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Efficient asymptotic frame selection for binary black hole spacetimes using asymptotic radiation

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Previous studies have demonstrated that gravitational radiation reliably encodes information about the natural emission direction of the source (e.g., the orbital plane). In this paper, we demonstrate that these orientations can be efficiently estimated by the principal axes of $\left\langle L_{(a}L_{b)}\right\rangle$, an average of the action of rotation group generators on the Weyl tensor at asymptotic infinity. Evaluating this average at each time provides the instantaneous emission direction. Further averaging across the entire signal yields an average orientation, closely connected to the angular components of the Fisher matrix. The latter direction is well-suited to data analysis and parameter estimation when the instantaneous emission direction evolves significantly. Finally, in the time domain, the average $\left\langle L_{(a}L_{b)}\right\rangle$ provides fast, invariant diagnostics of waveform quality.

I. INTRODUCTION

Ground-based gravitational wave detectors like LIGO and Virgo are likely to see many few-stellar-mass black hole (BH) binaries formed through isolated [1, 2] and dynamical [3–5] processes. Additionally, advanced detectors could see the merger signature of two intermediate-mass black holes (each $M \in [100, 10^3]M_\odot$), perhaps formed in dense globular clusters [6]. Black hole spin will significantly impact all phases of the signal to which these ground-based detectors are sensitive: the late-time inspiral, merger signal, and (through the final BH spin) ringdown. Misaligned spins break symmetry in the orbital plane, generally leading to strong modulations in the inspiral [7], merger, and ringdown. At early times, these modulations can be easily estimated through separation of timescales: the emitted inspiral waveforms are often well-described by quasistationary radiation from a circular orbit, slowly rotating with time modulated with time as the orbital plane precesses [7–9]. In the extreme but astrophysically important case of BH-NS binaries, the emission direction can precess about the total angular momentum through arbitrarily large angles, leading to strong amplitude and phase modulations [7, 10]. At late times, numerical relativity simulations of mergers can efficiently predict gravitational radiation from a variety of spins and orbits. A well-chosen static or time-evolving frame makes it easier to compare their outputs to one another and to these analytic models.

Previous studies have demonstrated the gravitational radiation carried to infinity encodes information about the instantaneous emission directions. For example, Schmidt et al. [8] track the precession of the orbital plane by selecting a frame that maximizes the amplitude of the $l = |m| = 2$ modes at each time. In this paper, we propose an invariant algebraic method to calculate the natural principal axes of asymptotic radiation. This method can be applied independently at each time, at each frequency, or at each binary mass. As with previous studies [8] the diagonalized time-domain representation may be useful in interpreting and modeling precessing signals as quasistationary circular orbits. Data analysts, however, will benefit most from choosing an optimal frame at each binary mass. For example, some signals like BH-NS binaries have an optimal time-domain emission orientation that evolves significantly. In these cases, as well as when higher harmonics become significant at high mass, the frame relevant to *data analysis* must suitably average over the relevant portion, frequency, and modal content of the signal. In this paper, we provide the first algorithm to select a suitable orientation for the average signal.

In Section II we describe our algebraic method for determining an optimal orientation versus time, frequency, or mass. In Section III we demonstrate our method provides a new way to efficiently and accurately determine the natural orientation of a quasistationary quasicircular inspiral versus time, using both artificially generated quasistationary sources (i.e., applying a rotation to a nonprecessing source) and waveforms from real precessing binaries. We also discuss numerical limitations in our time-domain method during the merger and ringdown, where multiple harmonics become increasingly important. In Section IV we demonstrate that a preferred frame versus *mass* arises naturally in

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data analysis applications, as the average of the Fisher matrix. Using a separation of timescales, we demonstrate this frame roughly corresponds to a time- and bandpass-weighted average of the optimal time-dependent frame. We briefly comment on the utility of mass-weighted frames for data analysis applications, such as mode-decomposed searches for spinning binaries [11].

II. NATURAL WAVEFORM FRAMES

Gravitational radiation carried away to infinity is well-known to encode certain preferred orientations, such as the radiated linear and angular momenta. More generally, given a one-parameter family of inner products $\langle a|b\rangle_\xi = \int a^*(t')K_\xi(t,t')b(t')dtdt'$ on pairs of complex-valued functions of time, generated by $K_\xi(t,t')$ and parameterized by ξ, and a tensor operator $Q_{a_1\ldots a_n}$ acting on the asymptotic Weyl scalar, the orientation average of $\langle\psi_4|Q_{a_1\ldots a_n}\psi_4\rangle$ defines an asymptotic tensor

$$\langle Q_{a_1\ldots a_n}\rangle_\xi \equiv \frac{\int d\Omega \int dtdt' K_\xi(t,t')\psi_4{}^* Q_{a_1\ldots a_n}\psi_4}{\int d\Omega \int dtdt' K_\xi(t,t')\psi_4{}^*\psi_4} . \tag{1}$$

The principal axes of this tensor in turn define preferred directions. Depending on the inner product and reference tensor, very different orientations can be selected; this formula, for example, includes as a special case both recoil kicks and radiated angular momentum.

In this paper we consider $\left\langle L_{(a}L_{b)}\right\rangle$, the average of products of *rotation group generators* L_k.[1] For example, adopting a kernel $K_t(\tau_1,\tau_2) = \delta(\tau_1 - t)\delta(\tau_2 - t)$ (i.e., using a quantum-mechanics-motivated state notation, $K = |t\rangle\langle t|$), an average orientation at each time t can be calcualted from the principal axes of

$$\begin{aligned}\left\langle L_{(a}L_{b)}\right\rangle_t &= \frac{\int d\Omega \psi_4{}^*(t) L_{(a}L_{b)}\psi_4(t)}{\int d\Omega |\psi_4|^2} \\ &= \frac{\sum_{lmm'} \psi_{4}{}^*_{lm'}\psi_{4lm}\left\langle lm'\left|L_{(a}L_{b)}\right|lm\right\rangle}{\int d\Omega|\psi_4|^2}\end{aligned} \tag{2a}$$

where in the second line we expand $\psi_4 = \sum_{lm}\psi_{4lm}(t)Y^{(-2)}_{lm}(\theta,\phi)$ and perform the angular integral. In this case, the average tensor $\left\langle L_{(a}L_{b)}\right\rangle_t$ at each time t can be calculated algebraically, from the mode amplitudes ψ_{4lm} and the well-known action of SU2 generators L_k on rotational eigenstates. The principal axes follow by diagonalizing this 3×3 matrix. The same construct can be applied to any kernel K. In particular, we define the average of L_aL_b at any frequency f or over all frequencies by

$$\left\langle L_{(a}L_{b)}\right\rangle_f = \frac{\int d\Omega \tilde{\psi_4}^*(f) L_{(a}L_{b)}\tilde{\psi_4}(f)}{\int d\Omega|\tilde{\psi_4}(f)|^2} \tag{2b}$$

$$\left\langle L_{(a}L_{b)}\right\rangle_M = \frac{\int d\Omega\int df \frac{\tilde{\psi_4}^*(f)L_{(a}L_{b)}\tilde{\psi_4}(f)}{(2\pi f)^4 S_h(f)}}{\int d\Omega\int df |\tilde{\psi_4}|^2/[(2\pi f)^4 S_h(f)]} \tag{2c}$$

where S_h is the strain noise power spectral density of a gravitational wave detector. The subscript M in the second expression denotes the binary mass, used to connect the scale-free output of a numerical relativity simulation to physical time t (i.e., $rM\psi_4(t)$ depends only on t/M). For context, in gravitational wave data analysis, the inner product $\langle a|b\rangle = 2\int a^*(f)b(f)/S_h(f)$ characterizes the natural optimal-filtering-induced metric for a noise power spectrum S_h; the denominator in the final expression thus corresponds to the inner product $\langle h_+|h_+\rangle + \langle h_\times|h_\times\rangle$ where the strain $\int\int \psi_4 dtdt' \propto h_+ - ih_\times$ is reconstructed from ψ_4 by integration.

The components, eigenspaces, and eigenvalues of this tensor characterize the emitted radiation, averaged over orientation and the specified quantity ξ $(= t, f, M, \ldots)$. As components of this matrix are associated with averages of mode orders weighted by $|\psi_4|^2$, such as

$$\left\langle L_z^2\right\rangle = \left\langle m^2\right\rangle \tag{3}$$

$$\mathrm{Tr}\left\langle L_aL_b\right\rangle = \left\langle l(l+1)\right\rangle \tag{4}$$

[1] Explicit coordinate forms for the generators L_k acting on functions with spin weight s are provided by [12], denoted as J_k. As the algebra of generators acting on the representation $Y^{(s)}_{lm}$ is unchanged, only the usual action of L_k on rotation eigenstates is needed to perform the calculations described in this paper.

the components of $\left\langle L_{(a}L_{b)}\right\rangle$ identify both the dominant emission direction and the dominant emission mode. In the special case of quasistationary emission along an $l = |m| = 2$ mode aligned with $\hat{z}$, the eigenspaces of $\left\langle L_{(a}L_{b)}\right\rangle_t$ are are $\hat{z}$ (eigenvalue $m^2 = 4$) and the $x - y$ plane (eigenvalue $\left\langle l(l+1) - m^2\right\rangle/2 = 1$). As quadrupolar emission dominates, these statements are a good approximation for all time-domain averages $\left\langle L_{(a}L_{b)}\right\rangle_t$.

Small errors in high-order modes can be amplified by the factor l^2, particularly during the merger phase where higher harmonics become significant both physically and through numerical error. However, since higher-order l subspaces can be invariantly removed from ψ_4, a similar average can also be constructed for any subset of l. To the extent that the dominant emission orientation is well-resolved, the directions derived from restricted averages should agree.

III. OPTIMAL ORIENTATION VERSUS TIME

The dominant principal axis of $\left\langle L_{(a}L_{b)}\right\rangle_t$ invariantly determines a preferred orientation $\hat{\mathcal{Z}}(t)$ versus time. Moreover, by design, any waveform that is well-approximated by quasistationary emission of a $l = |m| = 2$ mode along $\hat{Z}(t)$ must have $\hat{\mathcal{Z}}(t)$ correspond to that instantaneous emission symmetry direction. On the contrary, though a priori the eigensystem of the symmetric tensor $\left\langle L_{(a}L_{b)}\right\rangle$ provides a frame, empirically the directions associated with the two smaller eigenvalues cannot be trusted. During the inspiral the perpendicular directions are degenerate; small numerical asymmetries in each timestep cause the two smaller eigenvectors to jump chaotically in the plane perpendicular to the dominant eigenvector. Given the empirical degeneracy of the smaller two eigenvalues, we adopt a preferred frame at time t by a fiducial rotation: starting with a cartesian frame aligned with the initial simulation's coordinate frame, rotate by $R = R_z(\phi)R_y(\theta)$, which takes $\hat{z}$ to $\hat{Z}$ if θ, ϕ are the spherical polar coordinates for $\hat{Z}$. In terms of this rotation, the waveform in the instantaneous "aligned" frame at each time can be expressed in terms of the instantaneously "aligned" eigenstates $Y^{(-2)}_{LM}(R^{-1}n)$ and the simulation-frame expansion coefficients ψ_{4LM} using the representation theory of SU(2) [13]:

$$\psi_4(t, \hat{n}) = \sum_{LM} \psi_4{}'_{LM} Y^{(-2)}_{LM}(R^{-1}\hat{n}) \tag{5}$$

$$\psi_4{}'_{LM} = e^{2i\chi(R)} \sum_{M'} D^L_{MM'}(R)\psi_{4LM'} \tag{6}$$

The overall phase χ does not enter into any calculation we perform and will be ignored.

As a concrete example, in Figure 1 we compare the difference between a generic time-dependent rotation and the reconstructed orientation, obtained by applying that rotation to a nonprecessing black hole binary simulation and reconstructing the optimal direction versus time. The quasicircular orientation is recovered to $\delta\theta \simeq 10^{-6}$ deg (i.e., $\delta\theta^2$ comparable to working machine precision). As another example, in Figure 2, we compare the mode amplitudes for a spinning, precessing binary in an asymptotic inertial frame (top) and in the "corotating" frame implied by this transformation (bottom). The transformed modes resemble the modes of a spin-aligned binary (Fig 1, right panel). Note that in going from the generic-spin to "corotating" frame, the $m = 1$ and $m = 0$ modes have been reduced by more than an order of magnitude during the inspiral.

Schmidt et al. [8] have previously reconstructed optimal orientations from the emitted waveform. By contrast to their maximization-based method for tuning ψ_4, our algebraic method is fast, accurate, and invariant. This method can also be applied to any constant-l subspace. For example, using the GT/PSU equal-mass, generic spin simulation set summarized by O'Shaughnessy et al. [14] we have reconstructed $\left\langle L_{(a}L_{b)}\right\rangle$ for all $l \leq 4$ modes; for $l = 2$ only; and for $l = 4$ only.[2] All agree during the inspiral and at least some of the merger.[3]

Finally, as this method operates on each l subspace as well as independently and algebraically at each timestep, it provides valuable, fast, invariant diagnostics of the quality of numerical waveforms. For example, numerical simulations can evaluate their waveform quality by the following three constructs: (a) eigenvalues of $\left\langle L_{(a}L_{b)}\right\rangle_t$ should evolve smoothly and remain near their predicted values (i.e., near $4, 1, 1$ if the $l = 2$ subspace is included); (b) the recovered

[2] We have also tested the behavior of $\left\langle L_{(a}L_{b)}\right\rangle_t$ using only the $l = 3$ mode subspace. During inspiral, this tensor has a well-defined dominant principal axis that agrees with the $l = 2$ result. However, at and beyond merger, $\left\langle L_{(a}L_{b)}\right\rangle_t$ becomes nearly isotropic and does not provide a natural emission direction.

[3] A few tens of M after the merger, owing to their low and rapidly decaying amplitude, the $l = 4$ modes previously available are not resolved to the precision needed for directional reconstruction.

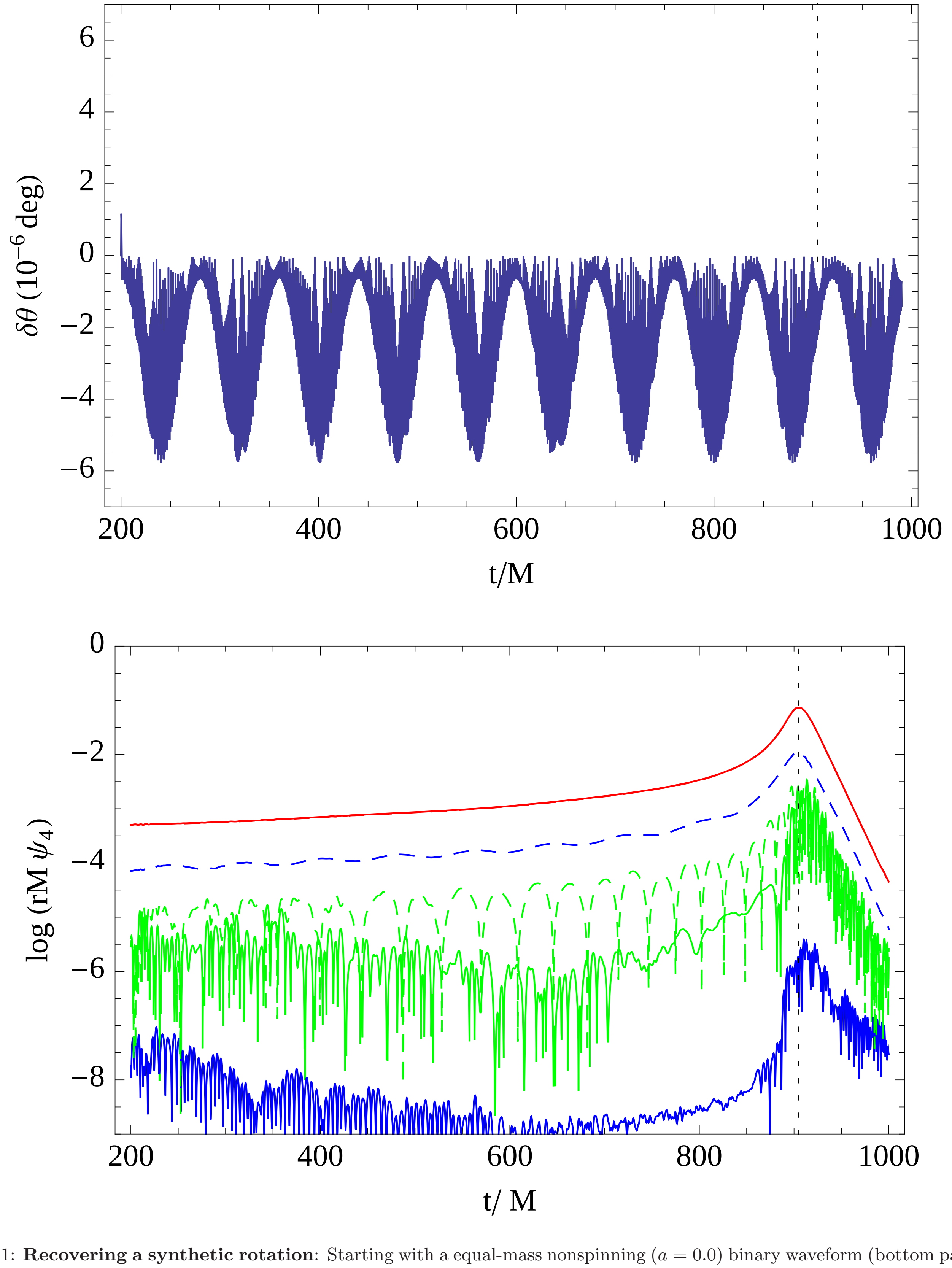

FIG. 1: **Recovering a synthetic rotation**: Starting with a equal-mass nonspinning ($a = 0.0$) binary waveform (bottom panel: ψ_{4lm} for $\ell \leq 2$, extracted at $r = 60M$), we apply a rotation taking $\hat{z}$ to the polar angles (θ, ϕ), where $\theta(t) = \theta_o(1+0.1\cos 2\pi t/P)$ and $\phi(t) = 2\pi t/P$ for $P = 80M$ and $\theta_o = \pi/20$. The bottom panel shows ψ_{4lm} after (dashed) and before (solid) this rotation, for the modes $l = 2$ and $m = 2$ (red), 1 (blue), and 0 (green). Using $\left\langle L_{(a}L_{b)}\right\rangle_t$, we recover an orientation $\hat{Z}(t)$ from the rotated waveform that agrees with the imposed rotation to within 5×10^{-6} deg (top panel: $\theta(Z(t)) - \theta(t)$). Applying this rotation recovers the unperturbed waveform. A vertical dotted line indicates the peak $l = |m| = 2$ emission in both figures.

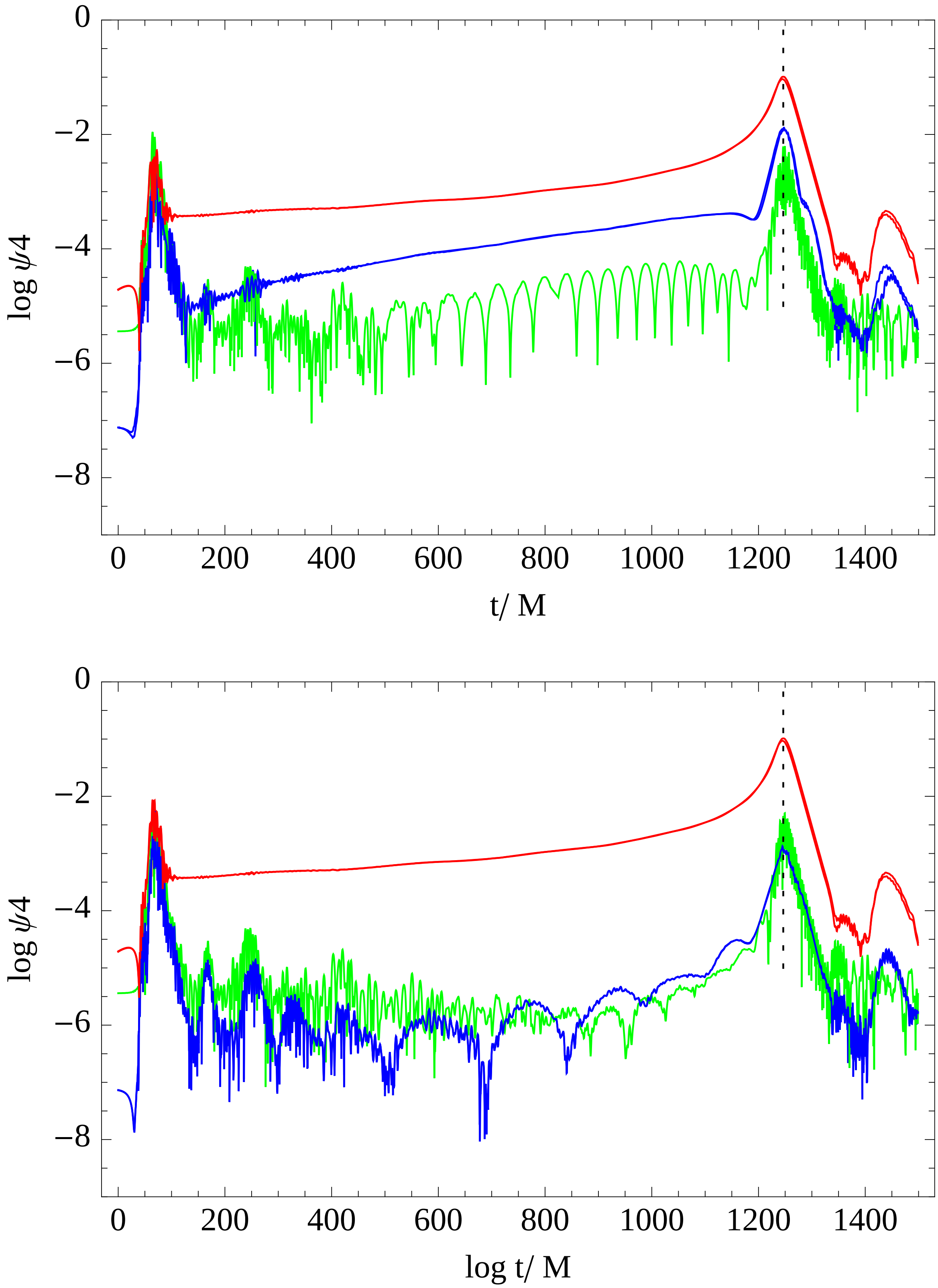

FIG. 2: **Aligning a precessing binary**: Starting with a precessing equal mass binary ($a_1 = 0.6\hat{z}, a_2 = 0.6(\hat{x} + \hat{z})/\sqrt{2}$; top panel), we derive the principal axis orientation from $\left\langle L_{(a}L_{b)}\right\rangle_t$, restricted to $l = 2$ modes. The bottom panel shows the transformed harmonics. The orientation transformation is derived and applied at all times shown. To illustrate this method's robustness, in this figure we retain initial transients and late-time errors.

dominant eigendirection of $\left\langle L_{(a}L_{b)}\right\rangle_t$ should be smooth in time; and (c) phenomenologically speaking, the recovered paths from different multipole orders should agree.

IV. OPTIMAL ORIENTATION VERSUS MASS

In many cases of astrophysical interest, the emission direction changes significantly during the merger. As an example, BH-NS binaries can have their dominant emission direction, the orbital angular momentum direction $\hat{L}$, precess one to several times in a cone around the total angular momentum [10]. In this case in particular and for high mass ratio binaries in general, the instantaneous emission direction averages out. Instead, the natural direction encoded by the signal is the total angular momentum. Generally, however, a suitable average is not intuitively self-evident.

A detector with comparable and coherent sensitivity to both gravitational wave polarizations is naturally characterized with a complex inner product that coherently accounts for both polarizations [15]. Specifically, two gravitational waveforms A, B expressed as curvature (ψ_4) along a particular line of sight $\hat{n}$ are naturally compared by a complex inner product

$$(A, B) \;\equiv\; \int_{-\infty}^{\infty} 2\frac{df}{(2\pi f)^4 S_h}\tilde{A}(f)^*\tilde{B}(f) \tag{7}$$

Taking derivatives with respect to angle and expanding out the generators of the rotation matrix, the Fisher matrix ($\Gamma_{ab} = |\,(\partial_a\psi_4, \partial_b\psi_4)\,|/|(\psi_4,\psi_4)|$) reduces to

$$\Gamma_{ab} = \frac{|(L_{(a}\psi_4, L_{b)}\psi_4)|}{|(\psi_4,\psi_4)|} \tag{8}$$

The tensor $\left\langle L_{(a}L_{b)}\right\rangle_M$ therefore corresponds to the orientation-averaged Fisher matrix.[4] In other words, $\left\langle L_{(a}L_{b)}\right\rangle_M$ characterizes how well we can determine a binary's orientation, averaged over all possible directions along which we could see it.

Moreover, for quasistationary precession dominated by a single mode at frequency $f(t)$, the signal-weighted average $\left\langle L_{(a}L_{b)}\right\rangle_M$ has a natural physical interpretation as a power-weighted average of $\left\langle L_{(a}L_{b)}\right\rangle_t$. Substituting the stationary phase approximation to the Fourier transform

$$\tilde{\psi_4}(f) \;=\; \frac{1}{\sqrt{idf/dt}}\psi_4(t(f))e^{-i2\pi f t(f)} \tag{9}$$

into the overlap $(L_a\psi_4, L_b\psi_4)$ leads to an amplitude-weighted integral over $\left\langle L_{(a}L_{b)}\right\rangle_t$

$$(L_a\psi_4, L_b\psi_4) \;\simeq\; \int \frac{2dt}{(2\pi f(t))^4 S_h(f(t))}(L_a\psi_4(t))^*(L_b\psi_4(t)) \tag{10}$$

The orientation average of the Fisher matrix is therefore

$$\int \Gamma_{ab} d\Omega/(4\pi) \;\simeq\; \frac{1}{|(\psi_4,\psi_4)|}\int 2dt\frac{\left\langle L_{(a}L_{b)}\right\rangle_t}{(2\pi f(t))^4 S_h(f(t))} \tag{11}$$

In other words, for sufficiently steady quasicircular precession, $\left\langle L_{(a}L_{b)}\right\rangle_M$ is precisely the power-weighted average of $\left\langle L_{(a}L_{b)}\right\rangle_t$. The principal eigenvector of $\left\langle L_{(a}L_{b)}\right\rangle_M$ therefore corresponds to our intuition about an "average frame".

V. CONCLUSIONS

In terms of orientation-averaged tensors, we have introduced an efficient algebraic method to extract fiducial orientations from waveforms extracted to infinity. Specifically, we have demonstrated that one natural orientation of

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[4] For brevity, we do not discuss maximization over coalescence time and phase; see [15] for a more extensive discussion of the Fisher matrix derived from a complex overlap.

the emitted beampattern is associated with the dominant principal axis of $\langle L_{(a}L_{b)}\rangle$. The average can be calculated at any time or averaged, at any mass. In the time domain, our method accurately, efficiently, and smoothly reconstructs the emission direction throughout the inspiral. As with [8], if we retroactively align the waveform with this optimal direction at each timestep, the dominant modes become smoother and the subdominant modes significantly smaller.

Conversely, if applied at each mass, our method also has close, provable connections to data analysis: it corresponds to the Fisher matrix for orientation angles, averaged over all possible lines of sight. In the case of extreme mass ratio, except for rare high-symmetry nonprecessing inclined orbits, our preferred direction reduces to the total angular momentum, about which the orbit precesses. This invariant approach to the preferred orientation will benefit search strategies for numerical relativity waveforms from strongly precessing binaries that coherently employ multiple modes, analagous to the proposal by Pan et al. [11] for strongly precessing BH-NS binaries.

We have proposed one of many possible definitions for the "instantaneous emission direction." Of course, other plausible generating tensors Q can be constructed with L_a alone, each corresponding to a different way of weighting the mode amplitudes ψ_{4lm}. Still more can be added by broadening the space of operators. That said, our definition offers a so-far unique feature: a provable connection to an astrophysically relevant quantity (the Fisher matrix).

Other astrophysically significant features of numerical relativity waveforms could be employed to determine preferred directions. For example, a "generalized equatorial plane" could consist of the set of directions along which gravitational radiation is locally linearly polarized (e.g., as the surface $\partial_t \ln \psi_4/\psi_4{}^* = 0$). Like the average $\langle L_{(a}L_{b)}\rangle$, these directions can be defined in the time, frequency, and mass domain. These emission directions lead to parameter estimation degeneracies: equal amount of left- and right-handed radiation imply two possible binary orientations are consistent with that signal. Though intriguing and invariant, these symmetry directions depend sensitively on delicate cancellations of all available modes. Less invariant but equally significant is a direct comparison ("overlap") of waveforms emitted in all possible directions, as would be seen by a gravitational wave detector. This method will be discussed in a subsequent paper [15], in the broader context of generic waveforms from spinning binaries.

Finally and as discussed previously by Schmidt et al. [8], both radiation and merger physics are more easily modeled in an "aligned" frame. For example, at present, hybrid waveforms have been constructed primarily for spin-aligned binaries for a handful of harmonics [16, 17] (cf. Sturani et al. [18]). By tabulating and modeling the "aligned"-frame waveforms and the corotating frame itself, hybrids can be constructed for generic precessing waveforms. As another example, previous studies of nonspinning and aligned-spin binaries suggest that their low-order modes evolve in phase with each other ($\arg \psi_{4lm} \propto m \arg \psi_{422}$) through inspiral and merger [19, 20]. By analogy, when expressed in a an "aligned" frame, low-order modes of generic precessing binaries also seem to evolve in phase. We will address these and other physical properties of "aligned"-frame waveforms in a subsequent publication.

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