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Natural Four-Generation Mass Textures in MSSM Brane Worlds

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Abstract

A fourth generation of Standard Model (SM) fermions is usually considered unlikely due to constraints from direct searches, electroweak precision measurements, and perturbative unitarity. We show that fermion mass textures consistent with all constraints may be obtained naturally in a model with four generations constructed from intersecting D6 branes on a $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. The Yukawa matrices of the model are rank 2, so that only the third- and fourth-generation fermions obtain masses at the trilinear level. The first two generations obtain masses via higher-order couplings and are therefore naturally lighter. In addition, we find that the third and fourth generation automatically split in mass, but do not mix at leading order. Furthermore, the SM gauge couplings automatically unify at the string scale, and all the hidden-sector gauge groups become confining in the range 10^{13}–10^{16} GeV, so that the model becomes effectively a four-generation MSSM at low energies.

I. INTRODUCTION

Current high-energy experimental data supports just three generations of chiral fermions. However, the existence of a fourth generation remains viable provided the mass of the fourth-generation neutrino ν' satisfies $m_{\nu'} > \frac{1}{2}M_Z$, and the corresponding charged fermion masses $m_{t'}$, $m_{b'}$, and $m_{\tau'}$ lie in the correct mass ranges to avoid constraints from direct searches and precision electroweak measurements [1]. Experimentally, various anomalies indicate possible manifestations of a fourth generation. Recent analysis from WMAP7 [2] suggests a number of relativistic neutrino flavors that is greater than three, $N_{\rm eff} = 4.34^{+0.86}_{-0.88}$. Explanations have been proposed based on sterile neutrinos with masses at sub-eV scales [3] (see also Refs. [4] for earlier discussion of astrophysical implications of the fourth generation). In addition, recent observations from MiniBooNE [5] and the observation of a reactor antineutrino anomaly [6] may indicate the existence of one or more light, sterile neutrinos. Furthermore, the existence of a fourth generation generically introduces additional sources of CP-violating effects [7–9], which are important for baryogenesis and could explain observed anomalies in flavor physics [10].

Although the existence of a fourth generation is still viable, it has long been considered unlikely. In part, this predisposition relies upon the perceived unnaturalness of the allowed fourth-generation fermion masses, which are strongly constrained by the limits provided by direct searches and precision electroweak measurements. In particular, these constraints mandate a large ν' mass in comparison to the tiny masses of the observed three neutrinos. Moreover, the fourth-generation quark masses are constrained by the perturbative unitarity of heavy-fermion scattering amplitudes [11] to be $\lesssim$ 500–600 GeV. Thus, the t' quark mass $m_{t'}$ can only lie between $\sim$ 2–3 times the m_t, a small difference when compared to the mass hierarchy $m_t/m_c \approx 135$. Indeed, for the known families of quarks and leptons, progressively larger mass splittings between each sequential family appears to be the rule, a pattern that cannot hold in four-generation models due to the limited allowed range for $m_{t'}$. Furthermore, contributions to the oblique S and T parameters from an additional chiral fermion generation are intolerably large unless the masses of the t' and b' quarks are equal to within $m_{t'} - m_{b'} \simeq (1 + \frac{1}{5}\ln\frac{m_h}{115}) \times 50$ GeV, where m_h is the Higgs mass [12]. Moreover, mixing between the third- and fourth-generation quarks is often presumed small to accommodate the unitarity of the CKM matrix. These features of four-family models appear to require significant fine tuning to explain, and therefore a fourth generation of chiral fermions is usually deemed improbable.

In fact, the pioneering work of Ref. [12] showed that fourth-generation fermions are phenomenologically perfectly viable, apart from mild tuning of the masses within the fermion $\mathrm{SU}(2)_L$ doublets.

The direct search limits quoted therein are $m_{\nu'}, m_{\tau'} > 100$ GeV and $m_{t'}, m_{b'} > 258$ GeV, and the electroweak precision-based constraints are given by $m_{\tau'} - m_{\nu'} \approx 30$–$60$ GeV and the $m_{t'} - m_{b'}$ splitting mentioned above, while the most stringent quark-mixing bounds are actually those with the first generation, $|V_{ub'}|$, $|V_{t'd}| \lesssim 0.04$. Subsequent bounds from both observation and more detailed calculations (*e.g.*, [13, 14]) modify these values slightly, but the possibility of a fourth chiral fermion generation remains vibrant and awaits the decisive verdict of the Large Hadron Collider.

Currently, string theory is the most promising approach for a unification of gravity with quantum mechanics. As such, it should provide a first-principles explanation of the detailed properties of our universe. However, as is well known, string theory exhibits a huge degeneracy of vacua, resulting in the so-called string landscape. For many string compactifications, replication of chiral matter seems to be a generic feature. At present, the principle that sets the precise number of observed chiral generations remains unknown. Indeed, many consistent string vacua can be constructed that closely resemble the Minimal Supersymmetric Standard Model (MSSM) but contain different numbers of generations [15].

One particularly promising class of string compactifications employs D branes on orientifolds (for reviews, see [16–19]). In such models chiral fermions—an intrinsic feature of the Standard Model (SM)—arise from configurations with D branes located at transversal orbifold/conifold singularities [20] and strings stretching between D branes intersecting at angles [21, 22] (or, in its T-dual picture, with magnetized D branes [23–25]). Within the framework of D-brane model building on toroidal orientifolds, the worldsheet areas A_{abc} spanning the region bounded by D branes (labeled by a, b, c) that support fermions and Higgses at their intersections give rise to Yukawa couplings $Y_{abc} \sim \exp(-A_{abc})$ [22, 26]. This pattern naturally encodes the hierarchy of Yukawa couplings. Indeed, models exist in the literature in which realistic mass matrices and CKM mixings for both three- and four- generation models may be obtained [15]. While accommodating full rank-3 or rank-4 Yukawa matrices is possible, one may nevertheless question whether the degree of fine-tuning of parameters required is too large to provide a completely natural explanation of the observed fermion mass and mixing hierarchy.

In this paper we construct a four-generation MSSM with intersecting D6 branes on a $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. We show that the Yukawa matrices are rank 2, and thus only the heavier two generations receive masses at the trilinear level, while the lighter two generations can receive masses from higher-order couplings.[1] Moreover, the third- and fourth-generation fermions

[1] In some cases, use of the term "naturally lighter" must be carefully considered: Note, for example, the second- to

automatically split in mass, but we find that they do not mix at the trilinear level and therefore do not generate $O(1)$ mixing angles. Thus, one can obtain fermion mass textures naturally consistent with fourth-generation constraints. Finally, the tree-level gauge couplings automatically unify at the string scale, and the hidden-sector gauge groups introduced to satisfy tadpole constraints become confining in the range 10^{13}–10^{16} GeV, leaving only the MSSM at low energies.

II. A FOUR-FAMILY MSSM

A. Model Building on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

Let us begin by considering Type IIA string theory compactified on a $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold [27, 28], where the six-torus is factorizable in terms of two-tori as $T^6 = T^2 \times T^2 \times T^2$, and the i^{th} two-torus, $i = 1, 2, 3$, has complex coordinates z_i. The θ and ω generators for the orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$ act on the complex coordinates of T^6 as

$$\begin{aligned} \theta : \ & (z_1, z_2, z_3) \to (-z_1, -z_2, z_3) \ , \\ \omega : \ & (z_1, z_2, z_3) \to (z_1, -z_2, -z_3) \ . \end{aligned} \tag{1}$$

The orientifold projection is implemented by gauging the symmetry ΩR, where Ω is worldsheet parity, and R is given by conjugation:

$$R : (z_1, z_2, z_3) \to (\overline{z}_1, \overline{z}_2, \overline{z}_3) \ . \tag{2}$$

As a result, one finds four kinds of orientifold 6-planes (O6 planes), corresponding to the actions ΩR, $\Omega R\theta$, $\Omega R\omega$, and $\Omega R\theta\omega$. Only two kinds of complex structures are consistent with orientifold projection for a two-torus: rectangular and tilted [27, 28], with $\beta_i = 0, 1$ for an untilted or tilted i^{th} torus, respectively. In the following, we only consider rectangular two-tori (but retain parameters allowing for tilt to permit the generalization of the model presented here). The homology three-cycles for a stack a of D6 branes and its orientifold image a' are given by

$$[\Pi_a] = \prod_{i=1}^{3} (n_a^i [a_i] + 2^{-\beta_i} l_a^i [b_i]), \quad [\Pi_{a'}] = \prod_{i=1}^{3} (n_a^i [a_i] - 2^{-\beta_i} l_a^i [b_i]) \, . \tag{3}$$

The Ramond-Ramond (R-R) tadpole cancellation requires the total homology-cycle charge of D6 branes and O6 planes to vanish [24], which may be as expressed as

$$\sum_a N_a [\Pi_a] + \sum_a N_a [\Pi_{a'}] - 4[\Pi_{O6}] = 0 \, . \tag{4}$$

third-generation mass ratio $m_c/m_\tau \approx 0.7$. However, a strong hierarchy remains when one compares only fermions of the same type, *e.g.*, $m_\mu \ll m_\tau$.

It is convenient to use the parameters

$$\begin{aligned}&\tilde{A}_a=-l_a^1l_a^2l_a^3\,, \quad \tilde{B}_a=l_a^1n_a^2n_a^3\,,\ \tilde{C}_a=n_a^1l_a^2n_a^3\,,\ \tilde{D}_a=n_a^1n_a^2l_a^3\,,\\ &A_a=-n_a^1n_a^2n_a^3\,,\ B_a=n_a^1l_a^2l_a^3\,,\ \ C_a=l_a^1n_a^2l_a^3\,,\ \ D_a=l_a^1l_a^2l_a^3\,,\end{aligned} \tag{5}$$

which were first introduced in [29]. The tadpole conditions can then be expressed as

$$\sum_a N_aA_a=\sum_a N_aB_a=\sum_a N_aC_a=\sum_a N_aD_a=-16\,. \tag{6}$$

Completely eliminating the R-R tadpoles actually requires the cancellation of D-brane charges as classified by K theory, which for tori of arbitrary tilt for the $\mathbb{Z}_2\times\mathbb{Z}_2$ orientifold requires [17, 30, 31]

$$\sum_a N_a\tilde{A}_a\in 4\mathbb{Z}\,,\ \ \sum_a N_a\tilde{B}_a\in 4\mathbb{Z}\,,\ \ \sum_a N_a\tilde{C}_a\in 4\mathbb{Z}\,,\ \ \sum_a N_a\tilde{D}_a\in 4\mathbb{Z}\,. \tag{7}$$

In order to preserve $\mathcal{N}=1$ supersymmetry in four dimensions, the rotation angle of any D brane with respect to the orientifold plane must be an element of SU(3) [21, 28]. Consider the angles θ_a^i between each brane a and the R-invariant axis of the $i^{\rm th}$ torus; one requires that $\theta_a^1+\theta_a^2+\theta_a^3=0$ mod 2π, which means $\sin(\theta_a^1+\theta_a^2+\theta_a^3)=0$ and $\cos(\theta_a^1+\theta_a^2+\theta_a^3)>0$. Define

$$\tan\theta_a^i=2^{-\beta_i}\frac{l_a^iR_2^i}{n_a^iR_1^i}=\frac{2^{-\beta_i}l_a^i}{n_a^i}\chi^i\,, \tag{8}$$

where $R_{1,2}^i$ are the radii of the $i^{\rm th}$ torus and $\chi_i=R_2^i/R_1^i$ are the complex-structure moduli. The supersymmetry conditions can then be recast as [29]

$$\begin{aligned}x_A\tilde{A}_a+x_B\tilde{B}_a+x_C\tilde{C}_a+x_D\tilde{D}_a=0\,,\\ A_a/x_A+B_a/x_B+C_a/x_C+D_a/x_D<0\,,\end{aligned} \tag{9}$$

where x_A, x_B, x_C, x_D are complex-structure parameters, which are given by

$$x_A=\lambda,\ \ x_B=\lambda\cdot 2^{\beta_2+\beta_3}/\chi_2\chi_3,\ \ x_C=\lambda\cdot 2^{\beta_1+\beta_3}/\chi_1\chi_3,\ \ x_D=\lambda\cdot 2^{\beta_1+\beta_2}/\chi_1\chi_2\,, \tag{10}$$

where λ is a positive parameter introduced [29] to put all the variables A,B,C,D on an equal footing. However, among the x_i, only three are independent.

The initial U(N_a) gauge group supported by a stack of N_a identical D6 branes is broken down by the $\mathbb{Z}_2\times\mathbb{Z}_2$ symmetry to a subgroup U($N_a/2$) [28]. Chiral matter fields are formed from open strings whose two ends attach to different stacks. By using the Grassmann algebra $[a_i][b_j]=-[b_j][a_i]=\delta_{ij}$ and $[a_i][a_j]=-[b_j][b_i]=0$, one can calculate the intersection numbers between stacks a and b and obtain the multiplicity ($\mathcal{M}$) of the corresponding bifundamental representation:

$$\mathcal{M}\left(\frac{N_a}{2},\frac{\overline{N_b}}{2}\right)=I_{ab}=[\Pi_a][\Pi_b]=2^{-k}\prod_{i=1}^{3}(n_a^il_b^i-n_b^il_a^i)\,, \tag{11}$$

where $k \equiv \beta_1 + \beta_2 + \beta_3$ is the total number of tilted tori. Likewise, stack a paired with the orientifold image b' of b yields

$$\mathcal{M}\left(\frac{N_a}{2}, \frac{N_b}{2}\right) = I_{ab'} = [\Pi_a][\Pi_{b'}] = -2^{-k}\prod_{i=1}^{3}(n_a^i l_b^i + n_b^i l_a^i)\,. \tag{12}$$

Strings between a brane in stack a and its orientifold image a' yield chiral matter in the antisymmetric a_A and symmetric a_S representations of the group $\mathrm{U}(N_a/2)$, with multiplicities

$$\mathcal{M}[(a_{A,a})_L] = \frac{1}{2}I_{aO6}, \quad \mathcal{M}[(a_{A,a} + a_{S,a})_L] = \frac{1}{2}\left(I_{aa'} - \frac{1}{2}I_{aO6}\right)\,, \tag{13}$$

so that the net numbers of antisymmetric and symmetric representations are given by

$$\begin{aligned}
\mathcal{M}(a_{A,\,a}) &= \frac{1}{2}\left(I_{aa'} + \frac{1}{2}I_{aO6}\right) = -2^{1-k}[(2A_a - 1)\tilde{A}_a - \tilde{B}_a - \tilde{C}_a - \tilde{D}_a]\,, \\
\mathcal{M}(a_{S,\,a}) &= \frac{1}{2}\left(I_{aa'} - \frac{1}{2}I_{aO6}\right) = -2^{1-k}[(2A_a + 1)\tilde{A}_a + \tilde{B}_a + \tilde{C}_a + \tilde{D}_a]\,,
\end{aligned} \tag{14}$$

where

$$I_{aa'} = [\Pi_a][\Pi_{a'}] = -2^{3-k}\prod_{i=1}^{3} n_a^i l_a^i\,, \tag{15}$$

$$I_{aO6} = [\Pi_a][\Pi_{O6}] = 2^{3-k}(\tilde{A}_a + \tilde{B}_a + \tilde{C}_a + \tilde{D}_a)\,. \tag{16}$$

Note that the expressions Eqs. (13) and (14) indicate the presence of non-chiral pairs of matter in the antisymmetric representation that are masked by Eq. (14), which gives only the net number.

A zero intersection number between two branes implies that the branes are parallel on at least one torus. At such types of intersection, additional non-chiral (vectorlike) multiplet pairs from $ab + ba$, $ab' + b'a$, and $aa' + a'a$ can arise [32][2]. The multiplicity of these non-chiral multiplet pairs is given by the remainder of the intersection product, neglecting the null sector. For example, if $(n_a^1 l_b^1 - n_b^1 l_a^1) = 0$ in $I_{ab} = [\Pi_a][\Pi_b] = 2^{-k}\prod_{i=1}^{3}(n_a^i l_b^i - n_b^i l_a^i)$, then

$$\mathcal{M}\left[\left(\frac{N_a}{2}, \frac{\overline{N_b}}{2}\right) + \left(\frac{\overline{N_a}}{2}, \frac{N_b}{2}\right)\right] = \prod_{i=2}^{3}(n_a^i l_b^i - n_b^i l_a^i)\,. \tag{17}$$

This result is useful since one can fill the spectrum with this matter without affecting the required global conditions, because the total effect of the pairs is zero. Typically in this type of model, the Higgs fields arises from this non-chiral matter. However, as we shall see for the model constructed here, the Higgs fields appear in the chiral sector of the model.

[2] Representations $(\mathrm{Anti}_a + \overline{\mathrm{Anti}}_a)$ occur at intersection of a with a' if they are parallel on at least one torus.

The total non-Abelian anomaly in intersecting brane-world models cancels automatically when the R-R tadpole conditions are satisfied, but additional mixed anomalies may be present. For example, the mixed gravitational anomalies that generate massive fields are not trivially zero [28]. These anomalies are cancelled by a generalized Green-Schwarz (G-S) mechanism that involves untwisted R-R forms. The couplings of the four untwisted R-R forms B_2^i to the U(1) field strength F_a of each stack a are given by

$$
\begin{aligned}
& N_a l_a^1 n_a^2 n_a^3 \int_{M4} B_2^1 \wedge \mathrm{tr} F_a, \quad N_a n_a^1 l_a^2 n_a^3 \int_{M4} B_2^2 \wedge \mathrm{tr} F_a\,, \\
& N_a n_a^1 n_a^2 l_a^3 \int_{M4} B_2^3 \wedge \mathrm{tr} F_a, \quad -N_a l_a^1 l_a^2 l_a^3 \int_{M4} B_2^4 \wedge \mathrm{tr} F_a\,.
\end{aligned}
\tag{18}
$$

These couplings then determine the exact linear combinations of U(1) gauge bosons that acquire string-scale masses via the G-S mechanism. If $\mathrm{U}(1)_X$ is a linear combination of the U(1)s from each stack,

$$
\mathrm{U}(1)_X \equiv \sum_a C_a \mathrm{U}(1)_a\,,
\tag{19}
$$

then the corresponding field strength must be orthogonal to those that acquire G-S mass. Thus, if a linear combination $\mathrm{U}(1)_X$ satisfies

$$
\begin{aligned}
& \sum_a C_a N_a \tilde{B}_a = 0, \quad \sum_a C_a N_a \tilde{C}_a = 0\,, \\
& \sum_a C_a N_a \tilde{D}_a = 0, \quad \sum_a C_a N_a \tilde{A}_a = 0\,,
\end{aligned}
\tag{20}
$$

the gauge boson of $\mathrm{U}(1)_X$ acquires no G-S mass and is anomaly-free.

B. The Model

We present the D6-brane configurations, intersection numbers, and complex-structure parameters of the model in Table I, and the resulting spectrum in Table II. With this configuration, all R-R tadpoles are cancelled, $\mathcal{N}=1$ supersymmetry is preserved, and K-theory conditions are satisfied. The resulting model has gauge symmetry $[\mathrm{U}(4)_C \times \mathrm{U}(2)_L \times \mathrm{U}(2)_R]_{\text{observable}} \times [\mathrm{USp}(8)^3]_{\text{hidden}}$, a left-right symmetric Pati-Salam symmetry that provides (as seen below) a convenient origin for the SM hypercharge $\mathrm{U}(1)_Y$.

In this model the anomalies from the three global U(1)s of $\mathrm{U}(4)_C$, $\mathrm{U}(2)_L$, and $\mathrm{U}(2)_R$ are each cancelled by the G-S mechanism, and the gauge fields of these U(1)s obtain masses via the linear $B \wedge F$ couplings, except for one linear combination $\mathrm{U}(1)_X = \mathrm{U}(1)_C + 2[\mathrm{U}(1)_{2L} + \mathrm{U}(1)_{2R}]$. The effective gauge symmetry of the observable sector is then $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_X$.

The Pati-Salam gauge symmetry must then be broken to the SM. The breaking is accomplished by brane splitting, which corresponds in the field theory description to giving VEVs along D-flat and F-flat directions to some of the adjoint scalars that arise via open-string moduli associated with each stack of D branes [33]. The a stack of D6 branes may be split on any of the three two-tori into $a1$ (quark) and $a2$ (lepton) stacks with $N_{a1} = 6$ and $N_{a2} = 2$ D6 branes, respectively. In same manner, the c stack of D6 branes is split into stacks $c1$ ($I_{3L} = +\frac{1}{2}$ fermions) and $c2$ ($I_{3L} = -\frac{1}{2}$ fermions) such that $N_{c1} = 2$ and $N_{c2} = 2$, respectively. After the splitting, the observable gauge symmetry is broken to $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_{I_{3R}} \times \mathrm{U}(1)_{B-L} \times \mathrm{U}(1)_X$, where the gauge bosons of $\mathrm{U}(1)_{I_{3R}}$, $\mathrm{U}(1)_{B-L}$, and $\mathrm{U}(1)_X$ remain massless. The $\mathrm{U}(1)_{I_{3R}} \times \mathrm{U}(1)_{B-L} \times \mathrm{U}(1)_X$ gauge symmetry may then be broken spontaneously if, for example, vectorlike particles with the quantum numbers $(\mathbf{1}, \mathbf{1}, \frac{1}{2}, -1, 3)$ and $(\mathbf{1}, \mathbf{1}, -\frac{1}{2}, 1, -3)$ under the $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_{I_{3R}} \times \mathrm{U}(1)_{B-L} \times \mathrm{U}(1)_X$ gauge symmetry from the $a_2 c_2'$ intersections (Φ, $\bar{\Phi}$ in Table II) receive VEVs,[3] leaving only the single U(1) corresponding to SM hypercharge,

$$\mathrm{U}(1)_Y = \frac{1}{6}[\mathrm{U}(1)_{a1} - 3\mathrm{U}(1)_{a2} - 3\mathrm{U}(1)_{c1} + 3\mathrm{U}(1)_{c2}] \,, \tag{21}$$

and the overall gauge group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{USp}(8)^3$.

The observable sector of the model then possesses the gauge symmetry and matter content of a four-generation MSSM with an extended Higgs sector. Note that the four pairs of Higgs fields in this model appear in the chiral spectrum, rather than arising as vectorlike matter from $\mathcal{N} = 2$ subsectors, as is the case for the models studied in [34], as well as most models in the literature. The extra matter in this model includes fields charged under the hidden-sector gauge groups, vectorlike matter between nonintersecting pairs of branes, and the chiral adjoints associated with each stack of branes. In addition, matter occurs in the symmetric triplet and antisymmetric singlet representations of $\mathrm{SU}(2)_L$.

In order for the model to reproduce just the MSSM at low energies, the gauge couplings must unify, and all extra matter besides the MSSM states must become massive at high-energy scales. Furthermore, since the MSSM contains just one pair $H_{u,d}$ of light Higgs doublets, the Higgs potential must be fine tuned. Since the four Higgs fields in this model appear in the chiral sector rather than as $\mathcal{N} = 2$ vectorlike multiplets, no intrinsic μ parameter (whose real part corresponds geometrically to the separation between stacks b and c) occurs since the b and c stacks intersect on all three two-tori. A single pair of light Higgs doublets may be obtained by fine-tuning the μ

[3] The $a_2 c_1'$ intersections were incorrectly indicated for this purpose in Ref. [15].

TABLE I: D6-brane configurations and intersection numbers for a four-family Pati-Salam model on a Type-IIA $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold, with no tilted tori. The complete gauge symmetry is $[\mathrm{U}(4)_C \times \mathrm{U}(2)_L \times \mathrm{U}(2)_R]_{\text{observable}} \times [\mathrm{USp}(8)^3]_{\text{hidden}}$, and the complex-structure parameters are $\chi_1 = 2\chi_2 = 2\chi_3$. The parameters β_i are the β-function coefficients for the $\mathrm{USp}(8)_i$ gauge groups.

	$\mathrm{U}(4)_C \times \mathrm{U}(2)_L \times \mathrm{U}(2)_R \times \mathrm{USp}(8)^3$											
	N	$(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)$	n_S	n_A	b	b'	c	c'	1	2	3
a	8	$(0, -1) \times (1, 1) \times (1, 1)$	0	0	4	0	-4	0	1	-1	0
b	4	$(2, 1) \times (1, -1) \times (1, 0)$	2	-2	-	-	4	0	0	1	-2
c	4	$(2, -1) \times (0, 1) \times (1, -1)$	-2	2	-	-	-	-	-1	0	2
1	8	$(1, 0) \times (1, 0) \times (1, 0)$	$\chi_1 = 2,\ \chi_2 = 1,\ \chi_3 = 1$								
2	8	$(1, 0) \times (0, -1) \times (0, 1)$	$\beta^g_1 = -3,\ \beta^g_2 = -3$								
3	8	$(0, -1) \times (1, 0) \times (0, 1)$	$\beta^g_3 = -2$								

term, which may be generated in the superpotential W via the higher-dimensional operators

$$W \supset \frac{y^{ijkl}_{\mu}}{M_{\rm St}} S^i_L S^j_R H^k_u H^l_d \ , \tag{22}$$

where y^{ijkl}_{μ} are Yukawa couplings, and $M_{\rm St}$ is the string scale. In this scenario, the singlets S^i_R may obtain string or GUT-scale VEVs while preserving the D-flatness of $\mathrm{U}(1)_R$, and the singlets S^i_L may obtain TeV-scale VEVs while preserving the D-flatness of $\mathrm{U}(1)_L$ (and hence are tied to electroweak symmetry breaking). The precise linear combinations that produce the two light Higgs eigenstates $H_{u,d}$ obtained by fine-tuning the Higgs potential via Eq. (22) are then correlated with the pattern of Higgs VEVs necessary to obtain Yukawa matrices for the quarks and leptons,

$$H_{u,d} = \sum_i \frac{v^i_{u,d}\, H^i_{u,d}}{\sqrt{\sum_j (v^j_{u,d})^2}}, \tag{23}$$

where $v^i_{u,d} = \left\langle H^i_{u,d} \right\rangle$.

The gauge coupling constant associated with a stack P is given by

$$g^{-2}_{D6_P} \ = \ |\mathrm{Re}\,(f_P)|\,, \tag{24}$$

where f_P is the holomorphic kinetic gauge function associated with stack P, given by [18, 35]

$$f_P \ = \ \frac{1}{4\kappa_P}\left(n^1_P\, n^2_P\, n^3_P\, s - n^1_P\, l^2_P\, l^3_P\, u^1 - n^2_P\, l^1_P\, l^3_P\, u^2 - n^3_P\, l^1_P\, l^2_P\, u^3\right)\ , \tag{25}$$

with $\kappa_P = 1$ for $\mathrm{SU}(N_P)$ and $\kappa_P = 2$ for $\mathrm{USp}(2N_P)$ or $\mathrm{SO}(2N_P)$ gauge groups, and s and u are moduli in the supergravity basis. The holomorphic gauge kinetic function associated with the SM

TABLE II: The chiral and vectorlike superfields, their multiplicities and quantum numbers under the gauge symmetry $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{USp}(8)_1 \times \mathrm{USp}(8)_2 \times \mathrm{USp}(8)_3$.

	Mult.	Quantum Number	Q_4	Q_{2L}	Q_{2R}	Field
ab	4	$(4,\overline{2},1,1,1,1)$	1	-1	0	$F_L(Q_L,L_L)$
ac	4	$(\overline{4},1,2,1,1,1)$	-1	0	1	$F_R(Q_R,L_R)$
bc	4	$(1,2,\overline{2},1,1,1)$	0	1	-1	H_u^i, H_d^i
$a1$	1	$(4,1,1,\overline{8},1,1)$	1	0	0	X_{a1}
$a2$	1	$(\overline{4},1,1,1,8,1)$	-1	0	0	X_{a2}
$b2$	1	$(1,2,1,1,\overline{8},1)$	0	1	0	X_{b2}
$b3$	2	$(1,\overline{2},1,1,1,8)$	0	-1	0	X_{b3}^i
$c1$	1	$(1,1,\overline{2},8,1,1)$	0	0	-1	X_{c1}
$c3$	2	$(1,1,2,1,1,\overline{8})$	0	0	1	X_{c3}^i
b_S	2	$(1,3,1,1,1,1)$	0	2	0	T_L^i
b_A	2	$(1,\overline{1},1,1,1,1)$	0	-2	0	S_L^i
c_S	2	$(1,1,\overline{3},1,1,1)$	0	0	-2	T_R^i
c_A	2	$(1,1,1,1,1,1)$	0	0	2	S_R^i
ab'	2	$(4,2,1,1,1,1)$	1	1	0	
	2	$(\overline{4},\overline{2},1,1,1,1)$	-1	-1	0	
ac'	2	$(4,1,2,1,1,1)$	1	0	1	Φ_i
	2	$(\overline{4},1,\overline{2},1,1,1)$	-1	0	-1	$\overline{\Phi}_i$
bb'	2	$(1,\overline{1},1,1,1,1)$	0	-2	0	s_L^i
	2	$(1,1,1,1,1,1)$	0	2	0	$\bar{s}_L^i$
cc'	2	$(1,1,1,1,1,1)$	0	0	2	s_R^i
	2	$(1,1,\overline{1},1,1,1)$	0	0	-2	$\bar{s}_R^i$
bc'	1	$(1,2,2,1,1,1)$	0	1	1	H'
	1	$(1,\overline{2},\overline{2},1,1,1)$	0	-1	-1	$\overline{H}'$

hypercharge $\mathrm{U}(1)_Y$ is then given for this model by the combination [36]

$$f_Y = \frac{1}{6}f_{a1} + \frac{1}{2}f_{a2} + \frac{1}{2}f_{c1} + \frac{1}{2}f_{c2}\,. \tag{26}$$

The complex-structure moduli U^i are obtained from the conditions to maintain $\mathcal{N}=1$ supersymmetry. Consistent with the complex-structure parameters $\chi_1=2$, $\chi_{2,3}=1$, this model has

$$U^1 = 2i\ , \quad U^2 = i\ , \quad U^3 = i\ . \tag{27}$$

Note that the conditions for preserving $\mathcal{N}=1$ supersymmetry do not actually fix one complex structure modulus, U^3, and we have chosen a value for it which results in gauge coupling unification.

In a completely realistic compactification, it would be necessary to find some mechanism to stabilize this modulus, as well as the Kähler and open-string moduli, and the dilaton. In the supergravity basis,

$$\begin{aligned}
\mathrm{Re}\,(s) &= \frac{e^{-\phi_4}}{2\pi}\left(\frac{\sqrt{\mathrm{Im}\,U^1\,\mathrm{Im}\,U^2\,\mathrm{Im}\,U^3}}{|U^1U^2U^3|}\right), \\
\mathrm{Re}\,(u^j) &= \frac{e^{-\phi_4}}{2\pi}\left(\sqrt{\frac{\mathrm{Im}\,U^j}{\mathrm{Im}\,U^k\,\mathrm{Im}\,U^l}}\right)\left|\frac{U^k\,U^l}{U^j}\right| \qquad (j,k,l)=(\overline{1,2,3}), \\
\mathrm{Re}(t^j) &= \frac{i\alpha'}{T^j}, \qquad (28)
\end{aligned}$$

where ϕ_4 is the four-dimensional dilaton, one finds

$$\mathrm{Re}(s)=\frac{1}{2\pi}\frac{e^{-\phi_4}}{\sqrt{2}}, \quad \mathrm{Re}(u^1)=\frac{1}{2\pi}\frac{e^{-\phi_4}}{\sqrt{2}}, \quad \mathrm{Re}(u^2)=\frac{1}{2\pi}\frac{2e^{-\phi_4}}{\sqrt{2}}, \quad \mathrm{Re}(u^3)=\frac{1}{2\pi}\frac{2e^{-\phi_4}}{\sqrt{2}}. \qquad (29)$$

Inserting these expressions into Eqs. (24)–(25), one finds that the gauge couplings are unified at the string scale,

$$g_s^2 = g_w^2 = \frac{5}{3}g_Y^2 = 2\sqrt{2}\pi e^{\phi_4}. \qquad (30)$$

Choosing $\phi_4 = -2.295$ so that $e^{\phi_4} = 0.101$, one obtains $g^2_{\mathrm{unif}} \approx 0.895$, the value of the unified gauge coupling at GUT scale for a four-generation MSSM. One further finds the string scale to be

$$M_{St} = \pi^{1/2}e^{\phi_4}M_{\mathrm{Pl}} \approx 4.35\times 10^{17}\ \mathrm{GeV}, \qquad (31)$$

where M_{Pl} is the reduced Planck scale, 2.44×10^{18} GeV.

Fixing the value of ϕ_4 also fixes the gauge couplings of the hidden-sector USp(8) groups at M_{St}:

$$g_1^2 = g_2^2 = 7.160, \qquad g_3^2 = 3.580. \qquad (32)$$

Using the β-function coefficients shown in Table I, the renormalization-group equations (RGEs) for the $\mathrm{USp}(8)_{1,2}$ gauge groups produce strong coupling at a scale $\Lambda_{1,2} \approx 4.5\times 10^{15}$ GeV, while the group $\mathrm{USp}(8)_3$ reaches strong coupling at $\Lambda_3 \approx 8.3\times 10^{12}$ GeV. Thus, matter charged under these groups becomes confined into composite states and decouples not far below the GUT scale.

Thus, at low energies this model is a four-generation MSSM, where the gauge couplings unify $\approx 2.2\times 10^{16}$ GeV. In addition, one finds matter charged under the hidden-sector gauge groups that becomes confined into massive bound states at high-energy scales. These matter representations all have fractional electric charges, $Q_{\mathrm{em}} = \pm\frac{1}{6},\ \pm\frac{1}{2}$. In particular, the fields charged under $\mathrm{USp}(8)_3$ all have half-integer electric charges and confine into bound states with integer electromagnetic charge at a scale $O(10^{13})$ GeV, similar to the “cryptons” studied in [37–40]. If the lightest of these bound states is electromagnetically neutral and is metastable with a sufficiently long lifetime, it provides a good candidate for superheavy cold dark matter (CDM). This observation is particularly true for states with masses of order Λ_3, since superheavy particles with masses in the range $M = O(10^{11-13})$ GeV might very well have been produced naturally through the interaction of the vacuum with the gravitational field during the post-inflationary reheating phase [41].

III. YUKAWA COUPLINGS

A. Formalism and Construction

A complete form for the Yukawa couplings arising from D6 branes wrapping on a full compact space $T^2 \times T^2 \times T^2$ can be expressed as [26, 42]:

$$Y_{ijk} \propto \prod_{r=1}^{3} \vartheta \begin{bmatrix} \delta^{(r)} \\ \phi^{(r)} \end{bmatrix} (\kappa^{(r)}) , \tag{33}$$

where the proportionality constant cancels in mass ratios, and

$$\vartheta \begin{bmatrix} \delta^{(r)} \\ \phi^{(r)} \end{bmatrix} (\kappa^{(r)}) = \sum_{l \in \mathbb{Z}} e^{\pi i (\delta^{(r)}+l)^2 \kappa^{(r)}} e^{2\pi i (\delta^{(r)}+l)\phi^{(r)}} , \tag{34}$$

with $r = 1, 2, 3$ denoting the three two-tori. The input parameters are given by

$$\begin{aligned} \delta^{(r)} &= \frac{i^{(r)}}{I_{ab}^{(r)}} + \frac{j^{(r)}}{I_{ca}^{(r)}} + \frac{k^{(r)}}{I_{bc}^{(r)}} + \frac{d^{(r)}(I_{ab}^{(r)}\epsilon_c^{(r)} + I_{ca}^{(r)}\epsilon_b^{(r)} + I_{bc}^{(r)}\epsilon_a^{(r)})}{I_{ab}^{(r)} I_{bc}^{(r)} I_{ca}^{(r)}} + \frac{s^{(r)}}{d^{(r)}}, \\ \phi^{(r)} &= \frac{I_{bc}^{(r)}\theta_a^{(r)} + I_{ca}^{(r)}\theta_b^{(r)} + I_{ab}^{(r)}\theta_c^{(r)}}{d^{(r)}}, \\ \kappa^{(r)} &= \frac{J^{(r)}}{\alpha'} \frac{|I_{ab}^{(r)} I_{bc}^{(r)} I_{ca}^{(r)}|}{(d^{(r)})^2}. \end{aligned} \tag{35}$$

where the indices $i^{(r)}$, $j^{(r)}$, and $k^{(r)}$ label the intersections on the r^{th} torus, $d^{(r)} = gcd(I_{ab}^{(r)}, I_{bc}^{(r)}, I_{ca}^{(r)})$, and the integer $s^{(r)}$ is a function of $i^{(r)}$, $j^{(r)}$, and $k^{(r)}$ corresponding to different ways of counting triplets of intersections. $J^{(r)}$ is the Kähler modulus on the r^{th} torus and α' is the string tension. The shift parameters $\epsilon_a^{(r)}$, $\epsilon_b^{(r)}$, and $\epsilon_c^{(r)}$ correspond to the relative positions of stacks a, b, and c, while the parameters $\phi_a^{(r)}$, $\phi_b^{(r)}$, and $\phi_c^{(r)}$ are Wilson lines associated with these stacks. For simplicity and to exhibit the flexibility of our model, we set the Wilson lines to zero. The brane shifts and Wilson lines together comprise the open-string moduli, which must be stabilized in a

TABLE III: Index assignments labeling the intersections at which each chiral field is localized.

Left-handed field	$i = \{i^{(1)}, i^{(2)}, i^{(3)}\}$	Right-handed field	$j = \{j^{(1)}, j^{(2)}, j^{(3)}\}$	Higgs field	$k = \{k^{(1)}, k^{(2)}, k^{(3)}\}$
$F_L^1(Q_L^1, L_L^1)$	$\{0,0,0\}$	$F_R^1(Q_R^1, L_R^1)$	$\{0,0,0\}$	$H^1(H_u^1, H_d^1)$	$\{0,0,0\}$
$F_L^2(Q_L^2, L_L^2)$	$\{0,1,0\}$	$F_R^2(Q_R^2, L_R^2)$	$\{0,0,1\}$	$H^2(H_u^2, H_d^2)$	$\{1,0,0\}$
$F_L^3(Q_L^3, L_L^3)$	$\{1,0,0\}$	$F_R^3(Q_R^3, L_R^3)$	$\{1,0,0\}$	$H^3(H_u^3, H_d^3)$	$\{2,0,0\}$
$F_L^4(Q_L^4, L_L^4)$	$\{1,1,0\}$	$F_R^4(Q_R^4, L_R^4)$	$\{1,0,1\}$	$H^4(H_u^4, H_d^4)$	$\{3,0,0\}$

complete model by some mechanism. In this paper we treat them as free parameters; however, we discuss the stabilization of these moduli in the next subsection. For simplicity, let us define

$$\epsilon^{(r)} \equiv \frac{d^{(r)}(I_{ab}^{(r)}\epsilon_c^{(r)} + I_{ca}^{(r)}\epsilon_b^{(r)} + I_{bc}^{(r)}\epsilon_a^{(r)})}{I_{ab}^{(r)} I_{bc}^{(r)} I_{ca}^{(r)}} \,. \tag{36}$$

Although these Yukawa coupling formulas originally refer to $T^6 = T^2 \times T^2 \times T^2$, they may be generalized to $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ simply by including all of the orbifold images. However, in the present case the cycles wrapped by the orbifold images of a stack of D branes a are homologically identical to the original cycle wrapped by the stack a. In addition, the intersection numbers between the cycles defined on the orbifold turn out to be the same as the intersection numbers between those on the ambient torus. Thus, the above formulas for $T^6 = T^2 \times T^2 \times T^2$ may be used without change on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$.

Since in this model the intersections are distributed over all three two-tori, they must be carefully labeled. Let us introduce the indices $i = \{i^{(1)}, i^{(2)}, i^{(3)}\}$, $j = \{j^{(1)}, j^{(2)}, j^{(3)}\}$, and $k = \{k^{(1)}, k^{(2)}, k^{(3)}\}$ to label the intersections at which the left-handed fields, right-handed fields, and Higgs fields, respectively, are localized. For this model, these indices can assume the values

$$\begin{aligned} &i^{(1)} = \{0,1\}\,, && i^{(2)} = \{0,1\}\,, && i^{(3)} = \{0\}\,, \\ &j^{(1)} = \{0,1\}\,, && j^{(2)} = \{0\}\,, && j^{(3)} = \{0,1\}\,, \\ &k^{(1)} = \{0,1,2,3\}\,, && k^{(2)} = \{0\}\,, && k^{(3)} = \{0\}\,, \end{aligned} \tag{37}$$

as obtained by considering the intersection numbers $I_{ab}^{(r)}$, $I_{ac}^{(r)}$, $I_{bc}^{(r)}$ on each torus. Then each left-handed, right-handed, and Higgs field is identified by a triplet of indices, as summarized in Table III. The independent Yukawa couplings $Y_{i^{(r)}j^{(r)}k^{(r)}}^{(r)}$ on each torus are then labeled:

$$r = 1: \; Y_{000}^{(1)}, \;\; Y_{011}^{(1)}, \;\; Y_{101}^{(1)}, \;\; Y_{110}^{(1)}, \;\; Y_{002}^{(1)}, \;\; Y_{013}^{(1)}, \;\; Y_{103}^{(1)}, \;\; Y_{112}^{(1)}, \tag{38}$$

$$r = 2: \qquad y_1 \equiv Y_{000}^{(2)}, \quad y_2 \equiv Y_{100}^{(2)}, \tag{39}$$

$$r = 3: \qquad z_1 \equiv Y_{000}^{(3)}, \quad z_2 \equiv Y_{010}^{(3)}, \tag{40}$$

being the only remaining components after applying the selection rules $i^{(r)}+j^{(r)}+k^{(r)} = 0 \bmod d^{(r)}$,

$$\begin{aligned} i^{(1)} + j^{(1)} + k^{(1)} &= 0 \bmod 2\,, \\ i^{(2)} + j^{(2)} + k^{(2)} &= 0 \bmod 1\,, \\ i^{(3)} + j^{(3)} + k^{(3)} &= 0 \bmod 1\,. \end{aligned} \tag{41}$$

The full Yukawa couplings are then given by a product of the couplings on each torus,

$$Y_{ijk} = \prod_{r=1}^{3} Y^{(r)}_{i^{(r)}j^{(r)}k^{(r)}}\,. \tag{42}$$

Thus, the Yukawa matrices (including VEVs) in this model are of the form

$$\mathcal{Y} \equiv \sum_k Y_{ijk} v_k = \begin{pmatrix} \left[Y^{(1)}_{000}v_0 + Y^{(1)}_{002}v_2\right] YZ & \left[Y^{(1)}_{011}v_1 + Y^{(1)}_{013}v_3\right] YZ \\ \left[Y^{(1)}_{101}v_1 + Y^{(1)}_{103}v_3\right] YZ & \left[Y^{(1)}_{110}v_0 + Y^{(1)}_{112}v_2\right] YZ \end{pmatrix}\,, \tag{43}$$

where YZ is the singular 2×2 matrix

$$YZ \equiv \begin{pmatrix} y_1 z_1 & y_1 z_2 \\ y_2 z_1 & y_2 z_2 \end{pmatrix}\,, \tag{44}$$

and the values $v_{k^{(1)}} = \langle H_{k^{(1)}+1}\rangle$ are the VEVs of the Higgs fields.

Since the same singular submatrix YZ appears in all sub-blocks of $\mathcal{Y}$, the full Yukawa matrices $\mathcal{Y}$ are only rank 2, so that at most two eigenvalues of $\mathcal{Y}$ can be different from zero. However, this result can be quite desirable, since the third- and fourth-generation quarks and leptons tend to be significantly heavier than those of the first and second generations. In principle, lifting the rank-2 degeneracies (to induce small masses and mixings) can be achieved by using higher-order couplings than included here, such as the fourth-order terms considered in [43, 44], or supersymmetry loop corrections [45]. For example, the couplings

$$W_4 \supset y_{ijk}\bar{s}^i_R F^j_L F^k_R H' + y'_{ijk}\bar{s}^i_L F^j_L F^k_R \bar{H}'. \tag{45}$$

can perturb the Yukawa matrices to break the rank degeneracy, or additional trilinear couplings with the vectorlike Higgs fields H and $\bar{H}'$ can be induced by D-brane instantons.

For this model, the δ parameters that enter the ϑ functions of Eq. (34) are given by

$$\delta^{(1)} = \frac{i^{(1)}}{2} - \frac{j^{(1)}}{2} - \frac{k^{(1)}}{4} - \frac{1}{4}(2\epsilon^{(1)}_a + \epsilon^{(1)}_b - \epsilon^{(1)}_c) + \frac{s^{(1)}}{2}\,, \tag{46}$$

$$\delta^{(2)} = -\frac{i^{(2)}}{2} + \frac{1}{2}(\epsilon^{(2)}_a - \epsilon^{(2)}_b - 2\epsilon^{(2)}_c)\,, \tag{47}$$

$$\delta^{(3)} = \frac{j^{(3)}}{2} - \frac{1}{2}(\epsilon^{(3)}_a - 2\epsilon^{(3)}_b + \epsilon^{(3)}_c)\,. \tag{48}$$

For simplicity, as mentioned above we have taken $\phi^{(1)} = \phi^{(2)} = \phi^{(3)} = 0$. Allowed choices for $s^{(1)}$ are restricted by the requirement that the parameters $\delta^{(1)}(i,j,k)$ must differ by an integer for triplets (i,j,k) forming triangles with the same area. In the present case, one finds two independent choices satisfying this constraint: $s^{(1)} = i^{(1)}$ and $s^{(1)} = j^{(1)}$. For definiteness, let us make the choice $s^{(1)} = j^{(1)}$. We then find

$$\begin{aligned} Y^{(1)}_{000} &= Y^{(1)}_{112} \equiv x_0\,, \quad Y^{(1)}_{011} = Y^{(1)}_{101} \equiv x_1\,, \\ Y^{(1)}_{002} &= Y^{(1)}_{110} \equiv x_2\,, \quad Y^{(1)}_{013} = Y^{(1)}_{103} \equiv x_3\,. \end{aligned} \tag{49}$$

The Yukawa matrix then takes the simple form

$$\mathcal{Y} = \begin{pmatrix} (x_0 v_0 + x_2 v_2)YZ & (x_1 v_1 + x_3 v_3)YZ \\ (x_1 v_1 + x_3 v_3)YZ & (x_2 v_0 + x_0 v_2)YZ \end{pmatrix}\,, \tag{50}$$

where we again see that the Yukawa matrices are rank 2, so that two eigenvalues are zero. The four eigenvalues of $\mathcal{Y}$ can be given in closed form as $\lambda_1 = \lambda_2 = 0$, and the two nonzero eigenvalues

$$\begin{aligned} \lambda_3 &= \frac{1}{2}(y_1 z_1 + y_2 z_2)[(x_0 + x_2)(v_0 - v_2) - \Delta]\,, \\ \lambda_4 &= \frac{1}{2}(y_1 z_1 + y_2 z_2)[(x_0 + x_2)(v_0 + v_2) + \Delta]\,, \end{aligned} \tag{51}$$

where

$$\Delta = \sqrt{(x_0 - x_2)^2 (v_0 - v_2)^2 + 4(x_1 v_1 + x_3 v_3)^2}\,, \tag{52}$$

from which one sees that a mass splitting between the third and fourth generations automatically occurs for generic nonzero values of the Higgs VEVs. In order to work with simple closed-form solutions, note that setting $v_0 = v_2$ simplifies Eq. (51) to just

$$\begin{aligned} \lambda_3 &= \frac{1}{2}(y_1 z_1 + y_2 z_2)\left[(x_0 + x_2)v_0 - (x_1 v_1 + x_3 v_3)\right]\,, \\ \lambda_4 &= \frac{1}{2}(y_1 z_1 + y_2 z_2)\left[(x_0 + x_2)v_0 + (x_1 v_1 + x_3 v_3)\right]\,. \end{aligned} \tag{53}$$

The ratio of the third- to fourth-generation fermion masses is then given by

$$\frac{m_3^{f_{u,d}}}{m_4^{f_{u,d}}} = \left| \frac{x_0^{f_{u,d}} v_0^{u,d} - x_1^{f_{u,d}} v_1^{u,d} + x_2^{f_{u,d}} v_0^{u,d} - x_3^{f_{u,d}} v_3^{u,d}}{x_0^{f_{u,d}} v_0^{u,d} + x_1^{f_{u,d}} v_1^{u,d} + x_2^{f_{u,d}} v_0^{u,d} + x_3^{f_{u,d}} v_3^{u,d}} \right|\,, \tag{54}$$

where $f_u = \{u, \nu\}$, $f_d = \{d, l\}$, couple to the appropriate Higgs VEVs $v_i^{u,d}$, respectively.

Finally, the eigenstates corresponding to the eigenvalues Eq. (51) or Eq. (53) may be expressed as linear combinations of the fields defined in Table III, which are localized at the intersections between D6 branes. One finds that the eigenfunctions assume the forms

$$|\lambda_3\rangle \propto \begin{pmatrix} -\alpha\chi \\ \chi \end{pmatrix}, \quad |\lambda_4\rangle \propto \begin{pmatrix} \chi \\ \alpha\chi \end{pmatrix}, \quad \text{where } \chi = \begin{pmatrix} \frac{y_1}{y_2} \\ 1 \end{pmatrix}, \tag{55}$$

where $\alpha \to 1$ in the special case of Eq. (53). Most significantly, the ratios of the third- to fourth-generation coefficients defining these linear combinations are both given by the same ratio of the Yukawa couplings on the second two-torus, y_1/y_2: Thus, so long as the Yukawa couplings y_1 and y_2 are the same for both the up- and down-type fermions, no mixing occurs between the third and fourth generations at the trilinear order. This condition can be easily satisfied if the D-branes are split only on the first and third two-tori.[4]

B. Analysis

A full comparison of model predictions to experimental data would require the full RGE evolution of the observed masses and CKM elements from the weak scale to the unification scale $M_{\rm GUT}$. However, the masses of the fourth-generation fermions, and their mixings with the other three, are of course unknown [46]. As discussed above, the combination of direct observation bounds, electroweak (EW) precision tests, and perturbative unitarity constraints place strong constraints on the possible masses, which is just as true in the SM as in the MSSM [47]. However, four-generation models, both supersymmetric and not, are plagued by the presence of several large Yukawa couplings that exhibit Landau poles at the TeV scale and above. While attempts have been made to stabilize the numerical evolution of the RGEs in four-generation models up to $M_{\rm GUT}$ by including new matter fields (*e.g.*, [48]), questions of the robustness of such models remain. For the following analysis, we assume that the effective running of the Yukawa couplings between the GUT scale and the EW scale may be stabilized by some mechanism, and we note that vectorlike matter similar to that introduced in [48] is present in the model, as seen in Table II. For simplicity, we further (crudely) assume that the running of the Yukawa couplings between the GUT and EW scales varies slowly, so that weak-scale values of observables may be used.

Let us recall that the Pati-Salam gauge symmetry has been broken to the SM by brane splitting. For the moment, assume that this splitting occurs only on the first two-torus. In fact, to forbid this mixing just for quarks, $c_1 = c_2$ on the second torus is sufficient. Then one can write the total

[4] Likewise, the two massless-state eigenvectors depend only upon the third-torus parameter combination z_1/z_2.

brane shifts on the first torus for each type of fermion, given by Eq. (36) with $r = 1$ as

$$\epsilon^\nu = \frac{2\epsilon_{a2} + \epsilon_b - \epsilon_{c1}}{4}, \qquad \epsilon^u = \frac{2\epsilon_{a1} + \epsilon_b - \epsilon_{c1}}{4},$$
$$\epsilon^l = \frac{2\epsilon_{a2} + \epsilon_b - \epsilon_{c2}}{4}, \qquad \epsilon^d = \frac{2\epsilon_{a1} + \epsilon_b - \epsilon_{c2}}{4}, \tag{56}$$

where quarks begin on stack $a1$ and leptons on stack $a2$, while the neutrinos and up-type quarks both end on stack $c1$, and the charged leptons and down-type quarks both end on stack $c2$. One then observes that

$$\epsilon^u - \epsilon^d = \frac{1}{4}(-\epsilon_{c1} + \epsilon_{c2}) = \epsilon^\nu - \epsilon^l. \tag{57}$$

Taking, for example, $\epsilon^\nu = 0$, one must then satisfy the constraint

$$\epsilon^d = \epsilon^u + \epsilon^l, \tag{58}$$

when choosing parameters to fit the mass ratios of third- and fourth-generation fermions. Note that although we are treating the brane-shifts as free parameters, these are open-string moduli that should be stabilized in a completely realistic compactification. This requirement can be satisfied if the D-branes wrap rigid cycles, which are available on several different backgrounds [49–51]. In addition, the Kähler moduli on each torus may be stabilized by including supergravity fluxes and/or by gaugino condensation in a hidden sector [17].

From the relations given in the previous subsection, we make certain general observations. First, Eq. (54) shows that a very light third-generation neutrino ν_τ can readily be obtained if

$$(x_0^\nu + x_2^\nu)v_0^u - x_1^\nu v_1^u - x_3^\nu v_3^u \approx 0. \tag{59}$$

where we take $v_0 = v_2$ so that the generic eigenvalues of the Yukawa matrices are given by Eq. (53). This condition can easily be satisfied if, for example, the Higgs VEVs take the values

$$v_0^u = 2C/(x_0^\nu + x_2^\nu), \qquad v_1^u = C/x_1^\nu, \qquad v_3^u = C/x_3^\nu \tag{60}$$

where C is constant. Using these values, Eq. (53) shows that m_{ν_τ} vanishes while $m_{\nu'}$ may be large. With these VEV choices, the neutrino Yukawa matrix takes the form

$$\mathcal{Y}^\nu \sim 2C \begin{pmatrix} YZ & YZ \\ YZ & YZ \end{pmatrix}, \tag{61}$$

and essentially degenerates to rank 1, so that only ν' receives mass at leading order. Note that this rank-1 condition is independent of the specific values assigned to the Kähler modulus and shift parameter ϵ^ν.

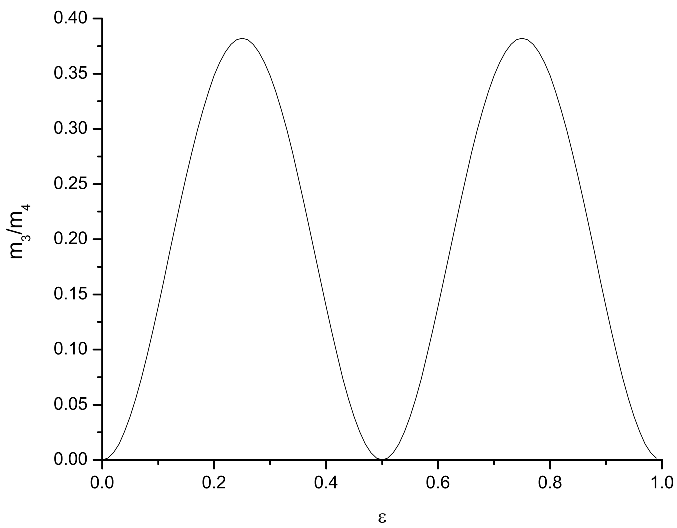

FIG. 1: Ratio of the t to t' quark masses as a function of the shift parameter $\epsilon \equiv \epsilon^u$, where the Higgs VEVs are chosen to satisfy Eq. (60), and $\kappa = \sqrt{3}\pi$.

Since the up-type quark masses depend upon the same Higgs VEVs as the neutrinos, the question now becomes whether it is possible to obtain masses for the t and t' quarks in the range

$$\frac{m_{t'}}{m_t} \approx 2\text{–}3\,. \tag{62}$$

At first glance, simultaneously satisfying these criteria may appear difficult. However, the Yukawa couplings x_i^u are in general different than x_i^ν since the position on the first two-torus of the $a1$ stack of branes upon which the quarks begin may be different than the position the $a2$ stack from which the leptons begin, so that $\epsilon_{a1} \neq \epsilon_{a2}$, and $\epsilon^u - \epsilon^\nu = \frac{1}{2}(\epsilon_{a1} - \epsilon_{a2})$.

We now show that one can obtain results consistent with Eq. (62) for generic nonzero values of $\kappa \equiv -i\kappa^{(1)}$ and ϵ^u. For example, let us choose $\epsilon^\nu = 0$ and $\kappa = \sqrt{3}\pi$ (an arbitrary value used purely for illustration), while the Higgs VEVs have the values given by Eq. (60). The resulting ratio $m_t/m_{t'}$ given by Eq. (60) as a function of the shift parameter $\epsilon \equiv \epsilon^u$ is plotted in Fig. 1. This plot shows that mass ratios satisfying Eq. (62) can be obtained for

$$\epsilon^u \approx 0.19\text{–}0.31, \qquad \text{and} \qquad \epsilon^u \approx 0.69\text{–}0.81\ . \tag{63}$$

Similarly, the down-type quark masses and charged leptons depend upon the Higgs VEVs v_k^d. However, in addition the shift parameters must satisfy Eq. (58). The Higgs VEVs v_k^d may be chosen

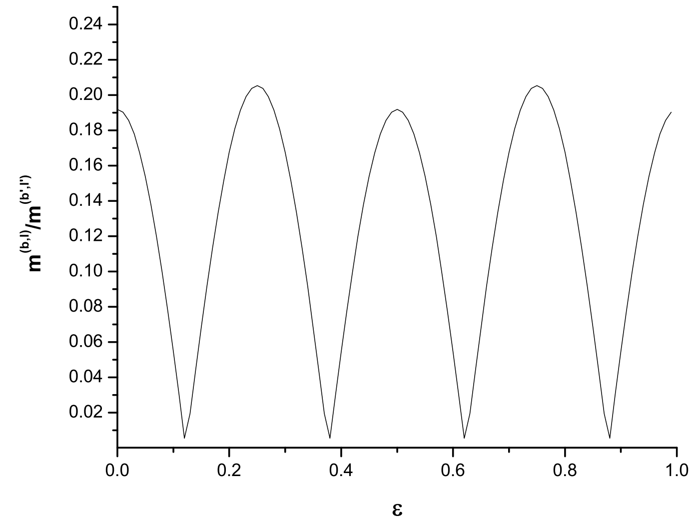

FIG. 2: Mass ratio between the third- and fourth-generation down-type quark (and charged-lepton masses) as functions of Kähler modulus $\kappa = \sqrt{3}\pi$ on the first torus and shift parameter $\epsilon \equiv \epsilon^d$.

freely, apart from overall normalization; for simplicity, let us choose

$$v_0^d = 2D/(x'^{\nu}_0 + x'^{\nu}_2), \qquad v_1^d = D/x'^{\nu}_1, \qquad v_3^d = D/x'^{\nu}_3, \tag{64}$$

where D is a constant and x'^{ν}_k are the same ϑ functions used in calculating the eigenvalues in the neutrino sector, but with a different value of the input parameter κ, which we label κ'. Note that the Yukawa couplings for the down-type quarks and charged leptons must be calculated using the same Kähler modulus $\kappa = \sqrt{3}\pi$ as used for the up-type quarks and neutrinos. However, the down-type Higgs VEVs are chosen by the ansatz Eq. (64), with $\kappa' < \kappa$. Obviously, many other suitable choices are possible.

In Fig. 2 we plot the ratio of third- to fourth-generation masses for the down-type quarks and charged leptons using $\kappa = \sqrt{3}\pi$ and $\kappa' = \sqrt{2}\pi/2$ (again, purely an illustrative choice), as a function of brane shift parameter $\epsilon \equiv \epsilon_d$. As one sees, the resulting function exhibits several maxima, with minima that are equally spaced about the maxima. The appearance of these alternating maxima and minima can, of course, be traced to the fact that the mass eigenvalues are essentially linear superpositions of the ϑ functions exhibiting well-defined symmetry properties in ϵ.

In fitting the third- to fourth-generation mass ratios for the down-type quarks and charged leptons, we require

$$\frac{m_{b'}}{m_b} \approx 80\text{–}130\,, \qquad\qquad \frac{m_{\tau'}}{m_\tau} \approx 60\text{–}700\,, \tag{65}$$

These mass ratios are clearly much larger than $m_{t'}/m_t$. Furthermore, one must impose Eq. (58) on the shift parameters. Fortunately, as noted previously, the mass ratio between third- and fourth-generation down-type quarks and neutrinos plotted in Fig. 2 exhibit minima near $\epsilon = 0.375$, $\epsilon = 0.125$, which are separated by $\Delta\epsilon \simeq 0.25$. By choosing values near $\epsilon^d \approx 0.375$ and $\epsilon^l \approx 0.625$, satisfying all constraints is possible. Indeed, if one chooses $\epsilon^d = 0.3737$ and $\epsilon^l = 0.1237$, for example, one finds

$$\frac{m_{b'}}{m_b} = 97.7 \qquad \text{and} \qquad \frac{m_{\tau'}}{m_\tau} = 267.3\,. \tag{66}$$

Choosing nearly symmetric shift values naturally provides nearly equal Yukawa couplings for the b and τ, so that b-τ unification may be naturally achieved. Since $\epsilon^u = \epsilon^d - \epsilon^l = 0.25$, one obtains a value of ϵ^u near a maximum of Fig 1. One then obtains the phenomenologically acceptable value

$$\frac{m_{t'}}{m_t} = 2.62\,. \tag{67}$$

Furthermore, noting that

$$\frac{m_{\nu'}}{m_{t'}} = \left| \frac{x_0^\nu v_0^u + x_1^\nu v_1^u + x_2^\nu v_0^u + x_3^\nu v_3^u}{x_0^u v_0^u + x_1^u v_1^u + x_2^u v_0^u + x_3^u v_3^u} \right| , \tag{68}$$

where x_i^ν and x_i^u differ only by the use of the shift parameters $\epsilon^\nu = 0$ and $\epsilon^u = 0.25$, respectively, one finds

$$\frac{m_{\nu'}}{m_{t'}} = 0.924\,. \tag{69}$$

Using the tabulated masses $m_{t,b,\tau}$ [46] and these ratios, one obtains $m_{t'} = 452.5$ GeV, $m_{b'} = 409.5$ GeV $\to m_{t'} - m_{b'} = 43.0$ GeV, and $m_{\tau'} = 474.9$ GeV, $m_{\nu'} = 418.1$ GeV $\to m_{\tau'} - m_{\nu'} = 56.8$ GeV, meaning that all the mass constraints listed in the Introduction are satisfied.

In addition, no mixing occurs between the third- and fourth-generation fermions since, as discussed above, the D-branes have only been split on the first two-torus. We also note that one can split the D-branes on the third two-torus as well, without inducing mixing between the third and fourth generations. Doing so does not affect the mass ratios m_3^f/m_4^f for each type f of fermion. However, splitting on the third two-torus does affect the relative mass hierarchy between the up-type quarks and the neutrinos, as well as the between the down-type quarks and charged leptons. The suppression of fourth-generation quark mixing implies long quark lifetimes and the possibility of hadronic bound states [52], as discussed in [53, 54].

As seen in this analysis, one can obtain mass splittings within each generation in a fairly natural way, since the mass ratios involve superpositions of ϑ functions that have convenient symmetry properties. As a result, the mass ratios between the third- and fourth-generation fermions exhibit maxima and minima as a function of shift parameter ϵ. Note that generic points reflect fully rank-2 Yukawa matrices, while the minima correspond [according to Eq. (54)] to points where the Yukawa matrices degenerate to rank 1. This tendency toward Yukawa matrices of lower rank was also observed in the rank-4 model of [15]. The mass ratio $m_t/m_{t'}$ is constrained to be much larger than the ratio between $m_b/m_{b'}$, $m_\tau/m_{\tau'}$, and especially $m_{\nu_\tau}/m_{\nu'}$. However, one sees that these ratios can be obtained if the shift parameter ϵ^u is chosen to lie near a maximum of the mass ratio curve, while the shift parameters ϵ^d, ϵ^l, and ϵ^ν correspond to minima. For the present model, the brane shift parameters ϵ^u, ϵ^d, ϵ^l, and ϵ^ν are continuous parameters constrained by Eq. (57).

IV. CONCLUSION

We have presented a globally consistent four-family MSSM constructed from intersecting D6 branes wrapping cycles on a $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. In contrast to other similar constructions, the Higgs fields appear in the chiral sector rather than as vectorlike matter that occurs between nonintersecting D branes. We find the gauge couplings to be unified at the string scale and the hidden-sector gauge groups to become confining in the range 10^{13}–10^{16} GeV. Thus, the model is a four-generation MSSM at low energies.

The Yukawa matrices of the model are rank 2, so that only the third- and fourth-generation fermions receive masses at trilinear order. The first two generations, in principle, receive masses via higher-order corrections, and so should be naturally lighter. This result contrasts sharply with that of our earlier rank-4 model [15], in which only small regions of the VEV parameter space accommodate the known fermion mass and mixing spectrum. In addition, the third- and fourth-generation fermions have nondegenerate masses but do not mix at leading order. Finally, a numerical analysis of the mass ratios between the third and fourth generations shows that simple, elegant, and fairly natural solutions satisfying all constraints are relatively easy to find. In particular, we have shown that one can obtain three light neutrinos and a heavy fourth, while simultaneously obtaining heavy t and t' quarks in a completely natural way. In addition, one may easily obtain masses for the b' quark and τ' lepton that are heavy in comparison to the b quark and τ lepton, respectively. Moreover, b-τ unification may be naturally realized. We conclude that our analysis demonstrates that the putative fourth-generation fermion masses required to satisfy the constraints placed upon them by direct experimental observations, precision electroweak measurements, and perturbative unitarity constraints can emerge in a completely natural fashion. The very real possibility that a fourth generation of chiral fermions exists deserves serious consideration, one that will be settled conclusively at the LHC.

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