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Constraints on Neutrino Velocities Revisited

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Abstract

With a minimally modified dispersion relation for neutrinos, we reconsider the constraints on superluminal neutrino velocities from bremsstrahlung effects in the laboratory frame. Employing both the direct calculation approach and the virtual Z -boson approach, we obtain the generic decay width and energy loss rate of a superluminal neutrino with general energy. The Cohen-Glashow's analytical results for neutrinos with a relatively low energy are confirmed in both approaches. We employ the survival probability instead of the terminal energy to assess whether a neutrino with a given energy is observable or not in the OPERA experiment. Moreover, using our general results we perform systematical analyses on the constraints arising from the Super-Kamiokande and IceCube experiments.

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I. INTRODUCTION

The OPERA collaboration has measured the velocities of muon neutrinos (ν_μ) with mean energy about 17.5 GeV, which travel from CERN to the Gran Sasso. It was found that the muon neutrinos travel at a speed faster than light (so called propagating superluminally), and the relative difference of the neutrino speed v_ν with respect to that of light c in the vacuum was measured to be

$$\delta'v_\nu \equiv \frac{v_\nu - c}{c} = (2.37 \pm 0.32 \text{ (stat.)}_{-0.24}^{+0.34} \text{ (sys.)}) \times 10^{-5}. \quad (1)$$

Also, the OPERA data indicate *no energy dependence* [1]. The OPERA result has been confirmed by a test performed using a beam with a short-bunch time-structure allowing to measure the neutrino flight time at the single interaction level [1]. Interestingly, this is compatible with the MINOS results [2] and the earlier short-baseline experiments [3]. From the theoretical point of view, many groups have already studied the possible solutions or pointed out the challenges to the OPERA anomaly [4–43]. For an early similar study, see Ref. [44].

The OPERA experiment would strongly imply new physics beyond the traditional special relativity if it could be confirmed by the future experiments. If Lorentz symmetry is broken hardly [45–47], *i.e.*, there is a preferred frame of reference, we do have two strong constraints. The first one comes from bremsstrahlung effects [16]. A superluminal muon neutrino with $\delta'v_\nu$ given in Eq. (1) would lose energy rapidly via Cherenkov-like processes on their way from CERN to the Gran Sasso, and the most important process is $\nu_\mu \rightarrow \nu_\mu e^+ e^-$. Thus, the OPERA experiment would not be able to observe muon neutrinos with energy in excess of about 12.5 GeV [16]. The second one arises from pion decays [17, 20, 22]. The superluminal muon neutrinos could not gain energy larger than about 5 GeV from the processes, $\pi^+ \rightarrow \mu^+ \nu_\mu$ and $\mu \rightarrow \nu_\mu e \bar{\nu}_e$ [20]. However, these constraints do not apply to other proposals which can explain the OPERA anomaly as well [24, 29, 34, 36, 39–42].

In Refs. [16, 17, 20, 22], the authors considered the following simple dispersion relation for neutrinos as a result of hard Lorentz violation

$$E_\nu^2 = \vec{p}_\nu^2 + m_\nu^2 + \delta \vec{p}_\nu^2, \quad (2)$$

where E_ν and \vec{p}_ν are respectively the neutrino energy and momentum, and $\delta \simeq 2\delta'v_\nu$. For simplicity, we shall use $\delta = 5 \times 10^{-5}$ in the following. The discussions on pion decays are

simple, and one can easily confirm the previous results [17, 20, 22]. However, the results for bremsstrahlung effects are a bit subtle. From private communications with our colleagues in October 2011, nobody seems to have succeeded in reproducing the results of Cohen and Glashow in the approximation of four-Fermi interactions (see also attempts in Refs. [37, 38]). In addition, for neutrinos with much larger energy $E > M_Z/\sqrt{\delta}$, their decay width and energy loss rate have not been studied as well.

In this paper, assuming the simple dispersion relation given in Eq. (2), we study in detail the electron-positron emission process of superluminal neutrinos $\nu_\mu \rightarrow \nu_\mu e^+ e^-$ for bremsstrahlung effects in the laboratory frame, and our results can be applied to the other processes as well. By a direct calculation, we confirm the Cohen-Glashow's results for the low energy neutrinos. We present a general formula for the emission process of superluminal neutrinos via a virtual Z -boson exchange, which applies at both high and low neutrino energies. Interestingly, we confirm the Cohen-Glashow's low-energy results as well. Moreover, we systematically analyze the constraints on the neutrino speed from the OPERA, Super-Kamiokande and IceCube experiments.

II. PAIR EMISSION DECAY WIDTH AND ENERGY LOSS RATE

We study the pair emission process for a superluminal neutrino in two approaches. In the first one, we work directly with the low-energy four-Fermi interaction that is appropriate for momentum transfer much smaller than the Z -boson mass M_Z . We recover the results reported in Ref. [16]. We also present a new result for the process of on-shell Z -boson emission. The latter will be employed in the second approach to work out a formula that applies also to neutrinos with general energy.

A. Direct Calculations

We start with some kinematical considerations of the pair emission process by a superluminal neutrino, $\nu_\mu(k) \rightarrow \nu_\mu(\ell) e^-(p_1) e^+(p_2)$, where the quantities in the parentheses denote the four-momenta

$$k = (E, \vec{k}), \quad \ell = (E_\nu, \vec{\ell}), \quad p_1 = (E_{e^-}, \vec{p}_1), \quad p_2 = (E_{e^+}, \vec{p}_2). \quad (3)$$

The neutrinos are assumed to satisfy the dispersion relation shown in Eq. (2), while the electrons fulfil the usual one

$$\begin{aligned} E^2 &= c_\nu^2 |\vec{k}|^2 + c_\nu^4 m_\nu^2, & E_\nu^2 &= c_\nu^2 |\vec{\ell}|^2 + c_\nu^4 m_\nu^2, \\ E_{e-}^2 &= |\vec{p}_1|^2 + m_e^2, & E_{e+}^2 &= |\vec{p}_2|^2 + m_e^2, \end{aligned} \quad (4)$$

where $m_{e,\nu}$ are respectively the electron and neutrino masses, $c_\nu \equiv \sqrt{1+\delta}$ is the limiting speed of neutrinos in units of the speed of light which we have set to be unity.

For the energy region of interest here, the lepton masses can be safely neglected, so that the dispersion relations are simplified to $E = c_\nu |\vec{k}|$, and $E_{e-} = |\vec{p}_1|$, etc. Noting $|\vec{p}_1 + \vec{p}_2| \leq |\vec{p}_1| + |\vec{p}_2|$, one gets the fraction of the initial neutrino energy carried away by the final one as follows

$$\ell_k \equiv \frac{|\vec{\ell}|}{|\vec{k}|} = \frac{E_\nu}{E} \in [0, \ell_k^{\max}], \quad (5)$$

where for a given angle θ_ν between the moving directions of the initial and final neutrinos we have

$$\ell_k^{\max} = \delta^{-1} \Delta [1 - \sqrt{1 - \delta^2 \Delta^{-2}}], \quad \Delta = c_\nu^2 - \cos \theta_\nu. \quad (6)$$

Fig. 1 shows in polar coordinates the relation between ℓ_k^{\max} and θ_ν , where for illustrative purpose we used an unrealistic value $\delta = 0.05$. The figure implies a factor of δ^2 suppression from the neutrino part of the phase space, which will contribute to the decay width.

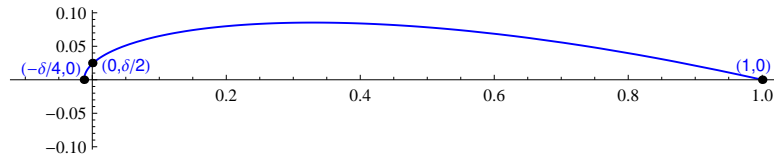


FIG. 1: ℓ_k^{\max} is shown in polar coordinates as a function of θ_ν for $\delta = 0.05$. The points $(1, 0)$, $(0, \delta/2)$, and $(-\delta/4, 0)$ (in rectangular coordinates) correspond to the three kinematical configurations: $\vec{k} \parallel \vec{\ell}$, $\vec{k} \perp \vec{\ell}$, and $-\vec{k} \parallel \vec{\ell}$, respectively.

We have done three independent calculations for the pair emission and Z -boson emission processes and reached consistent results. In what follows, we present some details about

the calculations. The amplitude for the pair emission process at momentum transfer much smaller than M_Z is

$$\mathcal{M} = \sqrt{2}G_F\bar{u}(\ell)\gamma_\mu P_L u(k)\bar{u}(p_1)\gamma^\mu((1-2s_W^2)P_L-2s_W^2P_R)v(p_2) , \quad (7)$$

where G_F is the Fermi constant, $P_{L,R} = (1 \mp \gamma^5)/2$, $s_W = \sin \theta_W$, and u, v are Dirac spinor wavefunctions for the particles. By the way, a modified dispersion relation for neutrinos may be formulated in Lagrangian field theory in terms of a modified metric tensor, which in turn can affect the spin sum of neutrinos and their effective interactions. For a careful analysis on such subtleties or model dependencies, see Ref. [43].

In this paper, we shall average over the initial neutrino spin states, similar to the Ref. [16]. We are aware that the OPERA neutrinos produced from the positively charged pion decay have a negative helicity, and the results are roughly the same if one does not average over the neutrino spin states [43]. Moreover, our results can be applied directly to the randomly polarized neutrinos whose astrophysical sources, for instance, are unknown. In short, the spin-summed and -averaged decay width and corresponding energy loss rate are

$$\Gamma = \frac{1}{2E} \int \sum_{\text{spins}} |\mathcal{M}|^2 d\text{PS}_3 , \quad (8)$$

$$\frac{dE}{dx} = \frac{1}{c_\nu} \int (E_\nu - E) d\Gamma , \quad (9)$$

where $d\text{PS}_3$ is the usual three-body phase space measure. Neglecting the neutrino and electron masses, which is appropriate for the OPERA experiment, the phase space of the electrons can be easily done. We obtain

$$\Gamma = \frac{8G_F^2}{96\pi} [(1-2s_W^2)^2 + (2s_W^2)^2] \frac{1}{2E} J , \quad (10)$$

where the phase space integral J reduces to ($q = k - \ell$)

$$J = \int \frac{d^3\vec{\ell}}{2E_\nu(2\pi)^3} 2(k \cdot \ell q^2 + 2k \cdot q \ell \cdot q) . \quad (11)$$

Using the kinematics defined earlier, we obtain, e.g., $k^2 = \delta|\vec{k}|^2$, and $k \cdot \ell = \Delta|\vec{k}||\vec{\ell}|$. We find that it is easier to use the variables ℓ_k and y instead of $|\vec{\ell}|$ and $\cos \theta_\nu$, where ℓ_k was defined in Eq. (5) and y is defined by $2\Delta = (y + y^{-1})\delta$. The integral becomes elementary

$$J = \frac{|\vec{k}|^6}{c_\nu(2\pi)^2} \int_{y_0}^1 dy \int_0^{\ell_k^{\max}} d\ell_k \frac{1}{2} \delta(y^{-2} - 1) \ell_k [3\delta\Delta\ell_k(1 + \ell_k^2) - 4\Delta^2\ell_k^2 - 2\delta^2\ell_k^2] , \quad (12)$$

where $y_0 = (c_\nu - 1)/(c_\nu + 1)$. Working out the integral we get

$$\Gamma = \frac{G_F^2 E^5 \delta^3}{192\pi^3 c_\nu^7} [(1 - 2s_W^2)^2 + (2s_W^2)^2] \left\{ \frac{1}{7} + \frac{y_0^7}{140} - \frac{y_0^5}{20} + \frac{3y_0^3}{20} - \frac{y_0}{4} \right\}. \quad (13)$$

This result is exact with the only approximation being $m_{e,\nu} = 0$. For the energy loss rate, one multiplies the integrand of J by $-c_\nu^{-1}E(1 - \ell_k)$ and obtains

$$\begin{aligned} \frac{dE}{dx} &= -\frac{G_F^2 E^6 \delta^3}{192\pi^3 c_\nu^8} [(1 - 2s_W^2)^2 + (2s_W^2)^2] \\ &\times \left\{ \frac{25}{224} - \left[\frac{1}{160}y_0^8 - \frac{1}{140}y_0^7 - \frac{3}{80}y_0^6 + \frac{1}{20}y_0^5 + \frac{7}{80}y_0^4 - \frac{3}{20}y_0^3 - \frac{7}{80}y_0^2 + \frac{1}{4}y_0 \right] \right\}. \end{aligned} \quad (14)$$

Putting $s_W^2 = 1/4$ and keeping only the leading term in $\delta \ll 1$ we recover the Cohen-Glashow results [16]

$$\Gamma \approx \frac{1}{14} \frac{G_F^2 E^5 \delta^3}{192\pi^3}, \quad (15)$$

$$\frac{dE}{dx} \approx -\frac{25}{448} \frac{G_F^2 E^6 \delta^3}{192\pi^3}. \quad (16)$$

For a superluminal neutrino with high enough energy, the process $\nu(k) \rightarrow \nu(\ell)Z(p)$ can also take place. Ignoring again the neutrino mass, the spin-summed and -averaged amplitude squared is

$$\overline{\sum_{\text{spins}}} |\mathcal{M}|^2 = \sqrt{2} G_F [k \cdot \ell M_Z^2 + 2k \cdot p \ell \cdot p]. \quad (17)$$

The energy-momentum conservation gives

$$z^2 - 2z\Delta\delta^{-1} + 1 - \ell_Z^2 = 0, \quad (18)$$

where $z = |\vec{\ell}|/|\vec{k}|$ is similar to ℓ_k in the pair emission process, Δ was defined in Eq. (6), and $\ell_Z = (c_\nu M_Z)/(E\sqrt{\delta})$. The above as an equation of z for given $\cos\theta_\nu \in [-1, 1]$ has always two real solutions for $c_\nu > 1$, but one of them being larger than unity is non-physical. Requiring the other solution to be in the physical region yields the threshold condition, $\ell_Z < 1$, *i.e.*, $E > c_\nu M_Z/\sqrt{\delta}$. Similarly to the case of pair emission, it is simpler to work in phase space with z instead of Δ or $\cos\theta_\nu$, $2\Delta = [z + (1 - \ell_Z^2)z^{-1}]\delta$, and the interval for $\cos\theta_\nu$ translates into $z \in [z_-, z_+]$ with

$$\begin{aligned} z_- &= (2 + \delta)\delta^{-1} \left[1 - \sqrt{1 - (2 + \delta)^{-2}\delta^2(1 - \ell_Z^2)} \right], \\ z_+ &= 1 - \ell_Z. \end{aligned} \quad (19)$$

Using the new variables, we have, e.g., $2k \cdot \ell = c_\nu^{-2} E^2 (1 + z^2 - \ell_Z^2) \delta$, and $2p \cdot k = c_\nu^{-2} E^2 (1 - z^2 + \ell_Z^2) \delta$. The decay width and the neutrino energy loss rate are found to be

$$\Gamma = \frac{1}{16\pi} \frac{G_F}{\sqrt{2}} \frac{E^3}{c_\nu^7} I_\Gamma, \quad (20)$$

$$\frac{dE}{dx} = -\frac{1}{16\pi} \frac{G_F}{\sqrt{2}} \frac{E^4}{c_\nu^8} I_R, \quad (21)$$

where the residual phase space integrals are elementary

$$I_\Gamma = c_\nu^2 \delta^2 \int_{z_-}^{z_+} dz [\ell_Z^2 (1 + z^2) + (1 - z^2)^2 - 2\ell_Z^4], \quad (22)$$

$$I_R = c_\nu^2 \delta^2 \int_{z_-}^{z_+} dz [\ell_Z^2 (1 + z^2) + (1 - z^2)^2 - 2\ell_Z^4] (1 - z). \quad (23)$$

Again the above results only assume $m_{e,\nu} = 0$ and are exact in parameters ℓ_Z and δ . Since $\delta \ll 1$, we can expand in δ while holding $\ell_Z < 1$ fixed and obtain the results in the leading order of δ

$$\Gamma \approx \frac{G_F E^3 \delta^2}{120 \sqrt{2} \pi} (4 + 10\ell_Z^2 - 25\ell_Z^3 + 11\ell_Z^5), \quad (24)$$

$$\frac{dE}{dx} \approx -\frac{G_F E^4 \delta^2}{960 \sqrt{2} \pi} (22 + 35\ell_Z^2 - 180\ell_Z^4 + 88\ell_Z^5 + 35\ell_Z^6), \quad (25)$$

where now $\ell_Z \approx M_Z / (E\sqrt{\delta})$.

B. Virtual Z Approach

In the previous subsection we computed the electron-positron emission in the low energy limit and the emission of a physical Z boson at sufficiently high energies. In the following we present our general results for the pair emission via a virtual Z exchange, $\nu(k) \rightarrow \nu(\ell) Z^*(q) \rightarrow \nu(\ell) f(p_1) \bar{f}(p_2)$, which can be applied at any initial neutrino energy E . One can do so purely numerically of course, but we would like to accomplish this in such a way that both low and high energy limits can be readily identified. After some manipulations of phase space and appropriate reorganization of amplitudes for the subprocesses, we obtain upon neglecting the neutrino masses

$$\Gamma = \frac{1}{\pi} \int_0^{\delta E^2} dm_*^2 \frac{m_* \Gamma_f(m_*)}{(m_*^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \Gamma_i(m_*), \quad (26)$$

$$\frac{dE}{dx} = \frac{1}{\pi} \int_0^{\delta E^2} dm_*^2 \frac{m_* \Gamma_f(m_*)}{(m_*^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \frac{dE_i(m_*)}{dx}, \quad (27)$$

where $m_* = \sqrt{q^2}$, Γ_Z is the usual total decay width of the Z boson, $\Gamma_i(m_*)$ is the decay width of the initial superluminal neutrino into a virtual Z boson with an effective mass m_* , and $dE_i(m_*)/dx$ is the corresponding energy loss rate. These functions are obtained from Eqs. (24) and (25) by replacing M_Z with m_* everywhere, *i.e.*, $\ell_Z \rightarrow \ell_{Z*} = \ell_Z m_*/M_Z$ and $G_F \rightarrow G_F^* = G_F M_Z^2/m_*^2$. Similarly, $\Gamma_f(m_*)$ is the decay width of a virtual Z into a pair of fermions. The above results can be used to individual channels that are kinematically allowed, although in numerical analysis we will show the results summing over all possible channels.

The above Eqs. (26) and (27) serve as a useful function interpolating between the low and high energy limits. At energies much lower than M_Z but still above the threshold for the electron-positron pair, the denominator is approximately equal to M_Z^4 , and the Cohen-Glashow's results in Eqs. (15) and (16) are recovered. In the high energy limit $E \gg M_Z/\sqrt{\delta}$, the integrals can be worked out in the narrow width approximation (NWA) (see, e.g., Appendix B in Ref. [48]). When summing over all decay channels of the virtual Z boson at sufficiently high energy, the results in Eqs. (24) and (25) for the physical Z -boson emission are recovered as well.

III. NUMERICAL ANALYSES FOR NEUTRINO VELOCITIES IN VARIOUS EXPERIMENTS

A. General Discussion

We first compare in Fig. 2 numerical results based on Eqs. (26) and (27) with those in the low-energy limit in Ref. [16] and in the high energy limit using NWA, Eqs. (24) and (25). To avoid misunderstanding, we note that while the low-energy result involves only the electron-positron emission, the other two include all possible channels that are kinematically allowed. Due to the opening of more and more channels one sees a continuous enhancement of the decay width (in the left panel) and energy loss rate (in the right panel). At very high energies the NWA result essentially coincides with the full calculation.

The difference to the low-energy limit can be more clearly seen in the right panel of Fig. 2 for the energy loss fraction. The mean fractional energy loss is about 0.79 for most energy scales until the Z threshold (main peak). At the threshold, the process is dominated by the

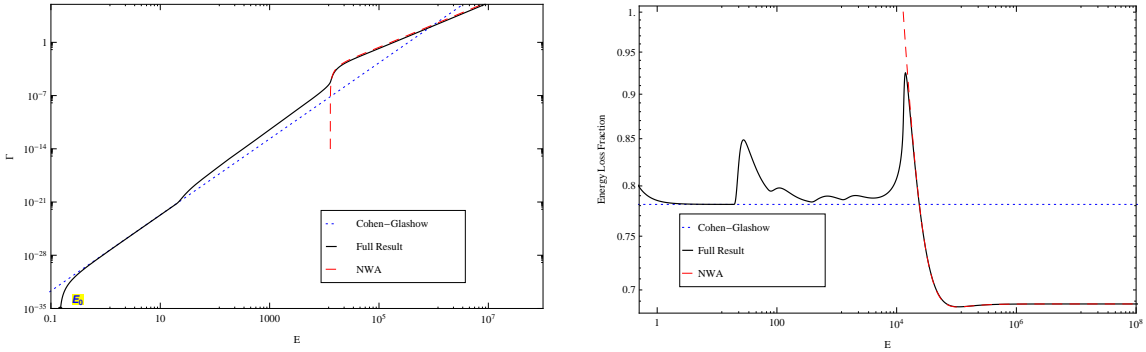


FIG. 2: Decay width Γ (in GeV, left panel) and energy loss fraction $-(dE/dx)/(E\Gamma)$ (right panel) of a superluminal neutrino are shown as a function of the initial energy E (in GeV), using the three results: full result (solid-black curve), Cohen-Glashow's result (dotted-blue) and NWA (dashed-red). Here $\delta = 5 \times 10^{-5}$, and $E_0 = 2m_e/\sqrt{\delta}$ is the threshold for the electron-positron emission.

production of an on-shell Z which takes away most of the initial neutrino energy. Beyond that point, there will be more energy left for the final neutrino. The secondary peak at about 30 GeV is due to the setting-in of the $u\bar{u}$, $d\bar{d}$, $\mu^+\mu^-$, and $s\bar{s}$ final states. It is worthy noticing that the energy loss rate decreases down to approximately 0.69 in the high energy limit where the low-energy approximation is not applicable.

B. The OPERA Experiment

Based on the energy loss rate at low energies in Eq. (16) Cohen and Glashow defined a terminal energy E_T for a superluminal neutrino after travelling a distance of L [16]

$$E_T^{-5} = \frac{125}{448} \frac{G_F^2 L \delta^3}{192\pi^3}, \quad (28)$$

meaning that a neutrino with energy less than E_T would not have travelled such a long distance. For the OPERA neutrinos with $L = 730$ km and $\delta = 5 \times 10^{-5}$, they got $E_T = 12.5$ GeV. We observe from Eq. (15) that a superluminal neutrino with energy E_T has a decay length, c/Γ , that is of the same order as the travel distance L . We thus propose to use a different statistical measure, the survival probability

$$P = e^{-L\Gamma}, \quad (29)$$

to decide whether a neutrino is observable or not at the OPERA detector.

To get some feeling of numbers, we notice that a superluminal neutrino of mean energy ~ 17.5 GeV has a lifetime of 1.9×10^{-3} s, which should be compared with its travel time of 2.4×10^{-3} s if it happens to have not yet decayed during the trip. The survival probability can tell in a statistically better way how likely such a neutrino can indeed be observable. From the left panel in Fig. 3 one can see that a neutrino has a survival probability of 0.79, 0.29, 0.09 respectively when it carries a terminal energy 12.5 GeV, mean energy 17.5 GeV, and an energy of 20 GeV. In particular, a neutrino with an initial energy of order E_T has a good chance to arrive at the OPERA detector. The right panel of Fig. 3 shows that the neutrino lifetime drops drastically as its energy increases. In that case the terminal energy serves as an appropriate measure on its observability.

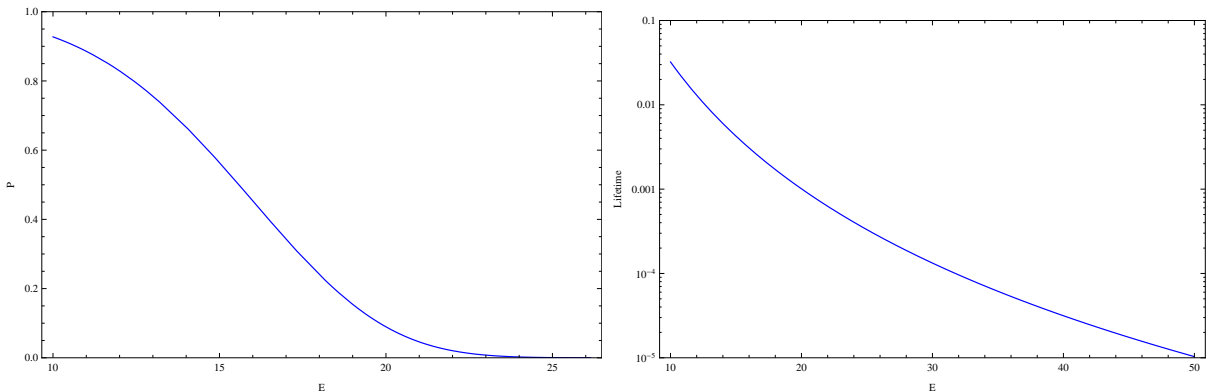


FIG. 3: Survival probability P (left panel) and lifetime (in seconds, right panel) as a function of energy E (in GeV) in the OPERA experiment with $\delta = 5 \times 10^{-5}$ and $L = 730$ km.

C. The Super-Kamiokande and IceCube Experiments

Besides OPERA, the Super-Kamiokande and IceCube experiments also reported observation of high energy neutrinos that travel a long distance. The Super-Kamiokande detected atmospheric neutrinos that traverse a distance of 10^4 km through the Earth (upward-going in the detector) over an energy range extending from 1 GeV to 1 TeV [49–51]. The IceCube collaboration observed upward-going showers with reconstructed shower energies above 16 TeV [52], with a baseline length estimated to be at least 500 km. Moreover, it reported the upward-going neutrinos from 100 GeV to 400 TeV [53]. For the latter we choose a rough travel distance about 10^4 km for the neutrinos within a zenith angle of $124^\circ - 180^\circ$.

We perform an analysis of constraints on the speed deviation δ of neutrinos from that of light in those two experiments using our general result in Eq. (26). We require that the neutrinos detected in Super-Kamiokande and IceCube experiments have travelled a distance L that is smaller than n times their decay length c_ν/Γ at the given energy E , *i.e.*, $L < nc_\nu/\Gamma$. Since Γ depends roughly on positive powers of E and δ (e.g., $E^5\delta^3$ at low energy), this gives an upper bound on δ as a function of E . The bound is not sensitive to n as long as n is not very different from unity, and a larger n yields a more conservative bound on δ using the same data. Our results assuming $n = 10$ are shown in Fig. 4 for three baseline lengths $L = 500, 730$, and 10^4 km. For instance, for $L = 500$ km the observations of neutrinos with energy in excess of 16 TeV imply $\delta < 6.6 \times 10^{-10}$, while for $L = 10^4$ km the observations of neutrinos with $E \geq 400$ TeV require $\delta < 7.8 \times 10^{-12}$. These bounds are indeed very stringent.

As both analytical results and Fig. 4 indicate, the bounds on δ are not sensitive to the baseline length L at a given neutrino energy E but are rather sensitive to E . The simple straight-line behavior implies that the extreme high energy regime has not yet been touched in those observations. To get some idea of the regime, we plot in Fig. 5 the decay width Γ as a function of E for four values of δ that correspond respectively to the bounds obtained from the experiments OPERA, Super-Kamiokande with $(L, E) = (10^4 \text{ km}, 1 \text{ TeV})$, and IceCube with $(L, E) = (500 \text{ km}, 16 \text{ TeV})$ and $(500 \text{ km}, 100 \text{ TeV})$. These experiments available on the Earth constrain the δ parameter to such tiny values that it is hard to reach the high energy regime with $E > M_Z/\sqrt{\delta}$.

IV. CONCLUSIONS

We revisited the constraints on superluminal neutrino velocities by assuming a minimally modified dispersion relation. We obtained the general decay width and energy loss rate of a superluminal neutrino with high or low energy from the bremsstrahlung process via both the direct calculation and virtual Z approaches in the laboratory frame. The analytical results by Cohen and Glashow were confirmed in the low energy limit. We used a different measure to assess whether a neutrino is observable or not in the OPERA experiment. We presented new results on the power law for the bremsstrahlung process in the high energy limit. Using our general results, we performed systematical analyses on the constraints arising from the

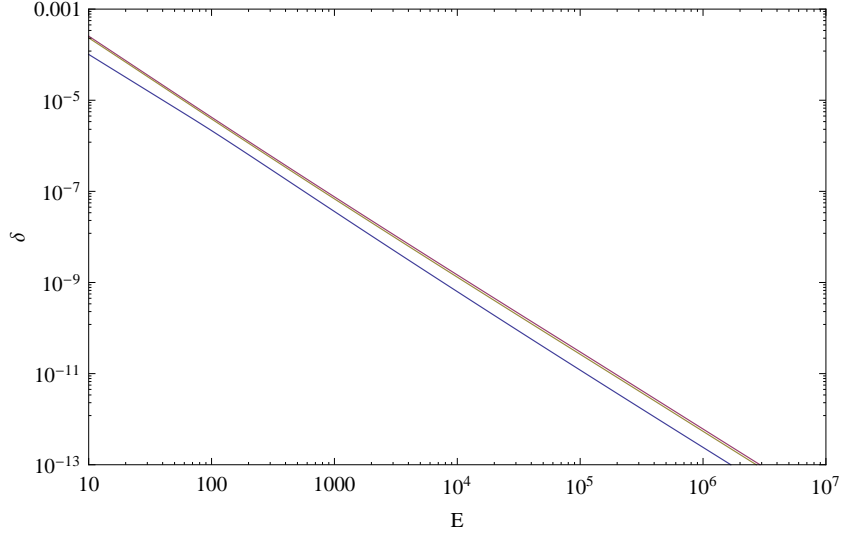


FIG. 4: Constraints on δ as a function of energy E (in GeV) for three baseline lengths $L = 500, 730$, and 10^4 km from the top to bottom lines.

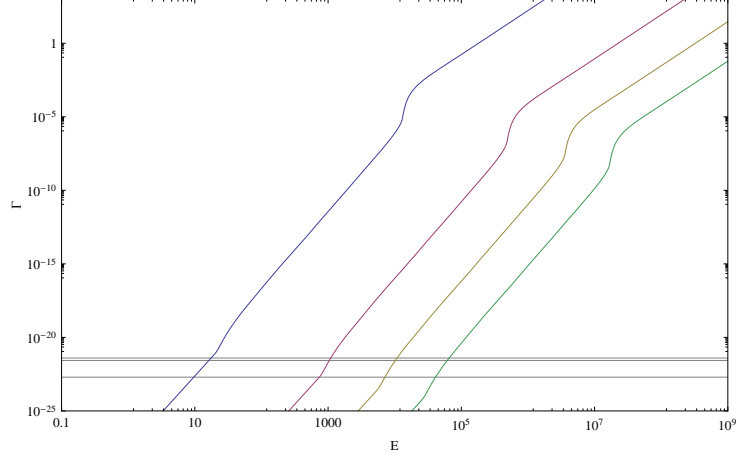


FIG. 5: Decay width Γ (in GeV) as a function of energy E (in GeV) for various $\delta = 5 \times 10^{-5}$, 3.6×10^{-8} , 6.6×10^{-10} , and 3.0×10^{-11} from the left to right curves. The horizontal lines indicate the decay length equal to the baseline lengths $L = 500, 730$, and 10^4 km from top to bottom.

Super-Kamiokande and IceCube experiments.

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