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Born-Infeld with Higher Derivatives

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We present new models of non-linear electromagnetism which satisfy the Noether-Gaillard-Zumino current conservation and are, therefore, self-dual. The new models differ from the Born-Infeld-type models in that they deform the Maxwell theory starting with terms like $\lambda(\partial F)^4$. We provide a recursive algorithm to find all higher order terms in the action of the form $\lambda^n \partial^{4n} F^{2n+2}$, which are necessary for the $U(1)$ duality current conservation. We use one of these models to find a self-dual completion of the $\lambda(\partial F)^4$ correction to the open string action. We discuss the implication of these findings for the issue of UV finiteness of $\mathcal{N}=8$ supergravity.

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I. INTRODUCTION

In this paper we discuss a method for constructing effective Lagrangians for non-linear theories with duality symmetries. This work builds on earlier papers by [1], [2], [3]. The hope is that this procedure may shed further light on counterterms in maximal supergravity theories. In particular it may improve our understanding of the role of $E_{7(7)}$ electro-magnetic duality symmetry in $\mathcal{N}=8$ supergravity.

Here we study a simplified class of models with only one vector field, no scalars and duality group $U(1)$. Although the $E_{7(7)}$ symmetry of $\mathcal{N}=8$ supergravity is a global continuous symmetry it has some unusual features which were uncovered for the first time in 1981 by Gaillard and Zumino [4] in the construction of extended supergravities (for a recent review see [5]). The familiar global continuous symmetries are defined by the Noether current conservation and are well known since 1918. However, duality symmetries have subtleties in the vector sector of the theory. Namely, the vector part of the action is not invariant under duality symmetry, but transforms in a specific way, so that the Bianchi identities and equations of motion transform into each other by duality symmetry. This feature is guaranteed by the conservation of the Noether-Gaillard-Zumino current and the corresponding NGZ identity.

Several theories with $U(1)$ duality are known. At the free, linear, level, there is Maxwell's electromagnetism and the higher-derivative generalizations constructed in [2]. At the interacting, non-linear, level, there is the Born-Infeld (BI) theory [6–8] and its generalizations [9, 10]. The fact that the original BI theory has electromagnetic duality was first noticed by Schrödinger [7]. The action of this model and of the generalizations constructed so far only contain powers of the Maxwell field strength F, and no higher derivatives. The BI Lagrangian had been derived by Fradkin and Tseytlin [8] as the low-energy spacetime effective Lagrangian for the vector field with a constant field strength, coupled to a string. The self-duality of Born-Infeld action and the relation to the D3-brane of type IIB superstring theory and its SL(2, Z)-symmetry was studied in [11]. For a review on BI action and open superstring theory we refer to [12].

The action of the BI model has a well-known closed form $\det^{1/2}(\eta_{\mu\nu}+F_{\mu\nu})$, while the actions of its generalization do not, so the Lagrangian has to be written as an infinite power series. Gibbons and Rasheed [9] have shown that there is a function of one variable's worth of Lagrangians admitting duality rotations and gave an explicit algorithm for their construction. These models were developed in more detail in [10] and more recently in [3]. The action of all these models is identical at the F^2, F^4, F^6 level, but they differ at the F^8 and higher levels.

In this paper we will construct two simple self-dual models of non-linear electrodynamics whose first deviation from the free Maxwell theory starts with a $(\partial F)^4$ term and contain terms of higher order in F and derivatives. We will present recursive procedure to construct all of them.

A term of this kind $\left((\partial F)^4\right)$ is known to arise in the 4-point amplitude of the open string[1] [13]. It was shown in [15] that, with this term (and other F^4 with higher derivatives present in the 4-point amplitudes), the theory satisfies the NGZ identity, and is consistent with electro-magnetic self-duality. Here we will show that a combination of the two simple self-dual models gives precisely the $(\partial F)^4$ term studied in [15] as well as higher-order terms required to satisfy the NGZ current conservation at the n-point level.

We will also describe a more general class of models where there are terms with F^n, without derivatives, as well as terms with derivatives $\partial^{2m}F^{2n}$. In all cases the algorithm for a construction of such actions satisfying the NGZ identity will be given.

II. $U(1)$ DUALITY, NO SCALARS

Our goal is to construct actions $S(F)$, where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Maxwell field strength, which have a non-linear $U(1)$ duality. Two classes of such actions are known in the literature: that of the Born-Infeld theory and its generalizations [9, 10], that depend only on F and not on its derivatives, and the action constructed in [2] which has higher derivatives but is quadratic in F.

As usual, we define the dual field strength $G(F)$ by

$$\tilde{G}^{\mu\nu} \equiv \tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}G_{\rho\sigma} \equiv 2\frac{\delta S(F)}{\delta F_{\mu\nu}}\,. \tag{2.1}$$

The infinitesimal $U(1)$ duality transformations that interchange the equations of motion $\partial_\mu \tilde{G}^{\mu\nu} = 0$ and Bianchi identities $\partial_\mu \tilde{F}^{\mu\nu} = 0$ are given by

$$\delta \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} 0 & B \\ -B & 0 \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}. \tag{2.2}$$

The necessary condition for the theory to be selfdual is conservation of the the NGZ current [4], which in $U(1)$ models without scalars requires that

$$\int d^4x(F\tilde{F} + G\tilde{G}) = 0\,. \tag{2.3}$$

The $U(1)$ case is a special case of a more general $Sp(2n, \mathbb{R})$ duality group

$$\delta \begin{pmatrix} F \\ G \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F \\ G \end{pmatrix}. \tag{2.4}$$

which also acts on scalars, $\delta\phi = \delta\phi(A, B, C, D)$.

In the general case, the NGZ identity requires the action to be of the form [4]

$$S = \tfrac{1}{4}\int d^4x\, F\tilde{G} + S_{\rm inv}\,, \tag{2.5}$$

where $S_{\rm inv}$ is exactly invariant under the duality group $\delta S_{\rm inv} = 0$. This is a reconstructive identity, since, in principle, it may be used to find the action from the knowledge of $S_{\rm inv}$ and $G(F)$. On the other hand,

$$\delta S = \tfrac{1}{2}\int d^4x\, \delta(F\tilde{G}) = \tfrac{1}{4}\int d^4x(\tilde{G}BG + \tilde{F}CF)\,. \tag{2.6}$$

Eqs. (2.5) and (2.6) are equivalent to NGZ current conservation, whereas eq. (2.3) is a particular form of the current conservation, valid only for $U(1)$ models without scalars. Indeed, only for $U(1)$ in absence of scalars $\delta S = \frac{1}{2}\tilde{G}BG$, with $B = -C$ and $A = 0$ and therefore

$$\tfrac{1}{2}\tilde{G}BG = \tfrac{1}{4}(\tilde{G}BG + \tilde{F}CF) \qquad \Rightarrow \qquad \int d^4x(F\tilde{F} + G\tilde{G}) = 0\,. \tag{2.7}$$

[1] The same type of terms have been considered in [14] as part of the effective action of a single D3-brane. They have been shown to fit elegantly into an $\mathrm{SL}(2,\mathbb{Z})$-invariant function that encodes both the perturbative and non-perturbative contributions to the amplitude.

A. New reconstructive identity in $U(1)$ models without scalars

As mentioned before, to use the generic reconstructive identity (2.5) one needs, in addition to $G(F)$ in each particular model, additional information on $S_{\rm inv}$. In the $U(1)$ models without scalars that we are considering here, this additional information comes form the following general observation: if λ is the coupling constant of the model (so the linear Maxwell term is independent of it), then, $S_{\rm inv}$ is related to the full action by

$$S_{\rm inv} = -\lambda \frac{\partial S}{\partial \lambda}\,. \tag{2.8}$$

This relation follows from the uniqueness of $S_{\rm inv}$ for a given non-linear theory, with given non-linear duality transformations and from the invariance of $\lambda \frac{\partial S}{\partial \lambda}$. The precise coefficient relating these two objects follows from the study of the linear and next-to-linear terms of a generic action. For example, it is well known [4] that in the BI model one has $S_{\rm inv} = -g^2 \frac{\partial S}{\partial g^2}$.

Using this general observation, one can derive a new, more useful, reconstructive identity:

$$S(F) = \frac{1}{4\lambda}\int d^4x\, d\lambda\, F\tilde{G}\,. \tag{2.9}$$

To prove is, we will first prove that $\lambda\frac{\partial S}{\partial \lambda}$ is duality invariant[2], using the NGZ identity and the definition (2.1) (2.3):

$$\lambda\frac{\partial}{\partial\lambda}\int d^4x (F\tilde{F} + G\tilde{G}) = 2\lambda \int d^4x \frac{\partial \tilde{G}}{\partial \lambda} G = \lambda \int d^4x \frac{\partial}{\partial\lambda}\left(\frac{\delta S}{\delta F}\right) G = 0\,. \tag{2.10}$$

Then, since the functional variation and the partial derivative with respect to λ commute, and using (2.2), we find that

$$0 = \int d^4x \frac{\delta}{\delta F}\left(\lambda\frac{\partial S}{\partial\lambda}\right) G = B^{-1}\int d^4x \frac{\delta}{\delta F}\left(\lambda\frac{\partial S}{\partial\lambda}\right)\delta F = B^{-1}\delta\left(\lambda\frac{\partial S}{\partial\lambda}\right)\,. \tag{2.11}$$

Now, using the observation (2.8) in (2.5) that

$$S + \lambda\frac{\partial S}{\partial\lambda} = \tfrac{1}{4}\int d^4x\, F\tilde{G}\,, \tag{2.12}$$

which can integrated immediately, leading to (2.9).

The new reconstructive identity (2.9) is particularly well-suited to find the action as a series expansion in λ when the dual field strength G is also available as a series expansion in λ: defining[3]

$$\tilde{G}(F) \;=\; -F + 2\sum_{n=1}^{\infty}\lambda^n \tilde{G}^{(n)}(F)\,, \tag{2.13}$$

$$S \;=\; -\tfrac{1}{2}\int d^4x\, F^2 + 2\sum_{n=1}^{\infty}\lambda^n S^{(n)}\,, \tag{2.14}$$

so that the $\lambda = 0$ free limit of the theory is the Maxwell theory, we find that each term in the expansion of the action is given by

$$S^{(n)} = \frac{1}{4(n+1)}\int d^4x\, F\tilde{G}^{(n)}(F)\,. \tag{2.15}$$

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[2]This is just a particular case of the general theorem proven in Appendix B of Ref. [4]. Note that the general proof in Ref. [4] is based on a condition that the duality transformation of scalars do not depend on a coupling associated with the deformation, meanwhile, the transformation law of vectors does depend on such a coupling. This raises the issue whether in extended supersymmetric theories, where scalars and vectors are in the same multiplet, the construction of this type is available.

[3]The global factors of 2 have been introduced for later convenience, since they lead to simpler expressions for the coefficients $T^{(n)\pm}$ to be introduced later.

In the models that we are going to consider $\tilde{G}(F)$ is given by a series expansion of the above form with all the terms of higher order in λ given by a simple recursion relation and these results can be checked explicitly order by order in λ.

B. NGZ identity with graviphoton convention

To proceed, we introduce the standard supergravity graviphoton conventions [16], employed in [3] in the covariant procedures for perturbative non-linear deformation of duality-invariant theories. In the complex basis we define

$$T = F - iG, \qquad T^* = F + iG\,, \tag{2.16}$$

which transform under finite $U(1)$ duality transformations with a phase, so, under (2.2)

$$\delta T = iBT\,. \tag{2.17}$$

We also introduce the self-dual notation,

$$T^{\pm} = \tfrac{1}{2}(T \pm i\tilde{T})\,. \tag{2.18}$$

and form 4 different combinations of the components of the graviphoton field, see Table II B. Observe that $T^{*+} = T^{-*}$. In this notation, the NGZ identity (2.3) takes the form

$$\int d^4x\, \left[T^{*+}T^{+} - T^{*-}T^{-}\right] = 0\,, \tag{2.19}$$

In the linear Maxwell theory $T^+ = 0$, so

$$T^+ = F^+ - iG^+ = 0\,, \tag{2.20}$$

which implies $\tilde{G} = -F$. In more general theories in which the dual field G is treated as independent of F, this constraint is used to eliminate the non-physical degrees of freedom and express G as a function of F and scalars, if any, and it is known as a linear twisted self-duality constraint.

In [2], Bossard and Nicolai proposed to use a non-linear deformation of the twisted self-duality constraint based on a manifestly duality invariant source of deformation $\mathcal{I}^{(1)}(T)$ to construct a self-dual theory. We will follow here the generalized procedure used in [3]. Let us assume that a manifestly duality-invariant $\mathcal{I}^{(1)}(T)$ is given. It was shown in [3] that if, instead of vanishing as required by the linear twisted self-duality condition, T^+ is given by the non-linear twisted self-duality condition

$$T^+_{\mu\nu} = \frac{\delta\mathcal{I}^{(1)}(T^-, T^{*+})}{\delta T^{*+}_{\mu\nu}}\,, \qquad (T^+_{\mu\nu})^* = T^{*-}_{\mu\nu} = \frac{\delta\mathcal{I}^{(1)}(T^-, T^{*+})}{\delta T^{-}_{\mu\nu}}\,, \tag{2.21}$$

it follows that the NGZ identity is satisfied automatically. One computes $T^{*+}T^{+} - T^{*-}T^{-}$, using (2.21) and finds that it vanishes since it is proportional to the variation of $\mathcal{I}^{(1)}(T)$ under duality, which vanishes since $\delta\mathcal{I}^{(1)} = 0$:

$$\int d^4x\, \left[T^{*+}T^{+} - T^{*-}T^{-}\right] = \int d^4x \left[T^{*+}\frac{\delta\mathcal{I}^{(1)}(T^-, T^{*+})}{\delta T^{*+}} - T^{-}\frac{\delta\mathcal{I}^{(1)}(T^-, T^{*+})}{\delta T^{-}}\right] = \frac{1}{B}\delta\mathcal{I}^{(1)} = 0\,. \tag{2.22}$$

Thus, once the eqs. (2.21) are solved for $G(F)$ there is no need to check the NGZ identity, it is satisfied and we have the $G(F)$ of a self-dual theory.

In the models that we are going to study, the non-linear, twisted, self-duality constraint can be solved as a power series in a parameter λ:

$$T^+ = -2\sum_{n=1}^{\infty}\lambda^n T^{(n)+}\,, \tag{2.23}$$

so

$$iG^+ = F^+ + 2\sum_{n=1}^{\infty}\lambda^n T^{(n)+}\,, \tag{2.24}$$

Graviphoton Components	Chirality	Charge
$T^{+} = F^{+} - iG^{+}$	+	+
$T^{*+} = F^{+} + iG^{+}$	+	-
$T^{-} = F^{-} - iG^{-}$	-	+
$T^{*-} = F^{-} + iG^{-}$	-	-

TABLE I: The 4 combinations of the graviphoton components have $\pm$ chirality and $\pm$ duality charge.

from which we can get the coefficients of the series (2.13) for $n > 0$

$$\tilde{G}^{(n)} = -(T^{(n)+} + \text{c.c.}) \,, \;\; (n > 0) \,. \tag{2.25}$$

and of (2.14) for $n > 0$ (for $n = 0$ they have been chosen to correspond to Maxwell's theory)

$$2S^{(n)} = -\frac{1}{2(n+1)} \int d^4x \, \left[F^{+}T^{(n)+} + \text{c.c.}\right] \,, \;\; (n > 0) \,. \tag{2.26}$$

III. BORN INFELD WITH HIGHER DERIVATIVES AND DUALITY CURRENT CONSERVATION

In this section we are going to construct two deformations of the Maxwell theory using two particularly simple manifestly duality invariant sources of deformation $\mathcal{I}_A^{(1)}(T)$ and $\mathcal{I}_B^{(1)}(T)$ given, respectively, by

$$\mathcal{I}_A^{(1)}(T) \;\equiv\; \frac{\lambda}{2^3} t^{(8)}{}_{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} \partial_\alpha T^{*+\,\mu_1\nu_1} \partial^\alpha T^{-\,\mu_2\nu_2} \partial_\beta T^{*+\,\mu_3\nu_3} \partial^\beta T^{-\,\mu_4\nu_4} \,, \tag{3.1}$$

$$\mathcal{I}_B^{(1)}(T) \;\equiv\; \frac{\lambda}{2^3} t^{(8)}{}_{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} \partial_\alpha T^{*+\,\mu_1\nu_1} \partial_\beta T^{-\,\mu_2\nu_2} \partial^\alpha T^{*+\,\mu_3\nu_3} \partial^\beta T^{-\,\mu_4\nu_4} \,, \tag{3.2}$$

where the tensor $t^{(8)}$ is defined in the Appendix, or, using the shorthand notation introduced in the Appendix,

$$\mathcal{I}_A^{(1)}(T) \;\equiv\; \frac{\lambda}{2^3} t^{(8)}{}_{abcd} \partial_\alpha T^{*+\,a} \partial^\alpha T^{-\,b} \partial_\beta T^{*+\,c} \partial^\beta T^{-\,d} \,, \tag{3.3}$$

$$\mathcal{I}_B^{(1)}(T) \;\equiv\; \frac{\lambda}{2^3} t^{(8)}{}_{abcd} \partial_\alpha T^{*+\,a} \partial_\beta T^{-\,b} \partial^\alpha T^{*+\,c} \partial^\beta T^{-\,d} \,. \tag{3.4}$$

At first order in λ the models that one obtains using the procedure described in the previous section are associated to the following deformations of the action

$$S_A^{(1)} \;=\; \tfrac{1}{4} \int d^4x \, t^{(8)}{}_{abcd} \partial_\alpha F^{+\,a} \partial^\alpha F^{-\,b} \partial_\beta F^{+\,c} \partial^\beta F^{-\,d} \,, \tag{3.5}$$

$$S_B^{(1)} \;=\; \tfrac{1}{4} \int d^4x \, t^{(8)}{}_{abcd} \partial_\alpha F^{+\,a} \partial_\beta F^{-\,b} \partial^\alpha F^{+\,c} \partial^\beta F^{-\,d} \,. \tag{3.6}$$

Alternative forms of these corrections which do not use the $t^{(8)}$ tensor are eqs. (A9) and (A10).

In what follows we are going to construct explicitly the model A, using $\mathcal{I}_A^{(1)}(T)$ in the non-linear twisted self-dual condition.

A. Model A

The simplest way to solve the non-linear twisted self-dual condition with $\mathcal{I}_A^{(1)}(T)$ is to plug the series expansion (2.23) into both sides of it and identify the terms with the same powers of λ. First, observe that the expansion (2.23) for T^+ implies for T^{*+} and T^-

$$T^{*+} \;=\; 2F^+ + 2\sum_{n=1}^{\infty}\lambda^n T^{(n)+}\,, \tag{3.7}$$

$$T^{-} \;=\; 2F^- + 2\sum_{n=1}^{\infty}\lambda^n T^{(n)+*}\,. \tag{3.8}$$

it is, then, convenient, to define[4]

$$T^{(0)+} = F^+\,, \tag{3.9}$$

so

$$T^{*+} \;=\; 2\sum_{n=0}^{\infty}\lambda^n T^{(n)+}\,, \tag{3.10}$$

$$T^{-} \;=\; 2\sum_{n=0}^{\infty}\lambda^n T^{(n)+*}\,. \tag{3.11}$$

With these definitions, the non-linear twisted self-dual condition for this model, which is

$$T^+_a = -\frac{\lambda}{2^2}t^{(8)}{}_{abcd}\partial_\alpha(\partial^\alpha T^{-b}\partial_\beta T^{*+c}\partial^\beta T^{-d})\,, \tag{3.12}$$

takes the form

$$\sum_{n=1}^{\infty}\lambda^n T^{(n)+}_a = t^{(8)}{}_{abcd}\sum_{p,q,r=0}\lambda^{p+q+r+1}\partial_\alpha(\partial^\alpha T^{(p)-b}\partial_\beta T^{(q)*+c}\partial^\beta T^{(r)-d})\,, \tag{3.13}$$

from which it follows that

$$T^{(n)+}_a = t^{(8)}{}_{abcd}\sum_{p,q,r=0}\delta_{p+q+r+1,n}\partial_\alpha(\partial^\alpha T^{(p)-b}\partial_\beta T^{(q)*+c}\partial^\beta T^{(r)-d})\,, \tag{3.14}$$

which can be solved recursively, given that $T^{(0)+}_a = F^+_a$. Thus,

$$T^{(1)+}_a \;=\; t^{(8)}{}_{abcd}\partial_\alpha(\partial^\alpha F^{-b}\partial_\beta F^{+c}\partial^\beta F^{-d})\,, \tag{3.15}$$

$$T^{(2)+}_a \;=\; t^{(8)}{}_{abcd}\left[\partial_\alpha(\partial^\alpha F^{-b}\partial_\beta F^{+c}\partial^\beta T^{(1)-d}) + \partial_\alpha(\partial^\alpha F^{-b}\partial_\beta T^{(1)+c}\partial^\beta F^{-d}) + \partial_\alpha(\partial^\alpha T^{(1)-b}\partial_\beta F^{+c}\partial^\beta F^{-d})\right] \tag{3.16}$$

$$T^{(3)+}_a \;=\; t^{(8)}{}_{abcd}\left[\partial_\alpha(\partial^\alpha F^{-b}\partial_\beta F^{+c}\partial^\beta T^{(2)-d}) + \partial_\alpha(\partial^\alpha F^{-b}\partial_\beta T^{(1)+c}\partial^\beta T^{(1)-d}) + \text{permutations}\right]\,, \tag{3.17}$$

etc. The action can be obtained immediately by using the power series expansion of reconstructive identity (2.26). Explicitly, we get:

$$2S^{(0)} \;=\; -\tfrac{1}{4}\int d^4x\, F^2\,, \tag{3.18}$$

$$2S^{(1)} \;=\; \tfrac{1}{2}\int d^4x\, t^{(8)}{}_{abcd}\partial_\alpha F^{+\,a}\partial^\alpha F^{-\,b}\partial_\beta F^{+\,c}\partial^\beta F^{-\,d}\,, \tag{3.19}$$

$$\begin{aligned} 2S^{(2)} \;&=\; -\tfrac{1}{2}\int d^4x\,\left[T^{(1)+}{}_a T^{(1)+\,a} + \text{c.c.}\right] \\ &=\; -\tfrac{1}{2}\int d^4x\,\left\{t^{(8)}{}_{abcd}t^{(8)}{}_{defg}\partial_\alpha\left(\partial^\alpha F^{-\,b}\partial_\beta F^{+\,c}\partial^\beta F^{-\,d}\right)\partial_\gamma\left(\partial^\gamma F^{-\,e}\partial_\delta F^{+\,f}\partial^\delta F^{-\,g}\right) + \text{c.c.}\right\}\,, \end{aligned} \tag{3.20}$$

---

[4]Notice, however, the expansion of T^+ is still given by (2.23) and has no term of zero order in λ.

etc., up to total derivatives. It can be checked order by order that this action is related to the dual field strength $iG^+ = F^+ - T^+$ by (2.1):

$$G^+{}_{\mu\nu} = 2i\frac{\delta S}{\delta F^{+\,\mu\nu}}\,, \tag{3.21}$$

as required.

B. Model B

The recursive algorithm for generating a complete action above produces the λ^n term from the previous ones. The derivation of this model follows the exact steps which we outlined in the case A. Each time the sequence of $(+-+-)$ has to be replaced by $(++--)$, the rest is the same. Therefore we will not provide more details on the derivation of the B model.

IV. SUPERSYMMETRIZABLE BORN-INFELD DUALITY SYMMETRIC MODEL WITH HIGHER DERIVATIVES

The model with derivatives of F known from the open superstring effective action [13] was shown to satisfy the NGZ current conservation condition (2.3) in [15]. In this model the first deformation of the Maxwell theory is given by the quartic coupling term

$$S^{(1)} = \frac{\lambda}{2^4}\int d^4x\, t^{(8)}_{abcd}\partial_\mu F^a\partial^\mu F^b\partial^\nu F^c\partial_\nu F^d\,, \tag{4.1}$$

in the notation introduced in the Appendix. As shown there, it can be rewritten in the form (eq. (A11))

$$S^{(1)} = \frac{\lambda}{2^2}\int d^4x\, t^{(8)}{}_{abcd}\,\left[\partial_\mu F^{+a}\partial^\mu F^{-b}\partial_\nu F^{+c}\partial^\nu F^{-d} + \tfrac{1}{2}\partial_\mu F^{+a}\partial_\nu F^{-b}\partial^\mu F^{+c}\partial^\nu F^{-d}\right]\,. \tag{4.2}$$

It is clear that, to reproduce this quartic term in the action we must take a combination of the models A and B studied above and use, as manifestly self-dual source of deformation

$$\mathcal{I}^{(1)}_{\text{string}}(T) = \mathcal{I}^{(1)}_A(T) + \tfrac{1}{2}\mathcal{I}^{(1)}_B(T) = \frac{\lambda}{2^3}\,t^{(8)}_{abcd}[\partial_\mu T^{*+\,a}\partial^\mu T^{-\,b}\partial_\nu T^{*+\,c}\partial^\nu T^{-\,d} + \frac{1}{2}\partial_\mu T^{*+\,a}\partial^\mu T^{*+\,b}\partial_\nu T^{-\,c}\partial^\nu T^{-\,d}]\,. \tag{4.3}$$

the resulting non-linear, twisted, self-duality constraint can be solved by the same recursive procedure we employed for the model A above and the dual field strength $G(F)$ and corresponding action can be found by the use of the new reconstructive identity.

It is interesting to observe that, after partial integration, the above quartic term is very close to the $(\partial F)^4$ term found in ref. [17], although the latter, corresponding to an amplitude calculation, is not real in its current form. It is likely that for the effective action one can produce the real expression dividing the one in ref. [17] by two and adding the Hermitean conjugate.

V. MORE GENERAL $U(1)$ DUALITY, NO SCALARS MODELS

Using the covariant procedures for perturbative non-linear deformation of duality-invariant theories [3] we can construct more general models with NGZ current conservation. For example, we may consider more general sources of deformation.

$$\mathcal{I}^{(1)}_{f_n}(T^-, T^{*+}) = \sum_{n=1} f_n\Big(\mathcal{I}^{(1)}(T^-, T^{*+})\Big)^n\,, \tag{5.1}$$

where $\mathcal{I}^{(1)}(T^-, T^{*+})$ is defined in (4.3) and f_n are arbitrary constants, and the model we described above in details has $f_1 = 1$ and $f_n = 0, n > 1$. In addition, we may add terms which depend only on F's without derivatives, for example, the ones studied in [3].

Any manifestly $U(1)$ duality invariant $\mathcal{I}(T^-, T^{*+})$ with space-time derivatives action of T's or without, has to have the same number of T^-' s as T^{*+}'s, and has to be Lorentz covariant. In such case, one expects a recursive equation, defining $G_{\mu\nu}(F)$ from the equation

$$T^+_{\mu\nu} = \frac{\delta \mathcal{I}(T^-, T^{*+})}{\delta T^{*+}_{\mu\nu}} \,, \tag{5.2}$$

as shown in the simple models defined in [3] without derivatives and in sec. III in case with derivatives. The solution is guaranteed to satisfy the NGZ $U(1)$ current conservation [3].

This eq. (5.2) in all cases provides a recursive procedure determining $G(F)$ as a powers series in λ. The action in these most general models of $U(1)$ duality without scalars is given by the new reconstructive identity (2.9).

VI. DISCUSSION

In this paper we have constructed explicitly the first complete model of Born-Infeld type with higher derivatives, which has an electromagnetic $U(1)$ duality. The model is given by the power series expansion of the Lagrangian and involves all powers of the Maxwell field strength and their derivatives.

$$S = S_{\text{Maxwell}} - \sum_{n=1}^{\infty} \frac{\lambda^n}{2(n+1)} \int d^4x \left[F^{+\mu\nu} T^{(n)+}{}_{\mu\nu} + \text{c.c.} \right] \,. \tag{6.1}$$

Here $T^{(n)} \sim \partial^{4n} F^{2n+1}$ and the explicit expression is given via a recursive algorithm in eq. (3.14), which defines $T^{(n)}$ in terms of $T^{(m)}$ with $m < n$ and starts with $T^{(0)+} = F^+$. We have also outlined the procedure to produce more complicated models where terms with and without derivatives on F are mixed.

Apart from the intrinsic motivation to discover a non-linear model with higher derivatives and with duality symmetry, which was not known in the past, our goal here was to test the Bossard-Nicolai proposal [2]. The authors conjectured that there is a straightforward algorithm which allows to construct $N = 8$ supergravity with higher derivatives, consistent with $E_{7(7)}$ duality. This conjecture was used in [2] to counter the argument of [1] suggesting that $E_{7(7)}$ duality symmetry predicts the finiteness of $\mathcal{N} = 8$ supergravity.

However, there is no actual construction of $\mathcal{N} = 8$ supergravity with higher derivatives in [2], which would be a formidable task. Therefore we performed a detailed investigation of this issue in applications to much simpler models, such as the Born-Infeld models and their generalizations. An investigation of this issue in [3] demonstrated that the algorithm of construction of $\mathcal{N} = 8$ supergravity with higher derivatives requires substantial modifications even in application to the simplest Born-Infeld model. Moreover, it was observed in [3] that the presence of the 4-point UV divergence $F^4 f(s,t,u)$ term in the $\mathcal{N} = 8$ supergravity would require to produce a theory of the Born-Infeld type with derivatives leads to a non-stop proliferation of the powers of the vector field strength with increasing number of derivatives.

A generalization of the results in [3] to the Born-Infeld model with higher derivatives required additional efforts. In this paper we were able to construct a toy model of a Born-Infeld $\mathcal{N} = 8$ supergravity, with $\mathcal{N} = 0$ supersymmetry replacing $\mathcal{N} = 8$ and $U(1)$ duality replacing $E_{7(7)}$. The model indeed has a full non-linearity in powers of $\lambda^n \partial^{4n} F^{2n+2}$ with $n \to \infty$, as predicted in [3].

Thus, whereas we were able to construct the Born-Infeld model with derivatives, and we are planning to develop a similar construction for supersymmetric models, which also have a $U(1)$ duality symmetry, at present we do not see any obvious way to extend this construction and develop the Born-Infeld version of $\mathcal{N} = 8$ supergravity along the lines of [2]. Until the existence of the Born-Infeld version of $\mathcal{N} = 8$ supergravity is demonstrated, the argument that $E_{7(7)}$ duality symmetry predicts the finiteness of $\mathcal{N} = 8$ supergravity [1] seems to us still valid. Moreover, even if one manages to construct a consistent 2-coupling model of $\mathcal{N} = 8$ supergravity with gravitational coupling κ^2 as well as Born-Infeld coupling λ, it will raise a new question whether the conjectured existence of this new theory predicts anything for the UV behaviour of the original one-coupling $\mathcal{N} = 8$ supergravity, which depends only on gravitational coupling.

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Appendix A: Some useful relations

Using the definition of the Hodge dual

$$\tilde{A}_{\mu\nu} \equiv \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}A^{\rho\sigma}\,, \qquad \tilde{\tilde{A}} = -A\,, \tag{A1}$$

and of the self- and anti-self-dual parts

$$A^{\pm} = \tfrac{1}{2}(A \pm i\tilde{A})\,, \tag{A2}$$

for 2-forms, one can easily prove the following identities involving arbitrary 2-forms A and B:

$$\tilde{A}\tilde{B} \;=\; BA - \tfrac{1}{2}\mathrm{tr}(AB)\mathbb{1}\,, \tag{A3}$$

$$\tilde{A}B \;=\; -\tilde{B}A + \tfrac{1}{2}\mathrm{Tr}(A\tilde{B})\mathbb{1}\,, \tag{A4}$$

$$A^{\pm}B^{\pm} \;=\; -B^{\pm}A^{\pm} + \tfrac{1}{2}\mathrm{Tr}(A^{\pm}B^{\pm})\mathbb{1}\,, \tag{A5}$$

$$A^{\pm}B^{\mp} \;=\; B^{\mp}A^{\pm}\,. \tag{A6}$$

The $t^{(8)}$ tensor [18] is totally symmetric in four pairs of antisymmetric indices. It is convenient to use only one Latin index $a, b, c\ldots$ to denote each of these four pairs and write ${t^{(8)}}_{abcd}$ instead of ${t^{(8)}}_{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} = {t^{(8)}}_{[\mu_1\nu_1][\mu_2\nu_2][\mu_3\nu_3][\mu_4\nu_4]}$. Then, in terms of these indices, $t^{(8)}$ is completely symmetric ${t^{(8)}}_{abcd} = {t^{(8)}}_{(abcd)}$.

$t^{(8)}$ can be defined by its contraction with 4 arbitrary 2-forms A, B, C, D:

$$\begin{aligned} {t^{(8)}}_{abcd}A^aB^bC^cD^d \;&=\; 8[\mathrm{Tr}(ABCD) + \mathrm{Tr}(ACBD) + \mathrm{Tr}(ACDB)] \\ &\quad -2[\mathrm{Tr}(AB)\mathrm{Tr}(CD) + \mathrm{Tr}(AC)\mathrm{Tr}(BD) + \mathrm{Tr}(AD)\mathrm{Tr}(BC)]\,. \end{aligned} \tag{A7}$$

Then, using the above relations, one can write

$${t^{(8)}}_{abcd}\,\partial_\mu F^a\partial^\mu F^b\partial_\nu F^c\partial^\nu F^d = 16\left\{\mathrm{Tr}\left(\partial_\mu F^+\partial_\nu F^+\right)\mathrm{Tr}\left(\partial^\mu F^-\partial^\nu F^-\right) + \tfrac{1}{2}\mathrm{Tr}\left(\partial_\mu F^+\partial^\mu F^+\right)\mathrm{Tr}\left(\partial_\nu F^-\partial^\nu F^-\right)\right\}\,, \tag{A8}$$

and

$${t^{(8)}}_{abcd}\,\partial_\mu F^{+a}\partial^\mu F^{-b}\partial_\nu F^{+c}\partial^\nu F^{-d} \;=\; 4\mathrm{Tr}\left(\partial_\mu F^+\partial_\nu F^+\right)\mathrm{Tr}\left(\partial^\mu F^-\partial^\nu F^-\right)\,, \tag{A9}$$

$${t^{(8)}}_{abcd}\,\partial_\mu F^{+a}\partial_\nu F^{-b}\partial^\mu F^{+c}\partial^\nu F^{-d} \;=\; 4\mathrm{Tr}\left(\partial_\mu F^+\partial^\mu F^+\right)\mathrm{Tr}\left(\partial_\nu F^-\partial^\nu F^-\right)\,, \tag{A10}$$

from which we find that

$${t^{(8)}}_{abcd}\,\partial_\mu F^a\partial^\mu F^b\partial_\nu F^c\partial^d = 4{t^{(8)}}_{abcd}\left[\partial_\mu F^{+a}\partial^\mu F^{-b}\partial_\nu F^{+c}\partial^\nu F^{-d} + \tfrac{1}{2}\partial_\mu F^{+a}\partial_\nu F^{-b}\partial^\mu F^{+c}\partial^\nu F^{-d}\right]\,. \tag{A11}$$

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