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Hidden-sector current-current correlators in holographic gauge mediation

Paul McGuirk*

Laboratory for Elementary-Particle Physics,

Cornell University, Ithaca, New York, 14853, USA and

Department of Physics, University of Wisconsin-Madison, Madison, Wisconsin, 53706, USA

Abstract

We discuss gauge mediation in the case where the hidden sector is strongly coupled but, via the gauge-gravity correspondence, admits a weakly-coupled description in terms of a warped higher-dimensional spacetime. In this framework, known as holographic gauge mediation, the visible-sector gauge group is realized in the gravitational description by probe D-branes and the non-supersymmetric state by normalizable perturbations to the geometry. Using the formalism of general gauge mediation, supersymmetry-breaking soft terms in the visible sector can be related to the two-point functions of the hidden-sector current superfield that couples to the visible-sector gauge group. Such correlation functions cannot be directly calculated in the strongly coupled field theory but can be determined using the gauge-gravity correspondence and holographic renormalization. We explore this procedure by considering a toy geometry where such two-point functions can be explicitly calculated. Unlike previous implementations of holographic mediation where sfermion masses were not calculable directly in a purely holographic framework, such terms are readily obtained via these correlators, while (due to the simplicity of the geometry considered) the visible-sector gauginos remain massless to leading order in the visible-sector coupling.

* mcguirk@cornell.edu

I. INTRODUCTION

The search for physics beyond the standard model is motivated in large part by the search for naturalness. In particular, the electroweak scale in the standard model is tied directly to the mass of a fundamental scalar and since the former exhibits an exponential hierarchy compared to the Planck scale ($m_Z \sim 10^{-16} m_{\mathrm{P}}$) while the latter is quadratically sensitive to short-wavelength physics, one is left either with the acceptance of an unnatural amount of fine tuning of classical effects against quantum effects or the acceptance of the existence of new physics.

One possibility for new physics is supersymmetry (SUSY) which addresses the electroweak hierarchy problem by tempering the quantum corrections to certain operators such as scalar masses. If the universe were in a supersymmetric state, then for every fermionic field there would be a bosonic field with the same charge and (in a Minkowski spacetime) same mass and vice-versa. The lack of discovery of such superpartners indicates that our universe is not in such a state. Nevertheless, if supersymmetry exists as a spontaneously broken symmetry, then the protection from quantum effects is to a large extent preserved.

In order for such a scenario to be phenomenologically viable, the spontaneous breaking of supersymmetry must not occur in the visible sector, e.g. the minimally supersymmetric standard model (MSSM), but within another set of fields. In a theory with finite m_{P}, this breaking will be communicated to the visible sector via quantum effects related to the superconformal anomaly [1] and classically by irrelevant operators which are generically expected to be present [2].

Whether or not gravity is present, the breaking of supersymmetry can also be mediated to the standard model via so-called messenger fields which transform under the visible-sector gauge group and couple to the sector in which supersymmetry is broken [3] (see also [4] for a review). This mechanism, known as gauge mediation, has the advantage that the effects of supersymmetry breaking respect the flavor structure of the visible sector, unlike mediation from irrelevant operators which will generically result in unacceptable flavor-changing neutral currents[1].

In any of these scenarios, the effects of the breaking of supersymmetry on the visible sector

[1] Although anomaly mediation also respects this flavor structure, it leads to spontaneous breaking of $\mathrm{U}(1)_{\mathrm{em}}$ unless supplemented by comparable contributions from gauge mediation [5], mediation from irrelevant operators [6], or both [7].

can be captured by so-called soft terms: operators in the effective Lagrangian of the visible-sector that do not reintroduce quadratic sensitivity on ultraviolet physics even though they do not respect supersymmetry. Since supersymmetry, if it exists as an underlying symmetry, is broken, all of the phenomenological implications of supersymmetry result from these soft terms (see [8] for reviews) and it is thus of clear importance to study them in various scenarios especially in light of the increasing experimental constraints on simple models.

In this work, we will focus on gauge mediation of supersymmetry breaking. Even within this class of scenarios, there are a number of possibilities based on the nature of the messenger sector. The minimal scenario involves messengers that are neutral under any gauge group of the SUSY-breaking sector, but couple to the SUSY-breaking-sector fields through superpotential operators. In direct gauge mediation scenarios, such as those in [9] the messenger fields are charged under SUSY-breaking-sector gauge groups, and indeed there is little distinction between the SUSY-breaking sector and the messenger sector. Semi-direct models are a compromise between these two scenarios in which the messenger sectors couple to the SUSY-breaking sector (in the original semi-direct proposal [10], the messengers had only gauge couplings to the SUSY-breaking sector) but do not participate in the breaking of supersymmetry.

In [11] (see also [12–14]), a general framework for gauge mediation scenarios was presented. In the limit where the visible-sector gauge couplings are taken to zero, the visible-sector gauge group is realized as a global symmetry of the messenger and SUSY-breaking sectors, which together compromise the hidden sector. In models of gauge mediation, the hidden sector and visible sector decouple in this limit. The conserved hidden-sector current j_μ corresponding to this global symmetry is a component field of a linear superfield $\mathcal{J}$ which contains also a scalar component and a spinor component. It was shown in [11] that once the visible-sector gauge group becomes weakly gauged, visible-sector soft terms arise and can be given in terms of two-point functions of these currents[2].

Although the couplings between the visible and hidden sectors are small, the hidden sector itself may be coupled and such current-current correlators cannot be directly calculated. However, certain strongly coupled gauge theories admit a weakly coupled dual description in terms of a classical gravitational theory on a curved spacetime of higher dimension [15]

[2] An important exception to this in the MSSM is the Lagrangian-level operator $B\mu H_u H_d$ (where H_u and H_d are the two Higgs doublets of the MSSM and μ appears in the analogous superpotential coupling), which must be treated differently as in [13].

(see [16] for reviews). This duality, known as (non-)AdS/(non-)CFT or the gauge-gravity correspondence, is the best understood example of the holographic principle [17]. If the gauge theory is supersymmetric, then a non-supersymmetric state can be constructed by considering particular perturbations to the geometry. For example, it was argued in [18] that the addition of a small number of anti-branes to the geometry of Klebanov and Strassler [19] is dual to the preparation of a metastable non-supersymmetric state in a particular[3] $\mathcal{N}_4 = 1$ gauge theory[4].

In the limit of vanishing visible-sector gauge coupling, the visible-sector gauge group becomes a global symmetry. If the hidden sector admits a dual gravity description, this global symmetry can be realized by D-branes, known as flavor branes, that extend along the holographic direction [21]. According to the AdS/CFT dictionary, some of the open-string excitations of these D-branes are dual to the components of the current superfield $\mathcal{J}$. The calculation of the classical two-point functions of these components thus corresponds to the calculation of the current-current correlators in the strongly-coupled dual field theory.

This paper will explore this procedure of calculating hidden-sector current-current correlators using holographic techniques. Such holographic models of supersymmetry breaking were first considered in [22] and further studied in [23, 24] (see also [25]) where certain soft terms (namely the mass of the visible-sector gaugino) were deduced via dimensional reduction to 4d with additional soft terms following from gaugino mediation [26]. The approach here differs from this previous work in that we make use of the formalism of general gauge mediation [11] to calculate soft terms in terms of current-current correlators. A drawback of this procedure is that it is difficult to precisely calculate such correlators in the types of geometries considered in [22, 23] and so in order to be able to calculate explicitly, we consider a toy geometry described below. We emphasize that the barrier to explicitly calculate two-point functions in the gravity picture is of an entirely different nature than the barrier in the direct gauge picture; in the former the complication is the inability to analytically solve in curved spacetime classical equations of motion, while in the latter the barrier is the inapplicability of perturbative techniques in a quantum theory. As a consequence of the simple geometry however, the visible-sector gauginos will remain massless in the construction we consider here, and we leave the analysis of more phenomenologically viable geometries

[3] $\mathcal{N}_D$ denotes the amount of supersymmetry in D spacetime dimensions. Hence, $\mathcal{N}_4 = 1$ has four supercharges while both $\mathcal{N}_4 = 2$ and $\mathcal{N}_5 = 1$ have eight supercharges.

[4] See, however, [20] for possible concerns with this procedure.

for future work.

Our paper is organized as follows. The formalism of general gauge mediation is reviewed in section II. In section III, we summarize the framework of holographic gauge mediation and introduce the geometry that we consider in this work. In section IV we deduce the classical 5d effective field theory (EFT) describing the open string fluctuations that is dual to the generating functional for current correlators in the dual field theory. We deduce this EFT in two different ways: in section IV A we find the on-shell action by dimensional reduction from the well-known action of a D-brane, and in section IV B we find the off-shell action by making use of the known off-shell action in the 5d Minkowski spacetime $R^{4,1}$. In section V, we calculate the current-current correlators in the supersymmetric case for the both the case of massless and massive messengers. This is done using the techniques of holographic renormalization which we also briefly review. In section VI, we extend this calculation to a non-supersymmetric example and in doing so we effectively determine the visible-sector soft terms which are the main subject of interest of this work. Section VII contains some concluding remarks and our conventions are presented in appendix A.

We note also that general gauge mediation has been considered together with warped geometries elsewhere in the literature [27]. The essential difference between [27] and the work below is that in the former, the SUSY-breaking sector is realized in an entirely field theoretic way in the warped geometry, while here the SUSY-breaking sector is realized by the geometry itself.

II. GENERAL GAUGE MEDIATION

As discussed in the introduction, general gauge mediation [11] relates visible-sector soft terms to hidden-sector current-current correlators. The underlying assumption in the formalism is that in the limit that the visible-sector gauge coupling g_{vis} vanishes, the visible sector and hidden sector decouple (this implicitly requires $m_{\mathrm{P}} \to \infty$). For simplicity of presentation, we consider the case in which the visible-sector gauge group is $\mathrm{U}(1)$. The hidden sector then possess a conserved current j^μ, e.g. a real vector satisfying the condition (here we are working on the Minkowski spacetime $R^{3,1}$)

$$\partial^\mu j_\mu = 0. \tag{2.1}$$

In an $\mathcal{N}_4 = 1$ theory, this is a component of a linear superfield $\mathcal{J}$ which in $\mathcal{N}_4 = 1$ superspace takes the form

$$\mathcal{J} = J + \mathrm{i}\theta j - \mathrm{i}\bar{\theta}\bar{j} - \theta\sigma^\mu\bar{\theta}j_\mu + \frac{1}{2}\theta^2\bar{\theta}\bar{\sigma}^\mu\partial_\mu j - \frac{1}{2}\bar{\theta}^2\theta\sigma^\mu\partial_\mu\bar{j} - \frac{1}{4}\theta^2\bar{\theta}^2\partial^2 J, \tag{2.2}$$

in which j is a two-component spinor and J is a real scalar. With these conditions, $\mathcal{J}$ satisfies $D^2\mathcal{J} = \bar{D}^2\mathcal{J} = 0$ where D is the usual supercovariant derivative

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + \mathrm{i}\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu, \quad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - \mathrm{i}\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu.$$

Upon weakly gauging the visible sector, this superfield couples to the visible-sector vector superfield which, in the Wess-Zumino gauge, takes the form

$$\mathcal{V} = -\theta\sigma^\mu\bar{\theta}A_\mu + \mathrm{i}\theta^2\bar{\theta}\bar{\lambda} - \mathrm{i}\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D. \tag{2.3}$$

The coupling between the current and vector superfields is

$$2g_{\text{vis}}\int_{R^{3,1}}\mathrm{d}^4x \int \mathrm{d}^4\theta\, \mathcal{V}\mathcal{J} = g_{\text{vis}}\int_{R^{3,1}}\mathrm{d}^4x \left\{DJ - \lambda j - \bar{\lambda}\bar{j} - A^\mu j_\mu\right\}, \tag{2.4}$$

in which g_{vis} is the visible-sector gauge coupling.

It is convenient to cast the two-point correlators as [11]

$$\begin{aligned}
\langle J(x)J(0)\rangle =& \frac{1}{x^4}C_0(x^2M^2),\\
\langle j_\alpha(x)\bar{j}_{\dot{\alpha}}(0)\rangle =& -\mathrm{i}\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\left(\frac{1}{x^4}C_{1/2}(x^2M^2)\right),\\
\langle j_\mu(x)j_\nu(0)\rangle =& (\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)\left(\frac{1}{x^4}C_1(x^2M^2)\right),\\
\langle j_\alpha(x)j_\beta(0)\rangle =& \epsilon_{\alpha\beta}\frac{1}{x^5}B_{1/2}(x^2M^2),
\end{aligned} \tag{2.5}$$

where M is some characteristic mass scale. In the supersymmetric limit [11],

$$C_0 = C_{1/2} = C_1, \quad B_{1/2} = 0. \tag{2.6}$$

The Fourier transforms take the form

$$\begin{aligned}
\langle J(k)J(q)\rangle =& C_0(k^2/M^2),\\
\langle j_\alpha(k)\bar{j}_{\dot{\alpha}}(q)\rangle =& -\sigma^\mu_{\alpha\dot{\alpha}}k_\mu C_{1/2}(k^2/M^2),\\
\langle j_\mu(k)j_\nu(q)\rangle =& -(k^2\eta_{\mu\nu} - k_\mu k_\nu)C_1(k^2/M^2),\\
\langle j_\alpha(k)j_\beta(q)\rangle =& \epsilon_{\alpha\beta}MB_{1/2}(k^2/M^2),
\end{aligned} \tag{2.7}$$

where we have, and will in what follows, suppressed the momentum-conserving delta function

$$\left(2\pi\right)^4\delta^4\left(k+q\right). \tag{2.8}$$

We use the same notation to denote the functions C_a and B and their Fourier transforms

$$\begin{aligned} C_a\left(k^2/M^2\right) &= \int_{R^{3,1}} \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}k\cdot x}\frac{1}{x^4}C_a\left(x^2M^2\right), \\ B_{1/2}\left(k^2/M^2\right) &= \int_{R^{3,1}} \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}k\cdot x}\frac{1}{Mx^5}B_{1/2}\left(x^2M^2\right). \end{aligned} \tag{2.9}$$

In general, these integrals require the introduction of a UV cutoff Λ, the dependence on which is suppressed in the above formulae.

A central result of [11] is that the visible-sector soft masses (except again for $B\mu$-like terms) can be expressed to leading order in g_{vis} in terms of these two-point functions. For the visible-sector gaugino corresponding to the partner of the U(1) gauge-boson,

$$m_{1/2} = g_{\mathrm{vis}}^2 M B_{1/2}\left(0\right). \tag{2.10}$$

For the sfermion masses, we now suppose that the visible-sector gauge group takes the form $G_{\mathrm{vis}} = \bigotimes_i G_i$ and that the sfermion transforms under the representations r_i for each of the G_i. Then,

$$m_{\tilde{f}}^2 = \sum_i g_i^4 c_2\left(r_i\right)\Gamma_i, \tag{2.11}$$

in which g_i is the coupling for G_i, $c_2\left(r_i\right)$ is the quadratic Casimir for the representation r_i of the group G_i and Γ_i is built from the current-current correlators for the corresponding group

$$\Gamma_i = -\frac{M^2}{16\pi^2}\int_0^\infty \mathrm{d}y\left\{3C_1\left(y\right) - 4C_{1/2}\left(y\right) + C_0\left(y\right)\right\}. \tag{2.12}$$

In the event of a vacuum expectation value for the scalar component of one the vector superfields (which does not violate any symmetries when the group is Abelian), there is an additional contribution which we will not consider here.

III. GEOMETRIC SETUP

We will now consider a special class of hidden sectors, namely those for which the SUSY-breaking sector is in the Maldacena limit [15]. In the simplest case of $\mathcal{N}_4 = 4$ SU(N) super Yang-Mills, this limit is obtained by first holding the 't Hooft coupling $\lambda_{\mathrm{t}} = g_{\mathrm{YM}}^2 N$ fixed and then taking the number of colors N to infinity (which of course requires taking the Yang-Mills coupling of the hidden sector $g_{\rm YM}$ to zero) and then taking $\lambda_{\rm t}$ to be large. The gauge theory in this limit is dual to classical type-IIB supergravity on the space $AdS^5 \times S^5$ where the 10d metric takes the form

$$ds_{10}^2 = \frac{r^2}{L^2}\eta_{\mu\nu}{\rm d}x^\mu {\rm d}x^\nu + \frac{L^2}{r^2}{\rm d}r^2 + L^2 {\rm d}\Omega_5^2, \tag{3.1}$$

in which ${\rm d}\Omega_5^2 = \hat{g}_{\phi\psi}{\rm d}y^\phi {\rm d}y^\psi$ is the metric for a unit S^5 and L is set by the 't Hooft coupling,

$$L^4 = 4\pi \ell_{\rm s}^4 g_{\rm s} N, \tag{3.2}$$

where $g_{\rm s} = g_{\rm YM}^2$ is the string coupling and $\ell_{\rm s}$ is the string length. The geometry is supported by a 5-form flux

$$F^{(5)} = \big(1 + *_{10}\big)\mathcal{F}^{(5)}, \tag{3.3}$$

in which $*_{10}$ is the 10d Hodge-$*$ and $\mathcal{F}^{(5)} = {\rm d}C^{(4)}$ with

$$C^{(4)} = \frac{r^4}{g_{\rm s} L^4}{\rm dvol}_{R^{3,1}}, \tag{3.4}$$

where ${\rm dvol}_{R^{3,1}}$ is the volume element of $R^{3,1}$. The duality can be motivated by considering a stack of N D3-branes in Minkowski spacetime $R^{9,1}$ which in this limit has an open-string description in terms of the gauge theory and a closed-string description in terms of this geometry.

A less symmetric example is the Klebanov-Strassler (KS) theory [19]. Although we will consider the simpler $AdS^5 \times S^5$ geometry in what follows, the breaking of supersymmetry has been recently studied in this geometry and so we will discuss it as an illustration of geometries suitable for holographic gauge mediation. The geometry is found by considering a collection of M fractional D3-branes (i.e. D5-branes wrapping a collapsing 2-cycle) at a conifold point. The geometry is similar to the above case in that it is a warped geometry

$$ds_{10}^2 = {\rm e}^{2A}\eta_{\mu\nu}{\rm d}x^\mu {\rm d}x^\nu + {\rm e}^{-2A}{\rm d}s_6^2, \tag{3.5}$$

in which ${\rm d}s_6^2$ is the Ricci-flat metric for a particular Calabi-Yau manifold over which the warp factor ${\rm e}^{4A}$ varies non-trivially. The conifold point additionally becomes deformed so that instead of there being a singularity, there is now a finite-sized S^3. In addition to the 5-form flux, the geometry is supported by an imaginary-self-dual 3-form flux. The dual gauge theory is no longer conformal but instead is an $\mathcal{N}_4 = 1$ theory that can be described by a series of Seiberg dualities [28]. If a number of $\overline{\text{D3}}$-branes are present on the finite S^3 where the conifold has been deformed, the geometry will no longer be supersymmetric. However, so long as the number p of $\overline{\text{D3}}$ branes is small compared to the amount of background flux, the geometry will be metastable [18]; without any D3-branes to directly annihilate against, the $\overline{\text{D3}}$s will decay only non-perturbatively, first puffing up via the Myers effect [29] to $\overline{\text{NS5}}$-branes which will then dissolve into flux and D3-branes. The influence of the $\overline{\text{D3}}$-branes on the geometry is involved [20, 30, 31], but by considering the geometry at distances far away from the tip where the geometry simplifies [32] it was argued in [30] that the perturbation to the geometry is such that the dual theory is in a non-supersymmetric state of the original theory, rather than a perturbation to the theory itself. More precisely, the duality states that for every operator $\mathcal{O}$ in the gauge theory, there is a corresponding operator Φ on the gravity side such that, if we imagine the gauge theory living on the boundary of, for example, AdS^5, the coupling of the bulk field to the field theory is

$$\int_{\delta AdS^5} \mathrm{d}^4x\, \sqrt{h}\mathcal{O}\Phi, \tag{3.6}$$

in which h is the metric induced on the boundary. Φ will satisfy a second-order differential equation and as $r \to \infty$ will behave as

$$\Phi \sim \phi_1 r^{-\Delta} + \phi_2 r^{\Delta-4}, \tag{3.7}$$

in which Δ is the mass dimension of $\mathcal{O}$. Solutions involving ϕ_2 are not normalizable and correspond to deformations of the Lagrangian in the gauge theory, $\delta\mathcal{L} \sim \phi_2\mathcal{O}$, while those involving just ϕ_1 are normalizable and correspond to a vacuum expectation value, $\langle\mathcal{O}\rangle \sim \phi_1$. The large-radius solution of [30] has only normalizable perturbations implying that the addition of $\overline{\text{D3}}$-branes produces a particular metastable state and does not change the underlying theory[5].

A global flavor group can be added to the gauge theory by adding a number of D-branes into the geometry [21]. For the warped geometries of type-IIB that we are considering here, the appropriate type of brane to add is a D7-brane that fills $R^{3,1}$ and wraps a non-compact 4-cycle in the transverse space. A stack of K such branes will produce an $\mathrm{SU}(K)$ flavor group. In addition, the matter content of the dual gauge theory will be modified by the addition of quarks: matter that transforms under a bifundamental of the flavor group and

[5] We again note the possible objections raised in [20].

the dual gauge group[6]. In the brane picture, these correspond to open strings that stretch from the (fractional or elementary) D3-branes that produce the geometry and the D7-branes so that the mass of the quarks is set by the position of the D7-branes. In the case when the number of flavor branes is much smaller than the number of color branes, the D7-branes may be considered in the probe approximation where the backreaction of the D7-branes can be neglected, an approximation which we make here. In addition to the gauge couplings, the quarks may possess superpotential couplings to other matter in the hidden sector [33]. One of the excitations of the D7-branes is the 1-form A_μ that acts as the 4d gauge field once the flavor group is weakly gauged and thus couples to a current on a boundary theory via $\mathcal{L} \sim j^\mu A_\mu$. That is, if we identify this flavor group as the visible-sector gauge group, the open-string field A_μ is dual to the current j^μ discussed in section II.

We now have the ingredients to put together a dual gravity description of gauge mediation as in [22]:

1. Begin with a theory of matter and gauge group $G_{\rm hid}$ that admits a geometric description via the gauge-gravity correspondence. This theory will function as the SUSY-breaking sector.

2. Add D7-branes[7] to the geometry, giving rise to a visible-sector group $G_{\rm vis}$ and quarks that transform under $G_{\rm vis}$ and $G_{\rm hid}$. These quarks (or rather their bound states) which will serve as messengers. The messengers and SUSY-breaking sector together constitute the hidden sector. At this point, $G_{\rm vis}$ is a global symmetry in the dual theory and there is a corresponding conserved current j_μ constructed from hidden-sector fields.

3. Prepare a SUSY-breaking state in the hidden sector. In the geometry, this corresponds to a SUSY-breaking normalizable perturbations to the geometry from the addition of non-SUSY sources. This state should be metastable, though we will not address this issue here.

4. Calculate the classical two-point functions of A_μ and other fields related to it via supersymmetry. This is equivalent to calculating the two-point functions for j_μ and its related fields in the dual theory.

[6] For clarity, we emphasize that the terms "flavor" and "quark" are used only in analogy with the standard model and not related to the corresponding concepts in the visible sector.

[7] Of course, in other classes of solutions, different sorts of branes would need to be used here.

5. Weakly gauge G_{vis} and introduce the visible-sector matter. On the gravity side, this requires gluing the warped geometry into a compact space and introducing (for example) a network of intersecting 7-branes. Fortunately, all that is necessary for calculating the soft considered here is knowledge of the representations and visible-sector gauge couplings. Such soft terms can be determined from the current-current correlators as in section II.

In the cases studied in [22, 23], the D7s were taken in the probe approximation and the breaking of supersymmetry occurs whether are not they are added. Such modes are thus closely related to models of semi-direct gauge mediation [10]: the messengers couple to the hidden sector via gauge and superpotential couplings but are not involved in the participation of the breaking of supersymmetry. As in section II, we will take the case of a single D7-brane, the extension to larger rank being a straightforward generalization.

A significant barrier to this procedure is the non-trivial geometries involved. In particular, even if the background corresponding to the SUSY-breaking sector is known before the breaking, the addition of the non-SUSY sources will backreact on the geometry and fluxes in a non-supersymmetric way and is difficult to compute. Once this is known, the equations of motion for the open-string modes have to be solved. Although the behavior along the radial direction will be under relatively good control, many of the relevant fields will transform non-trivially under the isometries of the angular space and so will not be constant along the internal directions (even for the lowest-lying state) and so the corresponding Laplace-Beltrami equation is difficult to solve[8]. For the sake of calculability, we will model the warped geometry as[9]

$$\mathrm{d}s_{10}^2 = \mathrm{e}^{2A}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \mathrm{e}^{-2\bar{B}}\left(\mathrm{d}r^2 + r^2\mathrm{d}\Omega_5^2\right), \tag{3.8}$$

supported by the RR-potential

$$C^{(4)} = g_{\mathrm{s}}^{-1}\mathrm{e}^{4C}\mathrm{dvol}_{R^{3,1}}. \tag{3.9}$$

The dilaton will be taken to be a constant, $\Phi = \log g_{\mathrm{s}}$, and the remaining closed string fields to vanish. The functions A, $\bar{B}$, and C, are taken to be functions of r alone. We will

[8] Note that this remains true even in the large-radius region of the KS solution where the internal Calabi-Yau is a cone over the homogeneous space $T^{1,1}$ [32, 34, 35]. Although a procedure exists for a harmonic analysis for such manifolds, the angular space wrapped by the D7 will not be as symmetric.

[9] Note that more generally we could have different factors multiplying the $\mathrm{d}r^2$ piece and the $\mathrm{d}\Omega_5^2$ piece, but by a redefinition of r they can be set equal.

in addition consider the case in which the warp factor takes the AdS-form $A = \log r/L$ before supersymmetry breaking. In this case the gauge theory is conformal before the addition of the flavor branes and, as argued in [30], such a theory cannot spontaneously break supersymmetry while preserving Lorentz symmetry. However, we will find below that the correlators of interest do not obey the relationships expected from supersymmetry, suggesting that SUSY is broken after the D7 is added.

Finally, we note that the dual theory will have extended supersymmetry. Our interest is only in the coupling to the visible sector which we will take to be only $\mathcal{N}_4 = 1$. We will therefore only couple part of the hidden sector to the visible sector, namely though the operator $\sim \int \mathrm{d}^4\theta\, \mathcal{J}\mathcal{V}$ where $\mathcal{V}$ is an $\mathcal{N}_4 = 1$ vector multiplet.

Explicitly, we take the coordinates on the S^5 to be

$$\begin{aligned}
x^4 =& r\sin(\varphi_5)\sin(\varphi_4)\sin(\varphi_3)\sin(\varphi_2)\sin(\varphi_1)\,,\\
x^5 =& r\cos(\varphi_5)\sin(\varphi_4)\sin(\varphi_3)\sin(\varphi_2)\sin(\varphi_1)\,,\\
x^6 =& r\cos(\varphi_4)\sin(\varphi_3)\sin(\varphi_2)\sin(\varphi_1)\,, \qquad (3.10)\\
x^7 =& r\cos(\varphi_3)\sin(\varphi_2)\sin(\varphi_1)\,,\\
x^8 =& r\cos(\varphi_2)\sin(\varphi_1)\,,\\
x^9 =& r\cos(\varphi_1)\,, \qquad (3.11)
\end{aligned}$$

so that $\varphi_5 \in [0, 2\pi)$ while $\varphi_{i\neq 5} \in [0, \pi)$. We place the D7 at a radial distance $r = \mu$ which we arrange by taking $x^8 = 0$, $x^9 = \mu$. The metric induced onto the D7-brane is

$$\begin{aligned}
\mathrm{d}s_8^2 =& \mathrm{e}^{2A}\eta_{\mu\nu} + \mathrm{e}^{-2B}\frac{L^2}{\rho^2}\mathrm{d}\rho^2 + \mathrm{e}^{-2B}L^2\mathrm{d}\Omega_3^2\\
=& \tilde{g}_{mn}\mathrm{d}x^m\mathrm{d}x^n + \mathrm{e}^{-2B}L^2\breve{g}_{\phi\psi}\mathrm{d}y^\phi\mathrm{d}y^\psi, \qquad (3.12)
\end{aligned}$$

in which $\mathrm{d}\Omega_3^2$ is the line element for a unit S^3, ρ is defined by the relationship $r^2 = \rho^2 + \mu^2$, and $B = \bar{B} + \log L/\rho$. We denote the non-compact 5d part of the worldvolume by $\mathcal{M}$.

IV. 5D EFFECTIVE FIELD THEORY

Our goal is to calculate the two-point correlation functions of the component fields of the current superfield (2.2). The duality relates the generating functional on the gauge theory side to the classical action on the gravity side. We will determine the latter in two ways.

In section IV A we perform a dimensional reduction of the 8d action that describes the low-energy excitations of the D7-brane. The resulting 5d theory will be useful in that it be valid whether or not supersymmetry is broken. In section IV B, the off-shell action is determined and written in $\mathcal{N}_4 = 1$ superspace language, using the flat spacetime result as a bootstrap. This method will only be effective when the closed string background is supersymmetric, since only the open-string modes are taken off-shell. However, this action is needed since the scalar component of the chiral superfield couples to the scalar component of the $\mathcal{N}_4 = 1$ vector superfield and the latter is an auxiliary field. Note that if we did not need to make use of the off-shell 5d theory, we could the 8d equations of motion and need not perform the intermediate dimensional reduction to get a 5d action.

A. On-shell theory from dimensional reduction

The starting place for the on-shell action is the Dirac-Born-Infeld (DBI) and Chern-Simons (CS) action describing the long-wavelength dynamics of a Dp-brane

$$S^{\mathrm{DBI}}_{\mathrm{D}p} = S^{\mathrm{DBI}}_{\mathrm{D}p} + S^{\mathrm{CS}}_{\mathrm{D}p}. \tag{4.1}$$

In the 10d Einstein frame, the DBI action takes the form

$$S^{\mathrm{DBI}}_{\mathrm{D}p} = -\tau_{\mathrm{D}p} \int_{\mathcal{W}} \mathrm{d}^{p+1}\xi \left(g_{\mathrm{s}}^{-1}\mathrm{e}^{\Phi}\right)^{\frac{p-3}{4}} \sqrt{\left|\det\left(M_{\alpha\beta}\right)\right|}, \tag{4.2}$$

in which

$$M_{\alpha\beta} = \mathrm{P}\big[g_{\alpha\beta} - g_{\mathrm{s}}^{1/2}\mathrm{e}^{-\Phi/2}B_{\alpha\beta}\big] + \lambda g_{\mathrm{s}}^{1/2}\mathrm{e}^{-\Phi/2}F_{\alpha\beta}. \tag{4.3}$$

P denotes the pullback from the 10d spacetime on to the worldvolume $\mathcal{W}$ of the Dp brane,

$$\mathrm{P}\big[v_\alpha\big] = v_M \frac{\partial x^M}{\partial \xi^\alpha}, \tag{4.4}$$

where ξ^α are coordinates on $\mathcal{W}$ and are in general dynamic. In what follows, we take the static gauge $\xi^\alpha = x^\alpha$. g_{MN} is the 10d metric and $B^{(2)}$ the NS-NS 2-form which vanishes for the backgrounds that we consider here. $F_{\alpha\beta}$ are the components of the field strength for the $\mathrm{U}(1)$ $(p+1)$-dimensional vector potential living on the brane, $F^{(2)} = \mathrm{d}A^{(1)}$. The tension of a D$p$-brane is given by $\tau_{\mathrm{D}p}^{-1} = (2\pi)^p\, \ell_{\mathrm{s}}^{(p+1)} g_{\mathrm{s}}$ and we have $\lambda = 2\pi\ell_{\mathrm{s}}^2$. The Chern-Simons action is

$$S^{\mathrm{CS}}_{\mathrm{D}p} = \tau_{\mathrm{D}p} g_{\mathrm{s}} \int_{\mathcal{W}} \mathrm{P}\left[\mathcal{C} \wedge \mathrm{e}^{-B^{(2)}}\right] \wedge \mathrm{e}^{\lambda F^{(2)}}, \tag{4.5}$$

in which $\mathcal{C}$ is the formal sum of all of the RR-potentials. The non-Abelian generalization of this action is more intricate [29]; however, to leading order in $\ell_{\rm s}$ and to quadratic order in the open string fields, it can be obtained by promoting $A^{(1)}$ and the fluctuations of the position to adjoint-valued fields and taking a trace over gauge indices.

To leading order in $\ell_{\rm s}$, the action for the gauge field on D7 in the above background can be found via a Taylor expansion

$$S = -\frac{1}{4g_8^2}\int_{\mathcal{W}} \mathrm{d}^8x\sqrt{g}\Big\{g^{\alpha\beta}g^{\gamma\delta}F_{\alpha\gamma}F_{\beta\delta} - \frac{g_{\rm s}}{2\cdot 4!\sqrt{g}}\epsilon^{\alpha\beta\gamma\delta\epsilon\eta\zeta\theta}C_{\alpha\beta\gamma\delta}F_{\epsilon\eta}F_{\zeta\theta}\Big\}, \tag{4.6}$$

in which $g_8^2 = 8\pi^3\ell_{\rm s}^4$ and $\epsilon^{0\cdots 7} = +1$. After integrating by parts, this can be written as

$$\begin{aligned} S = \frac{L^3}{g_8^2}\int_{\mathcal{W}} \mathrm{d}^8x\,\sqrt{\tilde{g}}\mathrm{e}^{-3B}\sqrt{\breve{g}}\Big\{ &-\frac{1}{4}\tilde{g}^{mn}\tilde{g}^{st}F_{ms}F_{nt} - \frac{\mathrm{e}^{2B}}{2L^2}\tilde{g}^{mn}\breve{g}^{\phi\psi}\partial_m A_\phi\partial_n A_\psi \\ &+ \frac{\mathrm{e}^{2B}}{2L^2}\tilde{g}^{mn}\breve{g}^{\phi\psi}\breve{\nabla}_\phi\breve{\nabla}_\psi A_m A_n - \frac{2\rho C'}{L^4}\mathrm{e}^{4C+4B-4A}\breve{\varepsilon}^{\phi\psi\zeta}A_\phi\breve{\nabla}_\psi A_\zeta \\ &+ \frac{\mathrm{e}^{4B}}{2L^4}\breve{g}^{\phi\psi}\breve{g}^{\zeta\xi}\big(\breve{\nabla}_\phi\breve{\nabla}_\psi A_\zeta - \breve{\nabla}_\zeta\breve{\nabla}_\phi A_\psi - \breve{R}_{\phi\zeta}A_\psi\big)A_\xi \\ &+ \frac{\mathrm{e}^{2B}}{L^2}\tilde{g}^{mn}\breve{g}^{\phi\psi}\tilde{\nabla}_m A_n\breve{\nabla}_\phi A_\psi - \frac{\rho^2 B'\mathrm{e}^{4B}}{L^4}\breve{g}^{\phi\psi}A_\rho\breve{\nabla}_\phi A_\psi\Big\}, \end{aligned} \tag{4.7}$$

in which $\tilde{\nabla}_m$ is the covariant derivative built from the metric $\tilde{g}_{mn}$ for the 5d space $\mathcal{M}$. Similarly, $\breve{\nabla}_\phi$ is the covariant derivative on S^3, and the associated Ricci tensor is $\breve{R}_{\phi\psi}$, and $\breve{\varepsilon}^{\phi\psi\zeta}$ is the antisymmetric tensor on a unit S^3. $'$ denotes a derivative with respect to ρ.

The components of the connection with the legs on $\mathcal{M}$ transform as scalars under rotations of the S^3 and thus can be expanded in terms of scalar spherical harmonics

$$A_m = \sum_{l=0}^{\infty} A_m^{(l)}(x^m)\mathcal{Y}_l(y^\theta), \tag{4.8}$$

where

$$\breve{\nabla}^2\mathcal{Y}_l = -l(l+2)\mathcal{Y}_l. \tag{4.9}$$

The harmonics satisfy the orthogonality relationship

$$\int_{S^3}\mathrm{dvol}_{S^3}\mathcal{Y}_l\mathcal{Y}_{l'} = \mathcal{V}_{S^3}\delta_{ll'}, \tag{4.10}$$

in which $\mathcal{V}_{S^3} = 2\pi^2$ is the volume of a unit S^3. We impose the gauge-fixing condition

$$\tilde{g}^{mn}\tilde{\nabla}_m A_n = 0 \Rightarrow \tilde{g}^{mn}\tilde{\nabla}_m A_n^{(l)} = 0. \tag{4.11}$$

Similarly, the angular components are expanded into the 1-form harmonics[10]

$$A_\phi = \sum_{l=0}^{\infty} B^{(l)}\left(x^m\right)\breve{\nabla}_\phi \mathcal{Y}_l\left(y^\theta\right) + L\sum_{l=1}^{\infty}\left\{a^{(l,+)}\left(x^m\right)\mathcal{Y}^+_{\phi,l}\left(y^\theta\right) + a^{(l,-)}\left(x^m\right)\mathcal{Y}^-_{\phi,l}\left(y^\theta\right)\right\}, \tag{4.12}$$

where the $\mathcal{Y}^\pm_{\phi,l}$ satisfy

$$\breve{\nabla}^2\mathcal{Y}^\pm_{\phi,l} - 2\mathcal{Y}^\pm_{\phi,l} = -\left(l+1\right)^2\mathcal{Y}^\pm_{\phi,l}, \quad \breve{\nabla}^\phi\mathcal{Y}^\pm_{\phi,l} = 0, \quad \varepsilon^{\phi\psi\zeta}\breve{\nabla}_\psi\mathcal{Y}^\pm_{\zeta,l} = \pm\left(l+1\right)\breve{g}^{\phi\xi}\mathcal{Y}^\pm_{\xi,l}, \tag{4.13}$$

and the orthogonality relationship

$$\int_{S^3}\mathrm{dvol}_{S^3}\breve{g}^{\phi\psi}\mathcal{Y}^\epsilon_{\phi,l}\mathcal{Y}^{\epsilon'}_{\psi,l'} \propto \delta_{ll'}\delta^{\epsilon\epsilon'}. \tag{4.14}$$

Owing to the various orthogonality relationships, the harmonics of different types decouple from each other. For the scalar harmonics, we get, after integrating over the S^3,

$$S = \frac{1}{g_5^2}\int_{\mathcal{M}}\mathrm{d}^5x\,\sqrt{\tilde{g}}\mathrm{e}^{-3B}\sum_{l=0}^{\infty}\left\{-\frac{1}{4}\tilde{g}^{mn}\tilde{g}^{st}F^{(l)}_{ms}F^{(l)}_{nt} - \frac{l\left(l+2\right)\mathrm{e}^{2B}}{2L^2}\tilde{g}^{mn}\partial_m B^{(l)}\partial_n B^{(l)} \right.$$
$$\left. - \frac{l\left(l+2\right)\mathrm{e}^{2B}}{2L^2}\tilde{g}^{mn}A^{(l)}_m A^{(l)}_n + \frac{l\left(l+2\right)\rho^2\mathrm{e}^{4B}}{L^4}A^{(l)}_\rho B^{(l)}\right\}. \tag{4.15}$$

in which $g_5^2 = 4\pi\ell_\mathrm{s}^4 L^{-3}$ is the 5d gauge coupling. For the 1-form sector,

$$S = \frac{1}{g_5^2}\int_{\mathcal{M}}\mathrm{d}^5x\,\sqrt{\tilde{g}}\sum_{l=1}^{\infty}\left\{-\frac{\mathrm{e}^{-B}}{2}\tilde{g}^{mn}\partial_m a^{(l,\pm)}\partial_n a^{(l,\pm)}\right.$$
$$\left. - \frac{\mathrm{e}^{B}}{2L^2}\left[\left(l+1\right)^2 \pm 4\left(l+1\right)\rho C'\mathrm{e}^{4C-4A}\right]a^{(l,\pm)}a^{(l,\pm)}\right\}. \tag{4.16}$$

The remaining bosonic degrees of freedom are the transverse fluctuations of the D7-brane. To leading order in ℓ_s they enter only through the pullback of the metric in this background

$$\mathrm{P}\left[g_{\alpha\beta}\right] = g_{\alpha\beta} + \lambda^2\frac{L^2}{\rho^2}\mathrm{e}^{-2B}\delta_{ij}\partial_\alpha\Phi^i\partial_\beta\Phi^j, \tag{4.17}$$

where $\Phi^{i=1,2}$ are related to the position of the D7 brane by

$$x^8 = \lambda\Phi^1, \quad x^9 = \mu + \lambda\Phi^2. \tag{4.18}$$

The action is

$$S = -\frac{1}{2g_8^2}\int_{\mathcal{W}}\mathrm{d}^8x\,\sqrt{g}g^{\alpha\beta}\frac{\rho^2}{L^2}\mathrm{e}^{-2B}\delta_{ij}\partial_\alpha\Phi^i\partial_\beta\Phi^j. \tag{4.19}$$

[10] These are related to the more familiar vector spherical harmonics by contraction with the metric. See, e.g. [36, 37] for discussions of tensor spherical harmonics.

Expanding in scalar spherical harmonics and integrating over the S^3,

$$S = \frac{1}{g_5^2} \int_{\mathcal{M}} \mathrm{d}^5x \, \sqrt{\tilde{g}} \sum_{l=0}^{\infty} \left\{ -\frac{L^2 \mathrm{e}^{-5B}}{2\rho^2} \tilde{g}^{mn} \partial_m \Phi^{i(l)} \partial_m \Phi^{i(l)} - \frac{l\,(l+2)\,\mathrm{e}^{-3B}}{2\rho^2} \Phi^{i(l)} \Phi^{i(l)} \right\}. \tag{4.20}$$

For the fermionic degrees of freedom, we begin with the Dirac-like action of [38], which in the Einstein frame reads [39]

$$S^{\mathrm{F}}_{\mathrm{D}p} = -\frac{\mathrm{i}}{g_8^2} \int_{\mathcal{W}} \mathrm{d}^8x \left(g_{\mathrm{s}}^{-1} \mathrm{e}^{\Phi} \right)^{\frac{p-3}{4}} \sqrt{\left| \det \left(M_{\alpha\beta} \right) \right|} \bar{\Theta} P_-^{\mathrm{D}p} \left\{ \left(\mathcal{M}^{-1} \right)^{\alpha\beta} \mathrm{P} \left[\Gamma_\beta \left(\mathcal{D}_\alpha + \frac{1}{4} \Gamma_\alpha \Delta \right) \right] - \Delta \right\} \Theta, \tag{4.21}$$

in which Θ is the bispinor

$$\Theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}, \tag{4.22}$$

where $\theta_{1,2}$ are 10d Majorana-Weyl spinors[11], $P_-^{\mathrm{D}p}$ is the projection operator

$$P_\pm^{\mathrm{D}p} = \frac{1}{2} \left(1 \pm \Gamma_{\mathrm{D}p} \right) = \frac{1}{2} \begin{pmatrix} 1 & \pm \breve{\Gamma}_{\mathrm{D}p}^{-1} \\ \pm \breve{\Gamma}_{\mathrm{D}p} & 1 \end{pmatrix}, \tag{4.23}$$

where

$$\breve{\Gamma}_{\mathrm{D}p} = \mathrm{i}^{(p-2)(p-3)} \Gamma^{(0)}_{\mathrm{D}p} L \left(\mathcal{F} \right), \tag{4.24}$$

with

$$\begin{aligned} \Gamma^{(0)}_{\mathrm{D}p} =& \frac{1}{(p+1)!} \varepsilon_{\alpha_1 \cdots \alpha_{p+1}} \Gamma^{\alpha_1 \cdots \alpha_{p+1}}, \\ L\left(\mathcal{F}\right) =& \frac{\sqrt{\left| \det \left(\mathrm{P}\left[g \right] \right) \right|}}{\sqrt{\left| \det \left(M \right) \right|}} \sum_q \frac{\left(g_{\mathrm{s}} \mathrm{e}^{-\Phi} \right)^{q/2}}{q! 2^q} \mathcal{F}_{\alpha_1 \alpha_2} \cdots \mathcal{F}_{\alpha_{2q-1} \alpha_{2q}} \Gamma^{\alpha_1 \cdots \alpha_{2q}}, \end{aligned} \tag{4.25}$$

with $\mathcal{F}^{(2)} = -\mathrm{P}\left[B^{(2)} \right] + \lambda F^{(2)}$ and $\varepsilon_{\alpha_1 \cdots \alpha_{p+1}}$ is the antisymmetric tensor. The operators $\mathcal{D}_M$ and Δ are involved in the SUSY-variations of the Einstein-frame gravitini and dilatini as in appendix A 2.

The action above is subject to a gauge redundancy known as κ-symmetry where we make the identification

$$\Theta \sim \Theta + \Gamma_-^{\mathrm{D}p} \kappa, \tag{4.26}$$

in which κ is an arbitrary 10d Majorana-Weyl bispinor. We choose the gauge

$$\Theta = \begin{pmatrix} \theta \\ 0 \end{pmatrix}, \tag{4.27}$$

[11] Our fermionic conventions are presented in appendix A.

and then in the above background, the action becomes

$$S_{\mathrm{D7}}^{\mathrm{F}} = -\frac{\mathrm{i}}{2g_8^2}\int_{\mathcal{W}} \mathrm{d}^8x\,\sqrt{g}\,\bar{\theta}\Big\{g^{\alpha\beta}\Gamma_\alpha\nabla_\beta + \frac{g_\mathrm{s}}{16}g^{\alpha\beta}\breve{\Gamma}_{\mathrm{D7}}^{-1}\Gamma_\alpha \not{F}^{(5)}\Gamma_\beta\Big\}\theta, \tag{4.28}$$

with

$$\breve{\Gamma}_{\mathrm{D7}} = -\mathrm{i}\sigma^{\underline{3}} \otimes \mathbb{I}_4 \otimes \sigma^{\underline{3}} \otimes \mathbb{I}_2. \tag{4.29}$$

θ is a 10d Majorana-Weyl spinor and thus can be written as

$$\theta = \begin{pmatrix}1\\0\end{pmatrix} \otimes \lambda \otimes \chi \otimes \psi - \mathrm{i}\begin{pmatrix}1\\0\end{pmatrix} \otimes \tilde{B}_5\lambda^* \otimes \sigma^{\underline{1}}\chi^* \otimes \mathrm{i}\sigma^{\underline{2}}\psi^*, \tag{4.30}$$

where λ, χ, and ψ are SO(4,1), SO(2), and SO(3) Dirac spinors respectively. That is, λ is a spinor on $\mathcal{M}$ and ψ is a spinor on the S^3 wrapped by the D7-brane. We additionally take the ansatz that λ depends only on the coordinates on $\mathcal{M}$, ψ depends only on the 3-cycle coordinates, and χ is a constant spinor. We have

$$g^{\alpha\beta}\Gamma_\alpha \not{F}^{(5)}\Gamma_\beta = 4\not{\mathcal{F}}^{(5)}, \tag{4.31}$$

where we have used the self-duality of $F^{(5)}$ and that this operator is acting on a 10d Weyl spinor. In this setup, we can take the aichtbein for the metric 3.12 to be

$$e_\alpha{}^{\underline{\beta}} = \begin{pmatrix} \mathrm{e}^A\delta_\mu{}^{\underline{\nu}} & & \\ & \mathrm{e}^{-B}\frac{L}{\rho} & \\ & & \mathrm{e}^{-B}L\breve{e}_{\underline{\theta}}{}^{\underline{\phi}} \end{pmatrix}, \tag{4.32}$$

where underlined indices denote the non-coordinate frame and $\breve{e}_{\underline{\theta}}{}^{\underline{\phi}}$ is the dreibein for a unit S^3. Then

$$4\not{\mathcal{F}} = \frac{16\mathrm{i}C'}{g_\mathrm{s}L}\mathrm{e}^{4C-4A+B}\sigma^{\underline{1}} \otimes \mathbb{I}_4 \otimes \mathbb{I}_2 \otimes \mathbb{I}_2, \tag{4.33}$$

where we have again made use of the fact that it acts on 10d Weyl spinor.

The covariant derivative can be written in terms of the spin connection

$$\nabla_\alpha = \partial_\alpha + \frac{1}{4}\omega_\alpha{}^{\underline{\beta\gamma}}\Gamma_{\underline{\beta\gamma}}, \tag{4.34}$$

where

$$\omega_\alpha{}^{\underline{\beta\gamma}} = \frac{1}{2}e_\alpha{}^{\underline{\delta}}\big(T_{\underline{\delta}}{}^{\underline{\beta\gamma}} - T^{\underline{\beta\gamma}}{}_{\underline{\delta}} - T^{\underline{\gamma}}{}_{\underline{\delta}}{}^{\underline{\beta}}\big), \quad T^{\underline{\alpha}}{}_{\underline{\beta\gamma}} = \big(e^\beta{}_{\underline{\beta}}e^\gamma{}_{\underline{\gamma}} - e^\gamma{}_{\underline{\beta}}e^\beta{}_{\underline{\gamma}}\big)\partial_\gamma e_\beta{}^{\underline{\alpha}}. \tag{4.35}$$

For the above choice of aichtbein, the non-vanishing components of the spin connection are

$$\omega_{\phi}{}^{\underline{\psi\zeta}} = \breve{\omega}_{\phi}{}^{\underline{\psi\zeta}}, \quad \omega_{\phi}{}^{\underline{\psi 4}} = -\rho B' \delta_{\phi}^{\underline{\psi}}, \quad \omega_{\mu}{}^{\underline{4\pi}} = -\frac{\rho A'}{L} \mathrm{e}^{A+B} \delta_{\mu}^{\underline{\pi}}. \tag{4.36}$$

To leading order in ℓ_{s}, the bosonic and fermionic fluctuations decouple and the action takes the form

$$\begin{aligned} S = -\frac{\mathrm{i}L^3}{2g_8^2} \int_{\mathcal{W}} \mathrm{d}^8 x \sqrt{\tilde{g}} \mathrm{e}^{-3B} \sqrt{\breve{g}} \Big\{ & \big(\bar{\lambda} \tilde{\nabla\!\!\!/} \lambda\big) \big(\chi^\dagger \chi\big) \big(\psi^\dagger \psi\big) + \frac{\rho C'}{L} \mathrm{e}^{4C-4A+B} \big(\bar{\lambda}\lambda\big) \big(\chi^\dagger \sigma^3 \chi\big) \big(\psi^\dagger \psi\big) \\ & - \frac{3\rho B'}{2L} \mathrm{e}^{B} \big(\bar{\lambda} \gamma_{(4)} \lambda\big) \big(\chi^\dagger \chi\big) \big(\psi^\dagger \psi\big) + \frac{\mathrm{i}}{L} \mathrm{e}^{B} \big(\bar{\lambda}\lambda\big) \big(\chi^\dagger \sigma^3 \chi\big) \big(\psi^\dagger \breve{\nabla\!\!\!/} \psi\big) \Big\} + \text{c.c.} \end{aligned} \tag{4.37}$$

Under SO (5) → SO (2) × SO (3), a Dirac spinor decomposes as (see, e.g. [40])

$$\eta \to \chi_+ \otimes \psi_+ + \chi_- \otimes \psi_-, \tag{4.38}$$

where $\chi_\pm$ are SO (2) Weyl spinors while $\psi_\pm$ are SO (3) Dirac spinors. As an SO (3) spinor, ψ can be expanded in spinor spherical harmonics[12] which satisfy

$$\breve{\nabla\!\!\!/} \mathcal{Y}_{l,\pm} = \pm \mathrm{i} \left(l + \frac{3}{2} \right) \mathcal{Y}_{l,\pm}, \tag{4.39}$$

where $l = 0, 1, 2, \ldots$. They again satisfy the orthogonality condition

$$\int_{S^3} \mathrm{dvol}_{S^3} \mathcal{Y}_{l,\epsilon}^\dagger \mathcal{Y}_{l',\epsilon} \propto \delta_{ll'} \delta_{\epsilon\epsilon'}. \tag{4.40}$$

Then we take the expansion

$$\theta = \sum_{l,\sigma,\epsilon} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \lambda^{(l,\sigma,\epsilon)} \otimes \chi_\sigma \otimes \mathcal{Y}_{l,\epsilon} + \cdots, \tag{4.41}$$

where $\cdots$ indicates the terms necessary to ensure that θ is Majorana. With this expansion,

$$\begin{aligned} S = \frac{1}{g_5^2} \int_{\mathcal{M}} \mathrm{d}^5 x \sqrt{\tilde{g}} \mathrm{e}^{-3B} \sum_{l=0}^{\infty} \sum_{\epsilon,\sigma} \Big\{ & -\mathrm{i} \bar{\lambda}^{l,\sigma,\epsilon} \tilde{\nabla\!\!\!/} \lambda^{l,\sigma,\epsilon} - \frac{3\mathrm{i}\rho B' \mathrm{e}^B}{2L} \bar{\lambda}^{l,\sigma,\epsilon} \gamma_{(4)} \lambda^{l,\sigma,\epsilon} \\ & + \frac{\mathrm{i}\sigma \mathrm{e}^B}{L} \left(\epsilon \big(l + \frac{3}{2}\big) - \rho C' \mathrm{e}^{4C-4A} \right) \bar{\lambda}^{l,\sigma,\epsilon} \lambda^{l,\sigma,\epsilon} \Big\} \end{aligned} \tag{4.42}$$

The degrees of freedom should be able to be organized into $\mathcal{N}_5 = 1$ super multiplets and our interest is in the lightest vector multiplet. We can identify this multiplying by comparing against the known results in the AdS^5 case which is recovered by taking $A = \bar{B} = C =$

[12] See e.g. [37].

$\log r/L$ and $\mu = 0$. In this case, the lightest (i.e. most negative m^2) comes from the $(1,-)$ sector of the scalar descending from A_ϕ, which saturates the BF bound [41],

$$m^2 = -\frac{4}{L^2}. \tag{4.43}$$

This is the expected mass for the scalar in the $\mathcal{N}_5 = 1$ massless vector hypermultiplet [42, 43]. The corresponding Dirac fermion has mass $\frac{1}{2L}$ and comes from the $(l, \epsilon, \sigma) = (0, -, +)$ mode of the fermions. The vector component of this multiplet comes from the $l = 0$ mode of A_m and so the action for these modes is

$$S = -\frac{1}{g_5^2}\int_{\mathcal{M}} \mathrm{d}^5x\,\sqrt{\tilde{g}}\mathrm{e}^{-3B}\bigg\{\frac{1}{4}\tilde{g}^{mn}\tilde{g}^{st}F_{ms}F_{nt} + \mathrm{i}\bar{\lambda}\tilde{\gamma}^m\tilde{\nabla}_m\lambda + \frac{\mathrm{e}^{2B}}{2}\tilde{g}^{mn}\partial_m a\partial_n a \\ + \mathrm{i}m_\lambda\bar{\lambda}\lambda + \mathrm{i}\alpha\bar{\lambda}\gamma_{(4)}\lambda + \frac{1}{2}m_a^2a^2\bigg\}, \tag{4.44}$$

in which

$$L^2m_a^2 = 4\mathrm{e}^{4B}\big(1 - 2\rho C'\mathrm{e}^{4C-4A}\big), \quad Lm_\lambda = \mathrm{e}^B\big(\frac{3}{2} - \rho C'\mathrm{e}^{4C-4A}\big), \quad L\alpha = -\frac{3\rho B'}{2}\mathrm{e}^B. \tag{4.45}$$

The higher modes give rise to $\mathcal{N}_5 = 1$ hyper multiplets and massive vector multiplets. Note that although not real by itself, when $\mathrm{i}\alpha\bar{\lambda}\gamma_{(4)}\lambda$ added to the kinetic term for the fermion, the entire action is real.

Since these are all components of a single supermultiplet, it will be convenient to perform a field redefinition so that they all have the same kinetic terms. Changing the kinetic term of A_m would spoil manifest gauge invariance, and so we redefine the scalar to match the gauge kinetic term. Defining $\Sigma = \mathrm{e}^A a$, we get

$$S = -\frac{1}{g_5^2}\int_{\mathcal{M}} \mathrm{d}^5x\,\sqrt{\tilde{g}}\mathrm{e}^{-3B}\bigg\{\frac{1}{4}\tilde{g}^{mn}\tilde{g}^{st}F_{ms}F_{nt} + \mathrm{i}\bar{\lambda}\tilde{\gamma}^m\tilde{\nabla}_m\lambda + \frac{1}{2}\tilde{g}^{mn}\partial_m\Sigma\partial_n\Sigma \\ + \mathrm{i}m_\lambda\bar{\lambda}\lambda + \mathrm{i}\alpha\bar{\lambda}\gamma_{(4)}\lambda + \frac{1}{2}m_\Sigma^2\Sigma^2\bigg\}, \tag{4.46}$$

in which

$$L^2m_\Sigma^2 = \mathrm{e}^{2B}\big(\rho^2B'' - \rho^2B'^2 + 4\rho^2A'B' + \rho B' - 8\rho C'\mathrm{e}^{4C-4A} + 4\big). \tag{4.47}$$

B. Off-shell theory from flat space

$\mathcal{N}_5 = 1$ supersymmetry is generated by eight real supercharges that can be arranged into a pair of symplectic-Majorana spinors $\mathcal{R}^{i=1,2}$. In addition to the connection $A^{(1)}$ and the gaugino (the degrees of freedom of which can again be expressed as a pair of symplectic-Majorana spinors $\lambda^{1,2}$) the off-shell theory contains three real auxiliary scalar fields $X^{I=1,2,3}$ (see e.g. [44]). Under the SU(2) R-symmetry that rotates the $\mathcal{R}^i$ into each other, the gauge field is inert, while the fermionic degrees transform as a **2** and the auxiliary fields as a **3**. Under a SUSY transformation parametrized by η^i, the fields transform in flat space as

$$\begin{aligned}
\delta_\eta A_m =& \mathrm{i}\bar{\eta}_i \tilde{\gamma}_m \lambda^i, \\
\delta_\eta \Sigma =& \bar{\eta}_i \lambda^i, \\
\delta_\eta X^I =& \bar{\eta}_i \left(\sigma^I\right)^i{}_j \tilde{\gamma}^m \partial_m \lambda^j, \\
\delta_\eta \lambda^i =& -\frac{1}{2} F_{mn} \tilde{\gamma}^{mn} \eta^i + \mathrm{i}\tilde{\gamma}^m \partial_m \Sigma \eta^i + \mathrm{i} X^I \left(\sigma^I\right)^i{}_j \eta^j,
\end{aligned} \tag{4.48}$$

where $\left(\sigma^I\right)^i{}_j$ are the components of the usual Pauli matrices. The algebra closes in the sense that

$$\left[\delta_\eta, \delta_\xi\right] = 2\mathrm{i}\bar{\xi}_i \tilde{\gamma}^m \partial_m \eta^i \partial_m, \tag{4.49}$$

except when acting on the gauge field which closes only up to a gauge transformation. The off-shell action is

$$S = -\frac{1}{g_5^2} \int_{R^{4,1}} \mathrm{d}^5 x \left\{ \frac{1}{4} F_{mn} F^{mn} + \frac{1}{2} \partial_m \Sigma \partial^m \Sigma + \frac{\mathrm{i}}{2} \bar{\lambda}_i \tilde{\gamma}^m \partial_m \lambda^i - \frac{1}{2} X^I X^I \right\}. \tag{4.50}$$

The vector multiplet can be written in $\mathcal{N}_4 = 1$ language as a vector superfield and a neutral chiral superfield [45]. In particular, we can embed an $\mathcal{N}_4 = 1$ into the higher supersymmetry by considering the transformations (4.48) and taking $\eta_\mathrm{R} = 0$. Under this restricted set, the fields transform as

$$\begin{aligned}
\delta_{\eta_\mathrm{L}} A_\mu =& \mathrm{i}\bar{\eta}_\mathrm{L} \bar{\sigma}_\mu \lambda_\mathrm{L} + \mathrm{i}\eta_\mathrm{L} \sigma_\mu \bar{\lambda}_\mathrm{L}, \\
\delta_{\eta_\mathrm{L}} A_4 =& \bar{\eta}_\mathrm{L} \bar{\lambda}_\mathrm{R} + \eta_\mathrm{L} \lambda_\mathrm{R}, \\
\delta_{\eta_\mathrm{L}} \Sigma =& -\mathrm{i}\bar{\eta}_\mathrm{L} \bar{\lambda}_\mathrm{R} + \mathrm{i}\eta_\mathrm{L} \lambda_\mathrm{R}, \\
\delta_{\eta_\mathrm{L}} X^1 =& \eta_\mathrm{L} \sigma^\mu \partial_\mu \bar{\lambda}_\mathrm{R} + \mathrm{i}\eta_\mathrm{L} \partial_4 \lambda_\mathrm{L} - \bar{\eta}_\mathrm{L} \bar{\sigma}^\mu \partial_\mu \lambda_\mathrm{R} - \mathrm{i}\bar{\eta}_\mathrm{L} \partial_4 \bar{\lambda}_\mathrm{R}, \\
\delta_{\eta_\mathrm{L}} X^2 =& \mathrm{i}\eta_\mathrm{L} \sigma^\mu \partial_\mu \bar{\lambda}_\mathrm{R} - \eta_\mathrm{L} \partial_4 \eta_\mathrm{L} + \mathrm{i}\bar{\eta}_\mathrm{L} \bar{\sigma}^\mu \partial_\mu \lambda_\mathrm{R} - \bar{\eta}_\mathrm{L} \partial_4 \bar{\lambda}_\mathrm{L}, \\
\delta_{\eta_\mathrm{L}} X^3 =& \bar{\eta}_\mathrm{L} \bar{\sigma}^\mu \partial_\mu \lambda_\mathrm{L} - \mathrm{i}\bar{\eta}_\mathrm{L} \partial_4 \bar{\lambda}_\mathrm{R} - \eta_\mathrm{L} \sigma^\mu \partial_\mu \bar{\lambda}_\mathrm{R} + \mathrm{i}\eta_\mathrm{L} \partial_4 \lambda_\mathrm{R}, \\
\delta_{\eta_\mathrm{L}} \lambda_\mathrm{L} =& F_{\mu\nu} \sigma^{\mu\nu} \eta_\mathrm{L} + \mathrm{i}\left(X^3 - \partial_4 \Sigma\right) \eta_\mathrm{L}, \\
\delta_{\eta_\mathrm{L}} \lambda_\mathrm{R} =& \mathrm{i}F_{\mu 4} \sigma^\mu \bar{\eta}_\mathrm{L} + \partial_\mu \Sigma \sigma^\mu \bar{\eta}_\mathrm{L} - \mathrm{i}\left(X^1 + \mathrm{i}X^2\right) \eta_\mathrm{L}.
\end{aligned} \tag{4.51}$$

It was recognized in [45] that these are the transformation rules for the components of a chiral superfield Φ and a vector superfield $\mathcal{V}$ in the Wess-Zumino gauge under the combination of a supersymmetry transformation and a complexified gauge transformation required to maintain the Wess-Zumino condition. The superfields take the form

$$\Phi = \phi + \sqrt{2}\theta\psi + \theta^2 F + \mathrm{i}\theta\sigma^\mu\bar{\theta}\partial_\mu\phi + \frac{\mathrm{i}}{\sqrt{2}}\theta^2\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi + \frac{1}{4}\theta^2\bar{\theta}^2\partial^2\phi, \tag{4.52a}$$

$$\mathcal{V} = -\theta\sigma^\mu\bar{\theta}A_\mu + \mathrm{i}\theta^2\bar{\theta}\bar{\lambda} - \mathrm{i}\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D, \tag{4.52b}$$

in which $\partial^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu$. The curl superfield $\mathcal{W}_\alpha = -\frac{1}{4}D^2\bar{D}_\alpha\mathcal{V}$ takes the standard form

$$\begin{aligned}\mathcal{W}_\alpha = & -\mathrm{i}\lambda_\alpha + \theta_\alpha D - \frac{\mathrm{i}}{2}\left(\sigma^\mu\bar{\sigma}^\nu\right)_\alpha^{\ \beta}\theta_\beta F_{\mu\nu} + \theta^2\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\bar{\lambda}^{\dot{\alpha}} + \theta\sigma^\mu\bar{\theta}\partial_\mu\lambda_\alpha \\ & + \mathrm{i}\theta_\alpha\theta\sigma^\mu\bar{\theta}\partial_\mu D + \frac{1}{2}\left(\sigma^\mu\bar{\sigma}^\nu\right)_\alpha^{\ \beta}\theta_\beta\theta\sigma^\kappa\bar{\theta}\partial_\kappa F_{\mu\nu} - \frac{\mathrm{i}}{4}\theta^2\bar{\theta}^2\partial^2\lambda_\alpha.\end{aligned} \tag{4.52c}$$

The component fields are related to the usual 5d fields through

$$\phi = \Sigma + \mathrm{i}A_4, \quad \psi = \mathrm{i}\sqrt{2}\lambda_{\mathrm{R}}, \quad F = X^1 + \mathrm{i}X^2, \qquad \lambda = \lambda_{\mathrm{L}}, \quad D = X^3 - \partial_4\Sigma, \tag{4.53}$$

where λ_{L} and λ_{R} are the left- and right-handed components of λ^1 defined as in (A12). Under the complexified gauge transformation

$$\mathcal{V} \to \mathcal{V} + \frac{1}{2}\left(\Lambda + \Lambda^*\right), \tag{4.54}$$

where Λ is a chiral superfield, Φ transforms as

$$\Phi \to \Phi + \partial_4\Lambda. \tag{4.55}$$

Then, the action (4.50) can be written in the language of $\mathcal{N}_4 = 1$ superspace

$$S = \frac{1}{g_5^2}\int_{R^{4,1}} \mathrm{d}^5x \left\{\frac{1}{4}\int \mathrm{d}^2\theta\, \mathcal{W}^\alpha\mathcal{W}_\alpha + \frac{1}{4}\int \mathrm{d}^2\bar{\theta}\, \overline{\mathcal{W}}_{\dot{\alpha}}\overline{\mathcal{W}}^{\dot{\alpha}} + \int \mathrm{d}^4\theta \left(\frac{1}{2}\left(\Phi + \Phi\right)^* - \partial_4\mathcal{V}\right)^2\right\}, \tag{4.56}$$

where the coefficients are chosen to recover the normalization of the action in (4.50). We can couple this $\mathcal{N}_5 = 1$ theory to an $\mathcal{N}_4 = 1$ theory localized at some point $x^4 = x^4_0$ by introducing an action

$$2\int_{x^4=x^4_0} \mathrm{d}^4x \int \mathrm{d}^4\theta\, \mathcal{V}\mathcal{J} = \int_{x^4=x^4_0} \mathrm{d}^4x \left\{JD - \lambda j - \bar{\lambda}\bar{j} - A^\mu j_\mu\right\}, \tag{4.57}$$

where $\mathcal{J}$ is a current superfield (2.2).

Given this flat space off-shell theory, we can produce the off-shell theory for the vector supermultiplet on AdS^5 [27, 46]. We can apply the same techniques to deduce the off-shell action for the $\mathcal{N}_5 = 1$ theory resulting from dimensional reduction onto a supersymmetric probe D7-brane in $AdS^5 \times X^5$ where X^5 is an Einstein-Sasaki manifold[13]. The procedure is to begin by identifying the Killing spinor for the 5d theory. We can begin by considering the Killing spinors of the 10d theory. For this geometry, the Killing spinor equations are

$$\mathcal{D}_M \varepsilon = \nabla_M \varepsilon + \frac{g_s}{16} \not{F}^{(5)} \Gamma_M \left(\mathrm{i}\sigma^2\right) \varepsilon, \tag{4.58}$$

where ε is a Majorana-Weyl bispinor

$$\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}. \tag{4.59}$$

Taking $\varepsilon_2 = -\mathrm{i}\varepsilon_1$, this becomes

$$0 = \nabla_M \varepsilon_1 + \frac{\mathrm{i} g_s}{16} \not{F}^{(5)} \Gamma_M \varepsilon_1. \tag{4.60}$$

Writing

$$\varepsilon = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \epsilon \otimes \beta - \mathrm{i} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \tilde{B}_5 \eta^* \otimes \hat{B}_5 \beta^*, \tag{4.61}$$

where ϵ is an $\mathrm{SO}(4,1)$ spinor and β is an $\mathrm{SO}(5)$ spinor, this equation becomes

$$\hat{\nabla}_\phi \beta = -\frac{\mathrm{i}}{2} \hat{\gamma}_\phi \beta, \quad \tilde{\nabla}_m \epsilon = +\frac{1}{2L} \tilde{\gamma}_m \epsilon, \tag{4.62}$$

where $\hat{\nabla}_\phi$ is the covariant derivative built from the metric on X^5. The covariant derivative can be easily deduced from the spin-connection on the D7 worldvolume (4.36). Since X^5 is an Einstein-Sasaki space, the first of (4.62) has a solution. The second does as well, the explicit form of which [41, 43, 48] will be useful for us. Since $\{\mathbb{I}_4, \tilde{\gamma}_{\underline{m}}, \tilde{\gamma}_{\underline{mn}}\}$ form a basis for 4×4 matrices, we can take the ansatz

$$\epsilon = \left(a + b^{\underline{\mu}} \tilde{\gamma}_{\underline{\mu}} + c^{\underline{\mu\nu}} \tilde{\gamma}_{\underline{\mu\nu}}\right) \eta^+ + \left(d + e^{\underline{\mu}} \tilde{\gamma}_{\underline{\mu}} + c^{\underline{\mu\nu}} \tilde{\gamma}_{\underline{\mu\nu}}\right) \eta^-, \tag{4.63}$$

where $\eta^\pm$ are constant Dirac spinors satisfying $\gamma_{(4)} \eta^\pm = \pm \eta^\pm$. The r-component of the Killing spinor equation is

$$\partial_r \epsilon = -\frac{1}{2} \gamma_{(4)} \epsilon, \tag{4.64}$$

[13] A similar process should in fact work for any of the $\mathcal{N}_4 \geq 1$ compactifications of [47]. However, once supersymmetry is broken by the background, one would have to consider the supergravity multiplet as well.

which implies that $c^{\underline{\mu}\nu} = 0$ while

$$a = \sqrt{\frac{L}{r}}\bar{a}, \quad b^{\underline{\mu}} = \sqrt{\frac{r}{L}}\bar{b}^{\underline{\mu}}, \quad d = \sqrt{\frac{r}{L}}\bar{d}, \quad e^{\underline{\mu}} = \sqrt{\frac{L}{r}}\bar{e}^{\underline{\mu}}, \tag{4.65}$$

where $\bar{a}$, $\bar{b}^{\underline{\mu}}$, $\bar{d}$, and $\bar{e}^{\underline{\mu}}$ are all functions of the $R^{3,1}$ coordinates x^μ. The μ components are

$$\partial_\mu \epsilon = -\frac{r}{2L^2}\delta^{\underline{\mu}}_{\mu}\big(1+\gamma_{(4)}\big)\epsilon, \tag{4.66}$$

which are solved by

$$\bar{a} = 1, \quad \bar{b}^{\underline{\mu}} = -\frac{1}{L}\delta^{\underline{\mu}}_{\mu}x^\mu, \quad \bar{d} = 1, \quad \bar{e}^{\underline{\mu}} = 0. \tag{4.67}$$

Hence, the Killing spinor takes the form [41, 43, 48]

$$\epsilon = \left(\frac{r}{L}\right)^{-\gamma_{(4)}/2}\left(1 - \frac{x^\mu}{L}\tilde{\gamma}_\mu\left(1+\gamma_{(4)}\right)\right)\eta = \begin{pmatrix} \sqrt{\frac{r}{L}}\left(\eta_{\mathrm{L}\alpha} - \frac{\mathrm{i}r}{L^2}x^\mu\delta^{\underline{\mu}}_{\mu}\bar{\sigma}^{\underline{\mu}}_{\alpha\dot{\alpha}}\bar{\eta}^{\dot{\alpha}}_{\mathrm{R}}\right) \\ \mathrm{i}\sqrt{\frac{L}{r}}\bar{\eta}^{\dot{\alpha}}_{\mathrm{R}} \end{pmatrix}. \tag{4.68}$$

where $\eta = \eta^+ + \eta^-$. Upon dimensional reduction, ϵ generates SUSY transformations for the 5d effective field theory on $\mathcal{M}$ where r is to be treated as a function of ρ.

The off-shell theory for the $\mathcal{N}_5 = 1$ vector multiplet can be determined by again considering the restricted supersymmetry transformations characterized by $\bar{\eta}_{\mathrm{R}} = 0$. The remaining components parametrize a rigid $\mathcal{N}_4 = 1$ transformation. An $\mathcal{N}_5 = 1$ transformation is induced by

$$\bar{\epsilon}_i\mathcal{R}^i = \mathrm{i}\epsilon_{\mathrm{L}}\mathcal{R}_{\mathrm{R}} - \mathrm{i}\bar{\epsilon}_{\mathrm{L}}\bar{\mathcal{R}}_{\mathrm{R}} + \mathrm{i}\bar{\epsilon}_{\mathrm{R}}\bar{\mathcal{R}}_{\mathrm{L}} - \mathrm{i}\epsilon_{\mathrm{R}}\mathcal{R}_{\mathrm{L}} = \mathrm{i}\sqrt{\frac{r}{L}}\eta_{\mathrm{L}}\mathcal{R}_{\mathrm{R}} - \mathrm{i}\sqrt{\frac{r}{L}}\bar{\eta}_{\mathrm{L}}\bar{\mathcal{R}}_{\mathrm{R}}, \tag{4.69}$$

where after the second equality we have set $\eta_{\mathrm{R}} = 0$. If we identify η_{L} as characterizing a rigid $\mathcal{N}_4 = 1$ transformation, then the corresponding generator is

$$Q = \sqrt{\frac{r}{L}}\mathcal{Q} = \mathrm{i}\sqrt{\frac{r}{L}}\mathcal{R}_{\mathrm{R}}. \tag{4.70}$$

Q induces translations in superspace

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - \mathrm{i}\sigma^{\underline{\mu}}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\delta_{\underline{\mu}}{}^{\mu}\partial_\mu, \tag{4.71}$$

and the restricted $\mathcal{N}_5 = 1$ transformations induce translations through a warped $\mathcal{N}_4 = 1$ superspace

$$\mathcal{Q}_\alpha = \frac{\partial}{\partial\vartheta^\alpha} - \mathrm{i}\sigma^{\underline{\mu}}_{\alpha\dot{\alpha}}\bar{\vartheta}^{\dot{\alpha}}\tilde{\epsilon}_{\underline{\mu}}{}^{\mu}\partial_\mu, \tag{4.72}$$

where

$$\vartheta^\alpha = \sqrt{\frac{r}{L}}\theta^\alpha = \mathrm{e}^{A/2}\theta^\alpha. \tag{4.73}$$

The $\mathcal{N}_5 = 1$ super-Maxwell theory on AdS^5 can be recovered from (4.56) by writing [27, 46]

$$\begin{aligned}\Phi =&\phi + \sqrt{2}\vartheta\psi + \vartheta^2 F + \mathrm{i}\vartheta\sigma^{\underline{\mu}}\bar{\vartheta}\tilde{e}_{\underline{\mu}}{}^{\mu}\partial_\mu\phi + \frac{\mathrm{i}}{\sqrt{2}}\vartheta^2\bar{\vartheta}\bar{\sigma}^{\underline{\mu}}\tilde{e}_{\underline{\mu}}{}^{\mu}\partial_\mu\psi + \frac{1}{4}\vartheta^2\bar{\vartheta}^2\tilde{g}^{\mu\nu}\partial_\mu\partial_\nu\phi \\ =&\phi + \sqrt{2}\mathrm{e}^{A/2}\theta\psi + \mathrm{e}^{A}\theta^2 F + \mathrm{i}\theta\sigma^{\underline{\mu}}\bar{\theta}\delta_{\underline{\mu}}{}^{\mu}\partial_\mu\phi + \frac{\mathrm{i}}{\sqrt{2}}\mathrm{e}^{A/2}\theta^2\bar{\theta}\bar{\sigma}^{\underline{\mu}}\delta_{\underline{\mu}}{}^{\mu}\partial_\mu\psi + \frac{1}{4}\theta^2\bar{\theta}^2\partial^2\phi, \quad (4.74a)\\ \mathcal{V} =& -\vartheta\sigma^{\underline{\mu}}\bar{\vartheta}\tilde{e}_{\underline{\mu}}{}^{\mu}A_\mu + \mathrm{i}\vartheta^2\bar{\vartheta}\bar{\lambda} - \mathrm{i}\bar{\vartheta}^2\vartheta\lambda + \frac{1}{2}\vartheta^2\bar{\vartheta}^2 D \\ =& -\theta\sigma^{\underline{\mu}}\bar{\theta}\delta_{\underline{\mu}}{}^{\mu}A_\mu + \mathrm{i}\mathrm{e}^{3A/2}\theta^2\bar{\theta}\bar{\lambda} - \mathrm{i}\mathrm{e}^{3A/2}\bar{\theta}^2\theta\lambda + \frac{1}{2}\mathrm{e}^{2A}\theta^2\bar{\theta}^2 D. \quad (4.74b)\end{aligned}$$

The curl chiral superfield is likewise

$$\begin{aligned}\mathcal{W}_\alpha =& -\frac{1}{4}\mathcal{D}^2\bar{\mathcal{D}}_\alpha\mathcal{V} \\ =& -\mathrm{i}\lambda_\alpha + \vartheta_\alpha D - \frac{\mathrm{i}}{2}\left(\sigma^{\underline{\mu}}\bar{\sigma}^{\underline{\nu}}\right)_\alpha{}^\beta \vartheta_\beta\tilde{e}_{\underline{\mu}}{}^{\mu}\tilde{e}_{\underline{\nu}}{}^{\nu}F_{\mu\nu} + \vartheta^2\sigma^{\underline{\mu}}_{\alpha\dot{\alpha}}\tilde{e}_{\underline{\mu}}{}^{\mu}\partial_\mu\bar{\lambda}^{\dot{\alpha}} + \vartheta\sigma^{\underline{\mu}}\bar{\vartheta}\tilde{e}_{\underline{\mu}}{}^{\mu}\partial_\mu\lambda_\alpha \\ &+ \mathrm{i}\vartheta_\alpha\vartheta\sigma^{\underline{\mu}}\bar{\vartheta}\tilde{e}_{\underline{\mu}}{}^{\mu}\partial_\mu D + \frac{1}{2}\left(\sigma^{\underline{\mu}}\bar{\sigma}^{\underline{\nu}}\right)_\alpha{}^\beta\vartheta_\beta\vartheta\sigma^{\underline{\kappa}}\bar{\vartheta}\tilde{e}_{\underline{\mu}}{}^{\mu}\tilde{e}_{\underline{\nu}}{}^{\nu}\tilde{e}_{\underline{\kappa}}{}^{\kappa}\partial_\kappa F_{\mu\nu} - \frac{\mathrm{i}}{4}\vartheta^2\bar{\vartheta}^2\tilde{g}^{\mu\nu}\partial_\mu\partial_\nu\lambda_\alpha \\ =& -\mathrm{i}\lambda_\alpha + \mathrm{e}^{A/2}\theta_\alpha D - \frac{\mathrm{i}}{2}\mathrm{e}^{-3A/2}\left(\sigma^{\underline{\mu}}\bar{\sigma}^{\underline{\nu}}\right)_\alpha{}^\beta\delta_{\underline{\mu}}{}^{\mu}\delta_{\underline{\nu}}{}^{\nu}\theta_\beta F_{\mu\nu} + \theta^2\sigma^{\underline{\mu}}_{\alpha\dot{\alpha}}\delta_{\underline{\mu}}{}^{\mu}\partial_\mu\bar{\lambda}^{\dot{\alpha}} + \theta\sigma^{\underline{\mu}}\bar{\theta}\delta_{\underline{\mu}}{}^{\mu}\partial_\mu\lambda_\alpha \\ &+ \mathrm{i}\mathrm{e}^{A/2}\theta_\alpha\theta\sigma^{\underline{\mu}}\bar{\theta}\delta_{\underline{\mu}}{}^{\mu}\partial_\mu D + \frac{1}{2}\mathrm{e}^{-3A/2}\left(\sigma^{\underline{\mu}}\bar{\sigma}^{\underline{\nu}}\right)_\alpha{}^\beta\theta_\beta\theta\sigma^{\underline{\kappa}}\bar{\theta}\delta_{\underline{\mu}}{}^{\mu}\delta_{\underline{\nu}}{}^{\nu}\delta_{\underline{\kappa}}{}^{\kappa}\partial_\kappa F_{\mu\nu} - \frac{\mathrm{i}}{4}\theta^2\bar{\theta}^2\partial^2\lambda_\alpha, \\ & \quad (4.74c)\end{aligned}$$

where $\mathcal{D}_\alpha = \mathrm{e}^{-A/2}D_\alpha$. The same result holds for $\mathcal{M}$ with $A \to A(\rho)$. The components of the chiral superfield are as in (4.53) except with

$$\phi = \Sigma + \mathrm{i}A_{\underline{4}} = \Sigma + \mathrm{i}\mathrm{e}^{A}A_\rho. \quad (4.75a)$$

Similarly for the vector superfield,

$$D = X^3 - \partial_{\underline{4}}\Sigma = X^3 - \mathrm{e}^{A}\partial_\rho\Sigma, \quad (4.75b)$$

where here we have used that the D7 is on $AdS^5 \times X^5$ so that $A = \bar{B}$. The off-shell action follows from (4.56)

$$S = \frac{1}{g_5^2}\int_{\mathcal{M}}\mathrm{d}^5x\,\sqrt{\tilde{g}}\mathrm{e}^{-3B}\left\{\frac{1}{4}\int\mathrm{d}^2\vartheta\,\mathcal{W}^\alpha\mathcal{W}_\alpha + \frac{1}{4}\int\mathrm{d}^2\bar{\vartheta}\,\overline{\mathcal{W}}_{\dot{\alpha}}\overline{\mathcal{W}}^{\dot{\alpha}} + \int\mathrm{d}^4\vartheta\left(\frac{1}{2}(\Phi+\Phi)^* - \mathrm{e}^{A}\partial_\rho\mathcal{V}\right)^2\right\}, \quad (4.76)$$

where the additional factor of e^{3B} in the measure is introduced to match with the gauge kinetic term of (4.46). In terms of components,

$$S = -\frac{1}{g_5^2}\int_{\mathcal{M}} \mathrm{d}^5x\,\sqrt{\tilde{g}}\frac{\rho^3}{L^3}\mathrm{e}^{-3A}\bigg\{\frac{1}{4}F_{mn}F^{mn} + \frac{\mathrm{i}}{2}\bar{\lambda}_i\tilde{\gamma}^m\nabla_m\lambda^i + \frac{1}{2}\partial_m\Sigma\partial^m\Sigma + \frac{\mathrm{i}}{2}m_\lambda\bar{\lambda}_i\big(\sigma^3\big)^i{}_j\lambda^j + \frac{1}{2}\bar{m}_\Sigma^2\Sigma^2 - \frac{1}{2}X^IX^I - \beta\Sigma X^3\bigg\}, \tag{4.77}$$

in which

$$\begin{aligned}\bar{m}_\Sigma^2 =& \frac{1}{\rho^2}\mathrm{e}^{2A}\big(\rho^2A'' + 2\rho^2A'^2 - 3\rho A' - 6\big),\\ m_\lambda =& \frac{1}{2\rho}\mathrm{e}^{A}\big(3 - 2\rho A'\big),\\ \beta =& \frac{1}{\rho}\mathrm{e}^{A}\big(3 - \rho A'\big),\end{aligned}$$

where we have used that in the supersymmetric case $B = A + \log L/\rho$. In the AdS^5 case, which can be recovered by taking $\mu \to 0$,

$$\bar{m}_\Sigma^2 = -\frac{8}{L^2}, \quad m_\lambda = \frac{1}{2L}, \quad \beta = \frac{2}{L}. \tag{4.78}$$

The auxiliary fields can be easily integrated out,

$$X^{1,2} = 0, \quad X^3 = -\beta\Sigma. \tag{4.79}$$

Then the action can be written as (4.46) with

$$\begin{aligned}m_\Sigma^2 =& \frac{1}{\rho^2}\mathrm{e}^{2A}\big(\rho^2A'' + 3\rho^2A'^2 - 9\rho A' + 3\big)\\ m_\lambda =& \frac{1}{2\rho}\mathrm{e}^{A}\big(3 - 2\rho A'\big),\\ \alpha =& \frac{3\mathrm{e}^A}{\rho}\big(1 - \rho A'\big),\end{aligned}$$

in which we have written $\lambda = \lambda_1$. Note that this agrees with the result in the previous subsection in supersymmetric limit $B = A + \log L/p$. In the AdS case, we recover $Lm_\lambda = 1/2$, $L^2m_\Sigma^2 = -4$, and $\alpha = 0$.

V. SUPERSYMMETRIC CURRENT-CURRENT CORRELATORS

We now turn to the calculation of the current-current correlators which, as discussed in section II, can be used to calculate visible-sector soft terms. These two-point functions can be calculated by considering the dual classical gravity solution. We will make use of the method of holographic renormalization [49–51]. The first step in the procedure is to solve the equations of motion near the boundary of the 5d spacetime $\mathcal{M}$. The resulting on-shell action will be divergent but can be regularized by cutting off the spacetime at finite $\rho = R$. The divergences can be subtracted by the addition of an appropriate boundary term action and then the renormalized action is defined in the $R \to \infty$ limit. The solution to the field equation for a particular field Φ that is dual to a operator $\mathcal{O}$ will be given in terms of two coefficients that are set by boundary conditions. One of these coefficients can be fixed by determining a boundary condition at $\rho = \infty$ while the other requires a boundary condition at small ρ. The former coefficient gives the leading behavior at large ρ and corresponds to a source for $\mathcal{O}$ while the second corresponds to the resulting point-point function for $\mathcal{O}$. Higher-point functions can then be determined by differentiation of the one-point with respect to the source.

To employ these methods, it is convenient to define a coordinate u by

$$\frac{\mathrm{d}u}{\mathrm{d}\rho} = -\frac{2u}{\rho}\mathrm{e}^{-B}. \tag{5.1}$$

Then the metric for $\mathcal{M}$ is written as

$$\mathrm{d}s_5^2 = \mathrm{e}^{2A}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \frac{L^2}{4u^2}\mathrm{d}u^2. \tag{5.2}$$

A. Massless messengers

The gravity fields dual to the field theory operators can be inferred from (4.57). In order to fix the normalizations, we will first consider the case of AdS^5 with $\mu = 0$. Since the spacetimes that we are considering are asymptotically anti-de Sitter, the normalizations will apply also in these other cases. Many of the results of this subsection have been presented previously in the literature.

1. *Scalar current*

The interaction Lagrangian contains the term JD where D is the auxiliary component of the $\mathcal{N}_4 = 1$ vector multiplet and J is the scalar component of the current superfield. The action for D was determined only in the supersymmetric case in section IV B. The action after integrating it out in either the SUSY or non-SUSY case was determined in section IV A and since we are interested in only the on-shell action, this action is sufficient for calculating two-point functions. However, the off-shell action is required to determine the leading behavior of D and thus the duality between the boundary operator and the bulk field. Since the spacetimes we are considering are asymptotically AdS, we can apply the result of the AdS^5 case to the other spacetimes. The analysis of scalar correlators were performed early in the stages of AdS/CFT [15, 52, 53] though the analysis here follows the methods of holographic renormalization (see, e.g. [51]) with the slight difference that we consider an auxiliary field.

In the pure-AdS case, $B = 0$ and so (5.1) is solved by

$$u = \frac{L^2}{\rho^2}, \tag{5.3}$$

and the action for the scalar Σ is

$$S = -\frac{1}{2} \int_{AdS^5} \mathrm{d}^5 x \sqrt{\tilde{g}} \Big\{ \tilde{g}^{mn} \partial_m \Sigma \partial_n \Sigma + m^2 \Sigma^2 \Big\}, \tag{5.4}$$

in which $m^2 = -4/L^2$ and we have redefined the field to absorb a factor of g_5. The resulting equation of motion, using the coordinate u is

$$0 = 4u^2 \Sigma'' - 4u\Sigma' + 4\Sigma - u\kappa^2 \Sigma, \tag{5.5}$$

where we have performed a Fourier transformation on the Minkwoski spacetime and have defined $\kappa^2 = L^2 k^2$ where k^2 is the momentum. This is solved by the series expansion

$$\Sigma = u \sum_{n=0}^{\infty} \Big\{ \sigma_{(2n)} + \tilde{\sigma}_{(2n)} \log u \Big\} u^n, \tag{5.6}$$

in which $\sigma_{(0)}$ and $\tilde{\sigma}_{(0)}$ are undetermined while for $n > 0$,

$$\begin{aligned} 0 =& 4n^2 \sigma_{(2n)} + 8n \tilde{\sigma}_{(2n)} - \kappa^2 \sigma_{(2n-2)}, \\ 0 =& 4n^2 \tilde{\sigma}_{(2n)} - \kappa^2 \tilde{\sigma}_{(2n-2)}. \end{aligned} \tag{5.7}$$

The scalar current J is dual to the auxiliary field D

$$D = X^3 + \frac{2u}{L} \partial_u \Sigma = \frac{2u}{L^2} \tilde{\sigma}_{(0)} + \cdots , \tag{5.8}$$

where we have used the fact that on-shell $X^3 = -\frac{2}{L}\Sigma$ and have made a small u expansion. Since $\tilde{\sigma}_{(0)}$ is leading order term, we identify it as the source for the field theory operator J. The other undetermined coefficient $\sigma_{(0)}$ should then be identified with the response.

The regulated action is defined by cutting off the integral at some small $u = \epsilon$. On-shell, this gives

$$S_{\text{reg}} = -\frac{1}{2}\int_{u\geq\epsilon} \mathrm{d}^5x\,\sqrt{\tilde{g}}\Big\{\tilde{g}^{mn}\partial_m\Sigma\partial_n\Sigma + m^2\Sigma^2\Big\} = \frac{1}{2}\int_{u=\epsilon}\mathrm{d}^4x\,\sqrt{h}\frac{2u}{L}\Sigma\partial_u\Sigma, \tag{5.9}$$

where after the second equality we have integrated by parts, applied the equation of motion and have written the boundary metric as

$$\mathrm{d}s_4^2 = h_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu. \tag{5.10}$$

Inserting in the above solution, we find

$$S_{\text{reg}} = \frac{1}{L}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\Big\{\tilde{\sigma}_{(0)}^2\,(\log\epsilon)^2 + 2\sigma_{(0)}\tilde{\sigma}_{(0)}\log\epsilon + \tilde{\sigma}_{(0)}^2\log\epsilon + \cdots\Big\}, \tag{5.11}$$

where $\cdots$ indicates those terms that are finite or vanishing as $\epsilon\to 0$. The action diverges in the limit $\epsilon\to 0$, but the divergence can be removed by adding a counterterm action

$$S_{\text{ct}} = -\frac{1+(\log\epsilon)^{-1}}{L}\int_{u=\epsilon}\mathrm{d}^4x\,\sqrt{h}\Sigma^2. \tag{5.12}$$

The subtracted action

$$S_{\text{sub}} = S_{\text{reg}} + S_{\text{ct}}, \tag{5.13}$$

is then finite as $\epsilon\to 0$. Defining the renormalized action

$$S_{\text{ren}} = \lim_{\epsilon\to 0} S_{\text{sub}}, \tag{5.14}$$

the response of the current J to the source $\tilde{\sigma}_{(0)}$ is

$$\langle J\rangle_{\text{s}} = \frac{1}{\sqrt{\det(\eta_{\mu\nu})}}\frac{\delta S_{\text{ren}}}{\delta\tilde{\sigma}_{(0)}}. \tag{5.15}$$

Using (5.14)

$$\langle J\rangle_{\text{s}} = \lim_{\epsilon\to 0}\frac{1}{\epsilon^2\sqrt{h}}\epsilon\log\epsilon\frac{\delta S_{\text{sub}}}{\delta\Sigma}, \tag{5.16}$$

where we have used the fact that for small u

$$\Sigma = \tilde{\sigma}_{(0)}u\log u + \cdots. \tag{5.17}$$

Under $\Sigma\to\Sigma+\delta\Sigma$ at the boundary

$$\delta S_{\text{sub}} = \frac{2}{L}\int_{u=\epsilon}\mathrm{d}^4x\sqrt{h}\Big\{u\partial_u\Sigma - \left(1+(\log\epsilon)^{-1}\right)\Sigma\Big\}\delta\Sigma. \tag{5.18}$$

Inserting in the above solution the divergent parts cancel and we get

$$\big\langle J\big\rangle_{\rm s} = -\frac{2}{L}\sigma_{(0)}. \tag{5.19}$$

The two-point function is then

$$\big\langle J(k)J(-k)\big\rangle = -\left.\frac{\delta\left\langle J\left(k\right)\right\rangle_{\rm s}}{\delta\tilde{\sigma}_{(0)}\left(-k\right)}\right|_{\tilde{\sigma}_{(0)}=0} \rightarrow \frac{2D_0}{L}\left.\frac{\delta\sigma_{(0)}}{\delta\tilde{\sigma}_{(0)}}\right|_{\tilde{\sigma}_{(0)}=0}, \tag{5.20}$$

where the arrow indicates that have introduced a constant D_0 to account a possible normalization. In the AdS^5 case, it is possible to solve the equation of motion exactly and we get

$$\Sigma = N_0 u I_0\big(\kappa\sqrt{u}\big) + M_0 u K_0\big(\kappa\sqrt{u}\big), \tag{5.21}$$

where I_ν and K_ν are modified Bessel functions of the first and second kinds and $\kappa = \sqrt{\kappa^2}$ and in this expression, κ is taken to be the Euclidean momentum. Demanding that $\Sigma \to 0$ as $u \to \infty$ gives the condition $N_0 = 0$. Then a small u expansion gives

$$\Sigma = u\left[-M_0\left(\gamma + \log\kappa + \log 2\right) - \frac{M_0}{2}\log u + \cdots\right], \tag{5.22}$$

where γ is the Euler-Mascheroni constant. From this expression we read off

$$\sigma_{(0)}\left(k\right) = \tilde{\sigma}_{(0)}\left[\log\kappa^2 + 2\gamma - \log 4\right], \tag{5.23}$$

giving

$$\big\langle J\left(k\right)J\left(-k\right)\big\rangle = \frac{2D_0}{L}\bigg(\log\kappa^2 + 2\gamma - \log 4\bigg). \tag{5.24}$$

The non-analytic behavior in k is completely determined by conformal invariance and has the expected form. Additionally, we have suppressed a factor of $\left(2\pi\right)^4\delta^4\left(0\right)$ resulting from momentum conservation.

The correlator for the operator dual to a BF scalar in AdS^{d+1} was first calculated in [53] and in position space

$$\big\langle\mathcal{O}\big(x\big)\mathcal{O}\big(0\big)\big\rangle = \frac{2}{\pi^2}\frac{1}{x^4} + \left(\text{contact terms}\right). \tag{5.25}$$

Note that the dual operator J that we consider here is not precisely dual to the BF scalar Σ but is instead dual to the auxiliary field D. However, since D latter is closely related to the BF scalar we will use the result of [53] to fix the normalization. Fourier transforming and comparing to (5.24), we get $D_0 = -L$.

2. Vector current

A similar analysis applies for the vector correlators [53] though again we apply the method of holographic renormalization as in, e.g., [50]. The bulk field due to the vector current j_μ are the components A_μ of the 5d vector field. The action for the vector field is

$$S = -\frac{1}{4}\int_{AdS^5} \mathrm{d}^5x\,\sqrt{\tilde{g}}\tilde{g}^{mn}\tilde{g}^{st}F_{ms}F_{nt}, \tag{5.26}$$

resulting in the equations of motion

$$\begin{aligned} 0 =& 4u^2 A_\mu + uL^2\partial^2 A_\mu - uL^2\partial_\mu\big(\partial\cdot A\big) - 4u^2\partial_\mu\big(\partial_u A_u\big), \\ 0 =& u\partial^2 A_u - u\partial_u\big(\partial\cdot A\big), \end{aligned} \tag{5.27}$$

where $\partial\cdot A := \eta^{\mu\nu}\partial_\mu A_\nu$ and as before $\partial^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu$. In section IV A, we imposed the 5d Lorenz condition

$$\tilde{\nabla}^m A_m = 0. \tag{5.28}$$

For the metric (5.2), the non-vanishing Christoffel symbols are

$$\tilde{\Gamma}^u{}_{uu} = -\frac{1}{u}, \quad \tilde{\Gamma}^u{}_{\mu\nu} = \frac{2}{L^2}\eta_{\mu\nu}, \quad \tilde{\Gamma}^\mu{}_{u\nu} = -\frac{1}{2u}\delta^\mu_\nu, \tag{5.29}$$

and so the Lorenz condition becomes

$$0 = uL^2\partial\cdot A + 4u^2\partial_u A_u - 4uA_u. \tag{5.30}$$

Then the equations of motion become

$$4\mathrm{i}uk_\mu A_u = 4u^2\partial_u^2 A_\mu - u\kappa^2 A_\mu, \tag{5.31a}$$

$$0 = 4u^2\partial_u^2 A_u - u\kappa^2 A_u. \tag{5.31b}$$

The solution to (5.31b) is

$$A_u = \sum_{n=0}^{\infty}\big\{a_{(2n)} + \tilde{a}_{(2n)}\log u\big\}u^n, \tag{5.32}$$

in which $\tilde{a}_{(0)} = 0$ while $a_{(0)}$ and $a_{(2)}$ are undetermined. For $n > 0$,

$$\begin{aligned} 0 =& 4n\,(n-1)\,a_{(2n)} + 4\,(2n-1)\,\tilde{a}_{(2n)} - \kappa^2 a_{(2n-2)}, \\ 0 =& 4n\,(n-1)\,\tilde{a}_{(2n)} - \kappa^2\tilde{a}_{(2n-2)}. \end{aligned} \tag{5.33}$$

The homogeneous part of (5.31a) has a similar solution

$$A_\mu^{(\mathrm{H})} = \sum_{n=0}^{\infty} \{a_{\mu(2n)} + \tilde{a}_{\mu(2n)} \log u\} u^n, \tag{5.34}$$

with $\tilde{a}_{\mu(0)} = 0$, $a_{\mu(0)}$ and $a_{\mu(2)}$ undetermined, and for $n > 0$,

$$\begin{aligned} 0 =& 4n\,(n-1)\, a_{\mu(2n)} + 4\,(2n-1)\,\tilde{a}_{\mu(2n)} - \kappa^2 a_{\mu(2n-2)}, \\ 0 =& 4n\,(n-1)\,\tilde{a}_{\mu(2n)} - \kappa^2 \tilde{a}_{\mu(2n-2)}. \end{aligned} \tag{5.35}$$

For the inhomogeneous part, we write

$$A_\mu^{(\mathrm{I})} = -\mathrm{i}k_\mu \sum_{n=0}^{\infty} \{\alpha_{(2n)} + \tilde{\alpha}_{(2n)} \log u\} u^n. \tag{5.36}$$

This leads to $\tilde{\alpha}_{(0)} = 0$. For $n > 0$,

$$\begin{aligned} 0 =& 4n\,(n-1)\,\alpha_{(2n)} + 4\,(2n-1)\,\tilde{\alpha}_{(2n)} + 4a_{(2n-2)} - \kappa^2 \alpha_{(2n)}, \\ 0 =& 4n\,(n-1)\,\tilde{\alpha}_{(2n)} + 4\tilde{a}_{(2n-2)} - \kappa^2 \tilde{\alpha}_{(2n-2)}. \end{aligned} \tag{5.37}$$

We can add some of the homogeneous solution to set $\tilde{\alpha}_{(2)} = 0$.

The gauge-fixing condition in Fourier space is

$$0 = 4u^2 \partial_u - 4uA_u + \mathrm{i}Lu\kappa^\mu A_\mu, \tag{5.38}$$

where $\kappa^\mu := \eta^{\mu\nu}\kappa_\nu$. This imposes the relations

$$\begin{aligned} 0 =& \mathrm{i}L\kappa^\mu a_{\mu(2n)} + 4\,(n-1)\,a_{(2n)} + 4\tilde{a}_{(2n)} - \kappa^2 \alpha_{(2n)}, \\ 0 =& \mathrm{i}L\kappa^\mu \tilde{a}_{\mu(2n)} + 4\,(n-1)\,\tilde{a}_{(2n)} - \kappa^2 \tilde{\alpha}_{(2n)}. \end{aligned} \tag{5.39}$$

Note that the latter implies $\kappa^\mu \tilde{a}_{\mu(2)} = \kappa^\mu a_{\mu(0)} = 0$.

With this solution, the regulated action is

$$S_{\mathrm{reg}} = \frac{1}{L} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \eta^{\mu\nu} \frac{\kappa^2}{4} a_{\mu(0)} a_{\nu(0)} \log\epsilon + \cdots . \tag{5.40}$$

The divergence can be cancelled by adding the counterterm action

$$S_{\mathrm{ct}} = -\frac{L\log\epsilon}{8} \int_{u=\epsilon} \mathrm{d}^4 x\, \sqrt{h} h^{\mu\nu} h^{\sigma\tau} F_{\mu\sigma} F_{\nu\tau}. \tag{5.41}$$

Writing

$$A_\mu = \sum_{n=0} \{A_{\mu(n)} + \tilde{A}_{\mu(n)} \log\epsilon\} u^n, \tag{5.42}$$

the leading behavior of A_μ is

$$A_\mu = A_\mu^{(0)} + \mathcal{O}(u), \tag{5.43}$$

and so the response function in the dual field theory is

$$\langle j_\mu \rangle_{\rm s} = \frac{1}{\sqrt{\det(\eta_{\mu\nu})}} \frac{\delta S_{\rm ren}}{\delta A^\mu_{(0)}} = \lim_{\epsilon\to 0} \frac{1}{\epsilon^2 \sqrt{h}} \frac{\delta S_{\rm sub}}{\delta A^\mu},$$

where as before, $S_{\rm sub} = S_{\rm reg} + S_{\rm sub}$ and $S_{\rm ren}$ is the limit of this sum as $\epsilon \to 0$ and the μ index has been raised with $\eta^{\mu\nu}$. Under $A_\mu \to A_\mu + \delta A_\mu$ at the boundary,

$$\delta S_{\rm sub} = \frac{2}{L} \int_{u=\epsilon} {\rm d}^4x \, \sqrt{h} \epsilon^2 \big\{ F_{u\mu} + \frac{L^2 \log \epsilon}{8} \eta^{\sigma\kappa} \partial_\sigma F_{\kappa\mu} \big\} A^\mu. \tag{5.44}$$

Inserting in the above solution gives

$$\langle j_\mu \rangle_{\rm s} = \frac{2}{L} \left(a_{\mu(2)} - \frac{k_\mu k^\nu}{k^2} a_{\nu(2)} + \frac{\kappa^2}{4} a_{\mu(0)} \right). \tag{5.45}$$

Using that $k^\mu a_{\mu(0)} = 0$, this gives

$$\langle j_\mu \rangle_{\rm s} = \frac{2}{L} (\delta_\mu^\nu - \frac{k_\mu k^\nu}{k^2}) \left(a_{\nu(2)} + \frac{\kappa^2}{4} a_{\nu(0)} \right). \tag{5.46}$$

Note that the longitudinal part is projected out, as expected for a conserved current. Since the solution to the inhomogeneous equation is transverse, we have

$$\langle j_\mu(k) j_\nu(-k) \rangle = - \frac{\delta \langle j_\mu(k) \rangle_{\rm s}}{\delta A^\nu_{(0)}(-k)} \bigg|_{a_{\nu(0)\kappa}=0} \to -\frac{2D_1}{L} (\delta_\mu^\lambda - \frac{k_\mu k^\lambda}{k^2}) \left(\frac{\delta a_{\lambda(2)}}{\delta a^\nu_{(0)}} + \frac{\kappa^2}{4} \eta_{\lambda\nu} \right). \tag{5.47}$$

In the AdS^5 case, we can again solve the equations of motion exactly, and the solution to the homogeneous part of (5.31a) is

$$A_\mu^{(H)} = N_\mu \sqrt{u} I_1(\kappa\sqrt{u}) + M_\mu \sqrt{u} K_1(\kappa \sqrt{u}). \tag{5.48}$$

Demanding that $A_\mu \to 0$ as $u \to \infty$ sets $N_\mu = 0$. Then expanding for small u,

$$A_\mu^{(\rm H)} = \frac{M_\mu}{\kappa} + \frac{\kappa M_\mu}{4} u (\log \kappa^2 + 2\gamma - \log 4 - 1) + \frac{\kappa M_\mu}{4} u \log u + \cdots, \tag{5.49}$$

so that

$$a_{\mu(2)} = \frac{\kappa^2 a_{\mu(0)}}{4} \left(\log \kappa^2 + 2\gamma - \log 4 - 1 \right), \tag{5.50}$$

giving

$$\langle j_\mu(k) j_\nu(-k) \rangle = -\frac{D_1 L}{2} (k^2 \delta_{\mu\nu} - k_\mu k_\nu) \left(\log \kappa^2 + 2\gamma - \log 4 \right), \tag{5.51}$$

where we have again moved into Euclidean space. Note that this satisfies

$$\langle j_\mu(k) j_\nu(-k) \rangle \propto -(k^2 \delta_{\mu\nu} - k_\mu k_\nu) \langle J(k) J(-k) \rangle, \tag{5.52}$$

as expected from supersymmetry.

3. Spinor current

We now turn to the analysis of the spinor current [53, 54], making use of holographic renormalization techniques as in [55]. It was argued in [56] that the action must be supplemented by a particular boundary term,

$$S = -\mathrm{i}\int_{AdS^5} \mathrm{d}^5x\, \sqrt{\tilde{g}}\Big\{\bar{\lambda}\tilde{\gamma}^m\tilde{\nabla}_m\lambda + m\bar{\lambda}\lambda\Big\} + \frac{\mathrm{i}}{2}\int_{\delta AdS^5} \mathrm{d}^4x\, \sqrt{h}\bar{\lambda}\lambda, \tag{5.53}$$

in which $m = 1/2L$. The non-vanishing components of spin connection are

$$\tilde{\omega}_\mu{}^{4\pi} = \frac{2}{L\sqrt{u}}\delta^{\pi}_{\underline{\mu}}, \tag{5.54}$$

and so the equation of motion resulting from this action is

$$0 = \mathrm{i}\kappa_\mu\sqrt{u}\tilde{\gamma}^{\underline{\mu}}\delta_{\underline{\mu}}{}^{\mu}\lambda - 2u\gamma_{(4)}\partial_u\lambda + 2\gamma_{(4)}\lambda + \frac{1}{2}\lambda. \tag{5.55}$$

Following [53], we apply $\tilde{\gamma}^m\partial_m$, giving

$$0 = 4u^2\partial_u^2\lambda - 6u\partial_u\lambda + \frac{1}{2}\gamma_{(4)}\lambda + \frac{23}{4}\lambda - u\kappa^2\lambda. \tag{5.56}$$

Writing λ as

$$\lambda = \begin{pmatrix} \lambda_{\mathrm{L}} \\ \mathrm{i}\bar{\lambda}_{\mathrm{R}} \end{pmatrix}, \tag{5.57}$$

we get the solutions

$$\begin{aligned} \lambda_{\mathrm{L}} =& u^{3/4}\sum_{n=0}^{\infty}\big\{\lambda_{\mathrm{L}(2n)} + \tilde{\lambda}_{\mathrm{L}(2n)}\log u\big\}u^n, \\ \lambda_{\mathrm{R}} =& u^{5/4}\sum_{n=0}^{\infty}\big\{\lambda_{\mathrm{R}(2n)} + \tilde{\lambda}_{\mathrm{R}(2n)}\log u\big\}u^n, \end{aligned} \tag{5.58}$$

in which $\lambda_{\mathrm{L}(0)}$ and $\lambda_{\mathrm{L}(2)}$ are undetermined, $\tilde{\lambda}_{\mathrm{L}(0)} = 0$ and for $n > 0$,

$$\begin{aligned} 0 =& 4n\,(n-1)\,\lambda_{\mathrm{L}(2n)} + 4\,(2n-1)\,\tilde{\lambda}_{\mathrm{L}(2n)} - \kappa^2\lambda_{\mathrm{L}(2n-2)}, \\ 0 =& 4n\,(n-1)\,\tilde{\lambda}_{\mathrm{L}(2n)} - \kappa^2\tilde{\lambda}_{\mathrm{L}(2n-2)}. \end{aligned} \tag{5.59}$$

Similarly $\lambda_{\mathrm{R}(0)}$ and $\tilde{\lambda}_{\mathrm{R}(0)}$ are (for the moment) unfixed and

$$\begin{aligned} 0 =& 4n^2\lambda_{\mathrm{R}(2n)} + 8n\tilde{\lambda}_{\mathrm{R}(2n)} - \kappa^2\lambda_{\mathrm{R}(2n-2)}, \\ 0 =& 4n^2\tilde{\lambda}_{\mathrm{R}(2n)} - \kappa^2\tilde{\lambda}_{\mathrm{R}(2n-2)}. \end{aligned} \tag{5.60}$$

Writing $\lambda_{\mathrm{L}} = u^{3/4}\chi_{\mathrm{L}}$ and $\lambda_{\mathrm{R}} = u^{5/4}\chi_{\mathrm{R}}$, the Dirac equation (5.55) gives the (not independent) relations

$$\begin{aligned} 0 =& 2\chi'_{\mathrm{L}} - \kappa_\mu \sigma^{\underline{\mu}} \delta_{\underline{\mu}}{}^{\mu} \bar{\chi}_{\mathrm{R}}, \\ 0 =& 2u\bar{\chi}'_{\mathrm{R}} + \kappa_\mu \bar{\sigma}^{\underline{\mu}} \delta_{\underline{\mu}}{}^{\mu} \chi_{\mathrm{L}}. \end{aligned} \tag{5.61}$$

Matching coefficients

$$\begin{aligned} 0 =& 2n\lambda_{\mathrm{L}(2n)} + 2\tilde{\lambda}_{\mathrm{L}(2n)} - \kappa_\mu \delta_{\underline{\mu}}{}^{\mu} \sigma^{\underline{\mu}} \bar{\lambda}_{\mathrm{R}(2n-2)}, \\ 0 =& 2n\lambda_{\mathrm{L}(2n)} - \kappa_\mu \delta_{\underline{\mu}}{}^{\mu} \sigma^{\underline{\mu}} \tilde{\bar{\lambda}}_{\mathrm{R}(2n-2)}, \\ 0 =& 2n\bar{\lambda}_{\mathrm{R}(2n)} + 2\tilde{\bar{\lambda}}_{\mathrm{R}(2n)} + \kappa_\mu \delta_{\underline{\mu}}{}^{\mu} \bar{\sigma}^{\underline{\mu}} \lambda_{\mathrm{L}(2n)}, \\ 0 =& 2n\tilde{\bar{\lambda}}_{\mathrm{R}(2n)} + \kappa_\mu \delta_{\underline{\mu}}{}^{\mu} \bar{\sigma}^{\underline{\mu}} \tilde{\lambda}_{\mathrm{L}(2n)}. \end{aligned} \tag{5.62}$$

The spinor current j on the boundary couples to λ_{L}. We have

$$\lambda_{\mathrm{L}} = \lambda_{\mathrm{L}(0)} + \mathcal{O}\big(u\big), \tag{5.63}$$

and so $\lambda_{\mathrm{L}(0)}$ is the source for the dual current j and $\lambda_{\mathrm{L}(2)}$ is the response.

On-shell, the bulk part of the action (5.53) vanishes, and so the regulated action comes only from the boundary term

$$S_{\mathrm{reg}} = \frac{1}{2} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \left\{ \tilde{\lambda}_{\mathrm{R}(0)} \lambda_{\mathrm{L}(0)} + \tilde{\bar{\lambda}}_{\mathrm{R}(0)} \bar{\lambda}_{\mathrm{L}(0)} \right\} \log\epsilon + \cdots, \tag{5.64}$$

where we have used the fact that $\tilde{\lambda}_{\mathrm{L}(0)} = 0$. Making use of (5.62), this is

$$S_{\mathrm{reg}} = -\frac{1}{2} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \bar{\lambda}_{\mathrm{L}(0)} \bar{\sigma}^{\underline{\mu}} \delta_{\underline{\mu}}{}^{\mu} \kappa_\mu \lambda_{\mathrm{L}(0)} \log\epsilon + \cdots. \tag{5.65}$$

The divergences can be canceled by the counterterm

$$S_{\mathrm{ct}} = -\frac{\mathrm{i}L\log\epsilon}{2} \int_{u=\epsilon} \mathrm{d}^4 x\, \sqrt{h} \bar{\lambda} \tilde{\gamma}^\mu \tilde{\nabla}_\mu \frac{1}{2} \big(1 - \gamma_{(4)}\big) \lambda. \tag{5.66}$$

The response function is

$$\big\langle j_\alpha \big\rangle = \frac{1}{\sqrt{\det\left(\eta_{\mu\nu}\right)}} \frac{\delta S_{\mathrm{ren}}}{\delta \lambda^\alpha_{\mathrm{L}(0)}} = \lim_{\epsilon\to 0} \frac{\epsilon^{3/4}}{\epsilon^2 \sqrt{h}} \frac{\delta S_{\mathrm{sub}}}{\delta \lambda^\alpha_{\mathrm{L}}}. \tag{5.67}$$

This gives

$$\big\langle j_\alpha \big\rangle_{\mathrm{s}} = \frac{1}{2} \lim_{\epsilon\to 0} \epsilon^{-5/4} \left\{ \lambda_{\mathrm{R}\alpha} + \bar{\lambda}_{\mathrm{L}\dot{\alpha}} \frac{\delta \bar{\lambda}^{\dot{\alpha}}_{\mathrm{R}}}{\delta \lambda^\alpha_{\mathrm{L}}} + \epsilon^{1/2} \log\epsilon\, \bar{\lambda}_{\mathrm{L}\dot{\beta}} \kappa_\mu \delta_{\underline{\mu}}{}^{\underline{\mu}} \bar{\sigma}^{\underline{\mu}\dot{\beta}\gamma} \epsilon_{\gamma\alpha} \right\}, \tag{5.68}$$

where we have used the fact that on shell, $\bar{\lambda}_{\rm R}$ and $\lambda_{\rm L}$ are not independent and indeed at small u

$$\bar{\lambda}_{\rm R} = -\frac{1}{2}u^{1/2}\log u\kappa_\mu \delta_{\underline{\mu}}{}^{\mu}\bar{\sigma}^{\underline{\mu}}\lambda_{\rm L} + \cdots . \tag{5.69}$$

Thus,

$$\big\langle j_\alpha \big\rangle_{\rm s} = \frac{1}{2}\lim_{\epsilon\to 0}\epsilon^{-5/4}\big\{\lambda_{{\rm R}\alpha} + \frac{1}{2}\epsilon^{1/2}\log\epsilon\bar{\lambda}_{{\rm L}\dot{\beta}}\kappa_\mu\delta_{\underline{\mu}}{}^{\mu}\bar{\sigma}^{\underline{\mu}\dot{\beta}\gamma}\epsilon_{\gamma\alpha}\big\} = \frac{1}{2}\lambda_{{\rm R}\alpha(0)}. \tag{5.70}$$

Making use of (5.62) and (5.58), this gives

$$\big\langle j_\alpha \big\rangle_{\rm s} = -\kappa_\mu\delta_{\underline{\mu}}{}^{\mu}\sigma^{\underline{\mu}}_{\alpha\dot{\alpha}}\bigg\{\frac{1}{\kappa^2}\bar{\lambda}^{\dot{\alpha}}_{{\rm L}(2)} + \frac{1}{4}\bar{\lambda}^{\dot{\alpha}}_{{\rm L}(0)}\bigg\}. \tag{5.71}$$

The two point functions are

$$\begin{aligned}\big\langle j_\alpha\big(k\big)j_\beta\big(-k\big)\big\rangle &= \frac{\delta\left\langle j_\alpha\left(k\right)\right\rangle}{\delta\lambda^{\beta}_{{\rm L}(0)}\left(-k\right)}\Bigg|_{\lambda_{{\rm L}(0)}=0},\\ \big\langle j_\alpha\big(k\big)\bar{j}_{\dot{\beta}}\big(-k\big)\big\rangle &= \frac{\delta\left\langle j_\alpha\left(k\right)\right\rangle}{\delta\bar{\lambda}^{\dot{\beta}}_{{\rm L}(0)}\left(-k\right)}\Bigg|_{\bar{\lambda}_{{\rm L}(0)}=0}.\end{aligned} \tag{5.72}$$

So in this case,

$$\big\langle j_\alpha\big(k\big)j_\beta\big(-k\big)\big\rangle = 0, \qquad \big\langle j_\alpha\big(k\big)\bar{j}_{\dot{\beta}}\big(-k\big)\big\rangle \to -D_{1/2}\kappa_\mu\delta_{\underline{\mu}}{}^{\mu}\sigma^{\underline{\mu}}_{\alpha\dot{\alpha}}\bigg\{\frac{1}{\kappa^2}\frac{\delta\bar{\lambda}^{\dot{\alpha}}_{{\rm L}(2)}}{\delta\bar{\lambda}^{\dot{\beta}}_{{\rm L}(0)}} + \frac{1}{4}\delta^{\dot{\alpha}}_{\dot{\beta}}\bigg\}. \tag{5.73}$$

In the AdS^5 case, we have the exact solution

$$\lambda_{{\rm L}\alpha} = N_\alpha u^{5/4}I_1\big(\kappa\sqrt{u}\big) + M_\alpha u^{5/4}K_1\big(\kappa\sqrt{u}\big) \tag{5.74}$$

Again imposing that $\lambda_{{\rm L}\to 0}$ as $u\to 0$ sets $N_\alpha = 0$, and so for small u

$$\lambda_{{\rm L}\alpha} = M_\alpha u^{3/4}\bigg[\frac{1}{\kappa} + \frac{\kappa}{4}\big(\log\kappa^2 + 2\gamma - \log 4 - 1\big)u + \frac{\kappa}{4}u\log u + \cdots\bigg], \tag{5.75}$$

from which we read off

$$\lambda_{{\rm L}(2)} = \frac{\kappa^2\lambda_{{\rm L}(0)}}{4}\bigg(\log\kappa^2 + 2\gamma - \log 4 - 1\bigg), \tag{5.76}$$

and so

$$\big\langle j_\alpha\big(k\big)\bar{j}_{\dot{\beta}}\big(-k\big)\big\rangle = -\frac{D_{1/2}}{4L}k_\mu\delta_{\underline{\mu}}{}^{\mu}\sigma^{\underline{\mu}}_{\alpha\dot{\beta}}\bigg(\log\kappa^2 + 2\gamma - \log 4\bigg). \tag{5.77}$$

The correlators satisfy the relations

$$\big\langle j_\alpha\big(k\big)j_\beta\big(-k\big)\big\rangle = 0 \tag{5.78}$$

$$\big\langle j_\alpha\big(k\big)\bar{j}_{\dot{\beta}}\big(-k\big)\big\rangle \propto -k_\mu\delta_{\underline{\mu}}{}^{\mu}\sigma^{\underline{\mu}}_{\alpha\dot{\beta}}\big\langle J\big(k\big)J\big(-k\big)\big\rangle, \tag{5.79}$$

as again expected from SUSY.

B. Massive messengers

The messengers can be made massive by taking $\mu > 0$. In this case, it turns out to be easier to work with the ρ coordinate. This system was considered also in [57] and we follow that work closely, but here we go beyond just determination of the spectrum to find the full two-point functions. We have

$$A = \frac{1}{2}\log\left(\frac{\rho^2+\mu^2}{L^2}\right), \quad B = \frac{1}{2}\log\left(\frac{\rho^2+\mu^2}{\rho^2}\right). \tag{5.80}$$

From (4.46), the equation of motion for the scalar is

$$\begin{aligned} 0 =& \frac{\mathrm{e}^{3B}}{\sqrt{\tilde{g}}}\left[\sqrt{\tilde{g}}\mathrm{e}^{-3B}\tilde{g}^{mn}\partial_m\Sigma\right] - m_\Sigma^2\Sigma \\ =& \mathrm{e}^{-2A+4B}\rho\partial_\rho\left[\mathrm{e}^{4A-2B}\rho\partial_\rho\Sigma\right] - \mathrm{e}^{2A}L^2m_\Sigma^2\Sigma - \kappa^2\Sigma. \end{aligned} \tag{5.81}$$

Writing $\rho = \mu p$,

$$0 = \frac{1+p^2}{p^3}\frac{\mathrm{d}}{\mathrm{d}p}\left[p^3\left(1+p^2\right)\frac{\mathrm{d}\Sigma}{\mathrm{d}p}\right] - \nu^2\Sigma - \frac{3-2p^2-4p^4}{p^2}\Sigma, \tag{5.82}$$

in which $\nu^2 = L^4k^2/\mu^2$. Taking

$$\Sigma = p^m\left(1+p^2\right)^n P\left(p\right), \tag{5.83}$$

This equation becomes

$$\begin{aligned} 0 =& \left(1+p^2\right)\frac{\mathrm{d}^2P}{\mathrm{d}p^2} + \frac{1}{p}\left[\left(5+2m+4n\right)p^2 + \left(3+2m\right)\right]\frac{\mathrm{d}P}{\mathrm{d}p} \\ &+ \frac{1}{p^2\left(1+p^2\right)}\left[\left(2+m+2n\right)^2p^4 + \left(2+8n+2m\left(3+m+2n\right)-\nu^2\right)p^2 + \left(m+3\right)\left(m-1\right)\right]P. \end{aligned} \tag{5.84}$$

Defining $y = -p^2$,

$$\begin{aligned} 0 =& y\left(1-y\right)\frac{\mathrm{d}^2P}{\mathrm{d}y^2} + \left[-\left(3+m+2n\right)y + \left(2+m\right)\right]\frac{\mathrm{d}P}{\mathrm{d}y} \\ &- \frac{1}{4\left(y-1\right)}\left[\left(2+m+2n\right)^2y - \left(2+8n+2m\left(3+m+2n\right)-\nu^2\right) + \frac{1}{y}\left(3+m\right)\left(m-1\right)\right]P. \end{aligned} \tag{5.85}$$

This can be cast as a hypergeometric differential equation

$$0 = y\left(y-1\right)\frac{\mathrm{d}^2P}{\mathrm{d}y^2} + \left[-\left(a+b+1\right)y + c\right]\frac{\mathrm{d}P}{\mathrm{d}y} - ab\,P, \tag{5.86}$$

by taking

$$m = 1, \quad n = -\frac{1}{2}\sqrt{1-\nu^2} =: \eta. \tag{5.87}$$

Then the full solution is

$$\Sigma = p\big(1+p^2\big)^{\eta}\bigg\{M_0\, F\bigg(\frac{3}{2}+\eta, \frac{3}{2}+\eta; 3; -p^2\bigg) + N_0\, p^{-4} F\bigg(-\frac{1}{2}+\eta, -\frac{1}{2}+\eta; -1; -p^2\bigg)\bigg\}, \tag{5.88}$$

in which $F\big(a,b;c;y\big)$ is the hypergeometric function. Demanding that Σ be regular at $y=0$ sets $N_0 = 0$. The solution to (5.1) is[14]

$$\rho = \frac{L}{\sqrt{u}}\bigg(1 - \frac{\mu^2 u}{4L^2}\bigg), \quad u \in \big[0, 4L^2/\mu^2\big], \tag{5.89}$$

and a small u expansion gives

$$\Sigma = \frac{-8M_0\mu^2}{L^2\pi\nu^2}\cos\big(\pi\eta\big)u\bigg[\psi\bigg(\frac{3}{2}+\eta\bigg) + \psi\bigg(\frac{3}{2}-\eta\bigg) + \log\bigg(\frac{\mu^2}{L^2}\bigg) + 2\gamma + \log u\bigg] + \cdots, \tag{5.90}$$

in which ψ is the digamma function. This gives the correlator function (c.f. (2.5)).

$$C_0 = \frac{2D_0}{L}\bigg[\psi\bigg(\frac{3}{2}+\eta\bigg) + \psi\bigg(\frac{3}{2}-\eta\bigg) + \log\bigg(\frac{\mu^2}{L^2}\bigg) + 2\gamma\bigg], \tag{5.91}$$

in which again

$$\nu^2 = \frac{L^4 k^2}{\mu^2}, \quad \eta = -\frac{1}{2}\sqrt{1-\nu^2}. \tag{5.92}$$

In the limit where $\mu \to 0$, this agrees with the result in the conformal case.

The digamma functions have poles corresponding to resonances located at

$$k^2 = -\frac{4\mu^2}{L^4}\big(\ell+1\big)\big(\ell+2\big), \quad \ell \in \mathbb{N}, \tag{5.93}$$

agreeing with the analysis of [57].

For the vectors, the equation of motion is

$$0 = \frac{\mathrm{e}^{3B}}{\sqrt{\tilde{g}}}\partial_m\big[\sqrt{\tilde{g}}\mathrm{e}^{-3B}\tilde{g}^{mn}\tilde{g}^{st}F_{nt}\big]. \tag{5.94}$$

[14] There is an overall multiplicative factor arising as an integration constant. A choice different from the one here would change the leading term of e^{2A} by a multiplicative constant, and the result of section V A would have to be re-performed. The end of result of course would be the same. With this choice of the integration constant, naive application of the expressions for the dual stress-energy tensor presented in [58] would imply that $\langle T_{\mu\nu}\rangle \neq 0$. However, since the analysis leading to those results makes use of the Einstein equations for the 5d metric and the metric above doesn't satisfy such equations (that is, the pullback of the 10d metric onto the worldvolume of course satisfies the pullback of the Einstein equations, but does not satisfy Einstein equations built from the pullback metric alone since generally the pullback of curvature tensors are different from the curvature of the pullback of the metric), the analysis does not apply.

So,

$$0 = e^{4B}\rho\,\partial_\rho\big[e^{2A-2B}\rho\big(\partial_\rho A_\mu - \partial_\mu A_\rho\big)\big] + L^2\partial^2 A_\mu - L^2\partial_\mu\big(\partial\cdot A\big), \qquad (5.95)$$
$$0 = \partial^2 A_\rho - \partial_\rho\big(\partial\cdot A\big).$$

The Christoffel symbols are

$$\tilde{\Gamma}^\rho{}_{\rho\rho} = -\frac{1}{\rho}\big(1+\rho B'\big), \quad \tilde{\Gamma}^\rho{}_{\mu\nu} = -\frac{\rho^2 A'}{L^2}e^{2A+2B}\eta_{\mu\nu}, \quad \tilde{\Gamma}^\mu{}_{\rho\nu} = A'\delta^\mu_\nu, \qquad (5.96)$$

and so the gauge-fixing condition becomes

$$0 = \tilde{g}^{mn}\tilde{\nabla}_m A_n = e^{-2A}\partial\cdot A + \frac{\rho}{L^2}e^{2B}\big(4\rho A' + \rho B' + 1\big)A_\rho + \frac{\rho^2}{L^2}e^{2B}\partial_\rho A_\rho. \qquad (5.97)$$

With this condition, the equation of motion for A_μ is

$$-\rho\, e^{2A+2B}\big(2\rho A' + 3\rho B'\big)\mathrm{i}k_\mu A_\rho = \rho\, e^{4B}\partial_\rho\big[\rho\, e^{2A-2B}\partial_\rho A_\mu\big] - \kappa^2 A_\mu, \qquad (5.98)$$

and that for A_ρ is

$$0 = \rho^2 e^{2A+2B}\partial_\rho^2 A_\rho + \big(6\rho A' + 3\rho B' + 3\big)e^{2A+2B}\rho\partial_\rho A_\rho$$
$$+ \big(4\rho^2 A'' + \rho^2 B'' + 8\rho^2 A'^2 + 2\rho^2 B'^2 + 10\rho^2 A'B' + 10\rho A' + 4\rho B' + 1\big)e^{2A+2B}A_\rho. \quad (5.99)$$

The homogeneous part of (5.98) is

$$0 = \frac{\big(1+p^2\big)^2}{p^3}\frac{\mathrm{d}}{\mathrm{d}p}\left[p^3\frac{\mathrm{d}A_\mu}{\mathrm{d}p}\right] - \nu^2 A_\mu, \qquad (5.100)$$

the general solution to which is

$$A_\mu = \big(1+p^2\big)^{\frac{1}{2}+\eta}\left\{\left(M_\mu F\left(\frac{3}{2}+\eta, \frac{1}{2}+\eta; 2; -p^2\right) + N_\mu\, p^{-2}F\left(\frac{1}{2}-\eta, -\frac{1}{2}-\eta; 0; -p^2\right)\right\}. \quad (5.101)$$

Regularity imposes $N_\mu = 0$ and at small u,

$$A_\mu = \frac{M_\mu\cos(\pi\eta)}{\pi\nu^2}\left\{4 + \frac{\mu^2\nu^2}{L^2}\left[\psi\left(\frac{3}{2}+\eta\right) + \psi\left(\frac{3}{2}-\eta\right) + \log\left(\frac{\mu^2}{L^2}\right) + 2\gamma - 1\right]u\right\} + \cdots. \qquad (5.102)$$

From this and (5.47), we find $C_1 = C_0$ where C_0 is as in (5.91).

The Dirac equation is

$$0 = \big(\tilde{\nabla}\!\!\!/ + m_\lambda + \alpha\gamma_{(4)}\big)\lambda \qquad (5.103)$$

$$0 = \left\{e^{-A}\tilde{\gamma}^{\underline{\mu}}\delta^\mu_{\underline{\mu}}\partial_\mu + \frac{\rho}{L}e^B\gamma_{(4)}\partial_\rho + \left(\alpha + \frac{2A'\rho}{L}e^B\right)\gamma_{(4)} + m_\lambda\right\}\lambda. \qquad (5.104)$$

Again applying $\tilde{\gamma}^m \partial_m$,

$$
\begin{aligned}
0 &= \rho^2 \mathrm{e}^{2B+2A} \partial_\rho^2 \lambda + \big(5\rho A' + \rho B' + 1 + 2\alpha L \mathrm{e}^{-B}\big)\rho\, \mathrm{e}^{2B+2A} \partial_\rho \lambda \\
&+ \Big[2\rho^2\big(A'' + 3A'^2 + A'B'\big) + \rho\big(2A' + \alpha' L \mathrm{e}^{-B} + 5A'\alpha L \mathrm{e}^{-B}\big) + \big(\alpha^2 - m_\lambda^2\big)L^2 \mathrm{e}^{-2B}\Big]\mathrm{e}^{2B+2A}\lambda \\
&+ \big(\rho A' m_\lambda L + \rho m_\lambda' L\big)\mathrm{e}^{B+2A}\gamma_{(4)}\lambda - \kappa^2 \lambda.
\end{aligned}
\tag{5.105}
$$

For the left-handed spinor,

$$
0 = \big(1+p^2\big)^2 \frac{\mathrm{d}^2 \lambda_{\mathrm{L}}}{\mathrm{d}p^2} + \frac{3\,(1+p^2)\,(1+2p^2)}{p} \frac{\mathrm{d}\lambda_{\mathrm{L}}}{\mathrm{d}p} + \frac{3\,(8+7p^2)}{4}\lambda_{\mathrm{L}} - \nu^2 \lambda_{\mathrm{L}}. \tag{5.106}
$$

This is solved by

$$
\lambda_{\mathrm{L}} = \big(1+p^2\big)^{\eta-\frac{1}{4}} \left\{ M_{\mathrm{L}}\, F\!\left(\frac{3}{2}+\eta, \frac{1}{2}+\eta; 2; -p^2\right) + N_{\mathrm{L}}\, p^{-2} F\!\left(\frac{1}{2}-\eta, -\frac{1}{2}-\eta; 0; -p^2\right) \right\}. \tag{5.107}
$$

Regularity again imposes that $N_{\mathrm{L}} = 0$, and then at small u,

$$
\lambda_{\mathrm{L}} = \frac{M_{\mathrm{L}} \mu^{3/2} \cos\big(\pi\eta\big) u^{3/4}}{\pi \nu^2 L^{3/2}} \left\{ 4 + \frac{\mu^2 \nu^2}{L^2}\left[\psi\!\left(\frac{3}{2}+\eta\right) + \psi\!\left(\frac{3}{2}-\eta\right) + \log\!\left(\frac{\mu^2}{L^2}\right) + 2\gamma - 1\right] u \right\} + \cdots, \tag{5.108}
$$

and so, using (5.73), we get $C_{1/2} = C_1 = C_0$ and $B_{1/2} = 0$.

VI. NON-SUPERSYMMETRIC CORRELATORS

We now turn to the analysis of non-supersymmetric cases which is our main interest in this work. In particular, we consider a normalizable non-supersymmetric perturbation to the above geometry and recalculate the correlators of section V in this perturbed background. As discussed in section III, these classical two-point functions correspond to the current-current correlators of the hidden sector after the latter has obtained a non-supersymmetric state and so calculation of these functions is tantamount to calculation of visible-sector soft terms resulting from the gauge mediation of supersymmetry breaking.

Inspired by the well-studied case in Klebanov-Strassler [20, 30, 31], the toy case that we consider is the addition of p D3-$\overline{\mathrm{D3}}$ pairs to $AdS^5 \times X^5$. The geometry was considered in [30] and (as argued in [30]) can be found by taking a near-horizon limit of geometries considered in [59]. In our notation, the solution is

$$
A = \log\!\left(\frac{r}{L}\right) - \frac{L^8 \mathcal{S}}{5r^8}, \quad \bar{B} = \log\!\left(\frac{r}{L}\right) - \frac{L^8 \mathcal{S}}{10r^8}, \quad C = \log\!\left(\frac{r}{L}\right) + \frac{3L^8 \mathcal{S}}{10r^8}, \tag{6.1}
$$

in which we have taken $\mathcal{S} = \frac{p}{N}$ to be a small parameter. It was pointed out in [30] that the perturbations are such that $\langle T_{\mu\nu}\rangle = 0$ and so the dual supersymmetry, which is rigid, is unbroken. This should be reflected in the correlators resulting from this background. Nevertheless, we find below that the messenger spectrum is split and therefore the correlators do not respect the relationships expected from supersymmetry. Presumably, this is related to the fact that a finite messenger mass spoils the conformal behavior of the dual theory and taking into account the backreaction of the D7s should result in a finite vacuum energy.

To leading order in $\mathcal{S}$, the equation of motion for the scalar field takes the form

$$0 = \left\{\frac{1+p^2}{p^3}\frac{\mathrm{d}}{\mathrm{d}p}\left[p^3\left(1+p^2\right)\left(1-\frac{\delta}{\left(1+p^2\right)^4}\right)\frac{\mathrm{d}}{\mathrm{d}p}\right] - \frac{3-2p^2-4p^4}{p^2}\left(1-\frac{\delta}{\left(1+p^2\right)^4}\right) - \nu^2\right\}\Sigma, \tag{6.2}$$

in which

$$\delta = \frac{3L^8}{5\mu^8}\mathcal{S}. \tag{6.3}$$

This is a Sturm-Liouville problem and so for a set of boundary conditions the solutions will be orthogonal with respect to the inner product

$$\left(\Sigma_\ell, \Sigma_{\ell'}\right)_0 = \int_0^\infty \mathrm{d}p\, \frac{p^3}{1+p^2}\Sigma_\ell\Sigma_{\ell'} \propto \delta_{\ell\ell'}. \tag{6.4}$$

Instead of attempting to solve (6.2) directly, we can apply perturbation theory. When $\delta = 0$, (6.2) is again a Sturm-Liouville problem and so the solutions are orthogonal with respect to the same inner product. The solutions that are regular were found in the last section, and the resulting correlation function (5.91) can be written as

$$C_0 = \frac{2D_0}{L}\sum_{\ell=0}^{\infty}\frac{-4\left(3+2\ell\right)}{\nu^2+4\left(\ell+1\right)\left(\ell+2\right)} + \cdots, \tag{6.5}$$

where we have omitted the contact terms that were specified by holographic renormalization. The simple structure of this correlator is a consequence of the fact that the messengers form mesonic bound states which are free in the large N limit [60].

The correlator in the perturbed geometry should be expressible in a similar way

$$C_0 = \frac{2D_0}{L}\sum_{\ell=0}^{\infty}\frac{Z_\ell}{\nu^2+L^4m_\ell^2/\mu^2} + \cdots. \tag{6.6}$$

The precise form of this requires the precise solution which we will not attempt to find. However, we can find the spectrum perturbatively by writing the equation of motion (6.2) for the modes corresponding to these poles as

$$\mathcal{H}_0\Sigma_\ell = \lambda_\ell\Sigma_\ell, \tag{6.7}$$

with $\lambda_\ell = -L^4 m_\ell^2/\mu^2$. When $\delta = 0$,

$$\mathcal{H}_0 = \mathcal{H}_0^{(0)} = \frac{1+p^2}{p^3}\frac{\mathrm{d}}{\mathrm{d}p}\left[p^3\left(1+p^2\right)\frac{\mathrm{d}}{\mathrm{d}p}\right] - \frac{3-2p^2-4p^4}{p^3}. \tag{6.8}$$

For $\delta \neq 0$, write $\mathcal{H}_0 = \mathcal{H}_0^{(0)} + \delta\mathcal{H}_0^{(1)}$ with

$$\mathcal{H}_0^{(1)} = \frac{8p}{\left(1+p^2\right)^3}\frac{\mathrm{d}}{\mathrm{d}p} - \frac{1}{\left(1+p^2\right)^4}\mathcal{H}_0^{(0)}. \tag{6.9}$$

The unperturbed spectrum is $\lambda_\ell^{(0)} = -4\left(\ell+1\right)\left(\ell+2\right)$ and the corresponding eigenfunctions,

$$\Sigma_\ell^{(0)} = M_\ell\, p\left(1+p^2\right)^{-\frac{3}{2}-\ell} F\left(-\ell,-\ell;3;-p^2\right), \tag{6.10}$$

form a complete set of functions that vanish as $p \to \infty$ and, by appropriate choice of M_ℓ, are orthonormal with respect to (6.4). We similarly expand $\lambda_\ell^{(0)} + \delta\lambda_\ell^{(1)}$ and $\Sigma_\ell = \Sigma_\ell^{(0)} + \delta\Sigma_\ell^{(1)}$ with

$$\Sigma_\ell^{(1)} = \sum_{\ell'} c_{\ell\ell'}\Sigma_{\ell'}^{(0)}. \tag{6.11}$$

Demanding that Σ_ℓ is normalized sets $c_{\ell\ell} = 0$. It follows then

$$\lambda_\ell^{(1)} = \left(\Sigma_\ell^{(0)}, \mathcal{H}_0^{(1)}\Sigma_\ell^{(0)}\right)_0. \tag{6.12}$$

Since ℓ is an integer, the solutions are a polynomial of order ℓ order and so these integrals can be easily performed. The resulting spectrum appears in table I. Meanwhile, for $\ell' \neq \ell$,

$$c_{\ell\ell'} = \frac{\left(\Sigma_{\ell'}^{(0)}, \mathcal{H}_0^{(1)}\Sigma_\ell^{(0)}\right)_0}{\lambda_\ell^{(0)} - \lambda_{\ell'}^{(0)}}. \tag{6.13}$$

The homogeneous equation for $A_\mu = a_\mu\mathcal{A}$ takes the form

$$0 = \left\{\frac{\left(1+p^2\right)^2}{p^3}\left(1-\frac{2}{3}\frac{\delta}{\left(1+p^2\right)^4}\right)\frac{\mathrm{d}}{\mathrm{d}p}\left[p^3\left(1-\frac{1}{3}\frac{\delta}{\left(1+p^2\right)^4}\right)\frac{\mathrm{d}}{\mathrm{d}p}\right] - \nu^2\right\}\mathcal{A}. \tag{6.14}$$

Here the appropriate inner product is

$$\left(\mathcal{A}_\ell, \mathcal{A}_{\ell'}\right)_1 = \int_0^\infty \mathrm{d}p\frac{p^3}{\left(1+p^2\right)^2}\left(1+\frac{2}{3}\frac{\delta}{\left(1+p^2\right)^4}\right)\mathcal{A}_\ell\mathcal{A}_{\ell'}. \tag{6.15}$$

The equation of motion takes the form $\mathcal{H}_1\mathcal{A}_\ell = \lambda_\ell\mathcal{A}_\ell$ with

$$
\begin{aligned}
\mathcal{H}_1 =& \mathcal{H}_1^{(0)} + \delta\mathcal{H}_1^{(1)},\\
\mathcal{H}_1^{(0)} =& \frac{\left(1+p^2\right)^2}{p^3}\frac{\mathrm{d}}{\mathrm{d}p}\left[p^3\frac{\mathrm{d}}{\mathrm{d}p}\right],\\
\mathcal{H}_1^{(1)} =& \frac{8p}{3\left(1+p^2\right)^3}\frac{\mathrm{d}}{\mathrm{d}p} - \frac{1}{\left(1+p^2\right)^4}\mathcal{H}_1^{(0)}.
\end{aligned}
\tag{6.16}
$$

The solutions to the $\delta = 0$ equation are

$$
\mathcal{A}_\ell^{(0)} = M_\ell\left(1+p^2\right)^{-1-n}F\left(-n,-1-n;2;-p^2\right). \tag{6.17}
$$

These are orthonormal with respect to

$$
\left\langle\mathcal{A}_\ell^{(0)},\mathcal{A}_{\ell'}^{(0)}\right\rangle_1 = \int_0^\infty \mathrm{d}p\,\frac{p^3}{\left(1+p^2\right)^2}\mathcal{A}_\ell\mathcal{A}_{\ell'}. \tag{6.18}
$$

Write $\mathcal{A}_\ell = \mathcal{A}_\ell^{(0)} + \delta\mathcal{A}_\ell^{(1)}$ with

$$
\mathcal{A}_\ell^{(1)} = \sum_{\ell'} c_{\ell\ell'}\mathcal{A}_{\ell'}, \tag{6.19}
$$

then imposing $\left(\mathcal{A}_\ell,\mathcal{A}_\ell\right)_1 = 1$ sets

$$
c_{\ell\ell} = -\frac{1}{3}\int_0^\infty \mathrm{d}p\,\frac{p^3}{\left(1+p^2\right)^2}\frac{1}{\left(1+p^2\right)^4}\mathcal{A}_\ell^{(0)}\mathcal{A}_\ell^{(0)}. \tag{6.20}
$$

While as with the scalar,

$$
\lambda_\ell^{(1)} = \left\langle\mathcal{A}_\ell^{(0)},\mathcal{H}_1^{(1)}\mathcal{A}_\ell^{(0)}\right\rangle_1, \tag{6.21}
$$

and for $\ell' \neq \ell$,

$$
c_{\ell\ell'} = \frac{\left\langle\mathcal{A}_{\ell'}^{(0)},\mathcal{H}_1^{(0)}\mathcal{A}_\ell^{(0)}\right\rangle_1}{\lambda_\ell^{(0)} - \lambda_{\ell'}^{(0)}}. \tag{6.22}
$$

Writing $\lambda_{\mathrm{L}}^\alpha = a_{\mathrm{L}}^\alpha\psi$, the equation of motion for the spinor takes the form

$$
0 = \left\{f(p)\frac{\mathrm{d}^2}{\mathrm{d}p^2} + g(p)\frac{\mathrm{d}}{\mathrm{d}p} + h(p) - \nu^2\right\}\psi, \tag{6.23}
$$

with

$$
f(p) = \left(1+p^2\right)^2\left(1 - \frac{\delta}{\left(1+p^2\right)^4}\right),
$$

$$
g(p) = \frac{3\left(1+p^2\right)\left(1+2p^2\right)}{p}\left(1 + \frac{\delta\left(-9+14p^2\right)}{9\left(1+p^2\right)^4\left(1+2p^2\right)}\right), \tag{6.24}
$$

$$
h(g) = \frac{3\left(8+7p^2\right)}{4}\left(1 + \frac{7\delta\left(8-9p^2\right)}{9\left(1+p^2\right)^4\left(8+7p^2\right)}\right). \tag{6.25}
$$

We can cast (6.23) as a Sturm-Liouville problem

$$0 = \left\{ f\mathrm{e}^{-\zeta} \frac{\mathrm{d}}{\mathrm{d}p} \left[\mathrm{e}^{\zeta} \frac{\mathrm{d}}{\mathrm{d}p} \right] + h - \nu^2 \right\} \psi, \tag{6.26}$$

in which

$$\zeta(p) = \int^p \mathrm{d}p' \, \frac{g(p')}{f(p')}. \tag{6.27}$$

Then,

$$\begin{aligned} 0 = \Bigg\{ & \frac{\sqrt{1+p^2}}{p^3} \left(1 + \frac{\delta}{3(1+p^2)^4} \right) \frac{\mathrm{d}}{\mathrm{d}p} \left[p^3 \left(1+p^2\right)^{3/2} \left(1 - \frac{4}{3} \frac{\delta}{(1+p^2)^4} \right) \frac{\mathrm{d}}{\mathrm{d}p} \right] \\ & + \frac{3(8+7p^2)}{4} \left(1 + \frac{7}{9} \frac{\delta(8-9p^2)}{(1+p^2)^4 (8+7p^2)} \right) - \nu^2 \Bigg\} \psi. \end{aligned} \tag{6.28}$$

The inner product is

$$\left(\psi_\ell, \psi_{\ell'}\right)_{1/2} = \int_0^\infty \mathrm{d}p \, \frac{p^3}{\sqrt{1+p^2}} \left(1 - \frac{\delta}{3(1+p^2)^4} \right) \psi_\ell \psi_{\ell'}. \tag{6.29}$$

The equation of motion is $\mathcal{H}_{1/2} \psi_\ell = \lambda_\ell \psi_\ell$ with

$$\begin{aligned} \mathcal{H}_{1/2} =& \mathcal{H}_{1/2}^{(0)} + \delta \mathcal{H}_1^{(1)}, \\ \mathcal{H}_{1/2}^{(0)} =& \frac{\sqrt{1+p^2}}{p^3} \frac{\mathrm{d}}{\mathrm{d}p} \left[p^3 \left(1+p^2\right)^{3/2} \frac{\mathrm{d}}{\mathrm{d}p} \right] + \frac{3(8+7p^2)}{4}, \\ \mathcal{H}_{1/2}^{(1)} =& \frac{32p}{3(1+p^2)^3} \frac{\mathrm{d}}{\mathrm{d}p} + \frac{7(8-9p^2)}{12(1+p^2)^4} - \frac{1}{(1+p^2)^4} \mathcal{H}_{1/2}^{(0)}. \end{aligned} \tag{6.30}$$

For $\delta = 0$, the solutions take the form

$$\psi_\ell^{(0)} = M_\ell \left(1+p^2\right)^{-7/4-\ell} F\left(-\ell, -1-\ell; 2; -p^2\right), \tag{6.31}$$

and are orthonormal with respect to the inner product

$$\left\langle \psi_\ell^{(0)}, \psi_{\ell'}^{(0)} \right\rangle_{1/2} = \int_0^\infty \mathrm{d}p \, \frac{p^3}{\sqrt{1+p^2}} \psi_\ell \psi_{\ell'} \tag{6.32}$$

Writing again

$$\psi_\ell^{(1)} = \sum_{\ell'} c_{\ell\ell'} \psi_{\ell'}^{(0)}, \tag{6.33}$$

and imposing $(\psi_\ell, \psi_\ell)_{1/2} = 0$ sets

$$c_{\ell\ell} = \frac{1}{6} \int_0^\infty \mathrm{d}p \, \frac{p^3}{\sqrt{1+p^2}} \frac{1}{(1+p^2)^4} \psi_\ell^{(0)} \psi_\ell^{(0)}. \tag{6.34}$$

Once again the perturbations to the masses are

$$\lambda_\ell^{(1)} = \left\langle \psi_\ell^{(0)}, \mathcal{H}_{1/2}^{(1)} \psi_\ell^{(0)} \right\rangle_{1/2}, \tag{6.35}$$

while for $\ell \neq \ell'$,

$$c_{\ell\ell'} = \frac{\langle \psi_{\ell'}^{(0)}, \mathcal{H}_{1/2}^{(0)} \psi_\ell^{(0)} \rangle_{1/2}}{\lambda_\ell^{(0)} - \lambda_{\ell'}^{(0)}}. \tag{6.36}$$

The residues Z_ℓ can be obtained as follows (see [61]). At large p, the solution for the scalar mode takes the form[15]

$$\Sigma = \sigma_1 p^{-2} + \sigma_2 p^{-2} \log p + \cdots . \tag{6.37}$$

When the 4-momentum is on a resonance, $\sigma_2 = 0$ which can be seen by expanding (5.91) for $\nu^2 = -4(\ell+1)(\ell+2)$. Then, when $\mathcal{S} = 0$ and the solution is normalized according to (6.4), we have

$$Z_\ell \sim \sigma_{\ell,1}^2. \tag{6.38}$$

As a check, the normalized $\ell = 0$ solution is

$$\Sigma_0^{(0)} = \frac{\sqrt{6}p}{(1+p^2)^{3/2}} = \frac{\sqrt{6}}{p^2} + \cdots , \tag{6.39}$$

comparing to (6.5) Z_ℓ, we get

$$Z_\ell = -2\sigma_{\ell,1}^2. \tag{6.40}$$

It is straightforward to check that this holds for higher ℓ as well[16]. When $\delta > 0$, we have $\sigma_{\ell,1} = \sigma_{\ell,1}^{(0)} + \delta\sigma_{\ell,1}^{(1)}$ with

$$\sigma_{\ell,1}^{(1)} = \sum_{\ell'} c_{\ell\ell'} \sigma_\ell^{(0)}. \tag{6.41}$$

The residue is then $Z_\ell = Z_\ell^{(0)} + \delta Z_\ell^{(1)}$ with

$$Z_\ell^{(0)} = -2\left(\sigma_{\ell,1}^{(0)}\right)^2, \qquad Z_\ell^{(1)} = -4\sigma_{\ell,1}^{(0)}\sigma_{\ell,1}^{(1)}. \tag{6.42}$$

The residues are also presented in table I. Since $c_{\ell,\ell'}$ vanishes if $|\ell - \ell'| \leq 4$, the correction to the residue can be calculated explicitly.

---

15 Note that this takes a different from from (3.7) since here we have $\Delta = 2$.

16 Alternatively, since the resonant solutions are polynomials of finite order, this should be possible to check for general ℓ. Note that the relationship presented in [61] between the residues Z_ℓ and the coefficient of the sub-dominant solutions must be modified for scalars satisfying the BF bound.

For the vectors, at large p,

$$\mathcal{A} = a_1 p^{-2} + a_2\big(1 + a(k) p^{-2}\log p\big) + \cdots . \tag{6.43}$$

For the mass eigenstates, $a_2 = 0$ and

$$Z_\ell = \frac{8}{\nu^2} a_{\ell,1}^2 . \tag{6.44}$$

Finally for the spinors, at large p

$$\psi = s_1 p^{-2} + s_2\big(1 + a(k) p^{-2}\log p\big) + \cdots , \tag{6.45}$$

and for mass eigenstates

$$Z_\ell = \frac{8}{\nu^2} s_{\ell,1}^2 . \tag{6.46}$$

We then find an expression for the perturbed residue that is similar to (6.42).

Now, the analytic terms in the correlators C_a correspond to contact terms and must cancel in (2.12). The non-analytic parts can be easily integrated. We have,

$$\int_0^{\Lambda^2} \mathrm{d}\nu^2 \sum_{\ell=0} \frac{Z_\ell}{\nu^2 - \lambda_\ell} \approx \sum_{\ell}^{\ell_\mathrm{m}} Z_\ell \big\{ \log\big(-\lambda_\ell\big) - \log\big(-\lambda_\ell - \lambda_{\ell_\mathrm{max}}\big) \big\}, \tag{6.47}$$

where ℓ_m is the largest ℓ satisfying $\lambda_\ell < \Lambda^2$. (2.12) then gives (suppressing the group index)

$$\Gamma = \frac{-\delta\mu^2}{16\pi^2 L^4} \left(3\Gamma^{(1)}_0 - 4\Gamma^{(1)}_{1/2} + \Gamma^{(1)}_1 \right), \tag{6.48}$$

in which for any particular spin,

$$\Gamma^{(0)}_a = \sum_{\ell}^{\ell_\mathrm{m}} \left\{ Z^{(1)}_\ell \log\left[\frac{\lambda^{(0)}_\ell - \lambda^{(0)}_{\ell_\mathrm{m}}}{\lambda^{(0)}_\ell} \right] - Z^{(1)}_\ell \lambda^{(1)}_\ell \left(\frac{1}{\lambda^{(0)}_\ell + \lambda^{(0)}_{\ell_\mathrm{m}}} - \frac{1}{\lambda^{(0)}_\ell} \right) \right\}. \tag{6.49}$$

As we observe from table I and as expected from supersymmetry, the residues and masses become degenerate as ℓ. The result is

$$\Gamma = \frac{3L^4\mathcal{S}}{80\pi^2\mu^6} c, \tag{6.50}$$

in which $c \approx 90$. We would like to be able to express this in terms of quantities on the gauge theory side, for example the messenger mass and the scale of supersymmetry breaking. The messenger mass is related to the D7-position by $m_\mu = \mu \ell_\mathrm{s}^{-2}$. However, the dual scale of supersymmetry breaking is not immediately obvious in this set up. The analysis

			Scalar		Vector		Spinor	
ℓ	$-\lambda_\ell^{(0)}$	$Z_\ell^{(0)}$	$-\lambda_\ell^{(1)}/\lambda_\ell^{(0)}$	Z_ℓ	$-\lambda_\ell^{(1)}/\lambda_\ell^{(0)}$	Z_ℓ	$-\lambda_\ell^{(1)}/\lambda_\ell^{(0)}$	Z_ℓ
0	8	−12	0.0143	0.474	0.104	2.33	−0.0780	0.538
1	24	−20	0.0767	3.36	0.201	5.37	0.0941	0.491
2	48	−28	0.145	6.72	0.236	7.62	0.178	7.50
3	80	−36	0.188	9.34	0.251	9.83	0.215	9.78
4	120	−44	0.214	11.8	0.258	12.0	0.234	12.0
5	168	−52	0.223	14.1	0.263	14.2	0.245	14.2
6	224	−60	0.234	16.3	0.265	16.4	0.252	16.4
7	288	−68	0.247	18.5	0.267	18.6	0.257	18.6
8	360	−76	0.252	20.7	0.268	20.8	0.260	20.8
9	440	−84	0.256	22.9	0.269	23.0	0.262	23.0
10	528	−92	0.259	25.1	0.270	25.2	0.264	25.2
20	1848	−172	0.269	47.0	0.272	47.0	0.271	47.0
50	10608	−412	0.273	113.	0.273	113.	0.273	113.
100	41208	−812	0.273	222.	0.273	222.	0.273	222.

TABLE I. Perturbed spectrum of mesonic messengers in the theory and state dual to the geometry (6.1). Note that although higher modes contribute increasingly large amounts to the correlators, the spectra also become degenerate as ℓ increases and so cancel out of (2.12). All of the entries in this table are approximations to rational numbers that can be determined for any ℓ, though the general expression is not simple.

of [30] indicates that (at least without the D7-brane) the dual state has no vacuum energy and supersymmetry is not broken. However, since the current-current correlators for the component fields of $\mathcal{J}$ do not satisfy supersymmetric relations, supersymmetry must be broken at least after the introduction of the flavor branes[17]. Relating the parameters of the gravitational theory to the gauge theory may then require calculation of the backreaction of the D7-brane, an analysis that we leave to future work.

[17] This indicates that the mediation of supersymmetry breaking is no longer precisely semi-direct.

VII. CONCLUSIONS

In this work, we considered models of supersymmetry breaking and mediation, using the language and techniques of the gauge/gravity correspondence. In this sense, this work is very much along the lines of [22, 23]. However, the approach used here differed from those works in that in the latter soft terms (in particular, gaugino masses) were inferred directly from dimensional reduction. In contrast, here we used the correspondence to calculate correlation functions in the gauge theory, much in the spirit of the some of the foundational works on AdS/CFT. Such correlators can be related to visible-sector soft terms via the formalism of general gauge mediation. The downside to this technique however is that it requires more explicit knowledge of solutions of the classical solutions of the equations of motion. In particular, the fact that some of the fields of interest had non-trivial angular dependence required us to consider particularly simple geometries.

One surprising result of the analysis performed concerns the geometry resulting from adding a small number of D3-$\overline{\text{D3}}$-branes to $AdS^5 \times S^5$. Although naively such a construction is supersymmetric, it was argued in [30] that supersymmetry was preserved in the state realized by the dual theory. However, we found that when a U (1) flavor group and a massive quark is added to the theory, the resulting spectrum of mesons is not supersymmetric. A possibility is that supersymmetry is broken only after the addition of the quarks. This could be confirmed by calculating the backreaction of the D7 on the geometry.

The advantage of the technique used here is that soft terms for chiral matter fields, which were not directly calculable from holography in the setups of [22, 23], are readily obtainable here. However, because of the large amount of symmetry, the gaugino in the dual gauge theory remains massless. That is, the response function for the spinor component of the dual current superfield took the form (5.70) and so the correlator function $B_{1/2}$ given in (2.5) vanish, leading to a vanishing $m_{1/2}$. This is directly related to the fact that the geometry does contain any 3-form flux which is necessary to give rise to gaugino masses in this class of constructions [22, 23, 62]. It would be of interest to extend the techniques considered here to geometries that are supported by fluxes such as [19, 32] and their non-supersymmetric perturbations [30, 31], especially since such constructions are perturbatively stable. However, in addition to the complications presented by an angular space[18], such

[18] See [63] for discussions on this point.

theories do not, strictly speaking, flow from a conformal fixed point but are instead cascading theories. Although the methods of holographic renormalization have been discussed for such theories [64], the geometry is such that the correlators cannot be explicitly calculated. Although this was also the case for the non-supersymmetric cases considered above, for asymptotically AdS spaces it is known how to infer the two-point functions from the 1-point functions and the spectrum. For cascading theories, this is less clear [61].

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Appendix A: Conventions

The index μ indicates an $R^{3,1}$ coordinate $x^{0,1,2,3}$ while m runs over all five non-compact coordinates (i.e. $R^{3,1}$ and the holographic direction). α runs over the coordinates of the probe D7-brane discussed in the text, with a, b, c, d running over coordinates transverse to $R^{3,1}$ and i, j running transverse to the brane. ϕ, ψ denote angular directions, either those in the full 10d space or along the worldvolume. M, N run over all 10 directions. Underlined indices are used to denote locally orthogonal non-coordinate bases for the (co-)vector spaces. The index α is also used to denote spinor indices, but context should allow these to be distinguished from worldvolume indices.

1. Fermion conventions

a. SO(3,1) spinors

We make use of the dotted and undotted notation of [65]. Such indices are raised and lowered with $\epsilon^{12} = \epsilon_{21} = 1$ and we define $\theta^2 = \theta\theta$. An $\mathrm{SO}(3,1)$ Dirac spinor takes the form

$$\psi = \begin{pmatrix} \psi_{\mathrm{L}\alpha} \\ \mathrm{i}\bar{\psi}_{\mathrm{R}}^{\dot{\alpha}} \end{pmatrix}, \tag{A1}$$

where we have chosen a Weyl basis for the $\mathrm{SO}(3,1)$ γ-matrices

$$\gamma^{\underline{\mu}} = \begin{pmatrix} 0 & \sigma^{\underline{\mu}} \\ -\bar{\sigma}^{\underline{\mu}} & 0 \end{pmatrix}, \tag{A2}$$

in which

$$\sigma^{\underline{\mu}} = \left(-\mathbb{I}_2, \boldsymbol{\sigma}\right), \quad \bar{\sigma}^{\underline{\mu}} = \left(-\mathbb{I}_2, -\boldsymbol{\sigma}\right), \tag{A3}$$

where $\bar{\sigma}$ are the usual Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A4}$$

and so the γ-matrices satisfy the Clifford algebra

$$\left\{\gamma^{\underline{\mu}}, \gamma^{\underline{\nu}}\right\} = 2\eta^{\underline{\mu\nu}}. \tag{A5}$$

The 4d chirality operator is

$$\gamma_{(4)} = -\mathrm{i}\gamma^{\underline{0}}\gamma^{\underline{1}}\gamma^{\underline{2}}\gamma^{\underline{3}} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{A6}$$

b. SO(4,1) spinors

For $\mathrm{SO}(4,1)$, we take

$$\tilde{\gamma}^{\underline{\mu}} = \gamma^{\underline{\mu}}, \quad \tilde{\gamma}^{\underline{4}} = \gamma_{(4)}. \tag{A7}$$

These then satisfy $\{\tilde{\gamma}^{\underline{m}}, \tilde{\gamma}^{\underline{n}}\} = 2\tilde{\eta}^{\underline{mn}}$. The Majorana matrix is

$$\tilde{B}_5 = \tilde{\gamma}^{\underline{2}}, \tag{A8}$$

which is imaginary, satisfies $\tilde{B}_5 \tilde{B}_5^* = -1$ and $\tilde{B}_5^{-1} \tilde{\gamma}^{\underline{m}} \tilde{B}_5 = -\tilde{\gamma}^{\underline{m}*}$. The charge-conjugation operator is

$$\tilde{C}_5 = \tilde{B}_5 \tilde{\gamma}^{\underline{0}} = \begin{pmatrix} \sigma^{\underline{2}} & 0 \\ 0 & -\sigma^{\underline{2}} \end{pmatrix}, \tag{A9}$$

which satisfies $\tilde{C}_5 \tilde{\gamma}^{\underline{m}} \tilde{C}_5^{-1} = \tilde{\gamma}^{\underline{m}\mathrm{T}}$. $\mathcal{N}_5 = 1$ is usefully parametrized in terms of symplectic-Majorana spinors. A pair of symplectic-Majorana spinors are Dirac spinors $\psi^{i=1,2}$ satisfying the property

$$\psi^i = \epsilon^{ij} \tilde{C}_5 \bar{\psi}_j^{\mathrm{T}}, \tag{A10}$$

where $\psi_j^* := \left(\psi^j\right)^*$ and again $\epsilon^{12} = \epsilon_{21} = 1$. Such spinors satisfy the identities

$$\bar{\chi}_i \tilde{\gamma}^{\underline{m}_1} \cdots \tilde{\gamma}^{\underline{m}_n} \psi^j = \epsilon_{il} \epsilon^{jk} \bar{\psi}_k \tilde{\gamma}^{\underline{m_n}} \cdots \tilde{\gamma}^{\underline{m_1}} \chi^l, \tag{A11a}$$

$$\left(\bar{\chi}_i \tilde{\gamma}^{\underline{m}_1} \cdots \tilde{\gamma}^{\underline{m}_n} \psi^j\right)^* = \left(-1\right)^{n+1} \epsilon^{ik} \epsilon_{jl} \bar{\chi}_k \tilde{\gamma}^{\underline{m}_1} \cdots \tilde{\gamma}^{\underline{m}_n} \psi^l. \tag{A11b}$$

For a pair of symplectic-Majorana spinors, we may write

$$\psi^1 = \begin{pmatrix} \psi_{\mathrm{L}\alpha} \\ \mathrm{i}\bar{\psi}_{\mathrm{R}}^{\dot{\alpha}} \end{pmatrix}, \qquad \psi^2 = \begin{pmatrix} -\psi_{\mathrm{R}\alpha} \\ \mathrm{i}\bar{\psi}_{\mathrm{L}}^{\dot{\alpha}} \end{pmatrix}, \qquad \bar{\psi}_1 = \begin{pmatrix} -\mathrm{i}\psi_{\mathrm{R}}^{\alpha} \\ -\bar{\psi}_{\mathrm{L}\dot{\alpha}} \end{pmatrix}^{\mathrm{T}}, \qquad \bar{\psi}_2 = \begin{pmatrix} -\mathrm{i}\psi_{\mathrm{L}}^{\alpha} \\ \bar{\psi}_{\mathrm{R}\dot{\alpha}} \end{pmatrix}^{\mathrm{T}}. \tag{A12}$$

c. SO(5) spinors

For SO (5) we choose a basis that is useful for the decomposition SO (5) → SO (2)×SO (3),

$$\hat{\gamma}^{\underline{1}} = \sigma^{\underline{1}} \otimes \mathbb{I}_2, \quad \hat{\gamma}^{\underline{2}} = \sigma^{\underline{2}} \otimes \mathbb{I}_2, \quad \hat{\gamma}^{\underline{3}} = \sigma^{\underline{3}} \otimes \sigma^{\underline{1}}, \quad \hat{\gamma}^{\underline{4}} = \sigma^{\underline{3}} \otimes \sigma^{\underline{2}}, \quad \hat{\gamma}^{\underline{5}} = \sigma^{\underline{3}} \otimes \sigma^{\underline{3}}. \tag{A13}$$

These satisfy

$$\left\{\hat{\gamma}^{\underline{\phi}}, \hat{\gamma}^{\underline{\psi}}\right\} = 2\delta^{\underline{\phi\psi}}. \tag{A14}$$

The Majorana matrix is

$$\hat{B}_5 = \hat{\gamma}^{\underline{2}} \hat{\gamma}^{\underline{4}} = \sigma^{\underline{1}} \otimes \mathrm{i}\sigma^{\underline{2}}. \tag{A15}$$

It satisfies $\hat{B}_5 = \hat{B}_5^*$, $\hat{B}_5^2 = -1$, and

$$\hat{B}_5^{-1} \hat{\gamma}^{\underline{\phi}} \hat{B}_5 = \hat{\gamma}^{\underline{\phi}*}. \tag{A16}$$

d. SO(9,1) spinors

On $R^{9,1}$ we take

$$\Gamma^{\underline{m}} = \sigma^{\underline{1}} \otimes \tilde{\gamma}^{\underline{m}} \otimes \mathbb{I}_4, \quad \Gamma^{\underline{\phi}} = \sigma^{\underline{2}} \otimes \mathbb{I}_4 \otimes \hat{\gamma}^{\underline{\phi}}, \tag{A17}$$

where the second equality should be understood to mean $\Gamma^{\underline{5}} = \sigma^{\underline{2}} \otimes \mathbb{I}_4 \otimes \hat{\gamma}^{\underline{1}}$, etc. The Γ-matrices satisfy $\left\{\Gamma^{\underline{M}}, \Gamma^{\underline{N}}\right\} = 2\eta^{\underline{MN}}$. The 10d chirality operator is

$$\Gamma_{(10)} = \mathrm{i}\Gamma^{\underline{0}}\Gamma^{\underline{1}} \cdots \Gamma^{\underline{9}} = \sigma^{\underline{3}} \otimes \mathbb{I}_4 \otimes \mathbb{I}_4, \tag{A18}$$

which satisfies $\Gamma_{(10)}^2 = 1$ and anti-commutes with all of the $\Gamma^{\underline{M}}$. The 10d Majorana matrix is

$$B_{10} = \Gamma^{\underline{2}}\Gamma^{\underline{5}}\Gamma^{\underline{7}}\Gamma^{\underline{9}} = -\mathrm{i}\sigma^3 \otimes \tilde{B}_5 \otimes \hat{B}_5, \tag{A19}$$

which satisfies $B_{10} = B_{10}^*$, $B_{10}B_{10}^* = 1$ and

$$B_{10}^{-1}\Gamma^{\underline{M}}B_{10} = \Gamma^{\underline{M}*}. \tag{A20}$$

In 10d, we generally work with Majorana-Weyl spinors satisfying $\Gamma_{(10)}\Psi = \Psi$, and $B_{10}\Psi = \Psi^*$. Additionally, we work with bispinors which are combination of two 10d spinors

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}. \tag{A21}$$

Γ-matrices act on bispinors as

$$\Gamma^{\underline{M}}\Psi = \begin{pmatrix} \Gamma^{\underline{M}}\Psi_1 \\ \Gamma^{\underline{M}}\Psi_2 \end{pmatrix}. \tag{A22}$$

2. Type-IIB supergravity

Our conventions for the bosonic modes of type-IIB supergravity are summarized by the pseudo-action which we write as

$$S_{\mathrm{IIB}} = S_{\mathrm{IIB}}^{\mathrm{NS}} + S_{\mathrm{IIB}}^{\mathrm{R}} + S_{\mathrm{IIB}}^{\mathrm{CS}}, \tag{A23}$$

in which

$$S_{\mathrm{IIB}}^{\mathrm{NS}} = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\,\sqrt{-\det(g)}\left\{R - \frac{1}{2}\partial_M\Phi\partial^M\Phi - \frac{g_{\mathrm{s}}}{2}\mathrm{e}^{-\Phi}\left(H^{(3)}\right)^2\right\},$$

$$S_{\mathrm{IIB}}^{\mathrm{R}} = -\frac{1}{4\kappa_{10}^2}\int \mathrm{d}^{10}x\,\sqrt{-\det(g)}\left\{\mathrm{e}^{2\Phi}\left(F^{(1)}\right)^2 + g_{\mathrm{s}}\mathrm{e}^{\Phi}\left(F^{(3)}\right)^2 + \frac{1}{2}g_{\mathrm{s}}^2\left(F^{(5)}\right)^2\right\}, \tag{A24}$$

$$S_{\mathrm{IIB}}^{\mathrm{CS}} = \frac{g_{\mathrm{s}}^2}{4\kappa_{10}^2}\int C^{(4)} \wedge H^{(3)} \wedge \mathrm{d}C^{(2)}, \tag{A25}$$

in which g_{MN} is the 10d Einstein-frame metric, Φ is the dilaton and the gauge-invariant field strengths of the NS-NS 2-form potential $B^{(2)}$ and R-R potentials $C^{(0)}$, $C^{(2)}$, and $C^{(4)}$ are

$$\begin{aligned} H^{(3)} &= \mathrm{d}B^{(2)}, & F^{(1)} &= \mathrm{d}C^{(0)}, \\ F^{(3)} &= \mathrm{d}C^{(2)} - C^{(0)} H^{(3)}, & F^{(5)} &= \mathrm{d}C^{(4)} + B^{(2)} \wedge \mathrm{d}C^{(2)}. \end{aligned} \tag{A26}$$

The dilaton is normalized such that $\langle \Phi \rangle = \log g_{\mathrm{s}}$ and for a p-form

$$\left(\Omega^{(p)}\right)^2 = \frac{1}{p!} \Omega_{M_1 \cdots M_p} \Omega^{M_1 \cdots M_p}. \tag{A27}$$

Type-IIB supergravity exhibits $\mathcal{N}_{10} = 2$ and so there are two Majorana-Weyl gravitini and two Majorana-Weyl dilatini which can be organized into bispinors

$$\Psi_M = \begin{pmatrix} \Psi_M^1 \\ \Psi_M^2 \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \Lambda^1 \\ \Lambda^2 \end{pmatrix}. \tag{A28}$$

SUSY transformations are parametrized by a Majorana-Weyl bispinor ϵ. For the fermionic fields

$$\delta_\epsilon \Psi_M = \mathcal{D}_M \epsilon, \quad \delta_\epsilon \Lambda = \Delta \epsilon, \tag{A29}$$

where, in the 10d Einstein frame,

$$\Delta = \frac{1}{2} \partial\!\!\!/ \Phi - \frac{1}{2} \mathrm{e}^{\Phi} F\!\!\!/^{(1)} \left(\mathrm{i}\sigma^2\right) - \frac{1}{4} \left(g_{\mathrm{s}} \mathrm{e}^{\Phi}\right)^{1/2} \mathcal{G}_3^+, \tag{A30a}$$

$$\mathcal{D}_M = \nabla_M + \frac{1}{2} \mathrm{e}^{\Phi} F_M \left(\mathrm{i}\sigma^2\right) + \frac{1}{16} g_{\mathrm{s}} F\!\!\!/^{(5)} \Gamma_M \left(\mathrm{i}\sigma^2\right) + \frac{1}{8} \left(g_{\mathrm{s}} \mathrm{e}^{\Phi}\right)^{1/2} \left(\mathcal{G}_3^- \Gamma_M + \frac{1}{2} \Gamma_M \mathcal{G}_3^-\right), \tag{A30b}$$

where Φ is the dilaton normalized such that $\langle \Phi \rangle = \log g_{\mathrm{s}}$, ∇_M is the covariant derivative built from the 10d metric, the Pauli matrices rotate that spinors constituting the bispinor into each other, and

$$\mathcal{G}_3^{\pm} = F\!\!\!/^{(3)} \sigma^1 \pm \mathrm{e}^{-\Phi} H\!\!\!\!/^{(3)} \sigma^3. \tag{A31}$$

For a p-form $\Omega^{(p)}$, we have defined

$$\Omega\!\!\!/^{(p)} = \frac{1}{p!} \Omega_{M_1 \cdots M_p} \Gamma^{M_1 \cdots M_p}. \tag{A32}$$

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