Holographic scalar field models of dark energy

Ahmad Sheykhi

Holographic Scalar Fields Models of Dark Energy

Ahmad Sheykhi*

Physics Department and Biruni Observatory, Shiraz University, Shiraz 71454, Iran
Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), P. O. Box 55134-441, Maragha, Iran

Many theoretical attempts toward reconstructing the potential and dynamics of the scalar fields have been done in the literature by establishing a connection between holographic/agegraphic energy density and a scalar field model of dark energy. However, in most of these cases the analytical form of the potentials in terms of the scalar field have not been reconstructed due to the complexity of the equations involved. In this paper, by taking Hubble radius as system's IR cutoff, we are able to reconstruct the analytical form of the potentials as a function of scalar field, namely $V = V(\phi)$ as well as the dynamics of the scalar fields as a function of time, namely $\phi = \phi(t)$, by establishing the correspondence between holographic energy density and quintessence, tachyon, K-essence and dilaton energy density in a flat FRW universe. The reconstructed potentials are quite reasonable and have scaling solutions. Our study further supports the viability of the holographic dark energy model with Hubble radius as IR cutoff.

I.   INTRODUCTION

Holographic dark energy (HDE) models have got a lot of enthusiasm recently, because they link the dark energy density to the cosmic horizon, a global property of the universe, and have a close relationship to the spacetime foam [1, 2]. For a recent review on different HDE models and their consistency check with observational data see [3]. There are also a number of theoretical motivations leading to the form of HDE, among which some are motivated by holography and others from other principles of physics. A fairly comprehensive motivations on HDE models can be seen in [4]. It is worthwhile to mention that in the literature, various models of HDE have been investigated via considering different system's IR cutoff. In the presence of interaction between dark energy and dark matter, the simple choice for IR cutoff could be the Hubble radius, $L = H^{-1}$ which can simultaneously drive accelerated expansion and solve the coincidence problem [5, 6]. Besides, it was argued that for an accelerating universe inside the event horizon the generalized second law does not satisfy, while the accelerating universe enveloped by the Hubble horizon satisfies the generalized second law [7, 8]. This implies that the event horizon in an accelerating universe might not be a physical boundary from the thermodynamical point of view. Thus, it looks that Hubble horizon is a convenient horizon for which satisfies all of our accepted principles in a flat Friedmann-Robertson-Walker (FRW) universe.

There has been a lot of interest in recent years in establishing a connection between holographic/agegraphic energy density and scalar field models of dark energy (see e.g. [9–12]). These studies lead to reconstruct the potential and the dynamics of the scalar fields according to the evolution of the holographic/agegraphic energy density. Unfortunately, in all of these cases ([9–12]) the analytical form of the potentials have not been constructed as

a function of scalar field, namely $V = V(\phi)$, due to the complexity of the equations involved. Recently, by implement a connection between scalar field dark energy and HDE density and introducing a new IR cutoff, namely $L^{-2} = \alpha H^2 + \beta \dot{H}$, the authors of [13] reconstructed explicitly the potentials and the dynamics of the scalar fields, which describe accelerated expansion. Our work differs from [13] in that we assume the pressureless dark matter and HDE do not conserve separately but interact with each other, while the authors of [13] have neglected the contributions from dark matter and consequently no interaction between two dark components. Besides, our system's IR cutoff differs from that of Ref. [13].

In this paper, by choosing Hubble radius $L = H^{-1}$ as system's IR cutoff, we implement the connection between the HDE and scalar fields models including quintessence, tachyon, K-essence and dilaton energy density in a flat FRW universe. This simple and most natural choice for IR cutoff allows us to reconstruct the explicit form of potentials, $V = V(\phi)$, and also the dynamics of the scalar fields as a function of time, namely $\phi = \phi(t)$.

II.   HDE WITH HUBBLE RADIUS AS AN IR CUT-OFF

For the flat FRW universe, the first Friedmann equation is

$$H^2 = \frac{1}{3M_p^2} \left(\rho_m + \rho_D \right), \tag{1}$$

where ρ_m and ρ_D are the energy density of dark matter and dark energy, respectively. Taking the interaction between dark matter and dark energy into account, the continuity equation maybe written as [14, 15]

$$\dot{\rho}_m + 3H\rho_m = Q, \tag{2}$$

$$\dot{\rho}_D + 3H\rho_D(1 + w_D) = -Q. \tag{3}$$

where $w_D = p_D/\rho_D$ is the equation of state (EoS) parameter of HDE, and Q stands for the interaction term.

*sheykhi@uk.ac.ir

It should be noted that the ideal interaction term must be motivated from the theory of quantum gravity. In the absence of such a theory, we rely on pure dimensional basis for choosing an interaction Q. It is worth noting that the continuity equations imply that the interaction term should be a function of a quantity with units of inverse of time (a first and natural choice can be the Hubble factor H) multiplied with the energy density. Therefore, the interaction term could be in any of the following forms: (i) $Q \propto H\rho_D$, (ii) $Q \propto H\rho_m$, or (iii) $Q \propto H(\rho_m + \rho_D)$. We find out that for all three forms of Q, the EoS parameter of HDE has a similar form, thus hereafter we consider only the first case, namely $Q = 3b^2 H\rho_D$, where b^2 is a coupling constant.

We assume the HDE density has the form

$$\rho_D = 3c^2 M_p^2 H^2, \tag{4}$$

where c^2 is a constant and we have set the Hubble radius $L = H^{-1}$ as system's IR cutoff. Inserting Eq. (4) in Eq. (1) immediately yields

$$u = \frac{1 - c^2}{c^2}. \tag{5}$$

where $u = \rho_m/\rho_D$ is the energy density ratio. From Eq. (5) we see that the ratio of the energy densities is a constant; thus the coincidence problem can be alleviated. It is worth noting that in general the term c^2 in holographic energy density can vary with time though very slowly [16]. By slowly varying we mean that $(\dot{c^2})/c^2$ is upper bounded by the Hubble expansion rate, H, i.e., [16]

$$\frac{(\dot{c^2})}{c^2} \leq H. \tag{6}$$

Note that this condition must be fulfilled at all times; otherwise the dark energy density would not even approximately be proportional to L^{-2}, something at the core of holography [16]. It was argued that c^2 depends on the infrared length, L [16]. For the case of $L = H^{-1}$, it was shown that one can take c^2 approximately constant in the late time where dark energy dominates ($\Omega_m < 1/3$) [16]. Since in the present work we study the late time cosmology, and also for later convenience, we assume the term c^2 to be a constant. Taking the time derivative of Eq. (4), after using Friedmann equation (1), we get

$$\dot{\rho}_D = -3c^2 H\rho_D(1 + u + w_D). \tag{7}$$

Substituting Eq. (7) in (3), after using relations $Q = 3b^2 H\rho_D$ and (5), we obtain

$$w_D = -\frac{b^2}{1 - c^2}. \tag{8}$$

Therefore for constant parameters c and b the EoS parameter becomes also a constant. In the absence of interaction, $b^2 = 0$, we encounter dust with $w_D = 0$. For the choice $L = H^{-1}$ an interaction is the only way to have an EoS different from that for dust [5, 6]. Let us note that in order to have $w_D < 0$ we should have $c^2 < 1$. Besides, the acceleration expansion ($w_D < -1/3$) can be achieved provided $c^2 > 1 - 3b^2$. Thus this model can describe the accelerated expansion if $1 - 3b^2 < c^2 < 1$. Moreover, w_D can cross the phantom line ($w_D < -1$) provided $b^2 > 1 - c^2$.

III.   CORRESPONDENCE WITH SCALAR FIELD MODELS

In this section we implement a correspondence between interacting HDE and various scalar field models, by equating the equations of state for this models with the equations of state parameter of interacting HDE obtained in (8).

A.   Reconstructing holographic quintessence model

In order to establish the correspondence between HDE and quintessence scaler field, we assume the quintessence scalar field model of dark energy is the effective underlying theory. The energy density and pressure of the quintessence scalar field are given by [17]

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{9}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{10}$$

Thus the potential and the kinetic energy term can be written as

$$V(\phi) = \frac{1 - w_\phi}{2}\rho_\phi, \tag{11}$$

$$\dot{\phi}^2 = (1 + w_\phi)\rho_\phi. \tag{12}$$

where $w_\phi = p_\phi/\rho_\phi$. In order to implement the correspondence between HDE and quintessence scaler field, we identify $\rho_\phi = \rho_D$ and $w_\phi = w_D$. Inserting Eqs. (4) and (8) in (12) we reach

$$\dot{\phi} = \sqrt{3\left(1 - \frac{b^2}{1 - c^2}\right)}cM_p\frac{\dot{a}}{a}. \tag{13}$$

Integrating yields

$$\phi(a) = \sqrt{3\left(1 - \frac{b^2}{1 - c^2}\right)}cM_p\ln a, \tag{14}$$

where we have set $\phi(a_0 = 1) = 0$ for simplicity. Next we want to obtain the scale factor as a function of t. Taking the time derivative of Eq. (1) and using (8) we find

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}\left[1 - \frac{b^2c^2}{1 - c^2}\right] \tag{15}$$

The fist integration gives

$$H = \frac{da}{adt} = \frac{2}{3\mathrm{k}t},$$ (16)

where $\mathrm{k} = 1 - \frac{b^2 c^2}{1-c^2}$. Integrating again we find

$$a(t) = t^{2/3\mathrm{k}}$$ (17)

Hence Eq. (14) can be rewritten

$$\phi(t) = \frac{2}{3\mathrm{k}} c M_p \sqrt{3\left(1 - \frac{b^2}{1-c^2}\right)} \ln t.$$ (18)

Next we obtain the potential as a function of ϕ. Combining Eq. (8) with Eq. (11) we reach

$$V(\phi) = \frac{3}{2} c^2 M_p^2 \left[1 + \frac{b^2}{1-c^2}\right] H^2.$$ (19)

Using Eqs. (16) and (18) we obtain the explicit expression for potential, namely

$$\begin{aligned} V(\phi) &= \frac{2c^2 M_p^2}{3\mathrm{k}^2} \left[1 + \frac{b^2}{1-c^2}\right] \times \\ &\quad \exp\left[-3\frac{\mathrm{k}}{cM_p}\left(3 - \frac{3b^2}{1-c^2}\right)^{-1/2}\phi\right]. \end{aligned}$$ (20)

Let us discuss the condition for which the scale factor (17), and hence the obtained potential, leads to an accelerated universe at the present time. Requiring $\ddot{a} > 0$ for the present time, leads to $\mathrm{k} < 2/3$, which can be translated into $c^2 > (1 + 3b^2)^{-1}$. Note that the condition $\mathrm{k} < 2/3$ valid only for the late time where we have a dark energy dominated universe. In general k depends on c, and for the matter dominated epoch where c is no longer a constant, then k is also not a constant and varies with time. The obtained exponential potential here is well-known in the literature for the quintessence scalar field [17]. The cosmological dynamics of this potential has been explored in detail [17]. In addition to the fact that exponential potentials can give rise to an accelerated expansion, they possess cosmological scaling solutions [17–19] in which the field energy density ρ_ϕ is proportional to the matter energy density ρ_m.

B. Reconstructing holographic tachyon model

The tachyon field has been proposed as a possible candidate for dark energy. A rolling tachyon has an interesting EoS whose parameter smoothly interpolates between -1 and 0 [20]. Thus, tachyon can be realized as a suitable candidate for the inflation at high energy [21] as well as a source of dark energy depending on the form of the tachyon potential [22]. Choosing different self-interacting potentials in the tachyon field model lead to different consequences for the resulting DE model. These give enough motivations us to reconstruct tachyon potential $V(\phi)$ from HDE model with Hubble radius as the IR cutoff. The correspondence between tachyon field and various dark energy models such as HDE [10] and agegraphic dark energy [11] has been already established. The extension has also been done to the entropy corrected holographic and agegraphic dark energy models [23]. However, in all of these cases [10, 11, 23] the explicit form of the tachyon potential, $V = V(\phi)$, has not been reconstructed due to the complexity of the equations involved.

The effective lagrangian for the tachyon field is given by [24]

$$L = -V(\phi)\sqrt{1 - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi},$$ (21)

where $V(\phi)$ is the tachyon potential. The corresponding energy momentum tensor for the tachyon field can be written in a perfect fluid form

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu},$$ (22)

where ρ and p are, respectively, the energy density and pressure of the tachyon and the velocity u_μ is

$$u_\mu = \frac{\partial_\mu\phi}{\sqrt{\partial_\nu\phi\partial^\nu\phi}}.$$ (23)

The energy density and pressure of tachyon field are given by

$$\rho = -T_0^0 = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}},$$ (24)

$$p = T_i^i = -V(\phi)\sqrt{1 - \dot{\phi}^2}.$$ (25)

Thus the EoS parameter of tachyon field is given by

$$w_T = \frac{p}{\rho} = \dot{\phi}^2 - 1.$$ (26)

To establish the correspondence between HDE and tachyon field, we equate w_D with w_T. From Eqs. (8) and (26) we find

$$\dot{\phi}^2 = 1 - \frac{b^2}{1-c^2}.$$ (27)

Integrating gives

$$\phi(t) = \left[1 - \frac{b^2}{1-c^2}\right]^{1/2} t,$$ (28)

where we set an integration constant to zero. Combining Eq. (24) with (27), the tachyon potential is obtained as

$$V(\phi) = 3c^2 M_p^2 H^2 \frac{b}{\sqrt{1-c^2}},$$ (29)

Using Eqs. (16) and (28) we obtain tachyon potential in terms of the scalar field

$$V(\phi) = \frac{4c^2 M_p^2}{3\text{k}^2} \frac{b}{\sqrt{1-c^2}} \left(1 - \frac{b^2}{1-c^2}\right) \frac{1}{\phi^2}, \quad (30)$$

From Eq. (28) we see that the evolution of the tachyon is given by $\phi(t) \propto t$. The above inverse square power-law potential corresponds to the one in the case of scaling solutions [17, 25, 26].

C. Reconstructing holographic K-essence model

The scalar field model called K-essence is also employed to explain the observed acceleration of the cosmic expansion. This model is characterized by a scalar field with a non-canonical kinetic energy. The most general scalar-field action which is a function of ϕ and $X = -\dot{\phi}^2/2$ is given by [27]

$$S = \int d^4 x \sqrt{-g} P(\phi, X), \quad (31)$$

where the lagrangian density $P(\phi, X)$ corresponds to a pressure density. According to this lagrangian the energy density and the pressure can be written as [17, 27]

$$\rho(\phi, X) = f(\phi)(-X + 3X^2), \quad (32)$$
$$p(\phi, X) = f(\phi)(-X + X^2). \quad (33)$$

Therefore the EoS parameter of the K-essence is given by

$$w_K = \frac{X-1}{3X-1}. \quad (34)$$

Equating w_K with the EoS parameter of HDE (8) one finds

$$X = \frac{1+b^2-c^2}{1+3b^2-c^2}. \quad (35)$$

which implies that X is a positive constant ($c^2 < 1$). Indeed the EoS parameter in Eq. (34) diverges for $X = 1/3$. Let us consider the cases with $X > 1/3$ and $X < 1/3$ separately. In the first case where $X > 1/3$, the condition $w_K < -1/3$ leads to $X < 2/3$. Thus we should have $1/3 < X < 2/3$ in this case. For example we obtain the EoS of a cosmological constant ($w_K = -1$) for $X = 1/2$. In the second case where $X < 1/3$, we have $X - 1 < -2/3 < 0$, thus $w_K = \frac{X-1}{3X-1} > 0$. This means that we have no acceleration at all. So this case is ruled out. As a result in K-essence model the accelerated universe can be achieved provided $1/3 < X < 2/3$, which translates into $1 - 3b^2 < c^2 < 1$. This is consistent with our previous discussions. Combining equation (35) with $X = -\dot{\phi}^2/2$, one gets

$$\dot{\phi}^2 = \frac{2(1+b^2-c^2)}{1+3b^2-c^2}, \quad (36)$$

and thus we obtain the expression for the scalar field in the flat FRW background

$$\phi(t) = \left[\frac{2(1+b^2-c^2)}{1+3b^2-c^2}\right]^{1/2} t, \quad (37)$$

where we have taken the integration constant ϕ_0 equal to zero. Taking the correspondence between HDE and K-essence into account, namely $\rho_D = \rho(\phi, X)$, after using Eqs. (16) and (37) we find

$$f(\phi) = \frac{4c^2 M_p^2}{3\text{k}^2} \left[\frac{1+3b^2-c^2}{1-c^2}\right] \frac{1}{\phi^2}. \quad (38)$$

Thus the K-essence potential $f(\phi)$ has a power law expansion. From Eq. (37) we see that $\dot{\phi} = \text{const}$. This means that the kinitic energy of K-essence becomes constant, though ϕ is not constant and evolves with time.

D. Reconstructing holographic dilaton field

The dilaton field may be used for explanation the dark energy puzzle and avoids some quantum instabilities with respect to the phantom field models of dark energy [28]. The lagrangian density of the dilatonic dark energy corresponds to the pressure density of the scalar field has the following form [29]

$$p = -X + \alpha e^{\lambda\phi} X^2, \quad (39)$$

where α and λ are positive constants and $X = \dot{\phi}^2/2$. Such a pressure (Lagrangian) leads to the following energy density [29]

$$\rho = -X + 3\alpha e^{\lambda\phi} X^2. \quad (40)$$

The EoS parameter of the dilaton dark energy can be written as

$$w_d = \frac{1 - \alpha e^{\lambda\phi} X}{1 - 3\alpha e^{\lambda\phi} X}. \quad (41)$$

To establish the correspondence between HDE and dilaton field we equate their EoS parameter, i.e. $w_d = w_D$. We reach

$$\frac{1 - \alpha e^{\lambda\phi} X}{1 - 3\alpha e^{\lambda\phi} X} = -\frac{b^2}{1-c^2}. \quad (42)$$

Using relation $X = \dot{\phi}^2/2$, and integrating with respect to t we find

$$\phi = \frac{2}{\lambda} \ln\left[\frac{\lambda}{\sqrt{2\alpha}} \left(\frac{1+b^2-c^2}{1+3b^2-c^2}\right)^{1/2} t\right] \quad (43)$$

The existence of scaling solutions for the dilaton was studied in [29] and was found that in this case the scaling solution corresponds to $Xe^{\lambda\phi} = \text{const.}$, which has the solution $\phi(t) \propto \ln t$. The results we found here by equating the EoS parameter of HDE and dilaton field are consistent with those obtained in [29].

IV. CONCLUSION AND DISCUSSION

In this paper by choosing the Hubble radius as system's IR cutoff for interacting HDE model, we established a connection between the scalar field model of dark energy including quintessence, tachyon, K-essence and dilaton energy density and holographic energy density. As a result, we reconstructed the analytical form of potentials namely $V = V(\phi)$ as well as the dynamics of the scalar fields as a function of time explicitly, namely $\phi = \phi(t)$ according to the evolutionary behavior of the interacting HDE model. The obtained expressions for the potentials are quite reasonable and lead to scaling solutions. Our studies favor the $L = H^{-1}$ IR cutoff as a viable phenomenological model of HDE.

Finally, I would like to mention that usually, for the sake of simplicity, the term c^2 in holographic energy density (4) is assumed constant. However, one should bear in mind that it is more general to consider it a slowly varying function of time, $c^2(t)$ [16]. In this case the EoS parameter given in (8) is no longer a constant. As a result we cannot integrate easily the resulting equations in section III and find the analytical form of the potentials. It is important to note that, although, with implement the correspondence between this HDE model and scalar field models, the EoS of scalar fields are assumed to be fixed, nevertheless, neither ϕ nor $V(\phi)$ are not constant and they still evolve with time. In the present work for simplicity we have taken $c = \text{const}$. The correspondence between HDE and scalar field models with varying c^2 term is under investigation and will be addressed elsewhere.

[1] Ng YJ 2001 Phys. Rev. Lett. 86 2946;
Arzano M, Kephart TW and Ng YJ 2007 Phys. Lett. B 649 243.
[2] I. Duran and Diego Pavon, arXiv:1012.4988
[3] S. del Campo, J. C. Fabris, R. Herrera, W. Zimdahl, arXiv:1103.3441.
[4] M. Li, X.D. Li, S. Wang, Y. Wang, arXiv:1103.5870.
[5] D. Pavon, W. Zimdahl, Phys. Lett. B 628 (2005) 206.
[6] W. Zimdahl and D. Pavon, Class. Quantum Grav. 24 (2007) 5461.
[7] J. Zhou, B. Wang, Y. Gong, E. Abdalla, Phys. Lett. B 652 (2007) 86.
[8] A. Sheykhi, Class. Quantum Grav. 27 (2010) 025007.
[9] X. Zhang, Phys. Lett. B 648 (2007)1.
[10] K. Karami, J. Fehri, Phys. Lett. B 684 (2010) 61.
[11] J. Cui, L. Zhang, J. Zhang, and X. Zhang, Chin. Phys. B 19 (2010) 019802;
A. Sheykhi, Phys. Lett. B 682 (2010) 329.
[12] A. Sheykhi, A. Bagheri, M.M. Yazdanpanah, JCAP 09 (2010) 017;
J. P. Wu, D. Z. Ma, Y. Ling, Phys. Lett. B 663 (2008) 152 ;
J. Zhang, X. Zhang, H. Liu, Eur. Phys. J. C 54 (2008) 303.
[13] L.N. Granda, A. Oliveros, Physics Letters B 671 (2009) 199.
[14] B. Wang, Y. Gong and E. Abdalla, Phys. Lett. B 624 (2005) 141;
B. Wang, C. Y. Lin and E. Abdalla, Phys. Lett. B 637 (2005) 357.
[15] N. Banerjee, D. Pavon, Phys. Lett. B **647** (2007) 477;
I. Duran, D. Pavon, Phys. Rev. D 83 (2011) 023504.
[16] N. Radicella and D. Pavon, JCAP 10 (2010) 005.
[17] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006).
[18] T. Barreiro, E. J. Copeland and N. J. Nunes, Phys. Rev. D 61, 127301 (2000).
[19] V. Sahni, H. Feldman and A. Stebbins, Astrophys. J. 385, 1 (1992)
[20] G. W. Gibbons, Phys. Lett. B 537 (2002) 1.
[21] A. Mazumdar, S. Panda and A. Perez-Lorenzana, Nucl. Phys. B 614, 101 (2001);
A. Feinstein, Phys. Rev. D 66, 063511 (2002);
Y. S. Piao, R. G. Cai, X. M. Zhang and Y. Z. Zhang, Phys. Rev. D 66, 121301 (2002).
[22] T. Padmanabhan, Phys. Rev. D 66, 021301 (2002);
J.S. Bagla, H.K.Jassal, T. Padmanabhan, Phys. Rev. D 67 (2003) 063504;
E. J. Copeland, M. R. Garousi, M. Sami and S. Tsujikawa, Phys. Rev. D 71, 043003 (2005).
[23] M. Jamil and A. Sheykhi, Int. J. Theor. Phys. 50 (2011) 625;
M. Umar Farooq, Muneer A. Rashid, Mubasher Jamil, Int. J. Theor. Phys. 49 (2010) 2278-2287;
E. Ebrahimi and A. Sheykhi, arXiv:1011.5005;
Kayoomars Karami, M. S. Khaledian, Mubasher Jamil, Phys. Scr.83, 2011, 025901.
[24] E.A. Bergshoeff, M. de Roo, T.C. de Wit, E. Eyras, S. Panda, JHEP 0005 (2000) 009;
A. Sen, JHEP 0204 (2002) 048;
A. Sen, Int. J. Mod. Phys. A 20 (2005) 5513.
[25] J. M. Aguirregabiria and R. Lazkoz, Phys. Rev. D 69, 123502 (2004).
[26] E. J. Copeland, M. R. Garousi, M. Sami and S. Tsujikawa, Phys. Rev. D 71, 043003 (2005).
[27] C. Armendariz-Picon, T. Damour, and V. F. Mukhanov, k-Inflation, Phys. Lett. B 458 (1999), 209;
C. Armendariz-Picon, V.F. Mukhanov, P.J. Steinhardt, Phys. Rev. D 63 (2001) 103510.
[28] S. M. Carroll, M. Hoffman and M. Trodden, Phys. Rev. D 68, 023509 (2003).
[29] F. Piazza and S. Tsujikawa, JCAP 0407, 004 (2004).