",

$$0 = \partial^2 B_\nu - \partial_\nu \partial \cdot B + \rho(x)^2 g' \psi_\nu(x) \ . \tag{9}$$

The quantity ψ_ν is given by

$$\psi_\nu(x) \equiv \partial_\nu \Theta - \frac{\sqrt{g^2+g'^2}}{2} Z_\nu \tag{10}$$

and satisfies the relationship

$$0 = \partial^\nu \rho(x)^2 \psi_\nu(x). \tag{11}$$

The gauge field W field satisfies two "W-equations". For $i = 3$,

$$\begin{aligned} \partial^2 W^3_\nu &- \partial_\nu \partial \cdot W^3 - g\rho(x)^2 \psi_\nu(x) \\ &= j^{(3)}_\nu(x) \ , \end{aligned} \tag{12}$$

and for $i = (1, 2)$,

$$\partial^2 W^i_\nu - \partial_\nu \partial \cdot W^i + m_W(x)^2 W^i_\nu = j^{(i)}_\nu(x) \ . \tag{13}$$

Here

$$\begin{aligned} j^{(i)}_\nu(x) &\equiv g\epsilon_{ijk}[W^k_\nu \partial \cdot W^j + 2W^j \cdot \partial W^k_\nu \\ &- W^j_\mu \partial_\nu W^{k\mu}] - g^2 \epsilon_{klm}\epsilon_{ijk} W^j_\mu W^{l\mu} W^m_\nu \ , \end{aligned} \tag{14}$$

and m_W, the mass of the W field, is given by

$$m_W(x)^2 = \frac{\rho(x)^2 g^2}{2} \ . \tag{15}$$

Taking the divergence of Eq. (14) and using the EOM for the W fields, we find

$$\begin{aligned} \partial \cdot j^{(i)}(x) &= -g\epsilon_{ij3} W^{j\mu}(x) \\ &\times \ (m^2_W(x) W^3_\mu(x) + g\rho(x)^2 \psi_\mu(x)) \\ &= \ -g^2 \rho(x)^2 \epsilon_{ij3} W^{j\mu}(x)(\frac{g}{2} W^3_\mu(x) + \psi_\mu(x)) \ . \end{aligned} \tag{16}$$

It will be seen that explicit solutions of Eqs. (12,13) will not be required to find the *em* current and hence the magnetic fields in the NAHM. Instead, the *em* current will be obtained indirectly from the EOM for the Z field given below taking advantage of a fortuitous connection between the two under the conditions of thermal erasure.

2.1.2 EOM for electromagnetic field A^{em}

Maxwell's equation for the *em* field $A^{em}_\nu(x)$ is found by taking the linear combination of the W^3 and B indicated in Eq (6). The EOM for A^{em} then becomes

$$\partial^2 A^{em}_\nu - \partial^\gamma \partial \cdot A^{em} \ = \ 4\pi j^{em}_\nu(x) \ . \tag{17}$$

The expression for the *em* current $j^{em}_\nu(x)$ deduced from this is proportional to j^3_ν,

$$4\pi j^{em}_\nu(x) = \frac{g'}{\sqrt{g^2 + g'^2}} j^{(3)}_\nu(x) \ , \tag{18}$$

consisting of terms quadratic and cubic in the three fields $W^i(x)$. That the *em* current is conserved,

$$\partial \cdot j^{em} = 0 \ , \tag{19}$$

follows from Eq. (16).

2.1.3 EOM for Z

Using the definition of the Z field in Eq. (6),

$$Z_\nu = \frac{1}{\sqrt{g^2+g'^2}}(gW^3_\nu - g'B_\nu) \ , \tag{20}$$

and using the EOM for B and W^3 we are able to find an EOM for the Z field,

$$\begin{aligned} &\partial^2 Z_\nu - \partial_\nu \partial\cdot Z - \rho(x)^2\sqrt{g^2+g'^2}\psi_\nu(x) \\ =\ & \frac{g}{g'}4\pi j^{em}_\nu(x) \ , \end{aligned} \tag{21}$$

similarly to determining Eq. (17).

In order to facilitate the solution of this equation, it is helpful to note that by taking the divergence of Eq. (21) and using Eq. (11) we obtain a consistency condition, the *auxiliary condition* for Z_ν,

$$\chi_Z(x) = 0 \ , \tag{22}$$

where

$$\chi_Z(x) \equiv \frac{\partial^\mu(m_Z^2 Z_\mu)}{m_Z^2} = \partial\cdot Z + \frac{Z\cdot\partial m_Z^2}{m_Z^2} \ , \tag{23}$$

introducing a mass for the Z field,

$$m_Z^2(t,\vec{r}) = \frac{g^2+g'^2}{2}\rho(t,\vec{r})^2 \ . \tag{24}$$

Equations (22,23) require

$$\partial\cdot Z = -\frac{Z\cdot\partial m_Z^2}{m_Z^2} \tag{25}$$

and that $m_Z(x)^2 > 0$ everywhere, as it is when averaged over the local distribution of bubbles. Using Eq. (25) the EOM in Eq. (21) may be rewritten as

$$\begin{aligned} \partial^2 Z_\nu &+ \partial_\nu \frac{Z\cdot\partial m_Z^2}{m_Z^2} - \rho(t,r)^2\sqrt{g^2+g'^2}\psi_\nu(x) \\ &= \frac{g}{g'}\frac{\sqrt{g^2+g'^2}}{2}4\pi j^{em}_\nu \ . \end{aligned} \tag{26}$$

The solution to this equation is equivalent Eq. (21) provided the auxiliary condition Eq. (22) is maintained for all x.

To see how the auxiliary condition Eq. (22) may be maintained for all $(t, \vec{x})$, note that Eq. (26) requires $\chi_Z(x)$ to satisfy the Klein-Gordon equation

$$\partial^2 \chi_Z(x) + m_Z(x)^2 \chi_Z(x) = 0. \tag{27}$$

By choosing the initial configuration of $Z_\nu(x)$, at time $t = t_0$, to satisfy

$$\chi_Z(t_0, \vec{r}) = 0 \tag{28}$$

and

$$\frac{\partial \chi_Z(t_0, \vec{r})}{\partial t} = 0 \ , \tag{29}$$

we assure that $\chi_Z(x) = 0$ for all future times since Eqs. (28, 29) are boundary conditions for the trivial solution of Eq. (27), $\chi_Z(t, \vec{r}) = 0$. In Ref. [25] the counterpart of t_0 was the point of first contact of the bubbles.

2.1.4 Solving the EOM: initial conditions

Issues just encountered in the discussion of Z are common to finding solutions of many of the EOM. For example, consider ψ_ν. Although it is not an independent field ψ_ν is a very useful adjunct quantity whose EOM,

$$\begin{aligned} \partial^2 \psi_\nu \ &- \ \partial_\nu \partial \cdot \psi + m_Z^2(t, \vec{r}) \psi_\nu \\ &= \ -\frac{g}{g'} \frac{\sqrt{g^2 + g'^2}}{2} j_\nu^{em} \ , \end{aligned} \tag{30}$$

follows from Eq. (21) by replacing Z by its expression in terms of ψ and Θ using Eq. (10)

$$Z_\nu = \frac{2}{\sqrt{g^2 + g'^2}} (\partial_\nu \Theta - \psi_\nu) \ , \tag{31}$$

and noting that the terms involving Θ cancel.

The EOM in Eq. (30) may be rewritten as

$$\begin{aligned} \partial^2 Z_\nu \ &+ \ \partial_\nu \frac{Z \cdot \partial m_Z^2}{m_Z^2} - \rho(t, r)^2 \sqrt{g^2 + g'^2} \psi_\nu(x) \\ &= \ \frac{g}{g'} \frac{\sqrt{g^2 + g'^2}}{2} 4\pi j_\nu^{em} \ , \end{aligned} \tag{32}$$

which is equivalent to Eq. (30) provided the corresponding auxiliary condition

$$\chi(x) = 0 \ , \tag{33}$$

where

$$\chi(x) \equiv \frac{\partial^\mu (m_Z^2 \psi_\mu)}{m_Z^2} = \partial \cdot \psi + \frac{\psi \cdot \partial m_Z^2}{m_Z^2} \ , \tag{34}$$

is maintained for all x. As in the case of Z, this requires choosing the initial configuration of $\psi_\nu(x)$, at time $t = t_0$, to satisfy

$$\chi(t_0, \vec{r}) = 0 \tag{35}$$

and

$$\frac{\partial \chi(t_0, \vec{r})}{\partial t} = 0 \ . \tag{36}$$

Because constraints such as those in Eq. (28,29) and Eq. (35,36) apply at the initial time t_0, they make it natural to distinguish two categories of initial conditions. The first category, which we will refer to as boundary conditions, consists of the initial fields that may be chosen freely. The second consists of the set determined by the constraints at $t = t_0$, which we will refer to as the constrained initial conditions.

3 The dynamical framework and gentle collisions

The Abelian Higgs model has been of interest as a prototype for the generation of magnetic fields in the early universe in collisions of bubbles during a first-order EW phase transition [16, 17, 18]. The Lagrangian of the Abelian Higgs model describes a complex scalar field coupled to the *em* field A^{em}_μ. It corresponds to the Lagrangian Eq. (1) in the Abelian sector, formed by eliminating the W fields, identifying A^{em}_μ with the field B_μ, and relating the electric charge e to coupling parameter g' as $e = g'/2$.

Results obtained in the Abelian Higgs model follow the original analysis of Kibble and Vilenkin [16], who expressed the EOM in coordinates appropriate to O(1,2) symmetry. With this choice of coordinates a point located a distance z from the origin along the axis of collision at a distance $r_\perp = \sqrt{x^2 + y^2}$ from this axis at time t is given by (τ, z), where

$$\tau = \sqrt{t^2 - r_\perp^2} \ . \tag{37}$$

They obtained $O(1,2)$ symmetric solutions with jump boundary conditions applied on a space-time surface defined by the time of collision $t = t_c$, assuming that for $t > t_c$ the Higgs field (and therefore the masses of the W and Z in the MSSM) is constant at ρ_0 within the region of bubble overlap. They demonstrated that when the phase of the Higgs fields is initially different within each bubble an axial magnetic field forms as the bubbles merge and that this field has the structure of an expanding ring of radius $b(t)$ encircling the overlap region of the colliding bubbles.

The main difference of our current work compared to the Abelian Higgs model is that we use the standard EW theory, rather than a model, and in our MSSM EW theory the source of the current is the charged gauge fields. The fact that the *em* current is given in terms of the W field in Eq. (18) suggests that to find the magnetic field one begins by solving the EOM for the W fields. This was the approach taken in Refs. [25, 27], where the initial $W^{\pm}$ fields in the bubble were argued to form a condensate with a coherence reflecting that of the Higgs field of the bubble.

3.1 Thermal Erasure

We characterize the thermal history of the early universe by a (time-dependent) temperature $T(t)$ and a corresponding equilibrium partition function. It is assumed that this partition function gives the probability that any of the gauge fields occurs at $t = t_0$ and is sufficient for thermal erasure of any coherence at the time of the phase transition. In the case of complete erasure all gauge fields and their time derivatives would, on average, initially vanish, *eg*,

$$\begin{aligned} W^i(t_0,\vec{r}) &= \left\langle W^i(t_0,\vec{r})\right\rangle = 0 \\ \partial W^i(t_0,\vec{r})/\partial t &= \left\langle \partial W^i(t_0,\vec{r})/\partial t\right\rangle = 0 \ . \end{aligned} \tag{38}$$

Of course, as time progresses nonlinearities in the theory may result in correlations that do not average out. Note in particular that the EOM for W is nonlinear and thus $\langle W^i(x)\rangle$ and $\langle j^{em}_\nu(x)\rangle$ calculated from W may develop finite expectation values as time increases.

Many field configurations arise in a thermal gas. Thus, to calculate any quantity, say the magnetic field $\vec{B}$, one should solve the EOM for each possible initial configuration, leading to an ensemble of solutions $W^{\pm}_\nu(t,\vec{r})$ and $Z_\nu(t,\vec{r})$ and their corresponding $\vec{B}$. To obtain the magnetic field, one would then average $\vec{B}$, $\vec{B}(x) \rightarrow < \vec{B}(x) >$, over the ensemble of configurations.

3.2 Gentle collisions

In Ref. [25] bubble collisions in the Coleman model [19] were studied by obtaining numerical solutions to the equation

$$\partial^2\phi(x) + \phi(x)\frac{\partial V}{\partial\phi^2} = 0 \ , \tag{39}$$

with the Higgs field a real scalar field ϕ ($\Theta = 0$), and an effective potential $V(\phi)$, which specifies the dependence of the energy of vacuum on ϕ. Fluctuations in $\rho(x)$ (see Eq(7)) are defined as $\rho(x) - \rho_0$, with ρ_0 a central value. It was found that these fluctuations became very small as the bubbles overlapped.

For the present paper, as well as Ref. [25], it is important that in the collision the expansion of $\rho(x)$, obtained as the solution of Eq. (8) have fluctuations which remain small compared to ρ_0. We refer to a collision of this character as a "gentle" collision. Thus for gentle collisions the solution $\rho(x)$ of Eq. (8) can be approximated by a simple function

$\bar{\rho}(x) \approx \rho(x)$. As in Ref. [25], an expansion in $a \propto \rho(x) - \bar{\rho}(x)$ shows that to leading order the equation for $\rho(x)$ becomes independent of the other fields allowing $\bar{\rho}(x)$ to be fixed in advance.

As in our earlier work based on the MSSM we achieve considerable simplification by applying the theory to gentle collisions following Kibble and Vilenkin. It follows that for the purpose of calculating the *em* current in the NAHM, and hence the corresponding magnetic field produced in bubble collisions, the EOM for the magnitude and phase of the Higgs field are the only ones required. The solutions are of course coupled to all the other fields through the scalar field, but for gentle collisions, as noted, the magnitude of the Higgs field decouples from these fields. As a result, in the NAHM for gentle collisions there is only one PDE to be solved in order to obtain the *em* current, that for the phase of the Higgs field in the collision.

3.3 The Non-abelian Higgs model

Although calculating the *em* current directly in terms of the W fields was relatively straightforward with the condensate boundary conditions of Refs. [25, 27], invoking thermal erasure would require obtaining these fields as solutions of the nonlinear equations for an ensemble of thermal boundary conditions and then averaging the current over this set of solutions. This would be a formidable undertaking.

Fortunately, thermal erasure admits an alternative to determining the *em* current that is actually quite simple and straightforward, the NAHM approach we take in this paper. The EOM leading to the calculation of the magnetic seed fields are obtained in Sect. 3.4.

3.4 EOM in the NAHM

In Sect. 3.4.1 we will obtain an expression for the *em* current of the NAHM defined entirely in terms of the gradient of the phase of the Higgs field (and its magnitude) for gentle collisions. Maxwell's equation for determining the magnetic field in bubble collisions including the effect of finite conductivity is also given here. The simple connection of the *em* current to the gradient of the Higgs phase arises from the linear, homogeneous character of the EOM for Z_ν, in contrast to the EOM for W, and thermal erasure of $< Z >= 0$ at $t = t_0$.

In Sect. 3.4.2 we will derive a PDE for this phase from current conservation and in Sect. 3.4.3 we discuss the significance of Z_ν for justifying the NAHM and derive a PDE for determining it. Both are linear and homogeneous.

3.4.1 Electromagnetic current and Maxwell's equation in the NAHM

Applying the ensemble averaging principle to Eq. (21) we find, at $t = t_0$,

$$4\pi \langle j^{em}_\nu(x) \rangle = -\frac{g'}{g}\sqrt{g^2 + g'^2}\rho(t,\vec{r})^2 \times \partial_\nu \Theta(x) \tag{40}$$

by setting $\langle Z_\nu(x)\rangle = 0$ in accord with our assumption that the gauge fields initially occupy thermal modes. The essence of the NAHM is that Eq. (40) should hold, not only at $t = t_0$, but also throughout the duration of the phase transition. This will clearly occur when $\langle Z_\nu(x)\rangle = 0$ for times $t > t_0$, a result that we show in Sect. 4.4 follows from the assumption of thermal erasure as given in Eq. (38). This result establishes that under the assumptions of thermal erasure, the NAHM provides a prediction for the magnetic seed fields following from our EOM that avoids having to determine the corresponding W fields. a result we show in Sect. 4.4. To the extent that this can hold, the solution of the EOM for the W fields is unnecessary determining the magnetic seed fields.

In what follows, we streamline the notation by understanding that $j_\nu^{em}(x)$ is short-hand for $\langle j^{em}(x)\rangle$, $Z_\nu(x)$ short-hand for $\langle Z_\nu(x)\rangle$, and $\psi_\nu(x)$ short-hand for $\langle \psi_\nu(x)\rangle$.

From Eq. (17), the expression for the *em* current averaged over the thermal partition function for the initial gauge field configurations, Eq. (40) leads to the Maxwell equation

$$\begin{aligned}\partial^2 A_\nu^{em} \quad &- \quad \partial^\gamma \partial \cdot A^{em} = 4\pi j_\nu^{em}(x) \\ &= \quad -\frac{g'}{g}\sqrt{g^2+g'^2}\rho^2(t,\vec{r})^2\partial_\nu\Theta(x) \ .\end{aligned} \tag{41}$$

To find Maxwell's equation for the magnetic field $\vec{B}$,

$$\vec{B} \quad = \quad \vec{\nabla}\times\vec{A}^{em} \ , \tag{42}$$

arising from the charged gauge bosons, we multiply Eq. (17) by $\epsilon_{ijk}\partial_j$, obtaining

$$\begin{aligned}\epsilon_{ijk}\partial_j\partial^2 A_k^{em} \quad &- \quad \epsilon_{ijk}\partial_j\partial_k\partial\cdot A^{em} \\ &= \quad 4\pi\epsilon_{ijk}\partial_j j_k^{em} \ ,\end{aligned} \tag{43}$$

Expressing Eq. (42) in components,

$$B_i \quad = \quad \epsilon_{ijk}\partial_j A_k^{em}, \tag{44}$$

we immediately find the basic result,

$$\partial^2\vec{B} \quad = \quad 4\pi\vec{\partial}\times\vec{j}^{em} \ . \tag{45}$$

In addition to the gradients of the Higgs phase, fermions also contribute to the current and have a significant impact on magnetic seed field production. One contribution was discussed recently in Ref. [23] and estimated there for the nucleation phase of the collision. Another occurs through the conductivity of the medium σ. This is taken into account through its associated current $\vec{j}_c(x)$,

$$\vec{j}_c(x) = \sigma\vec{E}(x) \ , \tag{46}$$

where the usual assumption that $\vec{j}_c(x)$ is proportional to the electric field $\vec{E}$ has been made. In the present work we do not consider fermions or the conductivity current, taking $\sigma = 0$. In this case, the quantity $\vec{B}(t,\vec{r})$ obtained by solving Eq. (45) is the expected magnetic seed field.

3.4.2 EOM for the Higgs Phase in the Non-abelian Higgs Model

A PDE for the Higgs phase $\Theta(x)$ follows immediately from current conservation, Eq. (19). Using Eq. (40) along with this, we find

$$\begin{aligned} 0 &= \frac{1}{m_Z^2(t,\vec{r})}\partial \cdot m_Z^2(t,\vec{r})^2 \partial\Theta(x) = 0 \\ &= \partial^2\Theta + (\partial\Theta)\cdot(\partial \ln m_Z^2) \ . \end{aligned} \tag{47}$$

To determine Θ from this, appropriate boundary conditions on Θ and its derivative on some initial space-time surface are required. When the Higgs fields of the colliding bubbles differ in phase before the collision occurs, $\partial_\nu\Theta(x)$ develops a non-zero value within the bubble overlap region after the collision. The *em* current $j_\nu^{em}(x)$ then develops a non-zero value there and magnetic fields will form. We will see explicitly how the *em* currents and associated magnetic fields begin to form in the collision once the bubbles begin to overlap when we solve the EOM in Sect. 5.

Note that the EOM for Θ in the NAHM contains no mass term and therefore differs in an essential way from the Abelian Higgs model. This difference can be traced to the fact that the only vector field available to define the covariant derivative in the Abelian Higgs model is A_ν^{em}, whereas the Non-abelian character of the MSSM means that this vector field becomes, instead, that of W_ν^i. One consequence is that the Z field plays the role of A_ν^{em} in the Abelian Higgs model [25]. Thus, thermal erasure can eliminate Z while preserving the possibility of generating a magnetic field. Thermal erasure is not only a meaningful concept in the MSSM, it is also the underlying reason for the absence of a mass term in the EOM for Θ and the source for the dramatically different time-dependence of Θ that we find in Sect. 5.

3.4.3 EOM for Z in the NAHM

Although not explicitly needed to calculate the magnetic field, Z plays an important role in the formulation of the NAHM. Specifically, in order to express the current in terms of the gradient of the Higgs phase, Eq. (40), requires that $\langle Z_\nu(x)\rangle = 0$. The results presented below form the basis for the argument.

Substituting the definition of ψ, Eq. (10), into Eq. (26), using the expression for the *em* current in the NAHM, Eq. (40), and taking the thermal average, we immediately find

$$\partial^2 Z_\nu + \partial_\nu \frac{Z\cdot\partial m_Z^2}{m_Z^2} + m_Z^2 Z_\nu = 0 \ . \tag{48}$$

The Z is subject to constrained initial conditions in Eqs. (28,29),

$$0 = \partial\cdot Z + Z\cdot\partial \ln m_Z^2 \tag{49}$$

and

$$0 = \frac{\partial}{\partial t}\partial\cdot Z + \frac{\partial}{\partial t} Z\cdot\partial\ln m_Z^2 \ . \tag{50}$$

Thus, the assumption of thermal erasure is consistent with $Z_\nu(x)$ being determined by a *linear, homogeneous* PDE even before invoking the assumption of gentle collisions, discussed in the next section.

We also see that the EOM of the NAHM for Z, Eq. (48), for $\partial_\nu\Theta$, Eq. (47), along with the electromagnetic current defined in Eq. (40), are all consequences of the EOM for Z in Eqs. (22,26) and current conservation, with Z defined in Eq. (6). Thus, obtaining j_ν^{em} from Eq. (40) by solving Eq. (47) is exactly equivalent to obtaining it by solving the EOM determined directly by minimizing the action.

4 The NAHM in O(1,2) Symmetry

In this section, following Kibble and Vilenkin, we express the EOM of the NAHM using the (τ, z) variables appropriate to O(1,2) symmetry. Although wall speeds $v_{wall} < c$ and electrical conductivity both break O(1,2) symmetry, it is still of interest to explore O(1,2) symmetric solutions in the NAHM. This is because of the relative transparency of the analysis and because it facilitates comparison with earlier work in the Abelian Higgs model [16, 17, 18] and the MSSM [24, 25].

The relevant EOM are those for Θ, Z, and $\vec{B}$. In this section we find the general solution of the EOM for a pair of colliding bubbles for jump boundary conditions applied on a space-time surface at fixed $\tau = t_c$, the current corresponding to these solutions, and the corresponding magnetic field by solving Maxwell's equations. Since the EOM are intended to give the fields inside the bubbles, our O(1,2) symmetric solutions are valid only within the region of overlap of the bubbles, that is for $\tau \leq t_c$.

4.1 Calculation of Z of the NAHM in O(1,2) Symmetry

As discussed in Sect. 3.4.3 the Z plays an important role in the formulation of the NAHM. Specifically, the EOM of the NAHM for Θ and Z in Eqs. (47,48) are consistent with the EOM determined by minimizing the action only if the solution of the EOM for Z, Eq. (48), with initial conditions consistent with thermal erasure lead to $\langle Z_\nu(\tau, z)\rangle = 0$ for all τ. Then and only then is it possible to express the *em* current in terms of the gradient of the Higgs phase, Eq. (40) throughout the phase transition. In this section we examine Z with this in mind.

To express the equations of motion, in terms of the (τ, z) coordinates, for the vector fields, for example Z_ν, we take

$$\begin{aligned} Z_\nu(x) &= Z_z(\tau, z), \quad \nu = 3 \\ Z_\nu(x) &= x_\nu Z(\tau, z), \quad \nu = (0, 1, 2)\ , \end{aligned} \tag{51}$$

with $Z_z = -Z^z$. In the remainder of the paper, we will find it convenient to use α to denote the Lorentz index for the values $\nu = (0, 1, 2)$. Also note that initial conditions for O(1,2) symmetric solutions are specified on a space-time surface defined by $\tau = t_c$.

Thus, $\partial^2 Z_\nu(\tau, z)$ becomes for $\nu = 3$

$$\partial^2 Z_z(\tau, z) = (\frac{\partial^2}{\partial\tau^2} + \frac{2}{\tau}\frac{\partial}{\partial\tau} - \frac{\partial^2}{\partial z^2})Z_z(x) \tag{52}$$

and

$$\partial^2 Z_\alpha(\tau, z) = x_\alpha(\frac{\partial^2}{\partial\tau^2} + \frac{4}{\tau}\frac{\partial}{\partial\tau} - \frac{\partial^2}{\partial z^2})Z_\alpha(x) \ . \tag{53}$$

Also,

$$\partial \cdot Z(x) \equiv \frac{\partial Z^z}{\partial z} + 3Z + \tau\frac{\partial Z}{\partial\tau} \ . \tag{54}$$

The EOM in Eqs. (48) for Z are then as follows,

$$0 = (\frac{\partial^2}{\partial\tau^2} + \frac{2}{\tau}\frac{\partial}{\partial\tau} - \frac{\partial^2}{\partial z^2} + m_Z^2)Z \ , \tag{55}$$

and

$$0 = (\frac{\partial^2}{\partial\tau^2} + \frac{4}{\tau}\frac{\partial}{\partial\tau} - \frac{\partial^2}{\partial z^2} + m_Z^2)Z_z \ . \tag{56}$$

To find the solution of these EOM, initial conditions are needed for Z and its time derivatives. Denoting by $Z_\nu(t_c, z)$ the profile of the Z_ν fields on the space-time surface when the collision occurs, the initial values for $Z_\nu(t_c, z)$ is determined from the partition function of a thermal gas at $\tau = t_c$. Since its distribution is incoherent and essentially random, consistent with a thermal distribution, we find for the initial conditions under the conditions of complete thermal erasure

$$\begin{aligned} \langle Z_z(t_c, z)\rangle &= 0 \\ \left\langle \frac{\partial Z_z(t_c, z)}{\partial\tau} \right\rangle &= 0 \\ \langle Z(t_c, z)\rangle &= 0 \\ \left\langle \frac{\partial Z(t_c, z)}{\partial\tau} \right\rangle &= 0 \ . \end{aligned} \tag{57}$$

It is easily shown that these are consistent the auxiliary condition of Eq. (23), which requires that Eq. (49) hold,

$$\begin{aligned} \chi_Z(t_c, z) &= \frac{\partial Z^z}{\partial z} + 3Z + t_c\frac{\partial Z}{\partial\tau} \\ &= 0 \ , \end{aligned} \tag{58}$$

and that Eq. (50) hold,

$$\begin{aligned}\frac{\partial \chi_Z(t_c,z)}{\partial \tau} &= \frac{\partial^2}{\partial z \partial \tau} Z^z + 4\frac{\partial}{\partial \tau} Z + t_c \frac{\partial^2}{\partial \tau^2} Z \\ &= \frac{\partial^2}{\partial z \partial \tau} Z^z + 4\frac{\partial}{\partial \tau} Z \\ &- t_c(\frac{4}{t_c}\frac{\partial}{\partial \tau} Z - \frac{\partial^2}{\partial z^2} Z + m_Z^2 Z) \\ &= 0 \ , \end{aligned} \tag{59}$$

where we have used the EOM of Eq. (55). As before the derivatives do not act on the θ functions. Equations (58,59) are clearly satisfied by Eq. (57).

With Z satisfying the linear, homogeneous EOM of Eq. (48) (Eqs. (55,56)in (τ, z) coordinates), and with the result that Z_ν and its τ derivatives vanish at $\tau = t_c$ under conditions of complete thermal erasure, it follows that the solution of these EOM, $\langle Z_\nu(\tau, z)\rangle$, must vanish for all τ. Consequently, the NAHM is established and we may proceed to the examination of its consequences for production of magnetic seed fields taking the *em* current to be related to gradients of the phase of the Higgs field as given in Eq. (40).

4.2 Calculation of Θ of the NAHM in O(1,2) Symmetry

The PDE for Θ in Eq. (47), in terms of which j^ϕ is defined, becomes, in (τ, z) coordinates

$$(\frac{\partial^2}{\partial \tau} + \frac{2}{\tau}\frac{\partial}{\partial \tau} - \frac{\partial^2}{\partial z^2})\Theta(t, \vec{r}) \ = \ 0 \ . \tag{60}$$

As indicated, the Non-abelian Higgs mechanism assumes that initially, at $\tau = t_c$, the Higgs phase for a single bubble is constant throughout the bubble, but this phase differs in the two colliding bubbles, as in the Abelian Higgs Model. Likewise, the boundary condition on the τ derivative is the same as it is in the Abelian Higgs model. Accordingly, the boundary condition on Θ is

$$\begin{aligned}\Theta(\tau = t_c, z) &= \Theta_0 \ \epsilon(z) \\ \frac{\partial}{\partial \tau}\Theta(\tau = t_c, z) &= 0 \ , \end{aligned} \tag{61}$$

where $\epsilon(z)$ is the sign of z and $0 < \Theta_0 < \pi/2$ is the initial Higgs phase in one of the colliding bubbles. Of course for $\tau > t_c$ the actual variation of the phase Θ in the bubble is determined by its EOM, Eq. (60).

The solution of Eqs. (60) is found by standard methods. Expressing $\Theta(x)$ as a Fourier transform in z, Eq. (60) gives an ordinary differential equation for the τ-dependence, yielding

$$\begin{aligned}\Theta(\tau, z) &= \frac{1}{\tau}\sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} \frac{e^{ikz}}{\sqrt{\omega_k}}(c_k \sin \omega_k \tau \\ &+ d_k \cos \omega_k \tau) dk \ , \end{aligned} \tag{62}$$

where $\omega_k = \sqrt{k^2 + m^2}$ where it is understood that $m \to 0$ at the end of the calculation.

The coefficients a_k and b_k in Eq. (62) are fixed by the boundary conditions on Θ in Eq. (61). Performing the sum over the modes k, we then obtain

$$\begin{aligned}
\Theta(\tau, z) &= \frac{\Theta_0 t_c}{\tau}\epsilon(z)[\theta(T-|z|)\int_0^{|z|} \\
&(\frac{1}{t_c} + \frac{\partial}{\partial T})J_0(m\sqrt{T^2 - z'^2})dz' \\
&+ \theta(|z|-T)\frac{1}{m}(\frac{1}{t_c} + \frac{\partial}{\partial T})\sin mT] \\
&\to \frac{\Theta_0 t_c}{\tau}\epsilon(z)[\theta(T-|z|)\int_0^{|z|}\frac{dz'}{t_c} \\
&+ \theta(|z|-T)(1+\frac{T}{t_c})] \\
&= \frac{\Theta_0}{\tau}\theta(T-|z|)z + \Theta_0\epsilon(z)\theta(|z|-T) \ , \qquad (63)
\end{aligned}$$

where $T = \tau - t_c$.

4.3 Calculation of $\vec{B}$ in NAHM with O(1,2) Symmetry

We first examine the calculation of the *em* current taking it to have the form

$$j_\nu^{em}(\tau, z)) = (j_z(\tau, z), x_\alpha j(\tau, z)) \ . \qquad (64)$$

in (τ, z) coordinates where the *em* current is given in the NAHM in Eq. (40). Thus, with O(1,2) symmetry we find the current in Eq. (40) expressed in (τ, z) coordinates becomes

$$\begin{aligned}
4\pi j_z &= -\frac{g'}{g}\sqrt{g^2 + g'^2}\rho_0^2\frac{\partial}{\partial z}\Theta(\tau, z) \\
&= -\frac{g'}{g}\sqrt{g^2 + g'^2}\rho_0^2\frac{\Theta_0}{\tau}\theta(T-|z|) \qquad (65)
\end{aligned}$$

and

$$\begin{aligned}
4\pi j &= -\frac{g'}{g}\sqrt{g^2 + g'^2}\rho_0^2\frac{\partial}{\partial \tau}\Theta(\tau, z) \\
&= \frac{g'}{g}\sqrt{g^2 + g'^2}\rho_0^2\frac{\Theta_0}{\tau^2}\theta(T-|z|)z \qquad (66)
\end{aligned}$$

where we have used Eq. (63) to obtain

$$\frac{\partial}{\partial \tau}\Theta(\tau, z) = -\frac{\Theta_0}{\tau^2}\theta(T-|z|)z \qquad (67)$$

and

$$\frac{\partial}{\partial z}\Theta(\tau,z) \;=\; \frac{\Theta_0}{\tau}\theta(T-|z|)\ . \tag{68}$$

When evaluating the partial derivatives in Eq. (65,66), and using the expressions in Eq. (63), we do not let the derivatives act on the step functions. We note that ignoring the surface derivatives is quite consistent with current conservation and should therefore lead to a valid expression for the magnetic field throughout the bubble interior.

To obtain the magnetic field we need to solve Maxwell's equation, Eq. (41), with the current given by Eq.(40), and with the Θ fields appearing in the current given by the solutions of Eqs. (60). The solution of Maxwell's equation is completely determined once we specify the boundary conditions on the A^{em} field with the Θ field given above.

Because the electromagnetic current has the form given in Eq. (64), the electromagnetic field has this form also,

$$A^{em}_\nu(\tau,z) = (a_z(\tau,z), x_\alpha a(\tau,z))\ . \tag{69}$$

Maxwell's equation becomes quite simple in the (τ,z) with the axial gauge, $a_z=0$, namely,

$$-\frac{\partial^2}{\partial z^2}a(\tau,z) = 4\pi j(\tau,z)\ . \tag{70}$$

Applying the boundary conditions, namely $a(\tau_0,z)=0$ and $\partial_z a(\tau=0,z)$, we find

$$a(\tau,z) = -4\pi\int_{-\infty}^{z}dz'\int_{-\infty}^{z'}j(\tau,z'')dz''\ . \tag{71}$$

The magnetic field, $\vec{B}=\vec{\partial}\times\vec{A}^{em}$, where $\vec{A}^{em}=(xa(\tau,z),ya(\tau,z),0)$ with $a(\tau,z)$ given in Eq. (71) is

$$\begin{aligned} B^z &= 0 \\ B^x &= -4\pi y\int_{-\infty}^{z}j(\tau,z')dz' \\ B^y &= 4\pi x\int_{-\infty}^{z}j(\tau,z')dz'\ , \end{aligned} \tag{72}$$

or

$$\vec{B} \;=\; \frac{(-y,x,0)}{r_\perp}B^\phi \tag{73}$$

with

$$B^\phi \;=\; 4\pi r_\perp\int_{-\infty}^{z}j(\tau,z')dz'\ . \tag{74}$$

Thus, the magnetic field generated in the bubble collision lies entirely in the azimuthal plane, with x and y components only, and encircles the z-axis, just as in the Abelian Higgs model and our earlier work [25].

To obtain the magnetic field we evaluate Eq. (74) with the current given by Eq. (66),

$$\begin{aligned}
B^{\phi} &= r_{\perp}\frac{g'}{g}\sqrt{g^2+g'^2}\rho_0^2\frac{\Theta_0}{\tau^2}\int_{-\infty}^{z}\theta(T-|z'|)z'\,dz' \\
&= r_{\perp}\frac{g'}{g}\sqrt{g^2+g'^2}\rho_0^2\frac{\Theta_0}{\tau^2}\theta(T-|z|)\int_{-T}^{z}z'\,dz' \\
&+ \frac{g'}{g}\sqrt{g^2+g'^2}\rho_0^2\frac{\Theta_0}{\tau^2}\theta(|z|-T)\int_{-T}^{T}z'\,dz' \\
&= r_{\perp}\frac{g'}{g}\sqrt{g^2+g'^2}\rho_0^2\frac{\Theta_0}{\tau^2}\theta(T-|z|) \\
&\times \frac{(|z|-T)^2}{2}\ .
\end{aligned} \tag{75}$$

4.4 The B, W, and ψ fields in the NAHM in O(1,2) Symmetry

Just as for Z, solutions of the EOM for the B, W^3, and ψ fields in the NAHM are not needed explicitly to calculate the magnetic seed fields. However, these solutions are easily obtained once the EOM for the NAHM are solved, for completeness we show how to do this in this section.

With $Z_\nu = 0$, Eq. (6) shows that the solutions of the EOM for B and W^3 must be proportional, specifically,

$$B_\nu = g' W^3_\nu / g\ , \tag{76}$$

with

$$A^{em}_\nu = \frac{\sqrt{g^2+g'^2}}{g'^2} W^3_\nu\ . \tag{77}$$

Thus, both W^3_ν and B_ν are determined once the *em* field is known, and it is unnecessary to solve their EOM to find them.

Accordingly, if we take $A^{em}_\nu = 0$ in an isolated bubble, the condition that the electric and magnetic fields vanish in the Lorentz gauge, this would imply $W^3_\nu = 0$. However, in colliding bubbles it is fortunately the case that $Z_\nu = 0$ does not rule out $A^{em}_\nu \neq 0$. The vanishing of the Z_ν field in this case rather implies that B_ν and W^3_ν must both be non-zero and proportional to A_ν. These considerations are completely consistent with the EOM for W^3, Eq. (12), and Maxwell's equation, Eq. (41).

The solution of the EOM for ψ, Eq. (30), can be determined directly from the definition of $\psi_\nu(t,\vec{r})$ in Eq. (10) and the solutions of the EOM for Θ and Z, Eqs. (47,48), respectively.

Thus, to obtain the solution of Eq. (30) it is not necessary to actually solve its PDE either. Since $\langle Z_\nu(t,\vec{r})\rangle = 0$ we find the simple result

$$\psi_\nu = \partial_\nu \Theta \; . \tag{78}$$

It is more difficult to solve the EOM for $W^\pm$ with thermal erasure because of its non-linear character and the number of (thermal) initial conditions for which it must be solved. Fortunately this solution is not needed for applying the NAHM to calculate the magnetic seed fields.

However, it may still be of interest to find a formulation in which the *em* current is expressed explicitly in terms of the charged gauge bosons. One way to do this would be to develop gentle collisions more thoroughly along the lines of Ref. [25, 27]. A critical element missing at this time is a means to find initial conditions of the $W^\pm$ fields consistent with full thermal erasure. A more direct approach would be to solve the full nonlinear theory. This is the approach taken in Ref. [23] with the "I-spin *ansatz*". Solutions have however been obtained only for the nucleation stage of the collision

5 Numerical Results

To facilitate the comparison with the Abelian Higgs model of Ref. [17] we assume that bubbles nucleate at points on the z-axis at $z = \pm R$ at time $t = 0$, that they expand from the point of nucleation with the radius of the bubble $R(t) = ct$ at the speed of light, and that they collide at time $t = t_c = R$, as in Refs. [17]. Results may be expressed in terms of the time after collision, δt, so that a point $(r_\perp, z)$ at $t = t_c + \delta t$ has the O(1,2) coordinates $(\tau, z) = (\sqrt{(t_c+\delta t)^2 - r_\perp^2}, z)$.

We will first show the magnetic field B^ϕ, the value of the azimuthal field, assuming a non-conducting medium $\sigma = 0$ and a terminal wall speed $v_{wall} = 1$, with distance and time expressed in units of $1/m_W$ and the magnetic field in units of m_W^2. Although the assumptions of $\sigma = 0$ and $v_{wall} = 1$ are unrealistic for the actual EWPT, the corrections are well-understood and thoroughly studied in the Abelian theory [16, 17, 18].

To compare to the Abelian Higgs model of Ref. [17] we take $t_c = R = 10$ with $\delta t = 5$, 10, 15, and 20, and with $\Theta_0 = 1$. We will note some striking differences that we link to the behavior of the Higgs phase in the Non-abelian character of our theory.

In the (τ, z) variables appropriate to O(1,2) symmetry, the point of collision on the z axis at $z = 0$ and $r_\perp = 0$ has the coordinates $(\tau, z) = (\sqrt{t^2 - b(t)^2}, 0)$ where $b(t)$ is the radius of the circle intersection in the plane of symmetry, the x-y plane at $z = 0$. Since the bubbles collide on the z-axis with radius $R_c = R$, $b(t)$ grows with time as $b(t) = \sqrt{R(t)^2 - R_c^2} = \sqrt{t^2 - R_c^2}$, or equivalently to $\tau = t_c$. The collision time t_c is determined by the condition $R_c = R(t_c)$ (or equivalently $b(t_c) = 0$) giving $t_c - t_n = \sqrt{R_c^2 - R_n^2}$. The fact that the speed of expansion can be superluminal is not in contradiction with relativity, as discussed in Ref. [18]

Figure 1 shows the region of bubble overlap in the x-z plane at time $\delta t = 20$ for this geometry. The circle of bubble intersection in the x-y plane is shown in Fig. 2. The so-

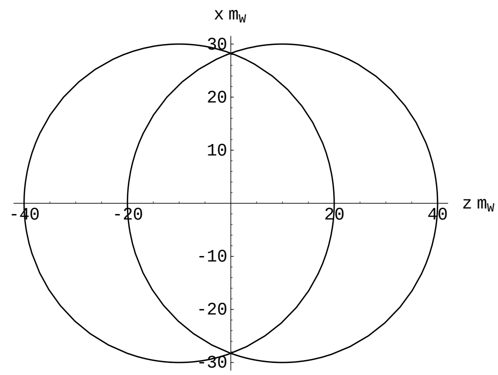

Figure 1: Bubble configuration in the $x - z$ plane at time $t_c + 20$ after collision.

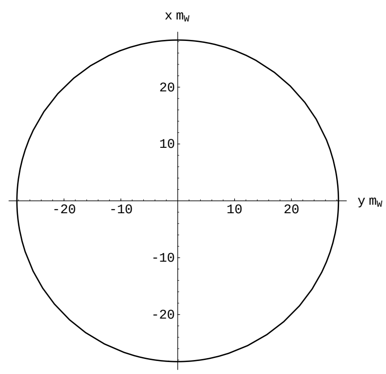

Figure 2: Bubble configuration in the $x - y$ plane at time $t_c + 20$ after collision.

lutions of the EOM are meaningful in O(1,2) symmetry only inside the region of overlap, corresponding to $T > 0$, as discussed above.

The B^ϕ field in the x-y plane at this time, calculated using Eq. (75), is shown in Fig. 3. This clearly shows that B^ϕ encircles the axis of collision.

Additionally, B^ϕ is narrowly confined longitudinally near $z = 0$ in the x-y plane as shown next in Fig. 4. This shows that B^ϕ is largest at intersection of the bubbles at $z = 0$, and that it falls off rapidly with z away from this point. The magnetic field is thus concentrated in a narrow ring encircling the axis of collision on the circle of intersection of the bubbles.

Figure 5 shows our calculated field B^ϕ at $\delta t = 5$, 10, 15, and 20. This is to be compared to the result shown in Fig. 6 calculated in the Abelian Higgs model with distance and time again expressed in units of inverse mass of the vector boson and the magnetic field in this mass squared. A convenient expression for the magnetic field is given in Eq. (29) of Ref. [18].

Thus, evaluating B^ϕ for parameters corresponding to numerical results of Ref. [17], we find that the peak magnetic fields in the MSSM for the NAHM as given in Eq. (75) occur along the axis of symmetry at the point of intersection of the bubbles, just as in the Abelian Higgs model. However, our fields are about a factor of 2 larger and do not change sign in contrast to that in the Abelian Higgs model, which displays oscillations. Additionally, it

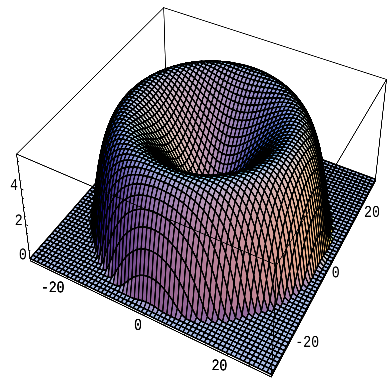

Figure 3: Magnetic field in the $x - y$ plane at time $t_c + 20$ after collision.

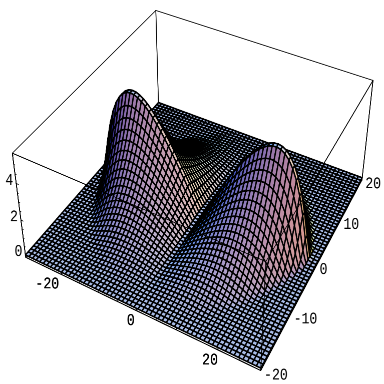

Figure 4: Magnetic field in the $x - z$ plane at time $t_c + 20$ after collision.

is clear that the magnetic field extends more deeply into the collision region in our theory, which makes the volume-averaged azimuthal field in the ring even larger than it is in the Abelian Higgs model.

Note that the magnetic field in the Abelian Higgs model drops to zero suddenly at the outer boundary of the expanding overlap region. This unphysical feature was discussed in Ref. [18] and was traced to the abrupt change in the boundary condition at the point of collision in that model.

Next we compare the magnetic field along the z direction at fixed radial distance ρ from the collision axis. Comparing the results for our theory and that of the Abelian Higgs model shown in Fig. 7 and Fig. 8, respectively, we see that near the center of the overlap region of the collision at $z = x = y = 0$ the magnetic field is quite small in both models. Because the field is small and sensitive to the distance off axis, the most significant difference is the lack of oscillations in the NAHM. The Abelian Higgs model again displays strong longitudinal oscillations in B^{ϕ}, particularly when we look deep within the overlap region.

In summary, from the comparison of Figs. 5,6 and Figs. 7,6 we see that the magnitude of the magnetic field in the present model is larger and smoother than the one obtained in the Abelian Higgs model. That the two should differ is not unexpected because the description

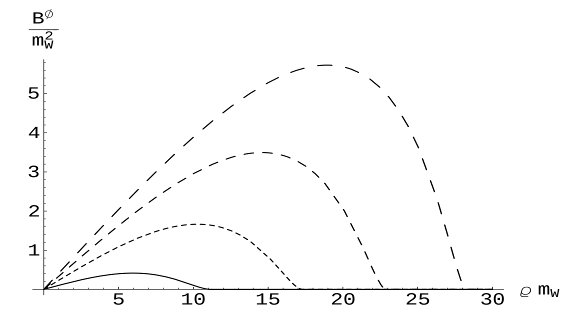

Figure 5: Magnitude of the azimuthal magnetic field calculated in the theory of the present paper. The field is shown as a function of distance $\rho = r_\perp$ from the axis of collision in the symmetry plane at time $\delta t = 5,\ 10,\ 15$, and 20.

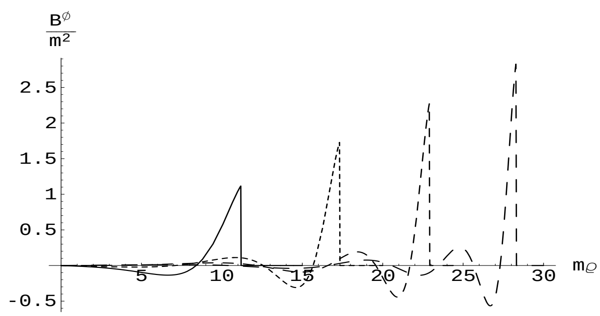

Figure 6: Magnitude of the azimuthal magnetic field calculated in the Abelian Higgs model. The field is shown as a function of distance $\rho = r_\perp$ from the axis of collision in the symmetry plane at time $\delta t = 5,\ 10,\ 15$, and 20.

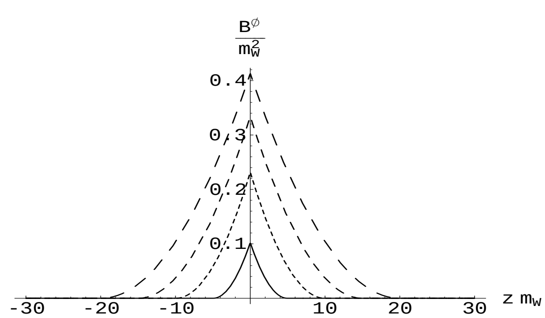

Figure 7: Magnitude of the azimuthal magnetic field calculated in the theory of the present paper. The field is shown as a function of distance z along the axis of collision at a distance $\rho = 1$ from the axis of collision for times $\delta t = 5,\ 10,\ 15$, and 20.

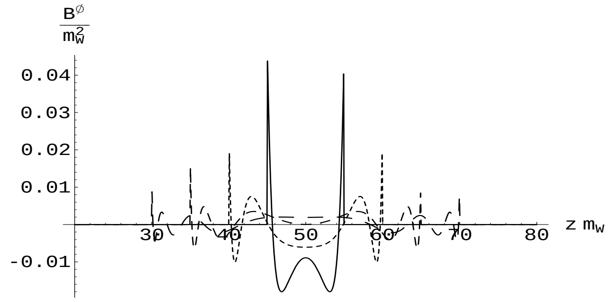

Figure 8: Magnitude of the azimuthal magnetic field calculated in the Abelian Higgs model. The field is shown as a function of distance z along the axis of collision at a distance $\rho = 1$ from the axis of collision for times $\delta t = 5,\ 10,\ 15$, and 20.

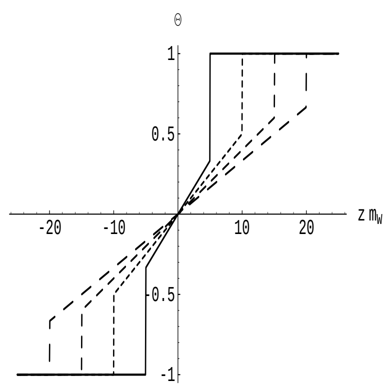

Figure 9: Higgs phase calculated in the theory of the present paper. The field is shown as a function of distance z along the axis of collision at time $\delta t = 5,\ 10,\ 15$, and 20.

of the physics is quite different in the two cases, with the source of the *em* current in the present model being charged gauge bosons of the MSSM. One consequence of the gauge bosons is the time dependence of the Higgs phase. We show the evolution of the Higgs phase in Fig. 9. As noted in Sect. 3.4.2, differences arise because a distinction is made between the *em* field and the Z field in the MSSM. Consequently, the phase in Fig. 9 approaches a constant independent of time outside the region of bubble overlap.

The differences, particularly the difference in scale, presumably are significant for the evolution of the magnetic field to the present era. Differences will of course be mitigated by the fact that the finite conductivity σ damps the magnetic field in the interior of the ring. Additionally, before we can make an estimate of present day magnetic fields, the effects of the speed of the bubble wall has to be calculated. These are most naturally calculated in an O(3) s ymmetric formulation [27].

The quantity we need to obtain from our calculation for determining the effectiveness of magnetic field creation during the electroweak phase transition seeding the galactic and extragalactic magnetic fields we see today is the integrated azimuthal magnetic field strength

$B(R)$ within the ring created within the colliding bubbles. In the Abelian Higgs model, the value of $B(R)$ is given by Eq. (24) of Ref. [17] as a function of v_{wall}, the radius of the bubbles R at the time of collision $t = t_c$, and the radius ρ_{ring} at the time of completion of the phase transition. This result depends only on B^ϕ in the surface of the because the conductivity damps out the magnetic field in the interior of the bubble but not at the outer surface of the ring where it is the largest. However, the finite wall speed does damp the magnetic field at the surface as shown in Eq. (24) of Ref. [17]. Because this damping is a kinematic effect, it scales with the value of the magnetic field at the surface of the ring.

Hence, from knowledge of the value of the magnetic field in the surface of the ring we can obtain the appropriate $B(R)$ for our theory by scaling it relative to the value of the magnetic field in the ring in the Abelian Higgs model. It is clear from Fig. 5, Fig. 6, Fig. 7, and Fig. 8 that this scaling indicates our seed fields would lead to a larger magnetic field than the estimate in Eq. (30) of Ref. [17] for the galactic dynamo. However, the helicity must be known for a complete calculation [5, 6]

6 Summary

We have developed a "Non-abelian Higgs model", our basic MSSM EW theory for "gentle collisions", taking advantage of the fact that the EOM for the Z field depends on the same current that determines the magnetic field. Under the conditions of complete thermal erasure this leads to a simple, although indirect, expression for the *em* current of charged gauge bosons through the gradient of the Higgs phase similar to that of the Abelian Higgs model in bubble collisions. As such, the NAHM applies in a different regime than the theory presented in Ref. [25] in which solutions of EOM for $W^\pm$ that evolved from condensate boundary conditions for gentle collisions drove the production of magnetic fields. It was shown that for gentle collisions the NAHM is consistent and leads to magnetic fields comparable to those found in the Abelian Higgs model. These results reinforce the hope that the EWPT is a promising source for production of seed for large-scale galactic and extra-galactic fields observed today, since the scale and magnitude of the magnetic fields is larger than those used previously in evolutionary models.

We would like to estimate the likelihood that present day magnetic fields could have been seeded by the fields generated during the EWPT that we calculate. To do this, however, would require a calculation of the helicity density h_r. Estimates of h_r thus becomes an important subject for future work.

Acknowledgments

The authors gratefully acknowledge Los Alamos National Laboratory for its hospitality and partial support of the PhD research of T. Stevens, upon which this work is based. MBJ and EMH thank the Department of Energy for partial support.

References

[1] D. Grasso and H.R. Rubinstein, Phys. Rep. **348** (2001) 163.

[2] L. S. Kisslinger, Phys. Rev. **D68** (2003) 043516

[3] L. S. Kisslinger, S. Walawalkar, and M. B. Johnson, Phys. Rev. **D71** (2005) 065017

[4] D. T. Son, Phys. Rev. **D59** (1999) 063008

[5] R. Banerjee and K. Jedamzik, Phys. Rev. Lett. **91** (2003) 251301; Phys. Rev. **D70** (2004) 123003.

[6] K. Dolag, M. Bartellmann, and A. V. Lesch, Astron. Astrophysics **348** (1999) 351; Astron. Astrophysics. **387** (2002) 383

[7] K. Kajantie, M. Laine, K. Rummukainen and M. Shaposhnikov, Phys. Rev. Lett. **77** (1996) 2887.

[8] J. Rosiek, Phys. Rev. **D41** (1990) 3464. Gives references to earlier work.

[9] M. Dine, R.G. Leigh, P. Huet, A. Linde, and D. Linde, *Phys. Rev. D* **46** (1992) 550.

[10] D. Bodeker, P. John, M. Laine and M.G. Schmidt, Nucl. Phys. **B497** (1997) 387.

[11] M. Laine, Nuc. Phys.**B481** (1996) 43; **B548** (1999) 637.

[12] J.M. Cline and K. Kianulainen, Nuc. Phys. **B482**(1996) 73; J.M. Cline and G.D. Moore, Phys. Rev. Lett. **81** (1998) 3315.

[13] M. Losada, Nucl. Phys. **B537** (1999) 3.

[14] V. Cirigliano, S. Profumo, and M. J. Ramsey-Musolf, hep-ph/0603246.

[15] K. Enqvist, J. Ignatius, K. Kajantie, and K. Rummukainen, Phys. Rev. **D45** (1992) 3415.

[16] T.W.B. Kibble and A. Vilenkin, Phys. Rev. **D52** (1995) 679.

[17] J. Ahonen and K. Enqvist, Phys. Rev. **D57** (1998) 664.

[18] E.J. Copeland, P.M. Saffin and O. Tornkvist, Phys. Rev. **D61** (2000) 105005.

[19] S. Coleman, Phys. Rev. **D15** (1977) 2929; **D16** (1977) 1248 (E); *Aspects of Symmetry* (Cambridge, 1985), Ch. 7.

[20] G. Baym and H. Heiselberg, Phys. Rev. **D56** (1997) 5254.

[21] E. M. Henley, M. B. Johnson, and L. S. Kisslinger, arXiv:0503247/hep-ph (unpublished)

[22] L. S. Kisslinger, S. Walawalkar, E. M. Henley, and M. B. Johnson, arXiv:050128/hep-ph (unpublished)

[23] E. M. Henley, M. B. Johnson, and L. S. Kisslinger, Phys. Rev **D81** (2010) 085035

[24] M. B. Johnson, L. S. Kisslinger, E. M. Henley, W-Y. P. Hwang, and T. Stevens, "Non-Abelian dynamics in First-Order Cosmological Phase Transitions", *Mod. Phys. Lett. A* **19**, 1187 (2004). (Contribution to the CosPA 2003 Cosmology and Particle Astrophysics Symposium, Nov.13 - 15, 2003.), hep-ph/0402198

[25] T. Stevens, M. B. Johnson, L. S. Kisslinger, E. M. Henley, W-Y. Pauchy Hwang, and M. Burkardt, Phys. Rev. **D77** (2008) 023501.

[26] T. Stevens, Thesis, New Mexico State University (2007).

[27] T. Stevens and M. B. Johnson, Phys. Rev. **D80** (2009) e083011; T. Stevens and M. B. Johnson, arXiv:1001.3694/astro-ph(2010) (unpublished).

[28] T. Kahniashvili, L. Kisslinger, and T. Stevens, Phys. Rev. **D81** (2010)