This is the accepted manuscript made available via CHORUS. The article has been published as:

Precise predictions for Z-boson +4 jet production at hadron colliders

H. Ita, Z. Bern, L. J. Dixon, F. Febres Cordero, D. A. Kosower, and D. Maître

Phys. Rev. D **85**, 031501 — Published 6 February 2012
DOI: 10.1103/PhysRevD.85.031501

Precise Predictions for $Z + 4$ Jets at Hadron Colliders

H. Ita[a], Z. Bern[a], L. J. Dixon[b,c], F. Febres Cordero[d], D. A. Kosower[e] and D. Maître[b,f]

[a]Department of Physics and Astronomy, UCLA, Los Angeles, CA 90095-1547, USA
[b]Theory Division, Physics Department, CERN, CH–1211 Geneva 23, Switzerland
[c]SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, USA
[d]Departamento de Física, Universidad Simón Bolívar, Caracas 1080A, Venezuela
[e]Institut de Physique Théorique, CEA–Saclay, F–91191 Gif-sur-Yvette cedex, France
[f]Department of Physics, University of Durham, Durham DH1 3LE, UK

We present the cross section for production of a Z boson in association with four jets at the Large Hadron Collider, at next-to-leading order in the QCD coupling. When the Z decays to neutrinos, this process is a key irreducible background to many searches for new physics. Its computation has been made feasible through the development of the on-shell approach to perturbative quantum field theory. We present the total cross section for pp collisions at $\sqrt{s} = 7$ TeV, after folding in the decay of the Z boson, or virtual photon, to a charged-lepton pair. We also provide distributions of the transverse momenta of the four jets, and we compare cross sections and distributions to the corresponding ones for the production of a W boson with accompanying jets.

PACS numbers: 12.38.-t, 12.38.Bx, 13.87.-a, 14.70.Hp

The Large Hadron Collider (LHC) is currently extending the energy frontier into uncharted territory, in the quest to identify new physics beyond the Standard Model of particle physics. Many signals of new physics, especially those containing dark matter candidates, lie in broad distributions with significant Standard Model backgrounds. A first-principles understanding of these backgrounds is provided by quantum chromodynamics (QCD) and the QCD-improved parton model. The leading perturbative order (LO) in the QCD coupling α_s gives a good qualitative prediction. Quantitatively reliable predictions require, at the least, next-to-leading-order (NLO) accuracy in the QCD coupling. For processes at a hadron collider with many-jet final states, NLO computations have long been a formidable challenge to particle theorists.

In this article we present the first NLO QCD results for Z boson production in association with four jets at a hadron collider, specifically at the LHC. We fold in the decay of the Z boson to an e^+e^- pair (or equivalently $\mu^+\mu^-$), and include contributions from virtual-photon exchange (collectively denoted by Z, γ^*). This process, containing identifiable charged leptons, is a benchmark for the closely related process in which the Z decays into neutrinos, which appear as missing transverse energy. The $Z \to \nu\bar{\nu}$ decay mode generates a key background process in the search for supersymmetry, as well as for other models that lead to dark-matter particle production at the end of a cascade of strongly-produced new particles. Fig. 1 shows a typical signal process, leading to the same signature of missing transverse energy with four jets and no sharp resonance. We note that another approach to estimating this process — combining a measurement of prompt-photon production with a theoretical estimate of the Z-to-photon ratio [1–3] — also benefits from NLO cross sections [4].

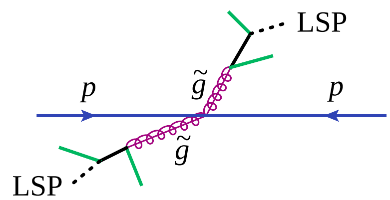

FIG. 1: Gluino pair production illustrates a typical signature of new physics scenarios: four jets plus a pair of lightest supersymmetric particles (LSPs) that escape the detector, yielding missing transverse energy.

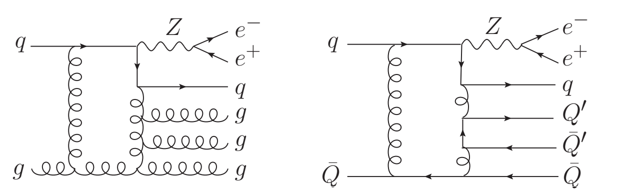

FIG. 2: Sample diagrams for the seven-point loop amplitudes for $qg \to Zqggg$ and $q\bar{Q} \to ZqQ'\bar{Q}'\bar{Q}$, followed by $Z \to e^+e^-$. There are also small contributions where the Z boson is replaced by a photon. This process is very similar theoretically to the case $Z \to \nu\bar{\nu}$ with missing transverse energy.

Recent years have witnessed a growing number of NLO QCD results using both traditional and on-shell approaches [5–11]. On-shell methods [12–15] exploit the analytic properties that all scattering amplitudes must satisfy, and generate new amplitudes from previously-computed ones. Computationally, they scale modestly with increasing numbers of external partons. We used these methods, as implemented in the BlackHat library [16], to compute the production of a W or Z boson in association with three jets at NLO [7, 9]. The

no. jets	Z LO	Z NLO	Z/W^+ LO	Z/W^+ NLO	$Zn/(n-1)$ LO	$Zn/(n-1)$ NLO
0	$323.1(0.1)^{+39.3}_{-44.3}$	$428.6(0.3)^{+6.2}_{-4.1}$	0.1209(0.0001)	0.1306(0.0003)	—	—
1	$66.69(0.04)^{+5.59}_{-5.30}$	$82.1(0.1)^{+3.3}_{-2.6}$	0.1674(0.0002)	0.166(0.001)	0.2064(0.0001)	0.1915(0.0004)
2	$19.10(0.02)^{+5.32}_{-3.82}$	$20.25(0.07)^{+0.31}_{-1.02}$	0.1636(0.0003)	0.166(0.002)	0.2864(0.0003)	0.247(0.001)
3	$4.76(0.01)^{+2.18}_{-1.35}$	$4.73(0.03)^{+0.05}_{-0.35}$	0.1634(0.0004)	0.169(0.002)	0.2494(0.0004)	0.234(0.002)
4	$1.116(0.002)^{+0.695}_{-0.390}$	$1.06(0.01)^{+0.05}_{-0.14}$	0.1618(0.0003)	0.172(0.002)	0.2343(0.0005)	0.223(0.002)

TABLE I: Total cross sections in pb for $Z, \gamma^* + n$-jet production at the LHC, using the anti-k_T jet algorithm with $R = 0.5$. The NLO result for $Z, \gamma^* +$ 4 jets uses the leading-color virtual approximation. The fourth and fifth columns give the cross-section ratios for $Z, \gamma^* \to e^+e^-$ production to $W^+ \to e^+\nu$ production. The final two columns give the ratios of the cross section for the given process to that with one fewer jet. The numerical integration uncertainty is in parentheses. The scale dependence is given in superscripts and subscripts.

predictions are generally in very good agreement with data from the Tevatron [17, 18]. (Earlier NLO results for $W +$ 3 jets were based on similar techniques and used various leading-color approximations [5, 6, 8].) We have also calculated [11] $W +$ 4-jet production at the LHC, making use of a leading-color approximation for the virtual terms that is known to be valid to about 3% for up to three associated jets [7, 9]. We will use a very similar approximation for $Z +$ 4-jet production.

The leading-color approximation we use here is the one described in ref. [19]. We drop certain contributions that are subleading in the number of colors N_c in the formal limit $N_c \to \infty$, with n_f/N_c held fixed. We retain the full color dependence in the Born terms, in the real-emission contributions (including dipole subtraction terms), in the integrated subtraction terms, and in the infrared-divergent terms in the virtual contributions. We drop the finite parts (ϵ^0 terms in dimensional regularization) of the subleading-color partial amplitudes. We also drop those finite parts of the leading-color partial amplitudes that are suppressed by explicit powers of $1/N_c$. In forming the color-summed interference of the surviving parts of the one-loop amplitudes with the tree amplitudes, we do not drop any further terms. The code performs the scheme shift from the four-dimensional helicity (FDH) scheme used internally in BlackHat to the 't Hooft–Veltman scheme [20] with full color dependence. The approximation differs in these last two aspects from the one used in the BlackHat calculation of $W +$ 4-jet production [11]. The $W +$ 4-jet results reported here treat subleading-color terms as in ref. [11]

The computation of $Z +$ 4-jet production is significantly more complex than that of $W +$ 4-jet production, because the quark flavor structure leads to more partonic subprocesses, especially those containing identical fermion pairs.

We incorporate a number of improvements to the evaluation of virtual contributions using BlackHat, compared to our earlier work [7, 9]. We have automated the assembly of subprocesses. Furthermore, to minimize the amount of higher-precision recomputation at points for which an instability is detected, only the unstable part, rather than the whole matrix element, is recomputed [7, 21]. Representative virtual diagrams are shown in fig. 2. We include all subprocesses, and as noted above make the leading-color approximation only in the virtual contributions to $Z +$ 4-jet production. As in ref. [9], we drop the axial- and vector-coupling loop contributions, along with the effects of top quarks in the loop. In $Z +$ 2-jet production at the Tevatron these contributions were less than 0.3%. We have also evaluated these contributions (in the two-quark subprocesses of $Z +$ 2-jet production), using a large-m_t approximation, at the LHC and find that they contribute less than 1%. The small numerical contribution is in line with the formal status of the single-flavor top-quark contribution as a subleading correction in our leading-color approximation with n_f/N_c held fixed.

The remaining NLO ingredients, the real-emission and dipole-subtraction terms [22], are computed using AMEGIC++ [23], which is part of the SHERPA package [24]. Here we retain the full color dependence. The SHERPA-based phase-space integration exploits QCD antenna structures [25, 26]. BlackHat supplies the real-emission tree amplitudes, using on-shell recursion relations [13] and efficient analytic forms extracted from $\mathcal{N} = 4$ super-Yang-Mills theory [27]. We have validated the code extensively. Previously, we compared many results against MCFM [28] for $W, Z +$ 2-jet production.

In the course of our study, we wish to investigate the effects of varying renormalization and factorization scales, and also to estimate the uncertainty due to uncertainties in our knowledge of the parton distribution functions (PDFs). We could do this by re-running the calculation for each scale or PDF error set independently; but this would be very wasteful of computer resources, because all contributions can be organized into sums of terms, where each term contains a simple function we wish to vary (for example, a logarithm of the renormalization scale) multiplied by a numerical coefficient independent of such variation, which is expensive to calculate. These coefficients can be calculated in one run, and stored for re-use. This is indeed how we organize a calculation: for each event we generate, we record the momenta for all partons in it along with its weight (the squared matrix element) and the coefficients of the various scale- or PDF-dependent

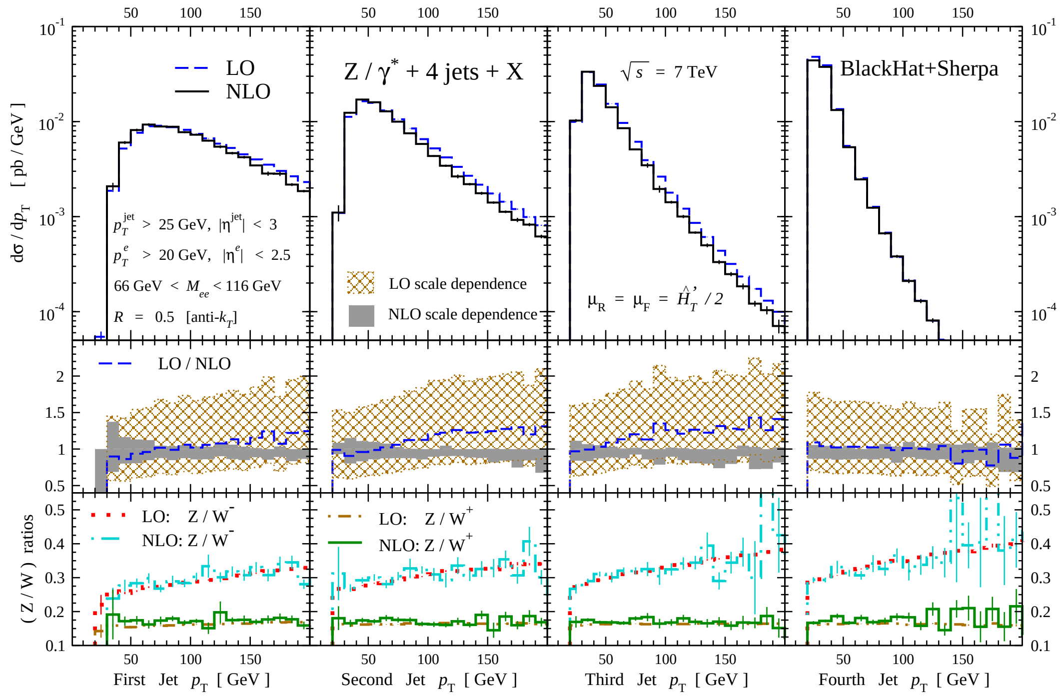

FIG. 3: A comparison of the p_T distributions of the leading four jets in $Z, \gamma^* + $ 4-jet production at the LHC. In the upper panels the NLO distribution is the solid (black) histogram and the LO predictions are shown as dashed (blue) lines. The thin vertical line in the center of each bin (where visible) gives its numerical (Monte Carlo) integration error. The middle panels show the LO distribution and LO and NLO scale-dependence bands normalized to the central NLO prediction. The bands are shaded (gray) for NLO and cross-hatched (brown) for LO. In the bottom panel, the dotted (red) line is the LO Z/W^- ratio, the dot-longer-dash (cyan) line the NLO Z/W^- ratio, the dot-shorter-dash (brown) line the LO Z/W^+ ratio and the solid (green) line the NLO Z/W^+ ratio.

functions. We store this information in ROOT-format n-tuple files [29]. The availability of these intermediate results in a standard format makes it computationally inexpensive for us to evaluate cross sections and distributions for different scales and PDF error sets. It also makes it easy to furnish our theoretical predictions to experimental collaborations, while allowing them to modify the cuts applied, or compute additional distributions [30]. Indeed, n-tuples corresponding to the $Z, \gamma^* + 1, 2, 3, 4$-jet production cross sections computed in this article were used to provide a theoretical prediction which has been compared by the ATLAS collaboration to their data [31].

Cross sections and distributions at LO suffer from strong sensitivity to the unphysical renormalization scale μ_R and factorization scale μ_F entering α_s and the parton distributions. This dependence is reduced at NLO. This issue is especially important at the LHC because of the wide range of kinematics probed. This wide range also obliges us to choose an event-by-event scale characteristic of the kinematics when we compute distributions. We choose $\mu_R = \mu_F \equiv \mu = \hat{H}'_T/2$ as our central scale [11], where $\hat{H}'_T \equiv \sum_i p_T^i + E_T^Z$. The sum runs over all final-state partons i, and $E_T^Z \equiv \sqrt{M_Z^2 + (p_T^{e^+e^-})^2}$; M_Z is fixed to its on-shell value. We follow standard procedure to assess scale dependence, varying the central scale up and down by a factor of two to construct scale-dependence bands, taking the minimum and maximum of any observable evaluated at five values: $\mu \times (1/2, 1/\sqrt{2}, 1, \sqrt{2}, 2)$.

The fixed-order perturbative expansion may break down in special kinematic regions, where large logarithms of ratios of physical scales emerge. Threshold logarithms can affect production at very large partonic center-of-mass energies. However, in ref. [11] it was argued, using results for inclusive single-jet production [32], that at the mass scales probed in $W, Z/\gamma^* +$ 4-jet production, such logarithms should remain quite modest. Tighter cuts can isolate regions subject to potentially large logarithms of either QCD or electroweak origin. In particular, cuts that force the vector boson to large p_T, the desired region for many searches for supersymmetry or dark-matter particles, can induce large electroweak Sudakov logarithms.

In our study, we consider the inclusive process $pp \to Z +$ 4 jets at an LHC center-of-mass energy of $\sqrt{s} = 7$ TeV. We incorporate the full Z, γ^* Breit-Wigner resonance and decay the intermediate boson into an electron–positron pair at the amplitude level, retaining all spin correlations. We impose the following cuts on the transverse momenta p_T, and pseudorapidities η: $p_T^e > 20$ GeV, $|\eta^e| < 2.5$, $p_T^{\rm jet} > 25$ GeV, $|\eta^{\rm jet}| < 3$, and 66 GeV $< M_{e^+e^-} < 116$ GeV. The lower cut on the lepton-pair invariant mass $M_{e^+e^-}$ eliminates the large contribution from the photon pole. Jets are defined using the infrared-safe anti-k_T algorithm [33] adopted by the LHC experiments. Here we present results for size parameter $R = 0.5$. We order the jets in p_T. In comparisons to W-boson cross sections we follow exactly the cuts of ref. [11]; the jet cuts are identical. We use the CTEQ6M [34] PDFs at NLO, and the CTEQ6L1 set at LO. Electroweak boson masses and couplings are chosen as in refs. [7, 9]. We also use the SHERPA six-flavor implementation of $\alpha_s(\mu)$ and the value of $\alpha_s(M_Z)$ provided by CTEQ.

In table I, we give LO and NLO parton-level inclusive cross sections for e^+e^- production via a Z, γ^* boson, and accompanied by zero through four jets. The NLO results exhibit a markedly reduced scale dependence compared to LO; the improvement becomes stronger as the number of jets increases. We also display the ratios of the Z to W^+ cross sections, and the "jet-production" ratios of $Z + n$-jet to $Z + (n-1)$-jet cross sections. Ratios to W^--boson cross sections can be obtained using the results of ref. [11]. Both kinds of ratios should be less sensitive to theoretical systematics than the absolute cross sections. Indeed, the Z/W ratios show relatively little difference between LO and NLO. This ratio changes very little under correlated variations of μ in numerator and denominator; hence we do not exhibit such scale variation. Varying the R parameter in the jet algorithm, we find very similar behavior as in the W case [11].

It has generally been expected that the jet-production ratio is roughly independent of the number of jets [35]. Other than the $Z+$ 1-jet/$Z+$ 0-jet ratio, which is smaller because of the restricted kinematics of the leading contribution to $Z +$ 0-jet production, the results shown in table I are consistent with this expectation. The ratios are, however, rather sensitive to the experimental cuts: for example, imposing large vector-boson p_T cuts makes them depend strongly on the number of jets [9].

In fig. 3, we show the p_T distributions of the leading four jets in $Z, \gamma^* +$ 4-jet production at LO and NLO. The predictions are normalized to the central NLO prediction in the middle panels. The NLO distributions display a much smaller dependence on the unphysical renormalization and factorization scales. For our central scale choice, the distributions for the first three leading jets soften noticeably from LO to NLO, while the fourth-jet distribution is virtually unchanged. The NLO corrections to the behavior of $Z, \gamma^* +$ 4-jet and $W +$ 4-jet production are quite similar in this respect [11].

The bottom panels in fig. 3 show the ratio of Z/W^+ and Z/W^- production both at LO and at NLO. The Z/W^- ratio rises with rising p_T while the Z/W^+ ratio is roughly flat. Both ratios reflect the rising dominance of the u quark distribution over the d quark with increasing parton fraction x. Because the Z has an appreciable coupling to an initial u quark (unlike the W^-), the shape of the p_T distribution follows more closely the W^+ case than the W^- case, which has a $d(x)/u(x)$ relative suppression. The excellent agreement between LO and NLO ratios for $Z/W^\pm$ production shows that these ratios are under solid perturbative control.

A comparison of parton-level results to experimental data requires estimating the size of non-perturbative effects, such as those induced by the underlying event or by fragmentation and hadronization of the outgoing partons. Standard LO parton-shower Monte Carlo programs can provide these estimates. As NLO parton-shower programs are developed [36], they can use virtual corrections computed with BlackHat. We expect non-perturbative effects to largely cancel in the $Z/W^\pm$ ratios.

In the present study of the $Z, \gamma^* +$ 4-jet process, we have imposed cuts typical of Standard-Model measurements at the LHC. The same code can be used to study the size of QCD corrections for observables under cuts used in new-physics searches. This will allow the study of backgrounds to missing energy signals of new physics, arising when a Z boson decays to a pair of neutrinos. Ratios such as the $Z/W +$ jets ratios offer highly-reliable theoretical predictions. Applying BlackHat along with SHERPA brings an unprecedented level of theoretical precision to Standard-Model backgrounds, aiding in the hunt for new-physics signals at the LHC.

We thank Giovanni Diana, Stefan Höche and Kemal Ozeren for many helpful discussions. We also thank Carola Berger, Darren Forde, and Tanju Gleisberg for contributing to earlier versions of BlackHat. We thank the Kavli Institute for Theoretical Physics, where part of this work was performed, for its hospitality. This research was supported by the US Department of Energy under contracts DE–FG03–91ER40662, DE–AC02–76SF00515 and DE–FC02–94ER40818. DAK's research is supported by the European Research Council under Advanced Investigator Grant ERC–AdG–228301. H.I.'s work is supported by a grant from the US LHC Theory Initiative through NSF contract PHY–0705682. This research was also supported in part by the National Science Foundation under Grant No. NSF PHY05-51164. This research used resources of Academic Technology Services at UCLA.

[1] S. Chatrchyan *et al.* [CMS Collaboration], JHEP **1108**, 155 (2011).

[2] G. Aad *et al.* [ATLAS Collaboration] *"Search for squarks and gluinos using final states with jets and missing transverse momentum with the ATLAS detector in $\sqrt{s}$ = 7 TeV proton-proton collisions"*, ATLAS–CONF–2011–086 (2011), unpublished; arXiv:1109.6572 [hep-ex].

[3] S. Ask, M. A. Parker, T. Sandoval, M. E. Shea and W. J. Stirling, JHEP **1110**, 058 (2011).

[4] Z. Bern, G. Diana, L. J. Dixon, F. Febres Cordero, S. Hoche, H. Ita, D. A. Kosower, D. Maître and K.J. Ozeren, arXiv:1106.1423 [hep-ph].

[5] C. F. Berger, Z. Bern, L. J. Dixon, F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D. A. Kosower and D. Maître, Phys. Rev. Lett. **102**, 222001 (2009).

[6] R. K. Ellis, K. Melnikov and G. Zanderighi, Phys. Rev. D **80**, 094002 (2009).

[7] C. F. Berger, Z. Bern, L. J. Dixon, F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D. A. Kosower and D. Maître, Phys. Rev. D **80**, 074036 (2009).

[8] K. Melnikov and G. Zanderighi, Phys. Rev. D **81**, 074025 (2010).

[9] C. F. Berger, Z. Bern, L. J. Dixon, F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D.A. Kosower and D. Maître, Phys. Rev. D **82**, 074002 (2010).

[10] A. Bredenstein, A. Denner, S. Dittmaier and S. Pozzorini, JHEP **0808**, 108 (2008); Phys. Rev. Lett. **103**, 012002 (2009); JHEP **1003**, 021 (2010); G. Bevilacqua, M. Czakon, C. G. Papadopoulos, R. Pittau and M. Worek, JHEP **0909**, 109 (2009); T. Binoth, N. Greiner, A. Guffanti, J. P. Guillet, T. Reiter and J. Reuter, Phys. Lett. B **685**, 293 (2010); G. Bevilacqua, M. Czakon, C. G. Papadopoulos and M. Worek, Phys. Rev. Lett. **104**, 162002 (2010); T. Melia, K. Melnikov, R. Rontsch and G. Zanderighi, Phys. Rev. D **83**, 114043 (2011); F. Campanario, C. Englert, M. Rauch and D. Zeppenfeld, Phys. Lett. B **704**, 515 (2011).

[11] C. F. Berger, Z. Bern, L. J. Dixon, F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D. A. Kosower and D. Maître, Phys. Rev. Lett. **106**, 092001 (2011).

[12] Z. Bern, L. J. Dixon, D. C. Dunbar and D. A. Kosower, Nucl. Phys. B **425**, 217 (1994); Nucl. Phys. B **435**, 59 (1995); Phys. Lett. B **394**, 105 (1997); Z. Bern and A. G. Morgan, Nucl. Phys. B **467**, 479 (1996); Z. Bern, L. J. Dixon and D. A. Kosower, Nucl. Phys. B **513**, 3 (1998); R. Britto, F. Cachazo and B. Feng, Nucl. Phys. B **725**, 275 (2005); C. Anastasiou, R. Britto, B. Feng, Z. Kunszt and P. Mastrolia, Phys. Lett. B **645**, 213 (2007); R. Britto and B. Feng, JHEP **0802**, 095 (2008).

[13] R. Britto, F. Cachazo, B. Feng and E. Witten, Phys. Rev. Lett. **94**, 181602 (2005).

[14] C. F. Berger, Z. Bern, L. J. Dixon, D. Forde and D. A. Kosower, Phys. Rev. D **74**, 036009 (2006).

[15] G. Ossola, C. G. Papadopoulos and R. Pittau, Nucl. Phys. B **763**, 147 (2007); D. Forde, Phys. Rev. D **75**, 125019 (2007); W. T. Giele, Z. Kunszt and K. Melnikov, JHEP **0804**, 049 (2008); S. D. Badger, JHEP **0901**, 049 (2009).

[16] C. F. Berger, Z. Bern, L. J. Dixon, F. Febres Cordero, D. Forde, H. Ita, D. A. Kosower and D. Maître, Phys. Rev. D **78**, 036003 (2008).

[17] V. M. Abazov *et al.* [D0 Collaboration], Phys. Lett. B **705**, 200 (2011).

[18] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **100**, 102001 (2008); Phys. Rev. D **77**, 011108 (2008); V. M. Abazov *et al.* [D0 Collaboration], Phys. Lett. B **678**, 45 (2009).

[19] H. Ita and K. Ozeren, arXiv:1111.4193 [hep-ph].

[20] G. 't Hooft and M. J. G. Veltman, Nucl. Phys. B **44**, 189 (1972).

[21] H. Ita, J. Phys. A **44**, 454005 (2011).

[22] S. Catani and M. H. Seymour, Nucl. Phys. B **485**, 291 (1997) [Erratum-ibid. B **510**, 503 (1998)].

[23] F. Krauss, R. Kuhn and G. Soff, JHEP **0202**, 044 (2002); T. Gleisberg and F. Krauss, Eur. Phys. J. C **53**, 501 (2008).

[24] T. Gleisberg, S. Höche, F. Krauss, M. Schönherr, S. Schumann, JHEP **0902**, 007 (2009).

[25] A. van Hameren and C. G. Papadopoulos, Eur. Phys. J. C **25**, 563 (2002).

[26] T. Gleisberg, S. Höche and F. Krauss, arXiv:0808.3672 [hep-ph].

[27] L. J. Dixon, J. M. Henn, J. Plefka and T. Schuster, JHEP **1101**, 035 (2011).

[28] J. M. Campbell and R. K. Ellis, Phys. Rev. D **65**, 113007 (2002).

[29] R. Brun and F. Rademakers, Nucl. Instrum. Meth. A **389**, 81 (1997).

[30] ATLAS Collaboration, *"Measurement of the production cross section for W-bosons in association with jets in pp collisions using 33 pb^{-1} of data at $\sqrt{s} = 7$ TeV with the ATLAS detector"*, ATLAS–CONF–2011–060 (2011), unpublished; G. Aad *et al.* [ATLAS Collaboration], arXiv:1201.1276 [hep-ex].

[31] G. Aad *et al.* [ATLAS Collaboration], arXiv:1111.2690 [hep-ex].

[32] D. de Florian and W. Vogelsang, Phys. Rev. D **76**, 074031 (2007).

[33] M. Cacciari, G. P. Salam and G. Soyez, JHEP **0804**, 063 (2008).

[34] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. M. Nadolsky and W.-K. Tung, JHEP **0207**, 012 (2002).

[35] S. D. Ellis, R. Kleiss and W. J. Stirling, Phys. Lett. B **154**, 435 (1985); F. A. Berends, W. T. Giele, H. Kuijf, R. Kleiss and W. J. Stirling, Phys. Lett. B **224**, 237 (1989); F. A. Berends, H. Kuijf, B. Tausk and W. T. Giele, Nucl. Phys. B **357**, 32 (1991); E. Abouzaid and H. J. Frisch, Phys. Rev. D **68**, 033014 (2003).

[36] S. Frixione and B. R. Webber, JHEP **0206**, 029 (2002); P. Nason, JHEP **0411**, 040 (2004); S. Frixione, P. Nason and C. Oleari, JHEP **0711**, 070 (2007); S. Alioli, P. Nason, C. Oleari and E. Re, JHEP **1006**, 043 (2010); K. Hamilton and P. Nason, JHEP **1006**, 039 (2010); S. Höche, F. Krauss, M. Schönherr and F. Siegert, JHEP **1104**, 024 (2011); JHEP **1108**, 123 (2011).