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# CP asymmetries in singly-Cabibbo-suppressed D decays to two pseudoscalar mesons 

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# CP ASYMMETRIES IN SINGLY-CABIBBO-SUPPRESSED $D$ DECAYS TO TWO PSEUDOSCALAR MESONS 

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The LHCb Collaboration has recently reported evidence for a CP asymmetry approaching the percent level in the difference between $D^{0} \rightarrow \pi^{+} \pi^{-}$and $D^{0} \rightarrow$ $K^{+} K^{-}$. We analyze this effect as if it is due to a penguin amplitude with the weak phase of the standard model $c \rightarrow b \rightarrow u$ loop diagram, but with a CPconserving enhancement as if due to the strong interactions. In such a case the magnitude and strong phase of this amplitude $P_{b}$ are correlated in order to fit the observed CP asymmetry, and one may predict CP asymmetries for a number of other singly-Cabibbo-suppressed decays of charmed mesons to a pair of pseudoscalar mesons. Non-zero CP asymmetries are expected for $D^{+} \rightarrow$ $K^{+} \bar{K}^{0}$ (the most promising channel for which a non-zero CP asymmetry has not yet been reported), as well as $D^{0} \rightarrow \pi^{0} \pi^{0}, D_{s}^{+} \rightarrow \pi^{+} K^{0}$, and $D_{s} \rightarrow \pi^{0} K^{+}$. No CP asymmetry is predicted for $D^{+} \rightarrow \pi^{+} \pi^{0}$ or $D^{0} \rightarrow K^{0} \bar{K}^{0}$ in this framework.

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## I Introduction

Although CP violation was first observed in neutral kaon decays and CP asymmetries have been seen at the tens of percent level in $B$ meson decays, the standard model describing these decays predicts naturally very small CP asymmetries in decays of charmed particles, of order $10^{-3}$ or less $[1,2,3]$. These decays are dominated by physics of the first two quark families, with the contribution of the third family suppressed both by smallness of elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and by the relatively small $b$ quark mass in the $c \rightarrow b \rightarrow u$ penguin diagram. This is in contrast to the $b \rightarrow t \rightarrow s$ penguin amplitude, which can profit from both a larger CKM factor and a much larger top quark mass. Following an early suggestion [4] that the penguin amplitude in $D$ decays may be enhanced by nonperturbative effects in analogy to the $s \rightarrow d$ penguin amplitude in
$K \rightarrow \pi \pi$, recent studies $[2,3,5]$ indicate that an order of magnitude enhancement is not impossible.

The LHCb Collaboration has reported $3.5 \sigma$ evidence for CP-violating charm decays in the difference between CP asymmetries in $D^{0} \rightarrow K^{-} K^{+}$and $D^{0} \rightarrow \pi^{-} \pi^{+}[6]$ :

$$
\begin{equation*}
\Delta A_{C P} \equiv A_{C P}\left(K^{+} K^{-}\right)-A_{C P}\left(\pi^{+} \pi^{-}\right)=[-0.82 \pm 0.21(\text { stat }) \pm 0.11(\text { syst })] \% \tag{1}
\end{equation*}
$$

For the decay of a charmed meson $D$ to any final state $f$ we are defining

$$
\begin{equation*}
A_{C P}(f) \equiv \frac{\Gamma(D \rightarrow f)-\Gamma(\bar{D} \rightarrow \bar{f})}{\Gamma(D \rightarrow f)+\Gamma(\bar{D} \rightarrow \bar{f})} . \tag{2}
\end{equation*}
$$

Although the CDF II Collaboration at the Fermilab Tevatron does not see statistically compelling evidence for CP violation in either of these two decays, their results are consistent with those of LHCb [7]:

$$
\begin{equation*}
A_{C P}\left(D^{0} \rightarrow K^{+} K^{-}\right)=(-0.24 \pm 0.22 \pm 0.09) \%, A_{C P}\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right)=(0.22 \pm 0.24 \pm 0.11) \% \tag{3}
\end{equation*}
$$

We calculate the corresponding $90 \%$ confidence level limits to be

$$
\begin{equation*}
-0.63 \% \leq A_{C P}\left(D^{0} \rightarrow K^{+} K^{-}\right) \leq 0.15 \%, \quad-0.21 \% \leq A_{C P}\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right) \leq 0.65 \% \tag{4}
\end{equation*}
$$

The LHCb results have led to numerous hypotheses of physics beyond the standard model (e.g., $[2,8,9,10]$ ) some of which had been studied earlier [11]. A more conservative approach, studying the above CP asymmetries within the Standard Model under relaxed assumptions about non-perturbative hadronic weak matrix elements, has been adopted recently in two papers applying flavor $\mathrm{SU}(3)$. Ref. [12] extended the hypothesis of triplet operator enhancement introduced in Ref. [4] by including in the effective weak Hamiltonian $\mathrm{SU}(3)$ breaking terms which are first order in the strange quark mass. A second work [13] (appearing while we were writing up our results), applying a diagramatic flavor $\mathrm{SU}(3)$ approach similar to the one discussed by us below, associated $W$-exchange and annihilation amplitudes with final state resonant effects [14]. While these two papers have some overlap with ours the specific assumptions and detailed predictions of the three studies, all based on flavor $\mathrm{SU}(3)$ analyses, are different.

In the present paper we explore a scenario in which the standard model $c \rightarrow b \rightarrow u$ penguin amplitude receives a sufficient enhancement from strong interaction physics to account for the effect observed by LHCb. This amplitude then must contribute to other direct CP asymmetries in decays of $D^{0}, D^{+}$, and $D_{s}^{+}$to pairs of pseudoscalar mesons. Nonzero CP asymmetries are expected for $D^{+} \rightarrow K^{+} \bar{K}^{0}$, as well as $D^{0} \rightarrow \pi^{0} \pi^{0}, D_{s}^{+} \rightarrow \pi^{+} K^{0}$, and $D_{s} \rightarrow \pi^{0} K^{+}$. No CP asymmetry is predicted for $D^{+} \rightarrow \pi^{+} \pi^{0}$ or $D^{0} \rightarrow K^{0} \bar{K}^{0}$ in this model. The former receives no penguin contribution, being a $\Delta I=3 / 2$ process, while the latter involves a different penguin amplitude than the one we are considering.

We perform the analysis in the context of a flavor- $\mathrm{SU}(3)$ model of charm decays presented previously $[15,16]$. We introduce notation in Sec. II and fit decay rates for SCS processes (including $\mathrm{SU}(3)$ breaking) in Sec. III. We then introduce a phenomenological penguin amplitude $P_{b}$ in Sec. IV to account for the CP violation observed by LHCb, and predict other CP asymmetries for SCS charmed meson decays. We summarize in Sec. V.


Figure 1: Relation between CF and SCS color-favored tree amplitudes.

## II Formalism and notation

Cabibbo-favored (CF) charm decays may be characterized by amplitudes $T, C, E$, and $A$, corresponding to color-favored tree, color-suppressed tree, exchange, and annihilation flavor topologies [15, 16]. A fit to CF decays of $D$ mesons to two pseudoscalar mesons leads to the following $(|T|>|C|)$ solution:

$$
\begin{align*}
T & =2.927  \tag{5}\\
C & =2.337 e^{-i 151.66^{\circ}}=-2.057-1.109 i  \tag{6}\\
E & =1.573 e^{i 120.56^{\circ}}=-0.800+1.355 i  \tag{7}\\
A & =0.33 e^{i 70.47^{\circ}}=0.110+0.311 i \tag{8}
\end{align*}
$$

quoted in units of $10^{-6} \mathrm{GeV}$. We note that complex conjugates of the amplitudes (6)-(8) give identical decay rates. (We neglect tiny phases of $V_{u d}$ and $V_{c s}$ ). To describe amplitudes corresponding to singly-Cabibbo-suppressed processes the above amplitudes are multiplied by $\pm \lambda$, where $\lambda=\tan \theta_{\text {Cabibbo }}=0.2317$. (Tiny phases of $V_{c d}$ and $V_{u s}$ are neglected.) The relation between CF and SCS color-favored tree amplitudes is illustrated in Fig. 1.

In order to account for $\mathrm{SU}(3)$ breaking in the $\operatorname{SCS} T$ amplitude we may use the following expressions:

$$
\begin{gather*}
T_{D^{0} \rightarrow \pi^{+} \pi^{-}}=T_{D^{+} \rightarrow \pi^{+} \pi^{0}}=T_{D_{s}^{+} \rightarrow \pi^{+} K^{0}}=T_{\pi},  \tag{9}\\
T_{D^{0} \rightarrow K^{+} K^{-}}=T_{D^{+} \rightarrow K^{+}}=T_{K} \tag{10}
\end{gather*}
$$



Figure 2: Relation between CF and SCS $A$ amplitudes.
where, neglecting the contribution of $f_{-}\left(q^{2}\right)$ at $q^{2}=m_{\pi, K}^{2}$,

$$
\begin{align*}
T_{\pi} & =T \cdot \frac{\left|f_{+\left(D^{0} \rightarrow \pi^{-}\right)}\left(m_{\pi}^{2}\right)\right|}{\left|f_{+\left(D^{0} \rightarrow K^{-}\right)}\left(m_{\pi}^{2}\right)\right|} \cdot \frac{m_{D}^{2}-m_{\pi}^{2}}{m_{D}^{2}-m_{K}^{2}},  \tag{11}\\
T_{K} & =T \cdot \frac{\left|f_{+\left(D^{0} \rightarrow K^{-}\right)}\left(m_{K}^{2}\right)\right|}{\left|f_{+\left(D^{0} \rightarrow K^{-}\right)}\left(m_{\pi}^{2}\right)\right|} \cdot \frac{f_{K}}{f_{\pi}} . \tag{12}
\end{align*}
$$

Similarly, the relation between CF and SCS $A$ amplitudes is illustrated in Fig. 2. Here, one may introduce $\mathrm{SU}(3)$ breaking as follows:

$$
\begin{align*}
& A_{D^{+} \rightarrow K^{+} \bar{K}^{0}}=A \cdot \frac{f_{D^{+}}}{f_{D_{s}^{+}}}=A_{D^{+}}  \tag{13}\\
& A_{D_{s}^{+} \rightarrow \pi^{+} K^{0}} \simeq A_{D_{s}^{+} \rightarrow K^{+} \pi^{0}}=A \tag{14}
\end{align*}
$$

We know the relevant decay constants [17] and meson masses [18] (in GeV ):

$$
\begin{gather*}
f_{\pi}=0.13041 ; \quad f_{K}=0.1561 ; \quad f_{D^{+}}=0.2067 ; \quad f_{D_{s}^{+}}=0.2575 ;  \tag{15}\\
m_{D^{0}}=1.8648 ; \quad m_{\pi}=0.13957018 ; \quad m_{K}=0.493677 \tag{16}
\end{gather*}
$$

The following approximate values are also known for the form factors from semileptonic $D^{0}$ decays [19, 20]:

$$
\begin{align*}
\left|f_{+\left(D^{0} \rightarrow \pi^{-}\right)}\left(m_{\pi}^{2}\right)\right| & \simeq 0.705,  \tag{17}\\
\left|f_{+\left(D^{0} \rightarrow K^{-}\right)}\left(m_{\pi}^{2}\right)\right| & \simeq 0.768,  \tag{18}\\
\left|f_{+\left(D^{0} \rightarrow K^{-}\right)}\left(m_{K}^{2}\right)\right| & \simeq 0.811 \tag{19}
\end{align*}
$$

After using relevant form factors and decay constants, we find, in units of $10^{-6} \mathrm{GeV}$,

$$
\begin{align*}
T_{\pi} & =2.87  \tag{20}\\
T_{K} & =3.70  \tag{21}\\
A_{D^{+}} & =(0.89+2.50 i) \times 10^{-1} \tag{22}
\end{align*}
$$



Figure 3: Penguin diagrams leading to a non-zero amplitude $P=P_{d}+P_{s}$ in the presence of imperfect cancellation between intermediate $d$ and $s$ quarks.


Figure 4: Penguin annihilation diagrams leading to a non-zero amplitude $P A=P A_{d}+P A_{s}$ in the presence of imperfect cancellation between intermediate $d$ and $s$ quarks.

The tree-level amplitudes for $D^{0} \rightarrow \pi^{+} \pi^{-}, D^{0} \rightarrow K^{+} K^{-}$, and $D^{0} \rightarrow \pi^{0} \pi^{0}$ involve the following respective combinations (in units of $10^{-7} \mathrm{GeV}$ ):

$$
\begin{align*}
-\lambda\left(T_{\pi}+E\right) & =-4.80-3.14 i  \tag{23}\\
\lambda\left(T_{K}+E\right) & =6.72+3.14 i  \tag{24}\\
\lambda(C-E) & =-2.91-5.71 i \tag{25}
\end{align*}
$$

as well as $\mathrm{SU}(3)$-breaking terms which we shall now introduce.

## III Fits to decay rates including $\mathrm{SU}(3)$ violation

The penguin amplitude $P$ for $c \rightarrow u$ transitions is normally thought to be very small because the contributions of $d$ and $s$ quarks in the intermediate state cancel one another [21]. If we regard this cancellation as inexact due to $\mathrm{SU}(3)$-violating masses of intermediate-state particles, we can regard the penguin amplitude $P$ as a proxy for $\mathrm{SU}(3)$ violation (see Fig. 3). It will then have the same weak phase as other standard model contributions to $D$ decays. The same can be said for a penguin annihilation $(P A)$ amplitude, contributing only to $D^{0}$ decays. It corresponds to the exchange processes $c \bar{u} \rightarrow s \bar{s}$ and $c \bar{u} \rightarrow d \bar{d}$ followed by $s \bar{s}$ or $d \bar{d}$ annihilation into a pair of charge-conjugate pseudoscalar mesons (Fig. 4).

The amplitudes for $D^{0} \rightarrow \pi^{+} \pi^{-}, D^{0} \rightarrow K^{+} K^{-}$, and $D^{0} \rightarrow \pi^{0} \pi^{0}$ then may be expressed as shown in the first three lines of Table I. Given the magnitudes of the relevant amplitudes determined from the decay rates $[16,22]$, in units of $10^{-7} \mathrm{GeV}$,

$$
\begin{equation*}
\left|\mathcal{A}\left(D^{0} \rightarrow \pi^{+} \pi^{-}\right)\right|=4.70 \pm 0.08 \tag{26}
\end{equation*}
$$

Table I: Representations and comparison of experimental and fit amplitudes for SCS decays of charmed mesons to two pseudoscalar mesons.

| Decay <br> Mode | Amplitude <br> representation | $\|\mathcal{A}\|\left(10^{-7} \mathrm{GeV}\right)$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Experiment | Theory Fit | $\chi^{2}$ |  |
| $D^{0} \rightarrow \pi^{+} \pi^{-}$ | $-\lambda\left(T_{\pi}+E\right)+(P+P A)$ | $4.70 \pm 0.08$ | 4.70 | 0 |
| $D^{0} \rightarrow K^{+} K^{-}$ | $\lambda\left(T_{K}+E\right)+(P+P A)$ | $8.49 \pm 0.10$ | 8.48 | 0.01 |
| $D^{0} \rightarrow \pi^{0} \pi^{0}$ | $-\lambda(C-E) / \sqrt{2}-(P+P A) / \sqrt{2}$ | $3.51 \pm 0.11$ | 3.51 | 0 |
| $D^{+} \rightarrow \pi^{+} \pi^{0}$ | $-\lambda\left(T_{\pi}+C\right) / \sqrt{2}$ | $2.66 \pm 0.07$ | 2.26 | 33 |
| $D^{0} \rightarrow K^{0} \bar{K}^{0}$ | $-(P+P A)+P$ | $2.39 \pm 0.14$ | 2.37 | 0.02 |
| $D^{+} \rightarrow K^{+} \bar{K}^{0}$ | $\lambda\left(T_{K}-A_{D^{+}}\right)+P$ | $6.55 \pm 0.12$ | 6.87 | 7 |
| $D_{s}^{+} \rightarrow \pi^{+} K^{0}$ | $-\lambda\left(T_{\pi}-A\right)+P$ | $5.94 \pm 0.32$ | 7.96 | 40 |
| $D_{s}^{+} \rightarrow \pi^{0} K^{+}$ | $-\lambda(C+A) / \sqrt{2}-P / \sqrt{2}$ | $2.94 \pm 0.55$ | 4.44 | 7 |

$$
\begin{align*}
\left|\mathcal{A}\left(D^{0} \rightarrow K^{+} K^{-}\right)\right| & =8.49 \pm 0.10  \tag{27}\\
\sqrt{2}\left|\mathcal{A}\left(D^{0} \rightarrow \pi^{0} \pi^{0}\right)\right| & =4.96 \pm 0.16 \tag{28}
\end{align*}
$$

one may then plot circles with these radii and centers defined by Eqs. (23-25) to solve for a common value of $P+P A$. The existence of a self-consistent solution for $P+P A$ is supported by a $\chi^{2}-$ minimization fit, which leads to

$$
\begin{equation*}
P+P A=[(0.44 \pm 0.23)+(1.41 \pm 0.36) i] \times 10^{-7} \mathrm{GeV} ; \quad \chi^{2} / \text { d.o.f. }=0.012 / 1=0.012 \tag{29}
\end{equation*}
$$

The construction and the corresponding $\Delta \chi^{2}=2.3$ error ellipse (corresponding to $68 \%$ probability) are shown in Fig. 5.

Using the extracted value of $P+P A$ we apply a similar construction technique to extract $P$. The relevant parts of the tree-level amplitudes that determine centers of the circles are as follows (in units of $10^{-7} \mathrm{GeV}$ ):

$$
\begin{align*}
-(P+P A) & =-0.44-1.41 i  \tag{30}\\
\lambda\left(T_{K}-A_{D^{+}}\right) & =8.37-0.58 i  \tag{31}\\
-\lambda\left(T_{\pi}-A\right) & =-6.40+0.72 i  \tag{32}\\
\lambda(C+A) & =-4.51-1.85 i \tag{33}
\end{align*}
$$

The relevant experimental rates $[16,22]$ lead to amplitudes once again determining the radii of the circles as follows (in units of $10^{-7} \mathrm{GeV}$ ):

$$
\begin{align*}
\left|\mathcal{A}\left(D^{0} \rightarrow K^{0} \bar{K}^{0}\right)\right| & =2.39 \pm 0.14  \tag{34}\\
\left|\mathcal{A}\left(D^{+} \rightarrow K^{+} \bar{K}^{0}\right)\right| & =6.55 \pm 0.12  \tag{35}\\
\left|\mathcal{A}\left(D_{s}^{+} \rightarrow K^{0} \pi^{+}\right)\right| & =5.94 \pm 0.32  \tag{36}\\
\sqrt{2}\left|\mathcal{A}\left(D_{s}^{+} \rightarrow K^{+} \pi^{0}\right)\right| & =2.94 \pm 0.55 \tag{37}
\end{align*}
$$

$\chi^{2}$-minimization gives us

$$
\begin{equation*}
P=\left[(-1.52 \pm 0.15)+\left(0.08_{-0.32}^{+0.38}\right) i\right] \times 10^{-7} \mathrm{GeV} ; \quad \chi^{2} / \text { d.o.f. }=54 / 2=27 \tag{38}
\end{equation*}
$$



Figure 5: Construction to determine $P+P A$. The relative sign between the left-hand and (magnified) right-hand panels is due to the fact that the vector $P+P A$ points toward the origin in the left-hand figure.


Figure 6: Construction to determine $P$. The relative sign between the left-hand and (magnified) right-hand panels is due to the fact that the vector $P$ points toward the origin in the left-hand figure.

The construction and the corresponding $68 \%$ error ellipse ( $\Delta \chi^{2}=2.3$ ) are shown in Fig. 6. In Table I we quote the representations and compare the experimental and fit amplitudes. Using the extracted values of $P$ and $P+P A$ we find

$$
\begin{equation*}
P A=[(1.95 \pm 0.38)+(1.34 \pm 0.71) i] \times 10^{-7} \mathrm{GeV} \tag{39}
\end{equation*}
$$

We recall the two-fold ambiguity permitting amplitudes which are complex conjugates of (30)-(33) and (38)-(39).

The poor $\chi^{2}$ in this fit is driven primarily by the large contribution from the $D_{s}^{+} \rightarrow \pi^{+} K^{0}$ amplitude. It is quite possible that our description of $\mathrm{SU}(3)$ breaking in this quantity is imperfect. In any case, the large experimental errors on the SCS decays of $D_{s}$ to two pseudoscalar mesons will hinder the study of CP-violating asymmetries in their decays for some time to come, so we shall not be greatly concerned with such decays for the present.

## IV Description and prediction of observed direct CP asymmetries

We now consider the effects of an additional phenomenological-penguin amplitude $P_{b}$, the weak phase of which differs from the weak phase of $P$ and $P A$ by the CKM-angle $\gamma$. (The subscript $b$ refers to a $b$ quark in the intermediate quark loop in Fig. 3.) In Table II we summarize the amplitudes for SCS processes obtained in the previous section, and extend the amplitude representations to include $P_{b}$. The quantities $\phi_{T}^{f}=\operatorname{Arg}\left[T_{f}\right]$ denote the strong phases of the non- $P_{b}$ amplitudes with respect to $T$.

Table II: Fit amplitudes for SCS charmed meson decays including $P$ and $P A$, and their representations including $P_{b}$.

| Decay <br> mode | Amplitude <br> representation | $\phi_{T}^{f}=\operatorname{Arg}\left[T_{f}\right]$ <br> (degrees) |
| :---: | :---: | :---: |
| $D^{0} \rightarrow \pi^{+} \pi^{-}$ | $-\lambda\left(T_{\pi}+E\right)+(P+P A)+P_{b}$ | -158.5 |
| $D^{0} \rightarrow K^{+} K^{-}$ | $\lambda\left(T_{K}+E\right)+(P+P A)+P_{b}$ | 32.5 |
| $D^{0} \rightarrow \pi^{0} \pi^{0}$ | $-\lambda(C-E) / \sqrt{2}-(P+P A) / \sqrt{2}-P_{b} / \sqrt{2}$ | 60.0 |
| $D^{+} \rightarrow \pi^{+} \pi^{0}$ | $-\lambda\left(T_{\pi}+C\right) / \sqrt{2}$ | 126.3 |
| $D^{0} \rightarrow K^{0} \bar{K}^{0}$ | $-(P+P A)+P$ | -145.6 |
| $D^{+} \rightarrow K^{+} \bar{K}^{0}$ | $\lambda\left(T_{K}-A_{D^{+}}\right)+P+P_{b}$ | -4.2 |
| $D_{s}^{+} \rightarrow \pi^{+} K^{0}$ | $-\lambda\left(T_{\pi}-A\right)+P+P_{b}$ | 174.3 |
| $D_{s}^{+} \rightarrow \pi^{0} K^{+}$ | $-\lambda(C+A) / \sqrt{2}-P / \sqrt{2}-P_{b} / \sqrt{2}$ | 16.4 |

In general, the amplitude for $D \rightarrow f$ may be written as follows:

$$
\begin{equation*}
\mathcal{A}(D \rightarrow f)=\left|T_{f}\right| e^{i \phi_{T}^{f}}\left(1+r_{f} e^{i\left(\gamma+\phi^{f}\right)}\right) \tag{40}
\end{equation*}
$$

where $T_{f}$ represents terms that have the same weak phase as the tree-level terms contributing to that amplitude, $\phi_{T}^{f}$ represents the strong phase of $T_{f}, r_{f}$ represents the ratio of the magnitude of the CP-violating penguin contribution to that of $T_{f}, \gamma$ represents the weak
phase of the CP-violating penguin (it is the same as the CKM angle), and $\phi^{f}$ is the strong phase of the CP-violating penguin relative to $T_{f}$. Let us take the example of the process $D^{0} \rightarrow \pi^{+} \pi^{-}$for clarity. Then

$$
\begin{align*}
T_{\pi^{+} \pi^{-}} & =-\lambda\left(T_{\pi}+E\right)+(P+P A)  \tag{41}\\
\phi_{T}^{\pi^{+} \pi^{-}} & =\operatorname{Arg}\left[T_{\pi^{+} \pi^{-}}\right]  \tag{42}\\
r_{\pi^{+} \pi^{-}} & =\frac{\left|P_{b}\right|}{\left|T_{\pi^{+} \pi^{-}}\right|}  \tag{43}\\
\phi^{\pi^{+} \pi^{-}} & =\operatorname{Arg}\left[P_{b}\right]-\phi_{T}^{\pi^{+} \pi^{-}}-\gamma . \tag{44}
\end{align*}
$$

The amplitude for $\bar{D} \rightarrow \bar{f}$ may be written as follows:

$$
\begin{equation*}
\mathcal{A}(\bar{D} \rightarrow \bar{f})=\left|T_{f}\right| e^{i \phi_{T}^{f}}\left(1+r_{f} e^{i\left(-\gamma+\phi^{f}\right)}\right) \tag{45}
\end{equation*}
$$

For a two-body decay, the rate is proportional to the absolute square of the amplitude. Thus, one may now define a CP asymmetry as follows:

$$
\begin{align*}
A_{C P}(f) & =\frac{\Gamma(D \rightarrow f)-\Gamma(\bar{D} \rightarrow \bar{f})}{\Gamma(D \rightarrow f)+\Gamma(\bar{D} \rightarrow \bar{f})} \\
& =-\frac{2 r_{f} \sin \gamma \sin \phi^{f}}{1+r_{f}^{2}+2 r_{f} \cos \gamma \cos \phi^{f}} \\
& =-\frac{2 p\left|T_{f}\right| \sin \gamma \sin \left(\delta-\phi_{T}^{f}\right)}{\left|T_{f}\right|^{2}+p^{2}+2 p\left|T_{f}\right| \cos \gamma \cos \left(\delta-\phi_{T}^{f}\right)} \tag{46}
\end{align*}
$$

where in the final step we have used $P_{b}=p e^{i(\delta+\gamma)}$.
The LHCb result (1) [6] may be used as a constraint on the magnitude and strong phase of the CP-violating penguin $P_{b}$. We use the following relationships:

$$
\begin{align*}
A_{C P}\left(K^{+} K^{-}\right) & =-\frac{2 p\left|T_{K^{+} K^{-}}\right| \sin \gamma \sin \left(\delta-\phi_{T}^{K^{+} K^{-}}\right)}{\left|T_{K^{+} K^{-}}\right|^{2}+p^{2}+2 p\left|T_{K^{+} K^{-}}\right| \cos \gamma \cos \left(\delta-\phi_{T}^{K^{+} K^{-}}\right)}  \tag{47}\\
A_{C P}\left(\pi^{+} \pi^{-}\right) & =-\frac{2 p\left|T_{\pi^{+} \pi^{-}}\right| \sin \gamma \sin \left(\delta-\phi_{T}^{\pi^{-} \pi^{-}}\right)}{\left|T_{\pi^{+} \pi^{-}}\right|^{2}+p^{2}+2 p\left|T_{\pi^{+} \pi^{-}}\right| \cos \gamma \cos \left(\delta-\phi_{T}^{\pi^{+} \pi^{-}}\right)} \tag{48}
\end{align*}
$$

We may use the theory fit results quoted in Table I for $\left|T_{f}\right|$. The strong phase $\phi_{T}^{f}$ can be taken from the results quoted in Table II. The CKM angle $\gamma$ may be taken to be $77^{\circ}$ [18]. Corresponding to each value of $\delta$ allowed by the $\Delta A_{C P}$ constraint, one may extract the allowed values of $p$. In addition we expect $\left|P_{b}\right|<\left|T_{f}\right|\left(r_{f}<1\right)$, which in turn restricts us to small values of $p$. In Fig. 7 we plot the allowed values of $p$ as a function of $\delta$ using Eqs. (47) and (48). For a wide range of $\delta$, a penguin amplitude of magnitude $0.01 \times 10^{-7} \mathrm{GeV}$, or $\mathcal{O}(0.1 \%)$ of the $D^{0} \rightarrow K^{+} K^{-}$amplitude, is sufficient to account for the observed value of $\Delta A_{C P}$. This is in accord with a conclusion reached in Ref. [3].

The constraint on $p$ as a function of $\delta$ now allows us to predict CP asymmetries in other channels such as $D^{+} \rightarrow K^{+} \bar{K}^{0}$ as a function of the angle $\delta$. In Fig. 8 we plot $A_{K^{+} \bar{K}^{0}}$ as a function of $\delta$. Values of $p$ are plotted only for the range of $\delta$ consistent with the limits (4), which will be specified shortly. In Fig. 9 we plot $A_{f}$ for the final states $K^{+} K^{-}, \pi^{+} \pi^{-}$, and $\pi^{0} \pi^{0}$. The limits (4) imply the following allowed ranges of $\delta$ :

$$
\begin{equation*}
A_{C P}\left(K^{+} K^{-}\right) \Rightarrow 0.50 \leq \delta \leq 3.57, \quad A_{C P}\left(\pi^{+} \pi^{-}\right) \Rightarrow 0.49 \leq \delta \leq 3.57 \tag{49}
\end{equation*}
$$



Figure 7: $p$ and $\delta$ allowed by the measured range of $\Delta A_{C P}$. The (red) line represents the central value, while inner (blue) and outer (green) bands respectively represent $68 \%$ confidence level $(1 \sigma)$ and $90 \%$ confidence level (1.64 $\sigma$ ) regions based on error in $\Delta A_{C P}$.


Figure 8: $A_{K^{+} \bar{K}^{0}}$ as a function of the allowed values of $\delta$. The (red) line represents the central value, while inner (blue) and outer (green) bands respectively represent $68 \%$ confidence level $(1 \sigma)$ and $90 \%$ confidence level $(1.64 \sigma)$ regions based on error in $\Delta A_{C P}$.


Figure 9: $A_{f}$ as a function of the allowed values of $\delta$. The (red) line represents the central value, while inner (blue) and outer (green) bands respectively represent $68 \%$ confidence level $(1 \sigma)$ and $90 \%$ confidence level $(1.64 \sigma)$ regions based on error in $\Delta A_{C P}$.

Figs. 7 and 8 are plotted only for values of $\delta$ consistent with both these limits. Note the correlation between the CP asymmetries in the channels $D^{0} \rightarrow \pi^{0} \pi^{0}$ and $D^{+} \rightarrow K^{+} \bar{K}^{0}$. More precise measurements of the individual asymmetries in $D^{0} \rightarrow \pi^{+} \pi^{-}$and $D^{0} \rightarrow K^{+} K^{-}$ can help to pin down the unknown strong phase $\delta$.

As mentioned in the preceding section, all the contributions to $T_{f}$ listed in Table II involve an ambiguity due to complex conjugation. Thus, the phase $\phi_{T}^{f}$ has a sign ambiguity, $\phi_{T}^{f} \rightarrow-\phi_{T}^{f}$, which is common to all final states $f$. The CP asymmetry (46) is approximately invariant under a joint transformation,

$$
\begin{equation*}
\phi_{T}^{f} \rightarrow-\phi_{T}^{f}, \quad \delta \rightarrow \pi-\delta, \tag{50}
\end{equation*}
$$

neglecting a very small contribution to the asymmetry quadratic in $p /\left|T_{f}\right|$. Thus, while plots similar to Figs. 7, 8 and 9 may be plotted with $\delta \rightarrow \pi-\delta$, the correlations between asymmetries in different decay modes are invariant under this redefinition.

We have left out $D_{s}^{+}$decay asymmetries since the corresponding branching ratios have large fractional errors. The process $D^{+} \rightarrow \pi^{+} \pi^{0}$ does not depend on $P_{b}$ in the isospin symmetry limit, and therefore its CP asymmetry is zero at this high level of approximation. The CP asymmetry in $D^{0} \rightarrow K^{0} \bar{K}^{0}$ depends only on a penguin annihilation diagram as there are no $u$ quarks in the final state. If it is found to be non-zero, our discussion must be expanded to include the possibility of CP violation due to interference between a $(P A)_{b}$ amplitude involving a $b$ quark in the loop and an $\mathrm{SU}(3)$ breaking term in $E$.

## V Discussion and summary

The observation by the LHCb Collaboration of a difference between the CP-violating asymmetries in $D^{0} \rightarrow K^{+} K^{-}$and $D^{0} \rightarrow \pi^{+} \pi^{-}$likely implies observable asymmetries in other decays of charmed mesons to pairs of pseudoscalar mesons. The present description of that difference assumes that a penguin amplitude with an intermediate $b$ quark, normally thought to provide a contribution below the observed effect, is amplified by CP-conserving physics (e.g., unforeseen QCD effects) to an extent which can account for the asymmetry. In that case several direct CP asymmetries are predicted as functions of a single strong phase difference $\delta$. These include asymmetries in the individual decays $D^{0} \rightarrow K^{+} K^{-}$and $D^{0} \rightarrow \pi^{+} \pi^{-}$, as well as $D^{0} \rightarrow \pi^{0} \pi^{0}$ and $D^{+} \rightarrow K^{+} \bar{K}^{0}$. These asymmetries are typically of order (a few) $\times 10^{-3}$, and the latter two are correlated with one another. Experimental limits (4) on the direct CP asymmetries in $D^{0} \rightarrow K^{+} K^{-}$and $D^{0} \rightarrow \pi^{+} \pi^{-}$[7] provide constraints on $\delta$. The observed asymmetry [23] $A_{C P}\left(D^{+} \rightarrow K^{+} \bar{K}^{0}\right)=(7.1 \pm 6.1 \pm 1.2) \%$ carries far too large an uncertainty at present to test its prediction.

In Fig. 9, while $A_{C P}\left(K^{+} K^{-}\right)$and $A_{C P}\left(\pi^{+} \pi^{-}\right)$are predicted to have opposite signs for a wide range of $\delta$, their relative magnitudes provide information about $\delta$, with the ratio $\left|A_{C P}\left(\pi^{+} \pi^{-}\right) / A_{C P}\left(K^{+} K^{-}\right)\right|$exceeding 1 for the mid-range of $\delta$ and behaving as a decreasing function of $\delta$. Thus, better measurements of these individual asymmetries will enable improved predictions of asymmetries such as $A_{C P}\left(K^{+} \bar{K}^{0}\right)$ and $A_{C P}\left(\pi^{0} \pi^{0}\right)$. We look forward to improvement of many of these determinations.

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