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Precise Predictions for $Z + 4$ Jets at Hadron Colliders

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We present the cross section for production of a $Z$ boson in association with four jets at the Large Hadron Collider, at next-to-leading order in the QCD coupling. When the $Z$ decays to neutrinos, this process is a key irreducible background to many searches for new physics. Its computation has been made feasible through the development of the on-shell approach to perturbative quantum field theory. We present the total cross section for $pp$ collisions at $\sqrt{s} = 7$ TeV, after folding in the decay of the $Z$ boson, or virtual photon, to a charged-lepton pair. We also provide distributions of the transverse momenta of the four jets, and we compare cross sections and distributions to the corresponding ones for the production of a $W$ boson with accompanying jets.

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The Large Hadron Collider (LHC) is currently extending the energy frontier into uncharted territory, in the quest to identify new physics beyond the Standard Model of particle physics. Many signals of new physics, especially those containing dark matter candidates, lie in broad distributions with significant Standard Model backgrounds. A first-principles understanding of these backgrounds is provided by quantum chromodynamics (QCD) and the QCD-improved parton model. The leading perturbative order (LO) in the QCD coupling gives a good qualitative prediction. Quantitatively reliable predictions require, at the least, next-to-leading-order (NLO) accuracy in the QCD coupling. For processes at a hadron collider with many-jet final states, NLO computations have long been a formidable challenge to particle theorists.

In this article we present the first NLO QCD results for $Z$ boson production in association with four jets at a hadron collider, specifically at the LHC. We fold in the decay of the $Z$ boson to an $e^+e^-$ pair (or equivalently $\mu^+\mu^-$), and include contributions from virtual-photon exchange (collectively denoted by $Z, \gamma^*$). This process, containing identifiable charged leptons, is a benchmark for the closely related process in which the $Z$ decays into neutrinos, which appear as missing transverse energy. The $Z \to \nu\bar{\nu}$ decay mode generates a key background process in the search for supersymmetry, as well as for other models that lead to dark-matter particle production at the end of a cascade of strongly-produced new particles. Fig. 1 shows a typical signal process, leading to the same signature of missing transverse energy with four jets and no sharp resonance. We note that another approach to estimating this process — combining a measurement of prompt-photon production with a theoretical estimate of the $Z$-to-photon ratio [1–3] — also benefits from NLO cross sections [4].

Recent years have witnessed a growing number of NLO QCD results using both traditional and on-shell approaches [5–11]. On-shell methods [12–15] exploit the analytic properties that all scattering amplitudes must satisfy, and generate new amplitudes from previously-computed ones. Computationally, they scale modestly with increasing numbers of external partons. We used these methods, as implemented in the BLACKHAT library [16], to compute the production of a $W$ or $Z$ boson in association with three jets at NLO [7, 9].
predictions are generally in very good agreement with data from the Tevatron [17, 18]. (Earlier NLO results for W + 4-jet production, as noted above make the leading-color approximation only in the virtual contributions to Z + 4-jet production. As in ref. [9], we drop the axial- and vector-coupling loop contributions, along with the effects of top quarks in the loop. In Z + 2-jet production at the Tevatron these contributions were less than 0.3%. We have also evaluated these contributions (in the two-quark subprocesses of Z + 2-jet production), using a large-m_{t} approximation, at the LHC and find that they contribute less than 1%. The small numerical contribution is in line with the formal status of the single-flavor top-quark contribution as a subleading correction in our leading-color approximation with n_{f}/N_{c} held fixed.

The remaining NLO ingredients, the real-emission and dipole-subtraction terms [22], are computed using AMEGIC++ [23], which is part of the SHERPA package [24]. Here we retain the full color dependence. The SHERPA-based phase-space integration exploits QCD antenna structures [25, 26]. BLACKHat supplies the real-emission tree amplitudes, using on-shell recursion relations [13] and efficient analytic forms extracted from N = 4 super-Yang-Mills theory [27]. We have validated the code extensively. Previously, we compared many results against MCFM [28] for W, Z + 2-jet production.

In the course of our study, we wish to investigate the effects of varying renormalization and factorization scales, and also to estimate the uncertainty due to uncertainties in our knowledge of the parton distribution functions (PDFs). We could do this by re-running the calculation for each scale or PDF error set independently; but this would be very wasteful of computer resources, because all contributions can be organized into sums of terms, where each term contains a simple function we wish to vary (for example, a logarithm of the renormalization scale) multiplied by a numerical coefficient independent of such variation, which is expensive to calculate. These coefficients can be calculated in one run, and stored for reuse. This is indeed how we organize a calculation: for each event we generate, we record the momenta for all partons in it along with its weight (the squared matrix element) and the coefficients of the various scale- or PDF-dependent

<table>
<thead>
<tr>
<th>no. jets</th>
<th>Z LO</th>
<th>Z NLO</th>
<th>Z/W+ LO</th>
<th>Z/W+ NLO</th>
<th>Zn/(n−1) LO</th>
<th>Zn/(n−1) NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>323.1(0.1)</td>
<td>539.3</td>
<td>428.6(0.3)</td>
<td>519.3</td>
<td>0.1209(0.0001)</td>
<td>0.1306(0.0003)</td>
</tr>
<tr>
<td>1</td>
<td>66.69(0.04)</td>
<td>55.3</td>
<td>82.1(0.1)</td>
<td>53.3</td>
<td>0.1674(0.0002)</td>
<td>0.166(0.001)</td>
</tr>
<tr>
<td>2</td>
<td>19.10(0.02)</td>
<td>31.2</td>
<td>20.25(0.07)</td>
<td>21.2</td>
<td>0.1636(0.0003)</td>
<td>0.166(0.002)</td>
</tr>
<tr>
<td>3</td>
<td>4.76(0.01)</td>
<td>2.18</td>
<td>4.73(0.03)</td>
<td>0.05</td>
<td>0.1634(0.0004)</td>
<td>0.169(0.002)</td>
</tr>
<tr>
<td>4</td>
<td>1.116(0.002)</td>
<td>0.985</td>
<td>1.06(0.01)</td>
<td>0.05</td>
<td>0.1618(0.0003)</td>
<td>0.172(0.002)</td>
</tr>
</tbody>
</table>

TABLE I: Total cross sections in pb for Z, γ∗ + n-jet production at the LHC, using the anti-k_{T} jet algorithm with R = 0.5. The NLO result for Z, γ∗ + 4 jets uses the leading-color virtual approximation. The fourth and fifth columns give the cross-section ratios for Z, γ∗ → e^+e^- + 1 jet to W^+ → e^+ν production. The final two columns give the ratios of the cross section for the given process to that with one fewer jet. The numerical integration uncertainty is in parentheses. The scale dependence is given in superscripts and subscripts.
functions. We store this information in ROOT-format n-tuple files [29]. The availability of these intermediate results in a standard format makes it computationally inexpensive for us to evaluate cross sections and distributions for different scales and PDF error sets. It also makes it easy to furnish our theoretical predictions to experimental collaborations, while allowing them to modify the cuts applied, or compute additional distributions [30]. Indeed, n-tuples corresponding to the Z, γ* + 1, 2, 3, 4-jet production cross sections computed in this article were used to provide a theoretical prediction which has been compared by the ATLAS collaboration to their data [31].

Cross sections and distributions at LO suffer from strong sensitivity to the unphysical renormalization scale \( \mu_R \) and factorization scale \( \mu_F \) entering \( \alpha_s \) and the parton distributions. This dependence is reduced at NLO. This issue is especially important at the LHC because of the wide range of kinematics probed. This wide range also obliges us to choose an event-by-event scale characteristic of the kinematics when we compute distributions. We choose \( \mu_R = \mu_F \equiv \mu = \mu_T/2 \) as our central scale [11], where \( \mu_T \equiv \sum_i p_T^i + E_T^Z \). The sum runs over all final-state partons \( i \), and \( E_T^Z \equiv \sqrt{M_Z^2 + (p_T^{\mu+} e^-)^2} \); \( M_Z \) is fixed to its on-shell value. We follow standard procedure to assess scale dependence, varying the central scale up and down by a factor of two to construct scale-dependence bands, taking the minimum and maximum of any observable evaluated at five values: \( \mu \times (1/2, 1/\sqrt{2}, 1, \sqrt{2}, 2) \).

The fixed-order perturbative expansion may break down in special kinematic regions, where large logarithms of ratios of physical scales emerge. Threshold logarithms can affect production at very large partonic center-of-mass energies. However, in ref. [11] it was argued, using results for inclusive single-jet production [32], that at the mass scales probed in \( W, Z/\gamma^* \) ± 4-jet production, such logarithms should remain quite modest. Tighter cuts can isolate regions subject to potentially large logarithms of either QCD or electroweak origin. In particular, cuts that force the vector boson to large \( p_T \), the desired region for many searches for supersymmetry or dark-matter particles, can induce large electroweak Sudakov logarithms.

In our study, we consider the inclusive process \( pp \to Z + 4 \) jets at an LHC center-of-mass energy of \( \sqrt{s} = 7 \) TeV. We incorporate the full \( Z, \gamma^* \) Breit-Wigner resonance and decay the intermediate boson into an electron–positron pair at the amplitude level, retaining all spin correlations. We impose the following cuts on the transverse momenta \( p_T \), and pseudorapidities \( \eta; p_T^{\mu+} > 20 \) GeV, \( |\eta^{\mu+}| < 2.5 \), \( p_T^{e^-} > 25 \) GeV, \( |\eta^{e^-}| < 3 \), and 66 GeV < \( M_{e^+ e^-} < 116 \) GeV. The lower cut on the lepton-pair invariant mass \( M_{e^+ e^-} \) eliminates the large contri-
bution from the photon pole. Jets are defined using the infrared-safe anti-
k_{T} algorithm [33] adopted by the LHC experiments. Here we present results for size parameter
\( R = 0.5 \). We order the jets in \( p_{T} \). In comparisons to \( W \)-boson cross sections we follow exactly the cuts of ref. [11];
the jet cuts are identical. We use the CTEQ6M [34] PDFs
at NLO, and the CTEQ6L1 set at LO. Electroweak boson masses and couplings are chosen as in refs. [7, 9]. We also use the SHERPA six-flavor implementation of \( \alpha_s(\mu) \) and the value of \( \alpha_s(M_Z) \) provided by CTEQ.

In table I, we give LO and NLO parton-level inclusive
cross sections for \( e^+e^- \) production via a \( Z, \gamma^* \) boson, and accompanied by zero through four jets. The NLO results
exhibit a markedly reduced scale dependence compared to LO; the improvement becomes stronger as the num-
ber of jets increases. We also display the ratios of the \( Z \) to \( W^+ \) cross sections, and the “jet-production” ratios
of \( Z + n \)-jet to \( Z + (n - 1) \)-jet cross sections. Ratios to \( W^- \)-boson cross sections can be obtained using the
results of ref. [11]. Both kinds of ratios should be less sen-
tive to theoretical systematics than the absolute cross
sections. Indeed, the \( Z/W \) ratios show relatively little
difference between LO and NLO. This ratio changes very
little under correlated variations of \( \mu \) in numerator and
denominator; hence we do not exhibit such scale vari-
tion. Varying the \( R \) parameter in the jet algorithm, we
find very similar behavior as in the W case [11].

It has generally been expected that the jet-production ratio is roughly independent of the number of jets [35].
Other than the \( Z + 1 \)-jet/\( Z + 0 \)-jet ratio, which is smaller
because of the restricted kinematics of the leading con-
tribution to \( Z + 0 \)-jet production, the results shown in
table I are consistent with this expectation. The ratios
are, however, rather sensitive to the experimental cuts;
for example, imposing large vector-boson \( p_{T} \) cuts makes
them depend strongly on the number of jets [9].

In fig. 3, we show the \( p_{T} \) distributions of the leading
four jets in \( Z, \gamma^* + 4 \)-jet production at LO and NLO. The
predictions are normalized to the central NLO prediction in
the middle panels. The NLO distributions display a
much smaller dependence on the unphysical renormaliza-
tion and factorization scales. For our central scale choice,
the distributions for the first three leading jets soften
noticeably from LO to NLO, while the fourth-jet distribution
is virtually unchanged. The NLO corrections to the
behavior of \( Z, \gamma^* + 4 \)-jet and \( W + 4 \)-jet production are
quite similar in this respect [11].

The bottom panels in fig. 3 show the ratio of \( Z/W^+ \)
and \( Z/W^- \) production both at LO and at NLO. The
\( Z/W^- \) ratio rises with rising \( p_{T} \) while the \( Z/W^+ \) ratio is
roughly flat. Both ratios reflect the rising dominance of
the \( u \) quark distribution over the \( d \) quark with increas-
ing parton fraction \( x \). Because the \( Z \) has an appreciable
coupling to an initial \( u \) quark (unlike the \( W^- \)), the shape
of the \( p_{T} \) distribution follows more closely the \( W^+ \) case
than the \( W^- \) case, which has a \( d(x)/u(x) \) relative sup-
pression. The excellent agreement between LO and NLO
ratios for \( Z/W^\pm \) production shows that these ratios are
under solid perturbative control.

A comparison of parton-level results to experimental
data requires estimating the size of non-perturbative ef-
ts, such as those induced by the underlying event or
by fragmentation and hadronization of the outgoing par-
tons. Standard LO parton-shower Monte Carlo programs
can provide these estimates. As NLO parton-shower pro-
grams are developed [36], they can use virtual corrections
computed with BLACKHAT. We expect non-perturbative
effects to largely cancel in the \( Z/W^\pm \) ratios.

In the present study of the \( Z, \gamma^* + 4 \)-jet process, we
have imposed cuts typical of Standard-Model measure-
ments at the LHC. The same code can be used to study
the size of QCD corrections for observables under cuts
used in new-physics searches. This will allow the study
of backgrounds to missing energy signals of new physics,
raising when a \( Z \) boson decays to a pair of neutrinos.
Ratios such as the \( Z/W + \) jets ratios offer highly-reliable
theoretical predictions. Applying BLACKHAT along with
SHERPA brings an unprecedented level of theoretical
precision to Standard-Model backgrounds, aiding in the
hunt for new-physics signals at the LHC.

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