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General covariant Horava-Lifshitz gravity without projectability condition and its applications to cosmology

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We consider an extended theory of Horava-Lifshitz gravity with the detailed balance condition softly breaking, but without the projectability condition. With the former, the number of independent coupling constants is significantly reduced. With the latter and by extending the original foliation-preserving diffeomorphisms symmetry $\text{Diff}(M, \mathcal{F})$ to include a local $U(1)$ symmetry, the spin-0 gravitons are eliminated. Thus, all the problems related to them disappear, including the instability, strong coupling and different speeds in the gravitational sector. When the theory couples to a scalar field, we find that the scalar field is not only stable in both the ultraviolet (UV) and infrared (IR), but also free of the strong coupling problem, because of the presence of high order spatial derivative terms of the scalar field. Furthermore, applying the theory to cosmology, we find that due to the additional $U(1)$ symmetry, the Friedmann-Robertson-Walker (FRW) universe is necessarily flat. We also investigate the scalar, vector, and tensor perturbations of the flat FRW universe, and derive the general linearized field equations for each kind of the perturbations.

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I. INTRODUCTION

Recently, Horava formulated a theory of quantum gravity, whose scaling at short distances exhibits a strong anisotropy between space and time [1],

$$\mathbf{x} \rightarrow b^{-1}\mathbf{x}, \quad t \rightarrow b^{-z}t. \quad (1.1)$$

In order for the theory to be power-counting renormalizable, in $(3+1)$ -dimensions the critical exponent z needs to be $z \geq 3$ [1, 2]. The gauge symmetry of the theory now is broken from the general covariance, $\tilde{x}^\mu = \tilde{x}^\mu(t, \mathbf{x})$ ($\mu = 0, 1, 2, 3$), down to the foliation-preserving diffeomorphisms $\text{Diff}(M, \mathcal{F})$,

$$\tilde{t} = t - f(t), \quad \tilde{x}^i = x^i - \zeta^i(t, \mathbf{x}). \quad (1.2)$$

Abandoning the Lorentz symmetry gives rise to a proliferation of independently coupling constants [3, 4], which could potentially limit the prediction powers of the theory. To reduce the number of these constants, Horava imposed two conditions, *the projectability and detailed balance* [1]. The former assumes that the lapse function N in the Arnowitt-Deser-Misner decompositions [5] is a function of t only,

$$N = N(t), \quad (1.3)$$

while the latter assumes that gravitational potential \mathcal{L}_V can be obtained from a superpotential W_g via the rela-

tions,

$$\mathcal{L}_{(V,D)} = E_{ij}\mathcal{G}^{ijkl}E_{kl}, \quad E^{ij} = \frac{1}{\sqrt{g}}\frac{\delta W_g}{\delta g_{ij}}, \quad (1.4)$$

where \mathcal{G}^{ijkl} denotes the generalized De Witt metric, defined as $\mathcal{G}^{ijkl} = \frac{1}{2}(g^{ik}g^{jl} + g^{il}g^{jk}) - \lambda g^{ij}g^{kl}$, and λ is a coupling constant.

However, with the detailed balance condition, the Newtonian limit does not exist [6], and a scalar field in the UV is not stable [7]. Thus, it is generally believed that this condition should be abandoned [8]. But, due to several remarkable features [9], Borzou, Lin and Wang recently studied it in detail, and found that the scalar field can be stabilized, if the detailed balance condition is allowed to be softly broken [10]. With such a breaking, all the other related problems found so far can be also resolved. For detail, we refer readers to [10].

On the other hand, with the projectability condition, the number of the independent coupling constants can be significantly reduced. In fact, together with the assumptions of the parity and time-reflection symmetry, it can be reduced from more than 70 to 11 [11] (See also [12]). But, the Minkowski spacetime now becomes unstable [11, 13, 14], although the de Sitter spacetime is [15, 16]. In addition, such a theory also faces the strong coupling problem [16, 17]¹. It should be noted that both

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¹ In the literature, the ghost problem was often mentioned [18, 19]. But, by restricting the coupling constant λ to the regions $\lambda \geq 1$ or $\lambda < 1/3$, this problem is solved (at least in the classical level) [1, 11, 13, 14]. In addition, when $\lambda \in (1/3, 1)$, the instability problem disappears. Therefore, one of these two problems can be always avoided by properly choosing λ . In this paper, we choose $\lambda \geq 1$, so the ghost problem does not exist.

of these two problems are closely related to the existence of a spin-0 graviton [18, 19], because of the foliation-preserving diffeomorphisms (1.2)². Another problem related to the presence of this spin-0 graviton is the difference of its speed from that of the spin-2 graviton. Since they are not related by any symmetry, it poses a great challenge for any attempt to restore Lorentz symmetry at low energies where it has been well tested experimentally. In particular, one needs a mechanism to ensure that in those energy scales all species of matter and gravity have the same effective speed and light cones.

To overcome these problems, so far three main different approaches have been taken. The first one is to provoke the Vainshtein mechanism, initially found in massive gravity [20]. In particular, Mukohyama studied spherically symmetric static spacetimes [19], and shown that the spin-0 gravitons decouple after nonlinear effects are taken into account. Similar considerations in cosmology were given in [21, 22] (See also [16]), where a fully nonlinear analysis of superhorizon cosmological perturbations was carried out, by adopting the so-called gradient expansion method [23]. It was found that the relativistic limit of the Horava-Lifshitz (HL) theory is continuous, and general relativity (GR) is recovered at least in two different cases: (a) when only the “dark matter as an integration constant” is present [21]; and (b) when a scalar field and the “dark matter as an integration constant” are present [22].

Another very attractive and completely different approach is to eliminate the spin-0 gravitons and meanwhile fix λ to its relativistic value, $\lambda_{GR} = 1$. This was done recently by Horava and Melby-Thompson (HMT) [24]. HMT first noticed that in the linearized theory a $U(1)$ symmetry exists only in the case $\lambda = 1$ [1]. Thus, to fix λ , one may extend the foliation-preserving diffeomorphism symmetry (1.2) to,

$$U(1) \ltimes \text{Diff}(M, \mathcal{F}). \quad (1.5)$$

To lift such a symmetry to the full nonlinear theory, HMT found that it is necessary to introduce a scalar field - the Newtonian prepotential, in addition to the $U(1)$ gauge field. Once this was done, HMT showed that the spin-0 graviton is eliminated [24]. This was further confirmed in [25]. Then, the instability and strong coupling problems of the spin-0 gravitons are out of question. In addition, since λ was fixed to one, these problems in the non-gravitational sectors are also resolved, as all of them are related to the fact that $\lambda \neq 1$ [26].

However, da Silva soon found that the introduction of the Newtonian prepotential is so strong that actions with $\lambda \neq 1$ also have the extended symmetry (1.5) [27]. The spin-0 gravitons are eliminated even with any given λ

[26–28], so that the strong coupling problem does not exist any longer in the pure gravitational sector. However, it still exists when matter is present. Indeed, in [26] it was shown that, for processes with energy higher than $\Lambda_\omega [\equiv |\lambda - 1|^{5/2} M_{pl}]$, the theory becomes strong coupling [26]. Together with Lin, three of the present authors [29] showed that this problem can be resolved by introducing a new energy scale M_* [30], so that $M_* < \Lambda_\omega$, where M_* denotes the suppression energy scale of high-order derivative terms of the theory.

Note that all the above two approaches assume the projectability condition (1.3). The third approach is to abandon this condition, by including the vector field [3]

$$a_i = \partial_i \ln(N), \quad (1.6)$$

into the action³. Although it also solves the instability and strong coupling problems, the presence of this vector field a_i gives rise to a proliferation of independent coupling constants [4], as mentioned above. When applying the theory to cosmology and astrophysics, this potentially limits its predictive powers. In addition, the problem of different speeds in the gravitational sector still exists, because the spin-0 graviton still exists in this setup, and its speed depends on the coupling constants λ and β_0 [3, 33, 34], while the one of the spin-2 graviton is independent of them, where β_0 is defined in Eq.(2.21) given below.

Recently, we proposed an extended version of HL gravity without the projectability condition (1.3) but with the enlarged symmetry (1.5) [35], with the purposes: (i) Reduce significantly the number of the independent coupling constants usually presented in the version of the HL theory without the projectability condition, by imposing the detailed balance condition. However, in order for the theory to be both UV complete and IR healthy, we allowed the detailed balance condition to be broken softly by adding all the low dimensional relevant terms. (ii) Eliminate the spin-0 gravitons even in the case without the projectability condition by implementing the enlarged symmetry (1.5)⁴, so that all the problems related to them disappear, including the instability, strong coupling and different speeds in the pure gravitational sector.

In this paper, we shall first provide a systematical study of this extended version of the HL gravity regarding to the above mentioned problems in the gravitational as well as matter sectors, and then apply it to cosmology. In particular, the paper is organized as follows. In Sec II, we construct the gravitational potential by imposing the detailed balance condition softly breaking. In Sec III, we

² Since the foliation-preserving diffeomorphisms (1.2) is also assumed in the version without the projectability condition [3], the spin-0 graviton exists there too.

³ It should be noted that the violation of the projectability condition often leads to the inconsistency problem [31]. However, as shown in [32], this is not the case in the setup of [3].

⁴ Note that the $U(1)$ symmetry in the case without projectability condition was also considered in the so-called $F(R)$ Horava-Lifshitz gravity [36].

extend the foliation-preserving diffeomorphism symmetry of HL gravity to include a local $U(1)$ symmetry, and with this enlarged symmetry, in Sec IV, we show that the spin-0 gravitons are indeed eliminated. In Sec V, we consider the coupling of the theory with a scalar field, and show that the scalar field is stable in both of the UV and IR. In addition, the strong coupling problem does not exist, because of the presence of the sixth-order spatial derivative terms of the scalar field, as long as their suppressed energy scale M_* is lower than the would-be strong coupling energy scale Λ_ω . In Sec VI we study cosmological models, and show that the FRW universe is necessarily flat in such a setup, while in Sec VII, we investigate the linear scalar, vector and tensor perturbations of the flat FRW universe, and present the general linearized field equations for each kind of the perturbations. Finally, in Sec VIII we present our main conclusions.

II. POTENTIAL WITH DETAILED BALANCE CONDITION SOFTLY BREAKING

To understand the consequence of the breaking of the projectability condition (1.3), let us start with counting the independent terms order by order. We first write the metric in the Arnowitt-Deser-Misner form [5],

$$ds^2 = -N^2 c^2 dt^2 + g_{ij} (dx^i + N^i dt) (dx^j + N^j dt), \quad (i, j = 1, 2, 3). \quad (2.1)$$

Under the rescaling (1.1) with $z = 3$, N , N^i and g_{ij} scale, respectively, as,

$$N \rightarrow N, \quad N^i \rightarrow b^{-2} N^i, \quad g_{ij} \rightarrow g_{ij}. \quad (2.2)$$

Under the foliation-preserving diffeomorphisms (1.2), they transform as

$$\begin{aligned} \delta g_{ij} &= \nabla_i \zeta_j + \nabla_j \zeta_i + f \dot{g}_{ij}, \\ \delta N_i &= N_k \nabla_i \zeta^k + \zeta^k \nabla_k N_i + g_{ik} \dot{\zeta}^k + \dot{N}_i f + N_i \dot{f}, \\ \delta N &= \zeta^k \nabla_k N + \dot{N} f + N \dot{f}, \end{aligned} \quad (2.3)$$

where $\dot{f} \equiv df/dt$, ∇_i denotes the covariant derivative with respect to the 3-metric g_{ij} and $N_i = g_{ik} N^k$.

Assuming that the engineering dimensions of space and time are [11],

$$[dx] = [k]^{-1}, \quad [dt] = [k]^{-3}, \quad (2.4)$$

we find that

$$\begin{aligned} [N^i] &= [c] = \frac{[dx]}{[dt]} = [k]^2, \quad [g_{ij}] = [N] = [1], \\ [K_{ij}] &= [k]^3, \quad [\Gamma_{jk}^i] = [k], \quad [R_{jkl}^i] = [k]^2. \end{aligned} \quad (2.5)$$

Then, to each order of $[k]$, we have the following independent terms that are all scalars under the transformations of the foliation-preserving diffeomorphisms (1.2)

[3, 4, 11],

$$\begin{aligned} [k]^6 &: K_{ij} K^{ij}, K^2, R^3, RR_{ij} R^{ij}, R_i^j R_k^l R_i^k, (\nabla R)^2, \\ &(\nabla_i R_{jk}) (\nabla^i R^{jk}), (a_i a^i)^2 R, (a_i a^i) (a_i a_j R^{ij}), \\ &(a_i a^i)^3, a^i \Delta^2 a_i, (a^i{}_i) \Delta R, \dots, \\ [k]^5 &: K_{ij} R^{ij}, \epsilon^{ijk} R_{il} \nabla_j R_k^l, \epsilon^{ijk} a_i a_l \nabla_j R_k^l, \\ &a_i a_j K^{ij}, K^{ij} a_{ij}, (a^i{}_i) K, \\ [k]^4 &: R^2, R_{ij} R^{ij}, (a_i a^i)^2, (a^i{}_i)^2, (a_i a^i) a^j{}_j, \\ &a^{ij} a_{ij}, (a_i a^i) R, a_i a_j R^{ij}, R a^i{}_i, \\ [k]^3 &: \omega_3(\Gamma), \\ [k]^2 &: R, a_i a^i, \\ [k]^1 &: \text{None}, \\ [k]^0 &: \gamma_0, \end{aligned} \quad (2.6)$$

where $\omega_3(\Gamma)$ denotes the gravitational Chern-Simons term, γ_0 is a dimensionless constant, $\epsilon^{ijk} \equiv e^{ijk}/\sqrt{g}$, ($e^{123} = 1$), $\Delta = g^{ij} \nabla_i \nabla_j$, and

$$\begin{aligned} K_{ij} &= \frac{1}{2N} (-\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i), \\ a_{i_1 i_2 \dots i_n} &= \nabla_{i_1} \nabla_{i_2} \dots \nabla_{i_n} \ln(N), \\ \omega_3(\Gamma) &= \text{Tr} \left(\Gamma \wedge d\Gamma + \frac{2}{3} \Gamma \wedge \Gamma \wedge \Gamma \right) \\ &= \frac{e^{ijk}}{\sqrt{g}} \left(\Gamma_{jl}^m \partial_j \Gamma_{km}^l + \frac{2}{3} \Gamma_{il}^n \Gamma_{jm}^l \Gamma_{kn}^m \right). \end{aligned} \quad (2.7)$$

In writing Eq.(2.6), we had not written down all the sixth order terms, as they are numerous [3, 4]. Then, the general action of the gravitational part will be given by

$$\hat{S}_g = \zeta^2 \int dt d^3 x N \sqrt{g} (\mathcal{L}_K - \mathcal{L}_V), \quad (2.8)$$

where the kinetic part \mathcal{L}_K is the linear combination of the first two sixth order derivative terms,

$$\mathcal{L}_K = g_T (K_{ij} K^{ij} - \lambda K^2). \quad (2.9)$$

Note that the coupling constant g_T can be absorbed into ζ^2 . So, without loss of generality, we can set it to one, $g_T = 1$. The potential part \mathcal{L}_V is the linear combination of all the other terms of Eq.(2.6), which are more than 70 terms, and could potentially weaken the prediction powers of the theory.

In the following, we look for conditions to reduce the number of the independent terms. First, since those with odd number of derivatives violate the spatial parity and time-reversal symmetry, they can be easily eliminated by imposing the parity conservation and time-reversal symmetry. To reduce the number of the sixth order derivative terms, following Horava we impose the ‘‘generalized’’ detailed balance condition,

$$\hat{\mathcal{L}}_{(V,D)} = \mathcal{L}_{(V,D)} - g_{ij} A^i A^j, \quad (2.10)$$

where A^i is defined by the superpotential W_a ,

$$A^i = \frac{1}{\sqrt{g}} \frac{\delta W_a}{\delta a_i}, \quad (2.11)$$

with

$$W_a = \frac{1}{2} \int d^3x \sqrt{g} a^i \left(\sum_{n=0}^{n=1} \mathbf{b}_n \Delta^n a_i \right), \quad (2.12)$$

where \mathbf{b}_n are arbitrary constants. Note that the term of $a_i \Delta^{1/2} a^i$ in principle can be included into W_a , which will give rise to fractional calculus, a branch of mathematics that has been well developed [37]. But this gives rise to fifth order derivative terms, and we shall discard these terms. Inserting Eq.(2.11) into Eq.(2.10), we find that its second term is involved only with a_i , and the corresponding action takes the form,

$$S_a = \int dt d^3x \sqrt{g} \left[\beta_0 (a_i a^i) + \eta_1 a_i \Delta a^i + \eta_2 (\Delta a^i)^2 \right], \quad (2.13)$$

where

$$\beta_0 \equiv \mathbf{b}_0^2, \quad \eta_1 \equiv 2\mathbf{b}_0 \mathbf{b}_1, \quad \eta_2 \equiv \mathbf{b}_1^2. \quad (2.14)$$

The superpotential W_g appearing in Eq.(1.4) is given by [1]

$$W_g = \int_{\Sigma} \left(\frac{1}{w^2} \omega_3(\Gamma) + \mu(R - 2\Lambda_W) \right), \quad (2.15)$$

where $\omega_3(\Gamma)$ is defined in Eq.(2.7), R (R_{ij}) is the Ricci scalar (tensor) built out of g_{ij} . Inserting Eq.(2.15) into Eq.(1.4), we find that

$$\begin{aligned} \mathcal{L}_{(V,D)} &= \zeta^2 \gamma_0 + \gamma_1 R + \frac{1}{\zeta^2} (\gamma_2 R^2 + \gamma_3 R_{ij} R^{ij}) \\ &+ \frac{\gamma_4}{\zeta^3} \epsilon^{ijkl} R_{il} \nabla_j R_k^l + \frac{\gamma_5}{\zeta^4} C_{ij} C^{ij}, \end{aligned} \quad (2.16)$$

where γ_n are dimensionless constants, given explicitly in terms of the five independent coupling constants ζ , w , μ , Λ_W and λ in [1]. C_{ij} denotes the Cotton tensor, defined by

$$C^{ij} = \frac{e^{ikl}}{\sqrt{g}} \nabla_k \left(R_l^j - \frac{1}{4} R \delta_l^j \right). \quad (2.17)$$

Using the Bianchi identities and the definition of the Riemann tensor, one can show that $C_{ij} C^{ij}$ can be written in terms of the five independent sixth order derivative terms in the form,

$$\begin{aligned} C_{ij} C^{ij} &= \frac{1}{2} R^3 - \frac{5}{2} R R_{ij} R^{ij} + 3 R_j^i R_k^j R_i^k + \frac{3}{8} R \Delta R \\ &+ (\nabla_i R_{jk}) (\nabla^i R^{jk}) + \nabla_k G^k, \end{aligned} \quad (2.18)$$

where

$$G^k = \frac{1}{2} R^{jk} \nabla_j R - R_{ij} \nabla^j R^{ik} - \frac{3}{8} R \nabla^k R. \quad (2.19)$$

When integrated, with the projectability condition (1.3), $\nabla_k G^k$ becomes a boundary term and can be discarded. However, in the case without this condition, this is no longer true, since now we have $N = N(t, x)$ and

$$\int_M dt d^3x N \sqrt{g} \nabla_k G^k = - \int_M dt d^3x N \sqrt{g} G^k a_k, \quad (2.20)$$

which in general is not zero.

As mentioned previously, in order for the theory to have a healthy IR limit, the detailed condition needs to be broken softly by adding all the lower (than six) dimensional relevant terms presented in Eq.(2.6), so that finally the potential is given by [35],

$$\begin{aligned} \mathcal{L}_V &= \gamma_0 \zeta^2 - (\beta_0 a_i a^i - \gamma_1 R) + \frac{1}{\zeta^2} (\gamma_2 R^2 + \gamma_3 R_{ij} R^{ij}) \\ &+ \frac{1}{\zeta^2} \left[\beta_1 (a_i a^i)^2 + \beta_2 (a^i{}_i)^2 + \beta_3 (a_i a^i) a^j{}_j \right. \\ &+ \beta_4 a^{ij} a_{ij} + \beta_5 (a_i a^i) R + \beta_6 a_i a_j R^{ij} + \beta_7 R a^i{}_i \left. \right] \\ &+ \frac{1}{\zeta^4} \left[\gamma_5 C_{ij} C^{ij} + \beta_8 (\Delta a^i)^2 \right], \end{aligned} \quad (2.21)$$

where $\beta_8 \equiv -\eta_2 \zeta^4$. All the coefficients, β_n and γ_n , are dimensionless and arbitrary, except for the ones of the sixth order derivative terms, γ_5 and β_8 , which must be,

$$\gamma_5 > 0, \quad \beta_8 < 0, \quad (2.22)$$

as can be seen from Eqs.(2.15) and (2.14). To be consistent with observations in the IR, we must set,

$$\zeta^2 = \frac{1}{16\pi G}, \quad \gamma_1 = -1, \quad (2.23)$$

where G denotes the Newtonian constant, and

$$\Lambda \equiv \frac{1}{2} \zeta^2 \gamma_0, \quad (2.24)$$

is the cosmological constant.

It can be shown that for quadratic action of the scalar perturbations in the Minkowski background the sixth-order spatial derivative terms of the potential (2.21) are absent. As a result, the gravitational sector is still strong coupling, and cannot be solved by the mechanism proposed in [30]. To solve this problem, one way is to eliminate the spin-0 gravitons, as HMT did in the case with the projectability condition. In the next section, we will show explicitly that this is possible by enlarging the $\text{Diff}(M, \mathcal{F})$ symmetry (1.2) to the one $U(1) \ltimes \text{Diff}(M, \mathcal{F})$ (1.5), even in the case without the projectability condition.

III. $U(1) \times \text{Diff}(M, \mathcal{F})$ SYMMETRY AND FIELD EQUATIONS

In order to eliminate the spin-0 gravitons, let us first consider the $U(1)$ gauge transformations [24],

$$\delta_\alpha N_i = N \nabla_i \alpha, \quad \delta_\alpha g_{ij} = 0 = \delta_\alpha N, \quad (3.1)$$

where α denotes the $U(1)$ generator. Under the above transformations, the variation of the HL action (2.8) is given by

$$\begin{aligned} \delta \hat{S}_g &= \zeta^2 \int dt d^3 x \sqrt{g} (\dot{\alpha} - N^i \nabla_i \alpha) R, \\ &+ 2\zeta^2 \int dt d^3 x \sqrt{g} N \alpha G^{ij} K_{ij}, \\ &+ 2\zeta^2 \int dt d^3 x \sqrt{g} N \hat{G}^{ijkl} K_{ij} a_{(l} \nabla_{k)} \alpha, \\ &+ 2(1-\lambda)\zeta^2 \int dt d^3 x \sqrt{g} N K (\nabla^2 \alpha + a^k \nabla_k \alpha), \end{aligned} \quad (3.2)$$

where $f_{(ij)} = (f_{ij} + f_{ji})/2$, $\hat{G}^{ijkl} = g^{il} g^{jk} - g^{ij} g^{lk}$, and $G^{ij} = R^{ij} - g^{ij} R/2$. In order for the theory to have the $U(1)$ symmetry, one can introduce a $U(1)$ gauge field A , which transforms as

$$\delta_\alpha A = \dot{\alpha} - N^i \nabla_i \alpha. \quad (3.3)$$

Then, by adding the new coupling term

$$\begin{aligned} S_A &= \zeta^2 \int dt d^3 x N \sqrt{g} \mathcal{L}_A \\ &= \zeta^2 \int dt d^3 x \sqrt{g} A (2\Lambda_g - R), \end{aligned} \quad (3.4)$$

to \hat{S}_g , one finds that its variation (for $\Lambda_g = 0$) with respect to α exactly cancels the first term given in Eq.(3.2). To repair the rest, we introduce the Newtonian prepotential φ , which transforms as,

$$\delta_\alpha \varphi = -\alpha. \quad (3.5)$$

Then, it can be shown that under Eq.(3.1) the variation of the term,

$$\begin{aligned} S_{(\varphi,1)} &= \zeta^2 \int dt d^3 x \sqrt{g} N \varphi G^{ij} \left[2K_{ij} + a_{(i} \nabla_{j)} \varphi \right. \\ &\quad \left. + \nabla_i \nabla_j \varphi \right], \end{aligned} \quad (3.6)$$

exactly cancels the second term in Eq.(3.2) as well as the term $2A\Lambda_g$ in (3.4), where

$$G^{ij} \equiv R^{ij} - \frac{1}{2} R g^{ij} + \Lambda_g g^{ij}. \quad (3.7)$$

The third and fourth terms in (3.2) can be canceled, respectively, by

$$\begin{aligned} S_{(\varphi,2)} &= \frac{\zeta^2}{3} \int dt d^3 x \sqrt{g} N \hat{G}^{ijkl} \left[6K_{ij} a_{(k} \nabla_{l)} \varphi \right. \\ &\quad + 4(\nabla_i \nabla_j) a_{(k} \nabla_{l)} \varphi + 5a_{(i} \nabla_{j)} \varphi a_{(k} \nabla_{l)} \varphi \\ &\quad \left. + 2\nabla_{(i} \varphi a_{j)(k} \nabla_{l)} \varphi \right], \end{aligned} \quad (3.8)$$

and

$$\begin{aligned} S_{(\varphi,3)} &= (1-\lambda)\zeta^2 \int dt d^3 x \sqrt{g} N \left\{ (a^k \nabla_k \varphi + \nabla^2 \varphi) \right. \\ &\quad \left. \times \left[2K + (a^k \nabla_k \varphi + \nabla^2 \varphi) \right] \right\}. \end{aligned} \quad (3.9)$$

Hence, the action

$$S_g = \hat{S}_g + S_A + S_\varphi, \quad (3.10)$$

is invariant under the $U(1) \times \text{Diff}(M, \mathcal{F})$ symmetry (1.5), where

$$S_\varphi = \sum_{n=1}^3 S_{(\varphi,n)} \equiv \zeta^2 \int dt d^3 x N \sqrt{g} \mathcal{L}_\varphi, \quad (3.11)$$

with

$$\begin{aligned} \mathcal{L}_\varphi &= \varphi G^{ij} (2K_{ij} + \nabla_i \nabla_j \varphi + a_i \nabla_j \varphi) \\ &+ (1-\lambda) \left[(\nabla^2 \varphi + a_i \nabla^i \varphi)^2 + 2(\nabla^2 \varphi + a_i \nabla^i \varphi) K \right] \\ &+ \frac{1}{3} \hat{G}^{ijkl} \left[4(\nabla_i \nabla_j \varphi) a_{(k} \nabla_{l)} \varphi + 5(a_{(i} \nabla_{j)} \varphi) a_{(k} \nabla_{l)} \varphi \right. \\ &\quad \left. + 2(\nabla_{(i} \varphi) a_{j)(k} \nabla_{l)} \varphi + 6K_{ij} a_{(l} \nabla_{k)} \varphi \right]. \end{aligned} \quad (3.12)$$

When coupling to the matter \mathcal{L}_M , the total action of the theory takes the form,

$$\begin{aligned} S &= \zeta^2 \int dt d^3 x \sqrt{g} N \left(\mathcal{L}_K - \mathcal{L}_V + \mathcal{L}_A + \mathcal{L}_\varphi \right. \\ &\quad \left. + \frac{1}{\zeta^2} \mathcal{L}_M \right). \end{aligned} \quad (3.13)$$

Then, the variations of S with respect to N and N^i give rise to the Hamiltonian and momentum constraints,

$$\mathcal{L}_K + \mathcal{L}_V^R + F_V - F_\varphi - F_\lambda = 8\pi G J^t, \quad (3.14)$$

$$\begin{aligned} \nabla_j \left\{ \pi^{ij} - \varphi G^{ij} - \hat{G}^{ijkl} a_l \nabla_k \varphi \right. \\ \left. - (1-\lambda) g^{ij} (\nabla^2 \varphi + a_k \nabla^k \varphi) \right\} = 8\pi G J^i, \end{aligned} \quad (3.15)$$

where

$$\begin{aligned} \mathcal{L}_V^R &= \gamma_0 \zeta^2 - R + \frac{\gamma_2 R^2 + \gamma_3 R_{ij} R^{ij}}{\zeta^2} + \frac{\gamma_5}{\zeta^4} C_{ij} C^{ij}, \\ J^i &= -N \frac{\delta \mathcal{L}_M}{\delta N_i}, \quad J^t = 2 \frac{\delta(N \mathcal{L}_M)}{\delta N}, \\ \pi^{ij} &= -K^{ij} + \lambda K g^{ij}, \end{aligned} \quad (3.16)$$

and F_V , F_φ and F_λ are given by Eqs.(A.1)-(A.3) in Appendix A. Note that in (refmom) we have separated \mathcal{L}_V

into two parts \mathcal{L}_V^R and \mathcal{L}_V^a . Variations of S with respect to φ and A yield, respectively,

$$\begin{aligned} & \frac{1}{2}\mathcal{G}^{ij}(2K_{ij} + \nabla_i \nabla_j \varphi + a_{(i} \nabla_{j)} \varphi) \\ & + \frac{1}{2N} \left\{ \mathcal{G}^{ij} \nabla_j \nabla_i (N\varphi) - \mathcal{G}^{ij} \nabla_j (N\varphi a_i) \right\} \\ & - \frac{1}{N} \hat{\mathcal{G}}^{ijkl} \left\{ \nabla_{(k} (a_l) N K_{ij}) + \frac{2}{3} \nabla_{(k} (a_l) N \nabla_i \nabla_j \varphi) \right. \\ & - \frac{2}{3} \nabla_{(j} \nabla_{i)} (N a_{(l} \nabla_{k)} \varphi) + \frac{5}{3} \nabla_j (N a_i a_k \nabla_l \varphi) \\ & \left. + \frac{2}{3} \nabla_j (N a_{ik} \nabla_l \varphi) \right\} \\ & + \frac{1-\lambda}{N} \left\{ \nabla^2 [N(\nabla^2 \varphi + a_k \nabla^k \varphi)] \right. \\ & - \nabla^i [N(\nabla^2 \varphi + a_k \nabla^k \varphi) a_i] \\ & \left. + \nabla^2 (NK) - \nabla^i (NK a_i) \right\} = 8\pi G J_\varphi, \end{aligned} \quad (3.17)$$

and

$$R - 2\Lambda_g = 8\pi G J_A, \quad (3.18)$$

where

$$J_\varphi = -\frac{\delta \mathcal{L}_M}{\delta \varphi}, \quad J_A = 2\frac{\delta(N\mathcal{L}_M)}{\delta A}. \quad (3.19)$$

On the other hand, the variation of S with respect to g_{ij} yields the dynamical equations,

$$\begin{aligned} & \frac{1}{\sqrt{g}N} \frac{\partial}{\partial t} (\sqrt{g} \pi^{ij}) + 2(K^{ik} K_k^j - \lambda K K^{ij}) \\ & + \frac{1}{N} \nabla_k (\pi^{ik} N^j + \pi^{kj} N^i - \pi^{ij} N^k) \\ & - F^{ij} - F_a^{ij} - F_\varphi^{ij} - \frac{1}{2} g^{ij} \mathcal{L}_K - \frac{1}{2} g^{ij} \mathcal{L}_A \\ & - \frac{1}{N} (A R^{ij} + g^{ij} \nabla^2 A - \nabla^j \nabla^i A) = 8\pi G \tau^{ij}, \end{aligned} \quad (3.20)$$

where

$$\begin{aligned} \tau^{ij} &= \frac{2}{\sqrt{g}N} \frac{\delta(\sqrt{g}N\mathcal{L}_M)}{\delta g_{ij}}, \\ F^{ij} &= \frac{1}{\sqrt{g}N} \frac{\delta(-\sqrt{g}N\mathcal{L}_V^R)}{\delta g_{ij}} \\ &= \sum_{s=0} \hat{\gamma}_s \zeta^{n_s} (F_s)^{ij}, \end{aligned} \quad (3.21)$$

$$\begin{aligned} F_a^{ij} &= \frac{1}{\sqrt{g}N} \frac{\delta(-\sqrt{g}N\mathcal{L}_V^a)}{\delta g_{ij}} \\ &= \sum_{s=0} \beta_s \zeta^{m_s} (F_s^a)^{ij}, \end{aligned} \quad (3.22)$$

$$\begin{aligned} F_\varphi^{ij} &= \frac{1}{\sqrt{g}N} \frac{\delta(\sqrt{g}N\mathcal{L}_\varphi)}{\delta g_{ij}} \\ &= \sum_{s=0} \mu_s (F_s^\varphi)^{ij}, \end{aligned} \quad (3.23)$$

the expressions of F_s , F_s^a and F_s^φ can be found in Appendix (A.4)-(A.6), and

$$\begin{aligned} \hat{\gamma}_s &= \left(\gamma_0, \gamma_1, \gamma_2, \gamma_3, \frac{1}{2}\gamma_5, -\frac{5}{2}\gamma_5, 3\gamma_5, \frac{3}{8}\gamma_5, \gamma_5, \frac{1}{2}\gamma_5 \right), \\ n_s &= (2, 0, -2, -2, -4, -4, -4, -4, -4, -4), \\ m_s &= (0, -2, -2, -2, -2, -2, -2, -2, -4), \\ \mu_s &= \left(2, 1, 1, 2, \frac{4}{3}, \frac{5}{3}, \frac{2}{3}, 1 - \lambda, 2 - 2\lambda \right). \end{aligned} \quad (3.24)$$

In addition, the matter components ($J^t, J^i, J_\varphi, J_A, \tau^{ij}$) satisfy the conservation laws of energy and momentum,

$$\begin{aligned} \int d^3x \sqrt{g}N \left[\dot{g}_{ij} \tau^{ij} - \frac{1}{\sqrt{g}} \partial_t (\sqrt{g} J^t) + \frac{2N_i}{\sqrt{g}N} \partial_t (\sqrt{g} J^i) \right. \\ \left. - \frac{A}{\sqrt{g}N} \partial_t (\sqrt{g} J_A) - 2\dot{\varphi} J_\varphi \right] = 0, \end{aligned} \quad (3.25)$$

$$\begin{aligned} \frac{1}{N} \nabla^i (N \tau_{ik}) - \frac{1}{\sqrt{g}N} \partial_t (\sqrt{g} J_k) - \frac{J_A}{2N} \nabla_k A - \frac{J^t}{2N} \nabla_k N \\ - \frac{N_k}{N} \nabla_i J^i - \frac{J_i}{N} (\nabla_i N_k - \nabla_k N_i) + J_\varphi \nabla_k \varphi = 0. \end{aligned} \quad (3.26)$$

IV. ELIMINATION OF SPIN-0 GRAVITONS

In this section, we show that the spin-0 gravitons are indeed eliminated for the theory described by the action (3.10). To this goal, we consider the scalar perturbations of the Minkowski background,

$$\begin{aligned} N &= 1 + \phi, \quad N_i = \partial_i B, \\ g_{ij} &= (1 - 2\psi) \delta_{ij} + 2E_{,ij}, \\ A &= \delta A, \quad \varphi = \delta \varphi. \end{aligned} \quad (4.1)$$

Using the gauge freedom, without loss of generality, one can choose the gauge,

$$E = 0, \quad \varphi = 0. \quad (4.2)$$

Then, after simple but tedious calculations, to second order we find that

$$\begin{aligned} S^{(2)} &= \zeta^2 \int dt d^3x \left\{ (1 - 3\lambda)(3\dot{\psi}^2 + 2\dot{\psi} \partial^2 B) \right. \\ &+ (1 - \lambda)(\partial^2 B)^2 - \left(\phi \bar{\partial} + \frac{4\beta_7}{\zeta^2} \partial^2 \psi \right) \partial^2 \phi \\ &\left. - 2(\psi - 2\phi + 2A + \alpha_1 \psi \partial^2) \partial^2 \psi \right\}, \end{aligned} \quad (4.3)$$

where

$$\begin{aligned} \alpha_1 &\equiv \frac{8\gamma_2 + 3\gamma_3}{a^2 \zeta^2}, \\ \bar{\partial} &\equiv \beta_0 + \frac{\beta_2 + \beta_4}{a^2 \zeta^2} \partial^2 - \frac{\beta_8}{a^4 \zeta^4} \partial^4, \\ \varphi &\equiv 1 - \frac{\beta_7}{a^2 \zeta^2} \partial^2. \end{aligned} \quad (4.4)$$

Here a is the scale factor of the FRW universe, which is one for the Minkowski background. Variations of $S^{(2)}$ with respect to A , ψ , B , and ϕ yield, respectively,

$$\partial^2\psi = 0, \quad (4.5)$$

$$\begin{aligned} \ddot{\psi} + \frac{1}{3}\partial^2\dot{B} + \frac{2}{3(1-3\lambda)}(\partial^2A + \partial^2\psi + \alpha_1\partial^4\psi) \\ = \frac{2}{3(1-3\lambda)}\partial^2\wp\phi, \end{aligned} \quad (4.6)$$

$$(\lambda-1)\partial^2B = (1-3\lambda)\dot{\psi}, \quad (4.7)$$

$$\ddot{\phi} = 2\wp\psi. \quad (4.8)$$

Eq.(4.5) clearly shows that ψ is not propagating, and, with proper boundary conditions, we can always set $\psi = 0$. Similarly, Eqs.(4.6), (4.7), and (3.17) show that B , A , and ϕ are also not propagating and can be set to zero by proper boundary conditions. Therefore, we finally obtain

$$\psi = B = A = \phi = 0. \quad (4.9)$$

Thus, the scalar perturbations indeed vanish identically in the Minkowski background, and, as a result, the spin-0 gravitons are eliminated. Then, all the problems related to the spin-0 gravitons disappear, including the ghost, instability and strong coupling problems [18, 19].

V. STABILITY AND STRONG COUPLING OF SCALAR FIELD

Since the spin-0 gravitons are eliminated, problems related to them, such as the ghost, instability, and strong coupling, in the gravitational sector do not exist. But, the self-interaction of matter fields and the interaction between a matter and a gravitational field can still lead to strong coupling, as shown in [29] for the theory with the projectability condition. In the following, we shall show that this is also the case here. However, it can be solved by the BPS mechanism [30], by simply introducing a new energy scale M_* , that suppresses the sixth-order spatial derivative terms. Let us first consider the stability of a scalar field in the Minkowski background.

A. Stability of Scalar Field

For a scalar field χ with the detailed balance conditions softly breaking, it is described by [10, 38],

$$\begin{aligned} \mathcal{L}_M &= \mathcal{L}_\chi^{(A,\varphi)} + \mathcal{L}_\chi^{(0)}, \\ \mathcal{L}_\chi^{(A,\varphi)} &= \frac{A-\mathcal{A}}{N} [c_1(\chi)\Delta\chi + c_2(\chi)(\nabla\chi)^2] \\ &\quad + \frac{f}{2}[(\nabla^k\varphi)(\nabla_k\chi)]^2 \\ &\quad - \frac{f}{N}(\dot{\chi} - N^i\nabla_i\chi)(\nabla^k\varphi)(\nabla_k\chi), \end{aligned} \quad (5.1)$$

$$\mathcal{L}_\chi^{(0)} = \frac{f}{2N^2}(\dot{\chi} - N^i\nabla_i\chi)^2 - \mathcal{V}, \quad (5.2)$$

where

$$\begin{aligned} \mathcal{V} &= V(\chi) + \left(\frac{1}{2} + V_1(\chi)\right)(\nabla\chi)^2 + V_2(\chi)\mathcal{P}_1^2 \\ &\quad + V_3(\chi)\mathcal{P}_1^3 + V_4(\chi)\mathcal{P}_2 + V_5(\chi)(\nabla\chi)^2\mathcal{P}_2 \\ &\quad + V_6\mathcal{P}_1\mathcal{P}_2, \end{aligned} \quad (5.3)$$

$$\mathcal{A} = -\dot{\varphi} + N^i\nabla_i\varphi + \frac{1}{2}N(\nabla_i\varphi)(\nabla^i\varphi), \quad (5.4)$$

and

$$\mathcal{P}_n \equiv \Delta^n\chi, \quad V_6 \equiv -\sigma_3^2, \quad (5.5)$$

where σ_3 is a constant. The coefficient f in (5.1) is a function of λ only. Then, it can be shown that the Minkowski spacetime $(\bar{N}, \bar{N}^i, \bar{g}_{ij}) = (1, 0, \delta_{ij})$ is a solution of the above theory, provided that

$$\bar{A} = \bar{\varphi} = 0, \quad \bar{\chi} = \bar{\chi}_0, \quad V(\bar{\chi}_0) = 0 = V'(\bar{\chi}_0), \quad (5.6)$$

where $\bar{\chi}_0$ is a constant. Without loss of generality, we set it to zero. Considering the perturbations (4.1), together with the one of the scalar field $\chi = \delta\chi$, we find that to the second order the total action is given by,

$$\begin{aligned} S^{(2)} &= \zeta^2 \int dt d^3x \left\{ (1-3\lambda)(3\dot{\psi}^2 + 2\dot{\psi}\partial^2B) \right. \\ &\quad + (1-\lambda)(\partial^2B)^2 - \left(\phi\ddot{\phi} + \frac{4\beta_7}{\zeta^2}\partial^2\psi \right) \partial^2\phi \\ &\quad - 2(\psi - 2\phi + 2A + \alpha_1\psi\partial^2)\partial^2\psi \\ &\quad + \frac{1}{\zeta^2} \left[\frac{f}{2}\dot{\chi}^2 - \frac{1}{2}V''\chi^2 + c_1A\partial^2\chi \right. \\ &\quad - \left(\frac{1}{2} + V_1 \right) (\partial\chi)^2 - V_2(\partial^2\chi)^2 \\ &\quad \left. \left. - V_4'\chi\partial^4\chi + \sigma_3^2\partial^2\chi\partial^4\chi \right] \right\}. \end{aligned} \quad (5.7)$$

Variations of this action with respect to A , ψ , B , ϕ , and χ , yield, respectively,

$$\psi = \frac{c_1}{4\zeta^2}\chi, \quad (5.8)$$

$$\begin{aligned} \ddot{\psi} + \frac{1}{3}\partial^2\dot{B} + \frac{2}{3(1-3\lambda)}(\partial^2A + \partial^2\psi + \alpha_1\partial^4\psi) \\ = \frac{2}{3(1-3\lambda)}\partial^2\wp\phi, \end{aligned} \quad (5.9)$$

$$(\lambda-1)\partial^2B = (1-3\lambda)\dot{\psi}, \quad (5.10)$$

$$\ddot{\phi} = 2\wp\psi, \quad (5.11)$$

$$\begin{aligned} f\ddot{\chi} + V''\chi - (1+2V_1)\partial^2\chi + 2(V_2+V_4')\partial^4\chi \\ = 2\sigma_3^2\partial^6\chi + c_1\partial^2A. \end{aligned} \quad (5.12)$$

From the above field equations, one can get a master equation for the scalar field χ , which in momentum space can be written in the form,

$$\ddot{\chi}_k + \omega_k^2\chi_k = 0, \quad (5.13)$$

where

$$\begin{aligned} \omega_k^2 = & \frac{1}{f + \frac{c_1^2}{4\zeta^2|c_\psi|^2}} \left\{ V'' + k^2 \left(1 + 2V_1 - \frac{c_1^2}{4\zeta^2} \right) \right. \\ & + k^4 \left(2V_2 + 2V_4' + \frac{c_1^2}{4\zeta^2} \frac{\lambda_3}{M_A^2} \right) + 2\sigma_3^2 k^6 \\ & \left. + \frac{c_1^2}{4\zeta^2} \frac{2 \left(1 + \frac{\lambda_2}{M_A^2} k^2 \right)^2 k^2}{\alpha - k^2 \frac{\lambda_1}{M_A^2} - k^4 \frac{\lambda_4}{M_B^4}} \right\}, \end{aligned} \quad (5.14)$$

with

$$\begin{aligned} \lambda_1 &\equiv \frac{\beta_2 + \beta_4}{\zeta^2} M_A^2, \quad \lambda_2 \equiv \frac{\beta_7}{\zeta^2} M_A^2, \\ \lambda_3 &\equiv \frac{8\gamma_2 + 3\gamma_3}{\zeta^2} M_A^2, \quad \lambda_4 \equiv \frac{\beta_8}{\zeta^4} M_B^4, \\ M_A^2 &\equiv \beta^2 \left(2\pi G c_1^2 \frac{8\gamma_2 + 3\gamma_3}{\zeta^2} + V_2 + V_4' \right)^{-1}, \\ M_B^4 &\equiv \frac{\beta^2}{\sigma_3^2}, \quad \beta^2 \equiv \frac{2\pi G c_1^2}{|c_\psi|^2} + \frac{f}{2}, \\ c_\psi^2 &\equiv \frac{1 - \lambda}{3\lambda - 1}. \end{aligned} \quad (5.15)$$

In the IR, we have $k \ll M_* = \text{Min.}(M_A, M_B)$. Then, we find that

$$\omega_k^2 = \frac{2\beta^2 m_\chi^2}{f + \frac{c_1^2}{4\zeta^2|c_\psi|^2}} > 0, \quad (5.16)$$

where

$$m_\chi^2 \equiv \frac{1}{2\beta^2} V'', \quad (5.17)$$

denotes the mass of the scalar field. Thus, it is stable for $f > 0$. In the UV, we have $k^2 \geq M_A, M_B$, and then we find that

$$\omega_k^2 \simeq \frac{2\sigma_3^2 k^6}{f + \frac{c_1^2}{4\zeta^2|c_\psi|^2}} > 0, \quad (5.18)$$

for $f > 0$. Therefore, in this regime the scalar field is also stabilized. In fact, it can be made stable in all the energy scales by properly choosing the coupling coefficients V_n , as can be seen from Eq.(5.14).

B. Strong Coupling of Scalar Field

To study the strong coupling problem, using Eqs.(5.8)-(5.12), we can integrate out ψ , B , ϕ and A , so $S^{(2)}$ finally takes the form,

$$\begin{aligned} S^{(2)} = & \beta^2 \int dt d^3x \left[\dot{\chi}^2 - \alpha_0 (\partial\chi)^2 - m_\chi^2 \chi^2 - \frac{\chi}{M_A^4} \partial^4 \chi \right. \\ & \left. + \frac{\chi}{M_B^4} \partial^6 \chi + \gamma \chi \partial^2 \left(\frac{\wp^2 \chi}{\partial} \right) \right], \end{aligned} \quad (5.19)$$

where

$$\begin{aligned} \alpha_0 &\equiv \frac{1}{2\beta^2} \left(1 + 2V_1 - 4\pi G c_1^2 \right), \\ \gamma &\equiv \frac{4\pi G c_1^2}{\beta^2}. \end{aligned} \quad (5.20)$$

As a consistency check, one can show that the variation of the action (5.19) with respect to χ yields the master equation (5.13). In addition, when λ satisfies the condition (2.16), the above expression shows clearly that the scalar field is ghost free for $f > 0$ and stable in all energy scales.

To study the strong coupling problem, let us first note that the corresponding cubic action is given by,

$$\begin{aligned} S^{(3)} = & \int dt d^3x \left\{ g_1 \left(\frac{1}{\partial^2} \ddot{\chi} \right) \chi \partial^2 \chi + g_2 \left(\frac{1}{\partial^2} \ddot{\chi} \right) \chi_{,i} \chi^{,i} \right. \\ & + g_3 \dot{\chi}^2 \left(\frac{2\wp}{\partial} - 1 \right) \chi + g_3' \chi \dot{\chi}^2 \\ & + g_4 \left(\frac{\partial^i \partial^j}{\partial^2} \dot{\chi} \right) \left(\frac{\partial_i}{\partial^2} \dot{\chi} \right) \partial_j \left(\frac{2\wp}{\partial} + 3 \right) \chi \\ & + g_5 \dot{\chi} \partial^i \left(\frac{2\wp}{\partial} - 1 \right) \chi \partial_i \left(\frac{\dot{\chi}}{\partial^2} \right) + g_5' \dot{\chi} \chi^{,i} \left(\frac{\partial_i}{\partial^2} \dot{\chi} \right) \\ & + g_6 \chi^3 + g_7 \chi^2 \partial^2 \chi + g_8 \chi^2 \partial^4 \chi + g_9 \chi^2 \partial^6 \chi \\ & \left. + \dots \right\}, \end{aligned} \quad (5.21)$$

where “...” represents the fourth- and sixth-order derivative terms, which are irrelevant to the strong coupling problem. It also contains terms like,

$$\begin{aligned} & \phi \dot{\chi}^2, \quad \phi \chi^2, \quad \phi \chi \partial^2 \chi, \quad \phi \chi \partial^4 \chi, \quad \phi \chi \partial^6 \chi, \quad \chi^2 \partial^2 \phi, \\ & \chi^2 \partial^4 \phi, \quad \chi^2 \partial^6 \phi, \quad \phi^2 \partial^2 \chi, \quad \phi^2 \partial^4 \chi, \quad \phi^2 \partial^6 \chi, \quad \phi \chi \partial^2 \phi \\ & \phi \chi \partial^4 \phi, \quad \phi \chi \partial^6 \phi, \quad \phi^2 \partial^2 \phi, \quad \phi^2 \partial^4 \phi, \quad \phi^2 \partial^6 \phi, \quad \dots \end{aligned} \quad (5.22)$$

Since these terms are also independent of λ , they are irrelevant to the strong coupling problem, too. The coefficients g_s are defined as,

$$\begin{aligned} g_1 &= \frac{c_1^3}{8\zeta^4|c_\psi|^2}, \quad g_2 = \frac{1}{|c_\psi|^2} \left(\frac{5c_1^3}{32\zeta^4} - \frac{c_1 c_2}{4\zeta^2} \right), \\ g_3 &= -\frac{c_1^3}{32\zeta^4|c_\psi|^2}, \quad g_3' = -\frac{3f c_1}{8\zeta^2}, \quad g_4 = \frac{c_1^3}{64\zeta^4|c_\psi|^4}, \\ g_5 &= -\frac{c_1^3}{64\zeta^4|c_\psi|^4}, \quad g_5' = \frac{c_1 f}{4\zeta^2|c_\psi|^2}, \quad g_6 = \frac{3c_1}{8\zeta^2} \ddot{V} - \frac{\ddot{V}}{6}, \\ g_7 &= \frac{\dot{V}_1}{2} + \frac{c_1^2 c_2}{8\zeta^2} - \frac{c_1}{16\zeta^2} - \frac{c_1 V_1}{8\zeta^2}, \\ g_8 &= \tilde{A}_1(\gamma_2, \gamma_3, c_1) + B_1(V_2, V_4), \\ g_9 &= \tilde{A}_2(\gamma_5, c_1) + B_2(V_3, V_5, V_6), \end{aligned} \quad (5.23)$$

where $\tilde{A}_i \equiv (4\pi G c_1)^3 A_i$. Depending on the energy scales, each of these terms will have different scalings. Thus, in the following we consider them separately.

$$1. \quad |\nabla| \ll M_*$$

When $|\nabla| \ll M_*$, where $M_* = \text{Min.}(M_A, M_B)$, we find that

$$\bar{\vartheta} \simeq \beta_0, \quad \varphi \simeq 1, \quad \phi \simeq \frac{2}{\beta_0} \psi. \quad (5.24)$$

Then, Eq.(5.19) reduces to

$$S^{(2)} \simeq \beta^2 \int dt d^3x \left[\dot{\chi}^2 - \tilde{\alpha} (\partial\chi)^2 \right], \quad (5.25)$$

where

$$\tilde{\alpha} = \alpha_0 + \frac{\gamma}{\beta_0}. \quad (5.26)$$

Note that in writing the above expression, without loss of generality, we had assumed that $|\nabla| \gg m_\chi$. By setting

$$t = b_1 \hat{t}, \quad x^i = b_2 \hat{x}^i, \quad \chi = b_3 \hat{\chi}, \quad (5.27)$$

Eq.(5.25) can be brought into the ‘‘canonical’’ form,

$$S^{(2)} \simeq \int dt d^3\hat{x} \left[(\dot{\hat{\chi}}^*)^2 - (\hat{\partial}\hat{\chi})^2 \right], \quad (5.28)$$

in which the coefficient of each term is order of one, for

$$b_2 = b_1 \sqrt{\tilde{\alpha}}, \quad b_3 = \frac{1}{b_1 \beta \tilde{\alpha}^{3/4}}, \quad (5.29)$$

where $\hat{\chi}^* \equiv d\hat{\chi}/d\hat{t}$. Note that the requirement that the coefficient of each term be order of one is important in order to obtain a correct coupling strength [3, 26, 29].

When $|\nabla| \ll M_*$, the third order action (5.21) can be expressed as

$$\begin{aligned} S^{(3)} = \int dt d^3x \left\{ g_1 \left(\frac{1}{\partial^2} \ddot{\chi} \right) \chi \partial^2 \chi + g_2 \left(\frac{1}{\partial^2} \ddot{\chi} \right) \chi_{,i} \chi^{,i} \right. \\ + \hat{g}_3 \chi \dot{\chi}^2 + \hat{g}_4 \left(\frac{\partial^i \partial^j}{\partial^2} \dot{\chi} \right) \left(\frac{\partial_i}{\partial^2} \dot{\chi} \right) \partial_j \chi \\ + \hat{g}_5 \dot{\chi} \chi^{,i} \left(\frac{\partial_i}{\partial^2} \dot{\chi} \right) + g_6 \chi^3 + g_7 \chi^2 \partial^2 \chi + g_8 \chi^2 \partial^4 \chi \\ \left. + g_9 \chi^2 \partial^6 \chi + \dots \right\}, \quad (5.30) \end{aligned}$$

where g_s are given by Eq.(5.23), and

$$\begin{aligned} \hat{g}_3 &= \left(1 - \frac{2}{\beta_0} \right) \frac{c_1^3}{32\zeta^4 |c_\psi|^2} - \frac{3f c_1}{8\zeta^2}, \\ \hat{g}_4 &= \left(3 + \frac{2}{\beta_0} \right) \frac{c_1^3}{64\zeta^4 |c_\psi|^4}, \\ \hat{g}_5 &= \left(1 - \frac{2}{\beta_0} \right) \frac{c_1^3}{64\zeta^4 |c_\psi|^4} + \frac{c_1 f}{4\zeta^2 |c_\psi|^2}. \quad (5.31) \end{aligned}$$

Inserting Eq.(5.27) into Eq.(5.30), we obtain

$$S^{(3)} = \frac{1}{b_1 \beta^3 \beta_0^{3/4}} \hat{S}^{(3)}, \quad (5.32)$$

where

$$\begin{aligned} \hat{S}^{(3)} \equiv \int dt d^3\hat{x} \left\{ g_1 \left(\frac{1}{\hat{\partial}^2} \hat{\chi}^{**} \right) \hat{\chi} \hat{\partial}^2 \hat{\chi} \right. \\ + g_2 \left(\frac{1}{\hat{\partial}^2} \hat{\chi}^{**} \right) \hat{\partial}_i \hat{\chi} \hat{\partial}^i \hat{\chi} + \hat{g}_3 \hat{\chi} \hat{\chi}^{*2} \\ + \hat{g}_4 \left(\hat{\partial}_i \hat{\chi} \right) \left(\frac{\hat{\partial}_j}{\hat{\partial}^2} \hat{\chi}^* \right) \left(\frac{\hat{\partial}^i \hat{\partial}^j}{\hat{\partial}^2} \hat{\chi}^* \right) \\ + \hat{g}_5 \hat{\chi}^* \left(\frac{\hat{\partial}_i}{\hat{\partial}^2} \hat{\chi}^* \right) \hat{\partial}_i \hat{\chi} \\ + g_6 b_1^2 \hat{\chi}^3 + \frac{g_7 b_1^2}{b_2^2} \hat{\chi}^2 \hat{\partial}^2 \hat{\chi} \\ + \frac{g_8 b_1^2}{b_2^4} \hat{\chi}^2 \hat{\partial}^4 \hat{\chi} + \frac{g_9 b_1^2}{b_2^6} \hat{\chi}^2 \hat{\partial}^6 \hat{\chi} \\ \left. + \dots \right\}. \quad (5.33) \end{aligned}$$

On the other hand, from Eq.(5.28) one finds that $S^{(2)}$ is invariant under the rescaling,

$$\hat{t} \rightarrow b^{-1} \hat{t}, \quad \hat{x}^i \rightarrow b^{-1} \hat{x}^i, \quad \hat{\chi} \rightarrow b \hat{\chi}. \quad (5.34)$$

Then, it can be shown that the terms of $g_{1,2,\dots,5}$ and g_7 in $S^{(3)}$ all scale as b , while the terms of $g_{6,8,9}$ scale as b^{-1} , b^3 , b^5 , respectively. Therefore, except for the g_6 term, all the others are irrelevant and nonrenormalizable [39]. For example, considering a process with an energy E , then we find that the fourth term has the contribution,

$$\int dt d^3\hat{x} \left(\hat{\partial}_i \hat{\chi} \right) \left(\frac{\hat{\partial}_j}{\hat{\partial}^2} \hat{\chi}^* \right) \left(\frac{\hat{\partial}^i \hat{\partial}^j}{\hat{\partial}^2} \hat{\chi}^* \right) \simeq E. \quad (5.35)$$

Since the action $S^{(3)}$ is dimensionless, we must have

$$\begin{aligned} \frac{\lambda_4}{b_1 \beta^3 \beta_0^{3/4}} \int dt d^3\hat{x} \left(\hat{\partial}_i \hat{\chi} \right) \left(\frac{\hat{\partial}_j}{\hat{\partial}^2} \hat{\chi}^* \right) \left(\frac{\hat{\partial}^i \hat{\partial}^j}{\hat{\partial}^2} \hat{\chi}^* \right) \\ \simeq \frac{E}{\Lambda_{SC}^{(4)}}, \quad (5.36) \end{aligned}$$

where $\Lambda_{SC}^{(4)}$ has the same dimension of E , and is given by,

$$\Lambda_{SC}^{(4)} = \frac{b_1 \beta^3 \tilde{\beta}_0^{3/4}}{g_4}. \quad (5.37)$$

Similarly, one can find $\Lambda_{SC}^{(n)}$ for all the other nonrenormalizable terms. But, when $\lambda \rightarrow 1$ (or $c_\psi \rightarrow 0$), the lowest one of the $\Lambda_{SC}^{(n)}$'s is given by $\Lambda_{SC}^{(4)}$, so we have

$$\Lambda_\omega \equiv \frac{b_1 \beta^3 \tilde{\beta}_0^{3/4}}{g_4}, \quad (5.38)$$

above which the nonrenormalizable \hat{g}_4 term becomes larger than unit, and the process runs into the strong

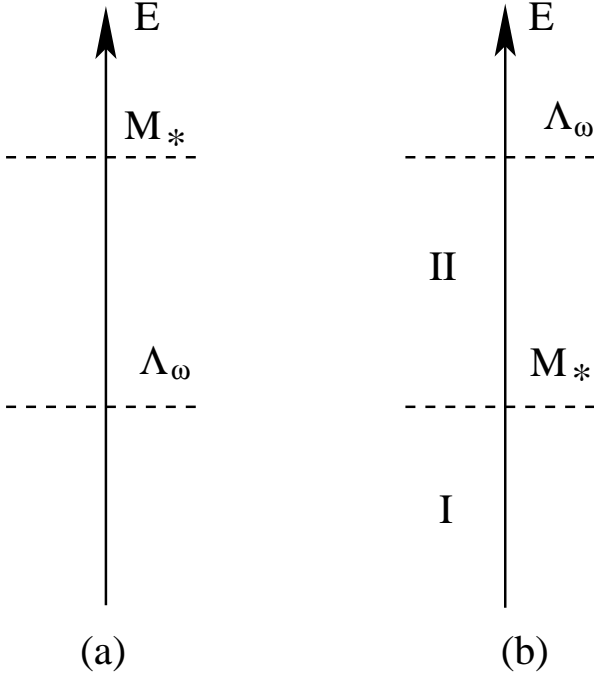


FIG. 1: The energy scales: (a) $\Lambda_\omega < M_*$; and (b) $\Lambda_\omega > M_*$.

coupling regime. Back to the physical coordinates t and x , the corresponding energy and momentum scales are given, respectively, by

$$\begin{aligned}\Lambda_\omega &= \frac{\Lambda_{\tilde{\omega}}}{b_1} \simeq \mathcal{O}(1) \left(\frac{\zeta}{c_1} \right)^{3/2} M_{pl} |c_\psi|^{5/2}, \\ \Lambda_k &= \frac{\Lambda_{\tilde{\omega}}}{b_2} \simeq \mathcal{O}(1) \left(\frac{\zeta}{c_1} \right)^{1/2} M_{pl} |c_\psi|^{3/2}.\end{aligned}\quad (5.39)$$

In particular, for $c_1 \simeq \zeta$, we find that $\Lambda_\omega \simeq M_{pl} |c_\psi|^{5/2}$, which is precisely the result obtained in [26].

It should be noted that the above conclusion is true only for $M_* > \Lambda_\omega$, that is,

$$M_* > \left(\frac{\zeta}{c_1} \right)^{3/2} M_{pl} |c_\psi|^{5/2}, \quad (5.40)$$

as shown by Fig. 1(a).

When $M_* < \Lambda_\omega$, the above analysis holds only for the processes with $E \ll M_*$ [Region I in Fig.1(b)]. However, when $E \gtrsim M_*$ and before the strong coupling energy scale Λ_ω reaches [cf. Fig.1(b)], the high order derivative terms of M_A and M_B in Eq.(5.19) cannot be neglected any more, and one has to take these terms into account. It is exactly because the presence of these terms that the strong coupling problem is cured [30]. In the following, we show that this is also the case here.

2. $M_* < \Lambda_\omega$, $M_A < M_B$

When $M_A < M_B$, we have $M_* = M_A$. In this case, we find that

$$\tilde{\delta} \simeq \frac{\lambda_1}{M_A^2} \partial^2, \quad \wp \simeq -\frac{\lambda_3}{M_A^2} \partial^2, \quad \phi \simeq -\frac{2\lambda_3}{\lambda_1} \psi. \quad (5.41)$$

For the processes with $E \gtrsim M_A$, Eq.(5.19) reduces to

$$S^{(2)} = \beta^2 \int dt d^3x \left(\dot{\chi}^2 - \frac{1}{\mu_A^2} \chi \partial^4 \chi \right), \quad (5.42)$$

$$\frac{1}{\mu_A^2} = \left(1 - \frac{\gamma\lambda_2}{\lambda_3} \right) \frac{1}{M_A^2}, \quad (5.43)$$

and the coefficients \hat{g}_3 , \hat{g}_4 , and \hat{g}_5 now are defined as,

$$\begin{aligned}\hat{g}_3 &= \left(1 + \frac{2\lambda_3}{\lambda_1} \right) \frac{c_1^3}{32\zeta^4 |c_\psi|^2} - \frac{3fc_1}{8\zeta^2}, \\ \hat{g}_4 &= \left(3 - \frac{2\lambda_3}{\lambda_1} \right) \frac{c_1^3}{64\zeta^4 |c_\psi|^4}, \\ \hat{g}_5 &= \left(1 + \frac{2\lambda_3}{\lambda_1} \right) \frac{c_1^3}{64\zeta^4 |c_\psi|^4} + \frac{c_1 f}{4\zeta^2 |c_\psi|^2}.\end{aligned}\quad (5.44)$$

Note that to have μ_A real, we must assume that

$$1 - \frac{\gamma\lambda_2}{\lambda_3} > 0. \quad (5.45)$$

To study the strong coupling problem, we shall follow what we did in the last case, by first writing $S^{(2)}$ in its canonical form,

$$S^{(2)} = \int dt d^3\hat{x} \left(\hat{\chi}^{*2} - \hat{\chi} \hat{\partial}^4 \hat{\chi} \right), \quad (5.46)$$

through the transformations (5.27). It can be shown that now b_2 and b_3 are given by

$$b_2 = \sqrt{\frac{b_1}{\mu_A}}, \quad b_3 = \frac{\mu_A^{3/4}}{b_1^{1/4} \beta}, \quad (5.47)$$

for which the cubic action $S^{(3)}$ takes the form,

$$S^{(3)} = \frac{\mu_A^{3/4}}{b_1^{1/4} \beta^3} \hat{S}^{(3)}, \quad (5.48)$$

where $\hat{S}^{(3)}$ is given by Eq.(5.33). Due to the nonrelativistic nature of the action (5.46), its scaling becomes anisotropic,

$$\hat{t} \rightarrow b^{-2} \hat{t}, \quad \hat{x}^i \rightarrow b^{-1} \hat{x}^i, \quad \hat{\chi} \rightarrow b^{1/2} \hat{\chi}. \quad (5.49)$$

Then, we find that the first five terms in Eqs.(5.48) and (5.33) scale as $b^{1/2}$, while the terms of g_6, \dots, g_9 scale, respectively, as $b^{-7/2}$, $b^{-3/2}$, $b^{1/2}$, $b^{5/2}$. Thus, except for the g_6 and g_7 terms, all the others are not renormalizable. It can be also shown that the processes with energy

higher than $\Lambda_\omega^{(A)}$ become strong coupling, where $\Lambda_\omega^{(A)}$ is given by,

$$\Lambda_\omega^{(A)} \simeq \left(\frac{M_{pl}}{\mu_A} \right)^3 M_{pl} |c_\psi|^4, \quad (M_A < M_B). \quad (5.50)$$

Therefore, when the fourth-order derivative terms dominate, the strong coupling problem still exists. This is expected, as power-counting tells us that the theory is renormalizable only when $z \geq 3$ [cf. Eq.(1.1)]. Indeed, as to be shown below, when the sixth order spatial derivative terms dominate, the strong coupling problem does not exist.

$$3. \quad M_* < \Lambda_\omega, \quad M_A \gtrsim M_B$$

In this case, we have $M_* = M_B$, and for processes with $E \gtrsim M_B$, Eq.(5.19) reduces to

$$S^{(2)} = \beta^2 \int dt d^3x \left(\dot{\chi}^2 - \frac{1}{M_B^4} \chi \partial^6 \chi \right). \quad (5.51)$$

Then, all the terms which contain ϕ in (5.21) can be neglected, and the coefficients \hat{g}_3 , \hat{g}_4 , and \hat{g}_5 in Eq.(5.30) now become

$$\begin{aligned} \hat{g}_3 &= \frac{c_1^3}{32\zeta^4 |c_\psi|^2} - \frac{3fc_1}{8\zeta^2}, \\ \hat{g}_4 &= \frac{c_1^3}{64\zeta^4 |c_\psi|^4}, \\ \hat{g}_5 &= \frac{c_1^3}{64\zeta^4 |c_\psi|^4} + \frac{c_1 f}{4\zeta^2 |c_\psi|^2}. \end{aligned} \quad (5.52)$$

Then, by the transformations (5.27) with

$$b_2 = \frac{b_1^{1/3}}{M_B^{2/3}}, \quad b_3 = \frac{M_B}{\beta}, \quad (5.53)$$

we obtain,

$$S^{(2)} = \int dt d^3\hat{x} \left(\hat{\chi}^{*2} - \hat{\chi} \hat{\partial}^6 \hat{\chi} \right), \quad (5.54)$$

while the cubic action $S^{(3)}$ becomes,

$$S^{(3)} = \frac{M_B}{\beta^3} \hat{S}^{(3)}. \quad (5.55)$$

Eq.(5.54) is invariant under the rescaling,

$$\hat{t} \rightarrow b^{-3}\hat{t}, \quad \hat{x}^i \rightarrow b^{-1}\hat{x}^i, \quad \hat{\chi} \rightarrow \hat{\chi}. \quad (5.56)$$

Then, it can be shown that the first five terms in Eqs.(5.55) and (5.33) are scaling-invariant, and so the last term. The terms of $g_{6,7,8}$, on the other hand, scale, respectively, as b^{-6} , b^{-4} , b^{-2} . Therefore, the first five terms as well as the last one now all become strictly renormalizable, while the g_6 , g_7 and g_8 terms become

superrenormalizable [39]. To have these strictly renormalizable terms to be weakly coupling, we require their coefficients be less than unit,

$$\frac{M_*}{\beta^3} g_n < 1, \quad (n = 1, \dots, 5, 9). \quad (5.57)$$

For $g \sim 1$ (or $|c_\psi| \sim 0$), we find that the above condition holds for

$$M_* < \frac{2}{3} M_{pl} |c_\psi|.$$

It can be shown that this condition holds identically, provided that $M_* < \Lambda_\omega$, that is,

$$M_* < \left(\frac{\zeta}{c_1} \right)^{3/2} M_{pl} |c_\psi|^{5/2}. \quad (5.58)$$

[Recall Λ_ω is given by Eq.(5.39) and $M_* = M_B$.] One can take $c_1 \simeq M_{pl}$, but now a more reasonable choice is $c_1 \simeq M_*$. Then, the condition (5.58) becomes

$$M_* < M_{pl} |c_\psi|^{1/2}, \quad (c_1 = M_*), \quad (5.59)$$

which is much less restricted than the one of $c_1 \simeq M_{pl}$. In addition, in order to have the sixth order derivative terms dominate, we must also require,

$$M_A \gtrsim M_*. \quad (5.60)$$

Therefore, it is concluded that, *provided that conditions (5.58) and (5.60) hold, the extended version of the HL gravity with the detailed balance condition softly breaking but without the projectability condition is absent of the strong coupling problem.*

VI. COSMOLOGICAL MODELS AND THE FLATNESS PROBLEM

One of the main motivations of inflation was to solve the horizon and flatness problems, encountered in the standard Big Bang model [40]. In the HL theory, the anisotropic scaling (1.1) provides a solution to the horizon problem and generation of scale-invariant perturbations even without inflation [41]. Clearly, these statements are also true in our current setup developed above. In this section, we shall show that the homogeneous and isotropic universe is also necessarily flat, when the enlarged symmetry (1.5) is introduced. This was first noted for a scalar field [26]. Here we argue that it is true for all the viable cosmological models. To this purpose, let us consider the general FRW universe,

$$\begin{aligned} ds^2 &= a^2(-d\eta^2 + \gamma_{ij} dx^i dx^j), \\ \gamma_{ij} &= \frac{\delta_{ij}}{(1 + kr^2/4)^2}, \end{aligned} \quad (6.1)$$

where

$$\delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j. \end{cases} \quad (6.2)$$

Then, we have

$$\begin{aligned}\hat{\Gamma}_{ij}^k &= -\frac{1}{2} \left(\frac{k}{1 + \frac{1}{4}kr^2} \right) (x_i \delta_j^k + x_j \delta_i^k - x^k \delta_{ij}), \\ \hat{R}_{ij} &= 2k\gamma_{ij}, \quad \hat{K}_{ij} = -a\mathcal{H}\gamma_{ij},\end{aligned}\quad (6.3)$$

where $\mathcal{H} = a'/a$. We use symbols with hats to denote quantities of the background in the conformal coordinates (6.1), following the conventions given in [10, 26, 29]. Using the $U(1)$ gauge freedom of Eqs.(3.3) and (3.5), we can always set one of \hat{A} and $\hat{\varphi}$ to zero. In this paper, we choose the gauge

$$\hat{\varphi}(\eta) = 0. \quad (6.4)$$

Then, we find that

$$\begin{aligned}\hat{\mathcal{L}}_K &= (1 - 3\lambda) \frac{3\mathcal{H}^2}{a^2}, \\ \hat{\mathcal{L}}_V &= 2\Lambda - \frac{6k}{a^2} + \frac{3\gamma_2 + \gamma_3}{\zeta^2} \frac{12k^2}{a^4}, \\ \hat{\pi}^{ij} &= (1 - 3\lambda) \frac{\mathcal{H}}{a} g^{ij}, \\ \hat{\mathcal{L}}_A &= -\frac{\hat{A}}{a} \left(\frac{6k}{a^2} - 2\Lambda_g \right), \\ \hat{\mathcal{L}}_\varphi &= \hat{F}^N = \hat{F}_{ij}^\varphi = \hat{F}_{ij}^a = 0, \\ \hat{F}_{ij} &= -a^2 \left[\Lambda - \frac{k}{a^2} - \frac{2k^2}{a^4} \frac{3\gamma_2 + \gamma_3}{\zeta^2} \right] \gamma_{ij}.\end{aligned}\quad (6.5)$$

Because of the spatial homogeneity, both $\hat{\mathcal{L}}_K$ and $\hat{\mathcal{L}}_V$ are independent of the spatial coordinates, and the matter sector takes the forms,

$$\hat{J}^t = -2\hat{\rho}, \quad \hat{J}^i = 0, \quad \hat{\tau}_{ij} = \hat{p}\hat{g}_{ij}, \quad (6.6)$$

where $\hat{\rho}$ and \hat{p} denote the total energy density and pressure, respectively. Then the Hamilton constraint (3.14) reduces to the super-Hamiltonian constraint $\hat{\mathcal{L}}_K + \hat{\mathcal{L}}_V = 8\pi G \hat{J}^t$ ⁵, which leads to the modified Friedmann Equation,

$$\frac{3\lambda - 1}{2} \frac{\mathcal{H}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3} \hat{\rho} + \frac{\Lambda}{3} + \frac{3\gamma_2 + \gamma_3}{\zeta^2} \frac{2k^2}{a^4}. \quad (6.7)$$

It can be shown that the super-momentum constraint (3.15) is satisfied identically, while Eq.(3.17) and Eq.(3.18) give, respectively,

$$\frac{\mathcal{H}}{a} \left(\Lambda_g - \frac{k}{a^2} \right) = -\frac{8\pi G}{3} \hat{J}_\varphi, \quad (6.8)$$

$$\frac{k}{a^2} - \frac{\Lambda_g}{3} = \frac{4\pi G \hat{J}_A}{3}. \quad (6.9)$$

The dynamical equation (3.20), on the other hand, reduces to

$$\begin{aligned}\frac{1 - 3\lambda}{2} (2\mathcal{H}' + \mathcal{H}^2) + a\hat{A} \left(\frac{k}{a^2} - \Lambda_g \right) \\ + a^2 \left(\Lambda - \frac{k}{a^2} - \frac{2k^2}{a^4} \frac{3\gamma_2 + \gamma_3}{\zeta^2} \right) = 8\pi G \hat{p} a^2.\end{aligned}\quad (6.10)$$

The conservation law of momentum (3.26) is satisfied identically, while the one of energy (3.25) reduces to,

$$\hat{\rho}' + 3\mathcal{H}(\hat{\rho} + \hat{p}) = \hat{A} \hat{J}_\varphi. \quad (6.11)$$

It is remarkable to note that when

$$\hat{J}_A = \hat{J}_\varphi = 0, \quad (6.12)$$

Eqs.(6.8) and (6.9) show that the universe is necessarily flat,

$$k = 0 = \Lambda_g. \quad (6.13)$$

As first noted in [26], this is true for the universe dominated by a single scalar field.

In general, the coupling of the gauge field A and the Newtonian prepotential φ to a matter field ψ_n is given by [27],

$$\int dt d^3x \sqrt{g} Z(\psi_n, g_{ij}, \nabla_k) (A - \mathcal{A}), \quad (6.14)$$

where \mathcal{A} is defined in Eq.(5.4), and Z is the most general scalar operator under the full symmetry of Eq.(1.5), with its dimension $[Z] = 2$. For a single scalar field, $Z(\chi, g_{ij}, \nabla_k)$ is given by $Z(\chi, g_{ij}, \nabla_k) = c_1 \Delta \chi + c_2 (\nabla \chi)^2$, as one can see from Eq.(5.1). In the multi-scalar field case, Z takes the form

$$\begin{aligned}Z &= \sum_{i=1}^n c_1^{(i)} \Delta \chi^{(i)} \\ &+ \sum_{i,j=1}^n c_{(i,j)} (\nabla \chi^{(i)}) (\nabla \chi^{(j)}),\end{aligned}\quad (6.15)$$

for which we have $\bar{J}_A = 0 = \bar{J}_\varphi$ with the gauge (6.4). Thus, in the case of multi-scalar fields, the universe is necessarily flat, too.

For a vector field (A_0, A_i) , we have $[A_0] = 2$, $[A_i] = 0$ [12, 42]. Then, we find

$$Z(A_0, A_i, g_{ij}, \nabla_k) = \mathcal{K} B_i B^i, \quad (6.16)$$

where \mathcal{K} is an arbitrary function of $A^i A_i$, and

$$B_i = \frac{1}{2} \frac{\varepsilon_i^{jk}}{\sqrt{g}} \mathcal{F}_{jk}, \quad \nabla^i B_i = 0, \quad (6.17)$$

with $\mathcal{F}_{ij} \equiv \partial_j A_i - \partial_i A_j$. This can be easily generalized to several vector fields, $(A_0^{(n)}, A_i^{(n)})$, for which we have

$$Z(\vec{A}_0, \vec{A}_i, g_{ij}, \nabla_k) = \sum_{m,n} \mathcal{K}_{mn} B_i^{(m)} B^{(n)i}, \quad (6.18)$$

⁵ Since now the Hamiltonian constraint is local, one cannot include a ‘‘dark matter component as an integration constant,’’ as in the case with the projectability condition [41].

where \mathcal{K}_{mn} is an arbitrary function of $A^{(k)i}A_i^{(l)}$. Then, in the FRW background, we have $\bar{J}_A = 0$, because $\bar{B}_i^{(m)} = 0$ [43]. With the gauge choice (6.4), it is easy to show that $\bar{J}_\varphi = 0$, too. Therefore, an early universe dominated by vector fields is also necessarily flat. This can be further generalized to the case of Yang-Mills fields [44].

For fermions, on the other hand, their dimensions are $[\psi_n] = 3/2$ [45]. Then, $Z(\psi_n, g_{ij}, \nabla_k)$ cannot be a functional of ψ_n . Therefore, in this case \bar{J}_A and \bar{J}_φ vanish identically.

In review of the above, it is not difficult to argue that, with the special form of the coupling given by Eq.(6.14), *the universe is necessarily flat for all the cosmologically viable models in the current setup.*

Similar conclusion is also obtained in the case with the projectability condition [46]. Therefore, in the rest of this paper, we shall consider only the flat FRW universe.

VII. COSMOLOGICAL PERTURBATIONS

In this section, we consider the linear perturbations in a flat FRW universe. Let us first write the linear perturbations in the form [14, 26, 49]

$$\begin{aligned}\delta N &= a\phi, & \delta N_i &= a^2(\partial_i B - S_i), \\ \delta g_{ij} &= a^2(-2\psi\delta_{ij} + 2\partial_i\partial_j E + 2\partial_{(i}F_{j)} + H_{ij}), \\ A &= \hat{A} + \delta A, & \varphi &= \hat{\varphi} + \delta\varphi,\end{aligned}\quad (7.1)$$

with $\hat{\varphi} = 0$, as one can see from Eq.(6.4), and

$$\partial^i S_i = \partial^i F_i = H^i{}_i = 0, \quad \partial^i H_{ij} = 0. \quad (7.2)$$

In the following, we shall consider the scalar, vector and tensor perturbations separately.

A. Scalar Perturbations

For the scalar perturbations ϕ , B , ψ , E , we choose the quasilongitudinal gauge [14],

$$E = \delta\varphi = 0. \quad (7.3)$$

Then, to first order we find that

$$\sqrt{g} = (1 - 3\psi)a^3, \quad (7.4)$$

$$\delta\Gamma_{ij}^k = -(\delta_i^k\partial_j\psi + \delta_j^k\partial_i\psi - \delta_{ij}\partial^k\psi), \quad (7.5)$$

$$\delta R_{ij} = \delta_{ij}\partial^2\psi + \partial_i\partial_j\psi, \quad (7.6)$$

$$\delta R = 2\psi\hat{R} + \frac{4\partial^2\psi}{a^2}, \quad (7.7)$$

$$\delta K_{ij} = a\left[(\phi\mathcal{H} + 2\psi\mathcal{H} + \psi')\delta_{ij} + \partial_i\partial_j B\right], \quad (7.8)$$

$$\delta K = \frac{1}{a}(3\phi\mathcal{H} + \partial^2 B + 3\psi'). \quad (7.9)$$

Other useful quantities are given in Appendix B. Thus, the field equations Eq.(3.17), Eq.(3.18), the momentum

constraint (3.15), the Hamiltonian constraint (3.14), the trace and traceless parts of dynamical equation (3.20) are given, respectively, by

$$\begin{aligned}\partial^2\left[2\mathcal{H}(\psi - \phi) + (1 - \lambda)(\partial^2 B + 3\psi' + 3\mathcal{H}\phi)\right] \\ = 8\pi G a^3 \delta J_\varphi,\end{aligned}\quad (7.10)$$

$$\partial^2\psi = 2\pi G a^2 \delta J_A, \quad (7.11)$$

$$(3\lambda - 1)(\psi' + \phi\mathcal{H}) + (\lambda - 1)\partial^2 B = 8\pi G a q, \quad (7.12)$$

$$\begin{aligned}\frac{3\lambda - 1}{2}\mathcal{H}(3\psi' + 3\phi\mathcal{H} + \partial^2 B) \\ - \wp\partial^2\psi + \frac{1}{2}\bar{\delta}\partial^2\phi = -4\pi G a^2 \delta\mu,\end{aligned}\quad (7.13)$$

$$\begin{aligned}\psi'' + 2\mathcal{H}\psi' + \mathcal{H}\phi' + (2\mathcal{H}' + \mathcal{H}^2)\phi \\ + \frac{\lambda - 1}{3\lambda - 1}\partial^2\left(\psi + \alpha_1\partial^2\psi - \wp\phi - \frac{\hat{A}\psi - \delta A}{a}\right) \\ = \frac{8\pi G a^2}{3\lambda - 1}\left(\delta\mathcal{P} + \frac{3\lambda - 1}{3}\partial^2\Pi\right),\end{aligned}\quad (7.14)$$

$$\begin{aligned}B' + 2\mathcal{H}B - \psi + \wp\phi \\ - \alpha_1\partial^2\psi + \frac{\hat{A}\psi - \delta A}{a} = -8\pi G a^2\Pi,\end{aligned}\quad (7.15)$$

where

$$\begin{aligned}\delta\mu &\equiv -\frac{1}{2}\delta J^t, & \delta J^i &\equiv \frac{1}{a^2}\partial^i q, \\ \delta\tau_{ij} &\equiv a^2[(\delta\mathcal{P} - 2\hat{p}\psi)\delta_{ij} + \Pi_{\langle ij\rangle}], \\ \Pi_{\langle ij\rangle} &\equiv \Pi_{,ij} - \frac{1}{3}\delta_{ij}\partial^2\Pi.\end{aligned}\quad (7.16)$$

In the above equations, α_1 , \wp and $\bar{\delta}$ are defined by Eq.(4.4). The conservation laws (3.25) and (3.26) to first order now read,

$$\begin{aligned}\int d^3x\left\{\delta\mu' + 3\mathcal{H}(\delta\mu + \delta\mathcal{P}) - 3(\hat{\rho} + \hat{p})\psi' \right. \\ \left. + \frac{1}{2a}\left[3\hat{A}\hat{J}_A\psi' - \hat{A}(\delta J'_A + 3\mathcal{H}\delta J_A)\right] \right. \\ \left. + \phi\hat{A}\hat{J}_\varphi - \hat{J}_\varphi\delta A\right\} = 0, \quad (7.17) \\ (v + B)' + (1 - 3c_s^2)(v + B) + \phi \\ + \frac{1}{\hat{\rho} + \hat{p}}\left(\delta\mathcal{P} + \frac{2}{3}\partial^2\Pi\right) = \frac{1}{2a(\hat{\rho} + \hat{p})}\left[\hat{J}_A\delta A \right. \\ \left. - 2a\hat{A}\hat{J}_\varphi(1 + c_s^2)\right], \quad (7.18)\end{aligned}$$

where $q \equiv -a(\bar{\rho} + \bar{p})(v + B)$ [14], and c_s^2 denotes the adiabatic speed of sound, defined as

$$c_s^2 \equiv \frac{\hat{p}'}{\hat{\rho}'}. \quad (7.19)$$

It is always useful to compare the above set of field equations with those given in GR. First, because of the presence of the gauge field A and the Newtonian prepotential φ , here we have two extra equations, Eqs.(7.10)

and (7.11), which are absent in GR. As shown in Sec. IV, it is exactly Eq.(7.11) in the vacuum that eliminates the spin-0 gravitons. The momentum constraint (7.12) reduces to that of GR given by Eq.(8.17) in [47] where $\lambda = 1$. Considering the gauge choice of Eq.(7.3), the Hamiltonian constraint (7.13) reduces to Eq.(8.16) of [47] for $\lambda = 1$ and $\beta_i = 0$, as expected. The same is true for the dynamical equations (7.14) and (7.15) and the conservation law of momentum (7.18), which will reduce, respectively, to Eqs.(8.27), (8.28) and (8.33) given in [47] for $\lambda = 1$, $\beta_i = \gamma_2 = \gamma_3 = \hat{A} = \delta A = 0$. However, because of the foliation-preserving diffeomorphisms $\text{Diff}(M, \mathcal{F})$ (1.2), the conservation law of energy (7.17) now takes an integral form. A direct consequence of it is that the gauge-invariant curvature perturbations

$$\zeta \equiv -\psi - \mathcal{H} \frac{\delta \rho}{\bar{\rho}'}, \quad (7.20)$$

is not necessarily conserved on large scales even the perturbations are adiabatic [14, 38]. In contrast, it was shown that ζ is conserved on large scales for adiabatic perturbations in any theory of relativistic gravity, as long as the conservation law of energy holds locally [48]. Note that ζ defined here should not be confused with that introduced in the action (2.8).

B. Vector Perturbations

For the vector perturbations, we have

$$\begin{aligned} \delta N &= 0, \quad \delta N^i = -S^i, \\ \delta g_{ij} &= 2a^2(\partial_{(i}F_{j)}), \\ \delta A &= \delta\varphi = 0, \end{aligned} \quad (7.21)$$

while the corresponding matter perturbations are given by

$$\begin{aligned} \delta J^i &= \frac{1}{a^2}q^i, \quad \delta J^t = 0, \\ \delta\tau_{ij} &= 2a^2(\Pi_{(i,j)} + \hat{p}F_{(i,j)}), \end{aligned} \quad (7.22)$$

where

$$\partial_i q^i = 0 = \partial_i \Pi^i. \quad (7.23)$$

Then, one finds that

$$\begin{aligned} \delta K_{ij} &= -a(F'_{(i,j)} + 2\mathcal{H}F_{(i,j)} + S_{(i,j)}), \\ \delta\Gamma_{ij}^k &= \partial_i \partial_j F^k, \\ \delta R_{ij} &= \delta\mathcal{L}_K = \delta\mathcal{L}_V = 0, \\ \delta F_{ij} &= -2a^2\Lambda F_{(i,j)}. \end{aligned} \quad (7.24)$$

Hence, to linear order, the momentum constraint (3.15) gives

$$\partial^2(F'_i + S_i) = 16\pi G a q_i, \quad (7.25)$$

and the dynamical equation (3.20) yields,

$$(F'_{(i,j)} + S_{(i,j)})' + 2\mathcal{H}(F'_{(i,j)} + S_{(i,j)}) = 16\pi G a^2 \Pi_{(i,j)}. \quad (7.26)$$

The conservation law of energy (3.25) does not give new constraint, while the conservation of momentum (3.26) yields,

$$q'_i + 3\mathcal{H}q_i = a\partial^2\Pi_i. \quad (7.27)$$

However, this equation is not independent, and can be obtained from Eqs. (7.25) and (7.26).

It is interesting to note that the vector perturbations given above are precisely the same as those presented in [49], in which the projectability condition $N = N(t)$ was assumed, but without the additional U(1) symmetry. Although one would expect some differences, because of the presence of the vector field a_i defined by Eq.(1.6), a closer examination shows it is not, this is simply because a_i is made of N , and perturbations of δN , as well as of δA and $\delta\varphi$, have no contributions to the vector perturbations.

C. Tensor Perturbations

The cosmological tensor perturbations are given by

$$\delta g_{ij} = a^2 H_{ij}, \quad \delta N^i = 0, \quad \delta N = \delta A = \delta\varphi = 0, \quad (7.28)$$

while the corresponding matter perturbations are given by

$$\delta\tau_{ij} = a^2(\Pi_{ij} + \hat{p}H_{ij}), \quad \delta J^t = 0 = \delta J^i, \quad (7.29)$$

where

$$\Pi^i{}_i = 0, \quad \partial^j \Pi_{ij} = 0. \quad (7.30)$$

Then, one finds that

$$\begin{aligned} \delta K_{ij} &= -a(\mathcal{H}H_{ij} + \frac{1}{2}H'_{ij}), \\ \delta\Gamma_{ij}^k &= \frac{1}{2}(\partial_i H_j^k + \partial_j H_i^k - \partial^k H_{ij}), \\ \delta R_{ij} &= -\frac{1}{2}\partial^2 H_{ij}, \quad \delta R = 0, \\ \delta\pi^{ij} &= -\frac{1}{a^3}[(1-3\lambda)\mathcal{H}H^{ij} - \frac{1}{2}H'^{ij}] \\ \delta F_{ij} &= -a^2\Lambda H_{ij} + \frac{1}{2}\partial^2 H_{ij} - \frac{\gamma_3}{2a^2\zeta^2}\partial^4 H_{ij} \\ &\quad + \frac{\gamma_5}{2a^4\zeta^4}\partial^6 H_{ij}. \end{aligned} \quad (7.31)$$

In this case, all the constraints and equations are satisfied identically, except for the dynamical one (3.20), which gives,

$$\begin{aligned} H''_{ij} + 2\mathcal{H}H'_{ij} - \left(1 - \frac{\hat{A}}{a}\right)\partial^2 H_{ij} \\ + \frac{\gamma_3}{a^2\zeta^2}\partial^4 H_{ij} - \frac{\gamma_5}{a^4\zeta^4}\partial^6 H_{ij} = 16\pi G a^2 \Pi_{ij}. \end{aligned} \quad (7.32)$$

When $\hat{A} = 0$, it reduces precisely to the one given in [49] for the case without the additional $U(1)$ symmetry.

This completes the general descriptions for the scalar, vector, and tensor perturbations in our current setup.

VIII. CONCLUSIONS

There are two major variants of Horava-Lifshitz gravity, which have the potential to solve all the problems found so far. One is the HMT generalization [24], which adopts the projectability condition and introduces a gauge field A and a Newtonian prepotential φ to eliminate the spin-0 gravitons. Another setup is due to BPS [3], who abandoned the projectability condition and improved the IR limit of the theory by introducing the vector field a_i , defined by Eq.(1.6). However, the inclusion of a_i gives rise to a proliferation of independent coupling constants.

In this paper, we have considered a new generalization of Horava-Lifshitz gravity without projectability condition but with detailed balance condition softly breaking. In order to reduce the number of independent coupling constants of the non-projectability Horava-Lifshitz gravity, in Sect II we have imposed the ‘‘generalized’’ detailed balance condition, so that the number of the independent coupling constants is dramatically reduced. However, for the theory to have a healthy IR limit, we have allowed the detailed balance condition to be broken softly, by adding all the low dimensional relevant terms. Even with those relevant terms, the number of independently coupling constants is still significantly reduced from more than 70 to 15.

However, it was found that this is not sufficient, because the detailed balance condition, even allowed to be broken softly, still prevents the existence of the sixth-order spatial derivative terms in the gravitational sector. As a result, the theory is not power-counting renormalizable and the strong coupling problem cannot be solved. To resolve this problem, in Sec III, we have extended the original foliation-preserving diffeomorphism symmetry to include a local $U(1)$ symmetry, i.e., $U(1) \times \text{Diff}(M, \mathcal{F})$. With this enlarged symmetry, in Sec IV, we have shown explicitly that the spin-0 gravitons are eliminated, and thus all the problems related to them in the gravitational sector disappear, including the ghost, instability, strong coupling, and different speeds.

In Sec V, we have considered the coupling of a scalar field to the theory, and found that in the Minkowski background it is stable in the both IR and UV, and becomes strong coupling for processes with energy higher than $\Lambda_\omega \equiv (M_{pl}/c_1)^{3/2} M_{pl} |c_\psi|^{5/2}$. However, this problem can be easily cured by introducing a new energy scale M_* , so that $M_* < \Lambda_\omega$, where M_* denotes the suppression energy scale of the sixth order derivative terms of the theory.

In Sec VI, we have considered cosmological applications, and found that the FRW universe is necessarily flat in such a setup. In Sec VII, we have studied the scalar,

vector, and tensor perturbations, and derived the general field equations for each kind of these perturbations. For the scalar perturbations, we have written the field equations closely following those given in GR [47], so one can see clearly the differences between these two theories. For the vector perturbations, they are the same as those given in [49] for the case with the projectability condition (1.3) but with only the foliation-preserving diffeomorphisms (1.2), while for the tensor perturbations, the only difference is the term proportional to \hat{A} in Eq.(7.32). This is simply because that the lapse function N , the gauge field A and the Newtonian prepotential φ all transform like scalars under the spatial coordinate transformations of Eq. (1.2), and hence their linear perturbations have no contributions to the vector and tensor perturbations of both gravitational and matter sectors.

It would be very interesting to apply those formulas to the studies of the early universe as well as to the ones of its large-scale structure formation.

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Appendix A: F_V , F_φ , F_λ , F_{ij} , F_{ij}^a and F_{ij}^φ

F_V , F_φ and F_λ , defined in Eq.(3.14), are given by,

$$\begin{aligned}
F_V = & \beta_0(2a_i^i + a_i a^i) - \frac{\beta_1}{\zeta^2} \left[3(a_i a^i)^2 + 4\nabla_i(a_k a^k a^i) \right] \\
& + \frac{\beta_2}{\zeta^2} \left[(a_i^i)^2 + \frac{2}{N} \nabla^2(N a_k^k) \right] \\
& - \frac{\beta_3}{\zeta^2} \left[(a_i a^i) a_j^j + 2\nabla_i(a_j^j a^i) - \frac{1}{N} \nabla^2(N a_i a^i) \right] \\
& + \frac{\beta_4}{\zeta^2} \left[a_{ij} a^{ij} + \frac{2}{N} \nabla_j \nabla_i(N a^{ij}) \right] \\
& - \frac{\beta_5}{\zeta^2} \left[R(a_i a^i) + 2\nabla_i(R a^i) \right] \\
& - \frac{\beta_6}{\zeta^2} \left[a_i a_j R^{ij} + 2\nabla_i(a_j R^{ij}) \right] \\
& + \frac{\beta_7}{\zeta^2} \left[R a_i^i + \frac{1}{N} \nabla^2(N R) \right] \\
& + \frac{\beta_8}{\zeta^4} \left[(\Delta a^i)^2 - \frac{2}{N} \nabla^i[\Delta(N \Delta a_i)] \right], \quad (\text{A.1})
\end{aligned}$$

$$F_\varphi = -\mathcal{G}^{ij} \nabla_i \varphi \nabla_j \varphi, -\frac{2}{N} \hat{\mathcal{G}}^{ijkl} \nabla_l(N K_{ij} \nabla_k \varphi),$$

$$\begin{aligned}
& -\frac{4}{3}\hat{G}^{ijkl}\nabla_l(\nabla_k\varphi\nabla_i\nabla_j\varphi) \\
& -\frac{5}{3}\hat{G}^{ijkl}\left[(a_i\nabla_j\varphi)(a_k\nabla_l\varphi) + \nabla_i(a_k\nabla_j\varphi\nabla_l\varphi) \right. \\
& \left. + \nabla_k(a_i\nabla_j\varphi\nabla_l\varphi)\right] \\
& +\frac{2}{3}\hat{G}^{ijkl}\left[a_{ik}\nabla_j\varphi\nabla_l\varphi + \frac{1}{N}\nabla_i\nabla_k(N\nabla_j\varphi\nabla_l\varphi)\right], \tag{A.2}
\end{aligned}$$

$$\begin{aligned}
F_\lambda = (1-\lambda)\left\{(\nabla^2\varphi + a_i\nabla^i\varphi)^2 - \frac{2}{N}\nabla_i(NK\nabla^i\varphi) \right. \\
\left. - \frac{2}{N}\nabla_i\left[N(\nabla^2\varphi + a_i\nabla^i\varphi)\nabla^i\varphi\right]\right\}. \tag{A.3}
\end{aligned}$$

$(F_n)_{ij}$, $(F_s^a)_{ij}$ and $(F_q^\varphi)_{ij}$, defined in Eq.(3.21), are given, respectively, by

$$\begin{aligned}
(F_0)_{ij} &= -\frac{1}{2}g_{ij}, \\
(F_1)_{ij} &= R_{ij} - \frac{1}{2}Rg_{ij} + \frac{1}{N}(g_{ij}\nabla^2N - \nabla_j\nabla_iN), \\
(F_2)_{ij} &= -\frac{1}{2}g_{ij}R^2 + 2RR_{ij} \\
&+ \frac{2}{N}[g_{ij}\nabla^2(NR) - \nabla_j\nabla_i(NR)], \\
(F_3)_{ij} &= -\frac{1}{2}g_{ij}R_{mn}R^{mn} + 2R_{ik}R_j^k \\
&- \frac{1}{N}\left[2\nabla_k\nabla_{(i}(NR_{j)}^k)\right. \\
&\left. - \nabla^2(NR_{ij}) - g_{ij}\nabla_m\nabla_n(NR^{mn})\right], \\
(F_4)_{ij} &= -\frac{1}{2}g_{ij}R^3 + 3R^2R_{ij} \\
&+ \frac{3}{N}(g_{ij}\nabla^2 - \nabla_j\nabla_i)(NR^2), \\
(F_5)_{ij} &= -\frac{1}{2}g_{ij}RR_{mn}R^{mn} \\
&+ R_{ij}R_{mn}R^{mn} + 2RR_{ik}R_j^k \\
&+ \frac{1}{N}\left[g_{ij}\nabla^2(NR_{mn}R^{mn}) \right. \\
&- \nabla_j\nabla_i(NR_{mn}R^{mn}) \\
&+ \nabla^2(NRR_{ij}) + g_{ij}\nabla_m\nabla_n(NRR^{mn}) \\
&\left. - 2\nabla_m\nabla_{(i}(R_{j)}^mNR)\right], \\
(F_6)_{ij} &= -\frac{1}{2}g_{ij}R_n^mR_l^nR_m^l + 3R_{mn}R_{mi}R_{nj} \\
&+ \frac{3}{2N}\left[g_{ij}\nabla_m\nabla_n(NR_a^mR^{na}) \right. \\
&+ \nabla^2(NR_{mi}R_j^m) - 2\nabla_m\nabla_{(i}(NR_{j)n}R^{mn})\left.], \\
(F_7)_{ij} &= -\frac{1}{2}g_{ij}R\nabla^2R + R_{ij}\nabla^2R + R\nabla_i\nabla_jR \\
&+ \frac{1}{N}\left[g_{ij}\nabla^2(N\nabla^2R) - \nabla_j\nabla_i(N\nabla^2R) \right.
\end{aligned}$$

$$\begin{aligned}
& + R_{ij}\nabla^2(NR) + g_{ij}\nabla^4(NR) - \nabla_j\nabla_i(\nabla^2(NR)) \\
& - \nabla_{(j}(NR\nabla_{i)}R) + \frac{1}{2}g_{ij}\nabla_k(NR\nabla^kR)\left.], \right. \\
(F_8)_{ij} &= -\frac{1}{2}g_{ij}(\nabla_mR_{nl})^2 + 2\nabla^mR_i^n\nabla_mR_{nj} \\
&+ \nabla_iR^{mn}\nabla_jR_{mn} + \frac{1}{N}\left[2\nabla_n\nabla_{(i}\nabla_m(N\nabla^mR_{j)}^n) \right. \\
&- \nabla^2\nabla_m(N\nabla^mR_{ij}) - g_{ij}\nabla_n\nabla_p\nabla_m(N\nabla^mR^{np}) \\
&- 2\nabla_m(NR_{l(i}\nabla^mR_{j)}^l) - 2\nabla_n(NR_{l(i}\nabla_j)R^{nl}) \\
&+ 2\nabla_k(NR_l^k\nabla_{(i}R_{j)}^l)\left.], \right. \\
(F_9)_{ij} &= -\frac{1}{2}g_{ij}a_kG^k + \frac{1}{2}\left[a^kR_{k((j}\nabla_{i)}R + a_{(i}R_{j)k}\nabla^kR\right] \\
&- a_kR_{mi}\nabla_jR^{mk} - a^kR_{in}\nabla^nR_{jk} - a_iR^{km}\nabla_mR_{kj} \\
&- \frac{3}{8}a_{(i}R\nabla_{j)}R + \frac{3}{8}\left\{R\nabla_k(Na^k)R_{ij} \right. \\
&+ g_{ij}\nabla^2\left[R\nabla_k(Na^k)\right] - \nabla_i\nabla_j\left[R\nabla_k(Na^k)\right]\left. \right\} \\
&+ \frac{1}{4N}\left\{-\frac{1}{2}\nabla^m\left[\nabla_i(Na_j\nabla_mR + Na_m\nabla_jR) \right. \right. \\
&+ \nabla_j(Na_i\nabla_mR + Na_m\nabla_iR)\left. \right] \\
&+ \nabla^2(Na_{(i}\nabla_{j)}R) + g_{ij}\nabla^m\nabla^n(Na_m\nabla_nR) \\
&+ \nabla^m\left[\nabla_i(Na_k\nabla_jR_m^k + Na_k\nabla_mR_j^k) \right. \\
&+ \nabla_j(Na_k\nabla_iR_m^k + Na_k\nabla_mR_i^k)\left. \right] \\
&- 2\nabla^2(Na_k\nabla_{(i}R_{j)}^k) - 2g_{ij}\nabla^m\nabla^n(Na_k\nabla_{(n}R_{m)}^k) \\
&- \nabla^m\left[\nabla_i\nabla_p(Na_jR_m^p + Na_mR_j^p) \right. \\
&+ \nabla_j\nabla_p(Na_iR_m^p + Na_mR_i^p)\left. \right] \\
&+ 2\nabla^2\nabla_p(Na_{(i}R_{j)}^p) \\
&+ 2g_{ij}\nabla^m\nabla^n\nabla^p(Na_{(n}R_{m)p})\left. \right\}, \tag{A.4}
\end{aligned}$$

$$\begin{aligned}
(F_0^a)_{ij} &= -\frac{1}{2}g_{ij}a^ka_k + a_ia_j, \\
(F_1^a)_{ij} &= -\frac{1}{2}g_{ij}(a_ka^k)^2 + 2(a_ka^k)a_ia_j, \\
(F_2^a)_{ij} &= -\frac{1}{2}g_{ij}(a_k^ka^k)^2 + 2a_k^ka_{ij} \\
&- \frac{1}{N}\left[2\nabla_{(i}(Na_j)a_k^k) - g_{ij}\nabla^l(a_lNa_k^k)\right], \\
(F_3^a)_{ij} &= -\frac{1}{2}g_{ij}(a_ka^k)a_l^l + a^ka_ia_j + a_ka^ka_{ij} \\
&- \frac{1}{N}\left[\nabla_{(i}(Na_j)a_ka^k) - \frac{1}{2}g_{ij}\nabla^l(a_lNa_ka^k)\right], \\
(F_4^a)_{ij} &= -\frac{1}{2}g_{ij}a^{mn}a_{mn} + 2a^ka_ia_j
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{N} \left[\nabla^k (2Na_{(i}a_{j)k}) - Na_{ij}a_k \right], \\
(F_5^a)_{ij} &= -\frac{1}{2}g_{ij}(a_k a^k)R + a_i a_j R + a^k a_k R_{ij} \\
& + \frac{1}{N} \left[g_{ij} \nabla^2 (Na_k a^k) - \nabla_i \nabla_j (Na_k a^k) \right], \\
(F_6^a)_{ij} &= -\frac{1}{2}g_{ij}a_m a_n R^{mn} + 2a^m R_{m(i}a_{j)} \\
& - \frac{1}{2N} \left[2\nabla^k \nabla_{(i}(a_{j)}Na_k) - \nabla^2 (Na_i a_j) \right. \\
& \left. - g_{ij} \nabla^m \nabla^n (Na_m a_n) \right], \\
(F_7^a)_{ij} &= -\frac{1}{2}g_{ij}Ra_k{}^k + a_k{}^k R_{ij} + Ra_{ij} \\
& + \frac{1}{N} \left[g_{ij} \nabla^2 (Na_k{}^k) - \nabla_i \nabla_j (Na_k{}^k) \right. \\
& \left. - \nabla_{(i}(NRa_{j)}) + \frac{1}{2}g_{ij} \nabla^k (NRa_k) \right], \\
(F_8^a)_{ij} &= -\frac{1}{2}g_{ij}(\Delta a_k)^2 + (\Delta a_i)(\Delta a_j) + 2\Delta a^k \nabla_{(i} \nabla_{j)} a_k \\
& + \frac{1}{N} \left[\nabla_k [a_{(i} \nabla^k (N\Delta a_{j)}) + a_{(i} \nabla_{j)} (N\Delta a^k) \right. \\
& \left. - a^k \nabla_{(i} (N\Delta a_{j)}) + g_{ij} Na^{lk} \Delta a_l - Na_{ij} \Delta a^k \right] \\
& \left. - 2\nabla_{(i} (Na_{j)k} \Delta a^k) \right], \tag{A.5} \\
(F_1^\varphi)_{ij} &= -\frac{1}{2}g_{ij} \varphi \mathcal{G}^{mn} K_{mn} \\
& + \frac{1}{2\sqrt{g}N} \partial_t (\sqrt{g} \varphi \mathcal{G}_{ij}) - 2\varphi K_{(i}^l R_{j)l} \\
& + \frac{1}{2} \varphi (KR_{ij} + K_{ij}R - 2K_{ij}\Lambda_g) \\
& + \frac{1}{2N} \left\{ 2\mathcal{G}_{k(i} \nabla^k (N_j) \varphi) - \mathcal{G}_{ij} \nabla^k (\varphi N_k) \right. \\
& + g_{ij} \nabla^2 (N\varphi K) - \nabla_i \nabla_j (N\varphi K) \\
& + 2\nabla^k \nabla_{(i} (K_{j)k} \varphi N), \\
& \left. - \nabla^2 (N\varphi K_{ij}) - g_{ij} \nabla^k \nabla^l (N\varphi K_{kl}) \right\}, \\
(F_2^\varphi)_{ij} &= -\frac{1}{2}g_{ij} \varphi \mathcal{G}^{mn} \nabla_m \nabla_n \varphi \\
& - 2\varphi \nabla_{(i} \nabla^k R_{j)k} + \frac{1}{2} \varphi (R - 2\Lambda_g) \nabla_i \nabla_j \varphi \\
& - \frac{1}{N} \left\{ -\frac{1}{2} (R_{ij} + g_{ij} \nabla^2 - \nabla_i \nabla_j) (N\varphi \nabla^2 \varphi) \right. \\
& - \nabla_k \nabla_{(i} (N\varphi \nabla^k \nabla_{j)} \varphi) + \frac{1}{2} \nabla^2 (N\varphi \nabla_i \nabla_j \varphi) \\
& + \frac{g_{ij}}{2} \nabla^k \nabla^l (N\varphi \nabla_k \nabla_l \varphi) \\
& \left. - \mathcal{G}_{k(i} \nabla^k (N\varphi \nabla_{j)} \varphi) + \frac{1}{2} \mathcal{G}_{ij} \nabla^k (N\varphi \nabla_k \varphi) \right\}, \\
(F_3^\varphi)_{ij} &= -\frac{1}{2}g_{ij} \varphi \mathcal{G}^{mn} a_m \nabla_n \varphi \\
& - \varphi (a_{(i} R_{j)k} \nabla^k \varphi + a^k R_{k(i} \nabla_{j)} \varphi) \\
& + \frac{1}{2} (R - 2\Lambda_g) \varphi a_{(i} \nabla_{j)} \varphi \\
& - \frac{1}{N} \left\{ -\frac{1}{2} (R_{ij} + g_{ij} \nabla^2 - \nabla_i \nabla_j) (N\varphi a^k \nabla_k \varphi) \right. \\
& - \frac{1}{2} \nabla^k \left[\nabla_{(i} (\nabla_{j)} \varphi N \varphi) + \nabla_{(i} (a_{j)} \varphi N \nabla_k \varphi) \right] \\
& + \frac{1}{2} \nabla^2 (N\varphi a_{(i} \nabla_{j)} \varphi) \\
& \left. + \frac{g_{ij}}{2} \nabla^k \nabla^l (N\varphi a_k \nabla_l \varphi) \right\}, \\
(F_4^\varphi)_{ij} &= -\frac{1}{2}g_{ij} \hat{\mathcal{G}}^{mnkl} K_{mn} a_{(k} \nabla_{l)} \varphi \\
& + \frac{1}{2\sqrt{g}N} \partial_t [\sqrt{g} \mathcal{G}_{ij}{}^{kl} a_{(l} \nabla_{k)} \varphi] \\
& + \frac{1}{2N} \nabla^l \left[a_l N_{(i} \nabla_{j)} \varphi + N_{(i} a_{j)} \nabla_l \varphi \right. \\
& \left. - N_l a_{(i} \nabla_{j)} \varphi + 2g_{ij} N_l a^k \nabla_k \varphi \right] \\
& + \frac{1}{N} \nabla_{(i} (N N_{j)} a^k \nabla_k \varphi) \\
& + a^k K_{k(i} \nabla_{j)} \varphi + a_{(i} K_{j)k} \nabla^k \varphi \\
& - K a_{(i} \nabla_{j)} \varphi - K_{ij} a^k \nabla_k \varphi, \\
(F_5^\varphi)_{ij} &= -\frac{1}{2}g_{ij} \hat{\mathcal{G}}^{mnkl} [a_{(k} \nabla_{l)} \varphi] [\nabla_m \nabla_n \varphi] \\
& - a_{(i} \nabla^k \nabla_{j)} \varphi \nabla_k \varphi - a_k \nabla^k \nabla_{(i} \varphi \nabla_{j)} \varphi \\
& + a_{(i} \nabla_{j)} \varphi \nabla^2 \varphi + a^k \nabla_k \varphi \nabla_i \nabla_j \varphi \\
& + \frac{1}{2N} \left\{ \nabla^k (N\varphi a_k \nabla_i \varphi \nabla_j \varphi) \right. \\
& - 2\nabla_{(i} (N \nabla_{j)} \varphi) \varphi a^k \nabla_k \varphi \\
& \left. + g_{ij} \nabla^l (\nabla_l \varphi a^k \nabla_k \varphi) \right\}, \\
(F_6^\varphi)_{ij} &= -\frac{1}{2}g_{ij} \hat{\mathcal{G}}^{mnkl} [a_{(m} \nabla_n) \varphi] [a_{(k} \nabla_l) \varphi] \\
& - \frac{1}{2} (a^k \nabla_i \varphi - a_i \nabla^k \varphi) (a_k \nabla_j \varphi - a_j \nabla_k \varphi), \\
(F_7^\varphi)_{ij} &= -\frac{1}{2}g_{ij} \hat{\mathcal{G}}^{mnkl} [\nabla_{(n} \varphi)] [a_{m)(k} \nabla_{l)} \varphi] \\
& - \frac{1}{2} a_k{}^k \nabla_i \varphi \nabla_j \varphi - \frac{1}{2} a_{ij} \nabla^k \varphi \nabla_k \varphi \\
& + a_{(i}^k \nabla_{j)} \varphi \nabla_k \varphi - \frac{1}{2N} \left\{ -\nabla_{(i} (Na_{j)} \nabla_k \varphi \nabla^k \varphi) \right. \\
& + \nabla^k (Na_{(i} \nabla_{j)} \varphi \nabla_k \varphi) \\
& + \frac{g_{ij}}{2} \nabla^k (Na_k \nabla^m \varphi \nabla_m \varphi) \\
& \left. - \frac{1}{2} \nabla^k (Na_k \nabla_i \varphi \nabla_j \varphi) \right\}, \\
(F_8^\varphi)_{ij} &= -\frac{1}{2}g_{ij} (\nabla^2 \varphi + a_k \nabla^k \varphi)^2 \\
& - 2(\nabla^2 \varphi + a_k \nabla^k \varphi) (\nabla_i \nabla_j \varphi + a_i \nabla_j \varphi) \\
& - \frac{1}{N} \left\{ -2\nabla_{(j} [N \nabla_{i)} \varphi] (\nabla^2 \varphi + a_k \nabla^k \varphi) \right\}
\end{aligned}$$

$$\begin{aligned}
& +g_{ij}\nabla^l[N(\nabla^2\varphi+a_k\nabla^k\varphi)\nabla_l\varphi]\Big\}, \\
(F_9^\varphi)_{ij} = & -\frac{1}{2}g_{ij}(\nabla^2\varphi+a_k\nabla^k\varphi)K \\
& -(\nabla^2\varphi+a_k\nabla^k\varphi)K_{ij} \\
& -(\nabla_i\nabla_j\varphi+a_i\nabla_j\varphi)K \\
& +\frac{1}{2\sqrt{g}N}\partial_t[\sqrt{g}(\nabla^2\varphi+a_k\nabla^k\varphi)g_{ij}] \\
& -\frac{1}{N}\left\{-\nabla_{(j}[N_{i)}(\nabla^2\varphi+a_k\nabla^k\varphi)]\right. \\
& +\frac{1}{2}g_{ij}\nabla^l[N_l(\nabla^2\varphi+a_k\nabla^k\varphi)] \\
& \left.-\nabla_{(j}(NK\nabla_{i)}\varphi)+\frac{1}{2}g_{ij}\nabla_k(NK\nabla^k\varphi)\right\}.
\end{aligned} \tag{A.6}$$

Appendix B: Some Quantities for Scalar Perturbations

To first order, the $(F_s)_{ij}$ are given by

$$\begin{aligned}
(F_0)_{ij} &= -\frac{1}{2}a^2\delta_{ij}+a^2\psi\delta_{ij}, \\
(F_1)_{ij} &= -(\partial^2\psi-\partial^2\phi)\delta_{ij}+\partial_i\partial_j(\psi-\phi), \\
(F_2)_{ij} &= -\frac{8}{a^2}(\partial_i\partial_j-\delta_{ij}\partial^2)\partial^2\psi,
\end{aligned}$$

$$\begin{aligned}
(F_3)_{ij} &= -\frac{3}{a^2}(\partial_i\partial_j-\delta_{ij}\partial^2)\partial^2\psi, \\
(F_7)_{ij} &= \frac{8}{a^4}(\delta_{ij}\partial^2-\partial_i\partial_j)\partial^4\psi, \\
(F_8)_{ij} &= -\frac{3}{a^4}(\delta_{ij}\partial^6\psi-\partial_i\partial_j\partial^4\psi),
\end{aligned} \tag{B.1}$$

and $(F_4)_{ij}=(F_5)_{ij}=(F_6)_{ij}=(F_9)_{ij}=0$. Thus, we obtain

$$\begin{aligned}
\delta F_{ij} &= 2\Lambda a^2\psi\delta_{ij}+\partial^2(\psi-\phi)\delta_{ij}-\partial_i\partial_j(\psi-\phi) \\
& -\alpha_1(\partial_i\partial_j+\delta_{ij})\partial^2\psi.
\end{aligned} \tag{B.2}$$

We also find that the only non-vanishing component of $(F_s^a)_{ij}$ is,

$$(F_7^a)_{ij}=-\frac{1}{a^2}(\partial_i\partial_j-\delta_{ij}\partial^2)\partial^2\phi. \tag{B.3}$$

In addition, we have the following,

$$\delta\mathcal{G}^{ij}=\frac{\partial^i\partial^j\psi-\delta^{ij}\partial^2\psi}{a^4}, \tag{B.4}$$

$$\delta(\mathcal{G}^{ij}K_{ij})=\frac{2\mathcal{H}}{a^3}\partial^2\psi, \tag{B.5}$$

$$\delta\left(\frac{1}{N}\mathcal{G}^{ijkl}\nabla_{(k}[Na_{l)}K_{ij}]\right)=\frac{2\mathcal{H}}{a^3}\partial^2\phi. \tag{B.6}$$

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