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Chiral Quark Dynamics and Topological Charge: The Role of the Ramond-Ramond $U(1)$ Gauge Field in Holographic QCD

H. B. Thacker*, Chi Xiong† , and Ajinkya S. Kamat‡

Department of Physics, University of Virginia,

P.O. Box 400714 Charlottesville, VA 22901-4714

The Witten-Sakai-Sugimoto construction of holographic QCD in terms of D4 color branes and D8 flavor branes in type IIA string theory is used to investigate the role of topological charge in the chiral dynamics of quarks in QCD. The QCD theta term arises from a compactified 5-dimensional Chern-Simons term on the D4 branes. This term couples the QCD topological charge to the Ramond-Ramond $U(1)$ gauge field of IIA string theory. For large N_c the contribution of instantons (D0 branes) is suppressed, and the nonzero topological susceptibility of pure-glue QCD is attributed to the presence of D6 branes, which constitute magnetic sources of the RR gauge field. The topological charge of QCD is required, by an anomaly inflow argument, to coincide in space-time with the intersection of the D6 branes and the D4 color branes. This clarifies the relation between D6 branes and the coherent, codimension-one topological charge membranes observed in QCD Monte Carlo calculations. Using open-string/closed-string duality, we interpret a quark loop (represented by a D4-D8 open string loop) in terms of closed-string exchange between color and flavor branes. The role of the RR gauge field in quark-antiquark annihilation processes is discussed. RR exchange in the s-channel generates a 4-quark contact term which produces an η' mass insertion and provides an explanation for the observed spin-parity structure of the OZI rule. The $(\log \mathrm{Det\ U})^2$ form of the $U(1)$ anomaly emerges naturally. RR exchange in the t-channel of the $q\overline{q}$ scattering amplitude produces a Nambu-Jona Lasinio interaction which may provide a mechanism for spontaneous breaking of $SU(N_f) \times SU(N_f)$.

I. INTRODUCTION

In the holographic construction of QCD-like gauge theories from type IIA string theory [1, 2], the elementary fields of the gauge theory are associated with the low-lying spectrum of open strings on N_c coincident D4 branes and N_f coincident D8 branes whose intersection is 4-dimensional spacetime. Gluons and quarks are represented by open D4-D4 and D4-D8 strings, respectively. In

* email: hbt8r@virginia.edu
† email: cx4d@virginia.edu
‡ email: ask4db@virginia.edu

the closed string sector, the $U(1)$ gauge field associated with the massless states of the Ramond-Ramond (RR) string plays a special role in reproducing the low energy chiral dynamics associated with topological charge in QCD [1]. In the holographic theory, the QCD theta term arises from a 5-dimensional Chern-Simons term which couples the RR gauge potential to the topological charge of the color gauge field. A very useful way of understanding the origin of this Chern-Simons term and its implications for low energy hadron physics is to interpret it in terms of anomaly inflow [3, 4]. By standard arguments, the coupling of the RR field to the color gauge field on the D4 branes is dictated by the requirement that chiral anomalies on the brane, associated with the topological charge of the gauge field, should be cancelled in the higher dimensional bulk theory by the inflow of Ramond-Ramond flux. Equivalently, in the presence of D-branes, the equations of motion and Bianchi identity for the RR field must be modified to include electric and magnetic source terms on the world volume of the branes. The form of these source terms is dictated by the anomaly inflow argument, which determines the $U(1)$ gauge variation of the RR field that must accompany a Yang-Mills gauge transformation in order to define a gauge invariant RR field strength. In terms of the RR potential C_1, the gauge invariant field strength is no longer just dC_1, but must include a Yang-Mills source term on the D4 branes proportional to the Chern-Simons current of the color gauge field.

In order to account for nonzero topological susceptibility, there must be a $q^2 = 0$ pole in the correlation function of two Chern-Simons currents $\langle K_\mu(x)\ K_\nu(0)\rangle$, with residue equal to χ_t [5], because $\partial^\mu K_\mu$ is the topological charge density. This does not imply a massless physical particle, since the Chern-Simons current is not gauge invariant. On the other hand, this pole does have direct physical implications, in that it gives rise to a delta-function contact term in the gauge invariant topological charge correlator. Such a contact term is expected on general principles [6]. Since all real intermediate state contributions to χ_t (as computed from the integrated Euclidean correlator) are negative, the correlator must be dominated by a positive contact term. In terms of a dispersion relation, the contact term represents the necessary subtractions required to render the sum over the tower of massive glueball states convergent, and can thus be viewed as a short-distance effect arising from the sum over massive closed string states. The sum over closed string states in the calculation of the η' mass has been studied in [7, 8]. In Ref. [8] it was also noted that the low-lying glueball states give a negative contribution to $m^2_{\eta'}$, so the contact term must be positive and quantitatively dominant at $q^2 \to 0$ in order to obtain positive susceptibility.

The presence of a contact term in the QCD topological charge correlator has been confirmed by lattice calculations [9, 10]. The Monte Carlo results showed that this contact term is the result of a laminated vacuum structure consisting of juxtaposed opposite-sign codimension one sheets of topological charge, essentially dipole layers [9, 11, 12]. Such a topological charge distribution is just what one would expect from the domain wall excitations and discrete "k=vacua" associated with D6 brane excitations in the holographic formulation [1]. In the 4-dimensional Yang-Mills framework, these are the "Wilson bag" excitations discussed by Luscher [5] which correspond to a codimension one surface of Chern-Simons 3-form flux. In this paper, we investigate the contribution of D6 brane/Wilson bag excitations to the topological susceptibility of QCD, the η' mass, and to short-range quark-quark interactions.

From a holographic viewpoint, the $q^2 = 0$ pole required in the Chern-Simons current correlator to obtain finite topological susceptibility is cancelled in gauge invariant amplitudes by the exchange of a massless (unphysical) longitudinal Ramond-Ramond gauge boson. This constitutes a 4D Yang-Mills generalization of the Kogut-Susskind dipole cancellation for the 2D Schwinger model. On the I2 brane intersection of a D6 brane with the D4 branes, a gauge invariant excitation consists of the D6 brane itself and the Yang-Mills gauge field excitation of the D4 branes that accompanies it. In the 2D Schwinger model, the cancellation takes place between the Chern-Simons pole and the pole of an auxiliary massless Goldstone field (defined in terms of the conserved but non-gauge invariant axial vector current without the anomaly term). In holographic QCD, the RR field serves a similar purpose to that of the massless Goldstone field in the covariant gauge Schwinger model solution. It introduces a separation between the charged object (a bare fermion in the Schwinger model or a D6 brane in QCD) and the Chern-Simons excitation that accompanies it. Since the spectrum of the 4-dimensional Yang-Mills theory should consist only of massive glueball states, there should be no massless poles in gauge invariant amplitudes. On the other hand, there must be a $q^2 = 0$ pole in the Chern-Simons correlator (because $\chi_t \neq 0$). This requires a cancelling pole in the Ramond-Ramond correlator on the D4 brane world volume. In pure glue QCD at low energy, the physical manifestation of these cancelling pole terms is the appearance of a contact term in the gauge invariant topological charge corrrelator, obtained from derivatives acting on the Chern-Simons pole.

It should be emphasized that, just as the $q^2 = 0$ pole in the C-S current correlator [5] does not imply a massless particle in 4-dimensional Yang-Mills theory, the pole in the RR correlator does not correspond to a physical massless closed string state in the near-horizon geometry of the D4 branes in the string theory. In both cases, the massless poles appear in the correlators of non-gauge-invariant operators. In this paper we study the dynamics of topological charge fluctuations in the QCD vacuum in the large N_c limit which correspond to the appearance of D6 brane and anti-D6 brane sources in the Witten-Sakai-Sugimoto model [1, 22]. As shown in Ref. [4] in this situation, the bulk Ramond-Ramond field is not by itself gauge invariant, but must include a Yang-Mills source term that is nonvanishing on the intersection between the D4 and D6 branes. (This takes the form of an anomalous term in the Bianchi identity for the RR field strength, similar to a Dirac monopole). There is no massless pole in the gauge invariant correlator by virtue of a cancellation between $q^2 = 0$ poles in the bulk RR correlator (on the I2 brane surface) and the Chern-Simons correlator. Note that, even though there is a mass gap, the $q^2 = 0$ pole in the CS current correlator must be present in order to have nonzero topological susceptibility. There must therefore also be a $q^2 = 0$ pole in the (non gauge invariant) RR correlator, in order to provide a Kogut-Susskind dipole cancellation [14] in gauge invariant amplitudes. The pole in the RR correlator arises not from the potential C_1 (which couples to gauge invariant topological charge density) but from the magnetic dual potential C_7, which couples minimally to D6 branes. As we will show, this RR pole couples to the chiral field only through derivative couplings, thereby reducing the exchange interaction to a local contact term. This is precisely analogous to the 2D Schwinger model. In that model (e.g. in Coulomb gauge) the local contact term that gives the η' a mass is generated by a long-range Coulomb potential with derivative couplings. Upon integration by parts, the long range interaction reduces to a local η' mass term because the linear potential satisfies $\partial_x \partial_y V(x-y) \propto \delta(x-y)$.

We find other interesting phenomenological effects of RR exchange by considering open-string/closed-string duality (OS/CS) for quark loops. The quark is represented by an open string attached at one end to a color D4 brane and at the other end to a flavor D8 brane. A closed quark loop in the field theory is represented by a one-loop open string diagram which can be reinterpreted via OS/CS duality as the tree-level exchange of a closed Ramond-Ramond string between the flavor branes and the color branes (see Fig. 1). This provides an interesting separation between the flavor and color structure of the quark loop. This closed-string description of a quark loop is particularly appropriate for discussing its ultraviolet behavior, which is determined in the string theory by low-energy structure in the closed string channel. Just as anomaly inflow arguments can be used to determine the coupling of the RR field to the color branes, similar arguments determine the RR coupling to the flavor branes [2]. In this case, anomaly inflow relates the RR gauge field to the $U(1)$ chiral Goldstone (η') field, giving the familiar equivalence between a shift of θ and a chiral rotation. By arguments similar to those leading to the contact term in the topological charge correlator, we show that RR exchange between quark lines induces an effective 4-quark contact term. In the flavor singlet channel for quark-antiquark scattering, s-channel RR exchange provides the η' mass insertion quark diagram ("double hairpin" graph, Fig. 3). In addition to reproducing the Witten-

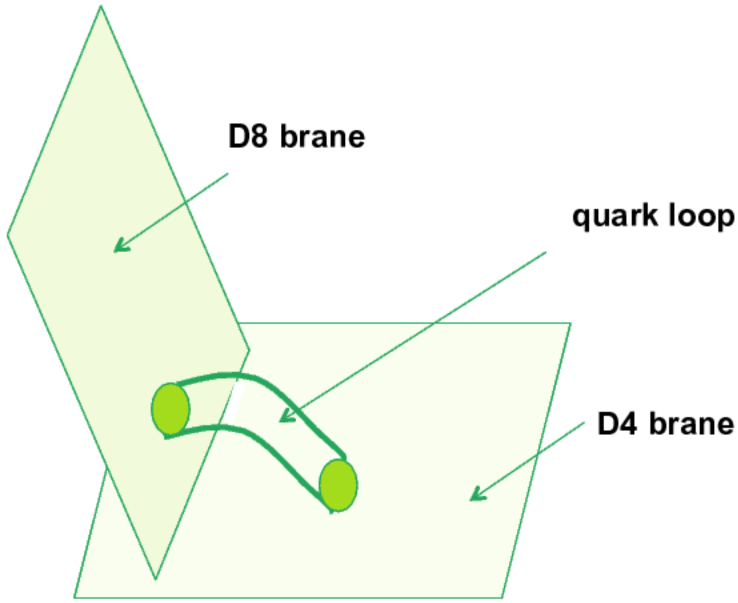


FIG. 1: An open string quark loop as a closed string exchange between D4 and D8 branes.

Veneziano relation for the η' mass [15, 16], the RR exchange picture provides an explanation of the pure double pole behavior observed in Monte Carlo calculations of the quenched double-hairpin correlator [17]. It has long been known that at large N_c, chiral Lagrangian arguments [18–20] lead to a specific form for the effective action term which correctly describes the axial $U(1)$ anomaly. This term must have the form of a pure η' mass insertion:

$$\mathcal{L}_a \propto -(\log \mathrm{Det} U - \log \mathrm{Det} U^\dagger)^2 \tag{1}$$

where $U_{ab} \propto \overline{q}_a(1+\gamma_5)q_b$ is the $N_f \times N_f$ chiral field. This large N_c expectation is quite distinct from the effective action induced by instantons, which would be proportional to the chiral determinant itself and therefore include OZI-violating six-quark interactions. We will show that an effective action of the form (1) is exactly what is expected from the 4-quark contact term generated by RR exchange. This follows essentially from the fact that the RR field couples, by anomaly inflow, to the $U(1)$ phase of the chiral field. Thus, the anomaly term (1) is interpreted as describing a contact term generated by RR exchange between two chiral densities.

In addition to its role in QCD topological susceptibility and the η' mass insertion, RR exchange may also play a direct role in spontaneous chiral symmetry breaking and the formation of the chiral condensate. The η' mass insertion arises from the q-$\overline{q}$ annihilation diagram (Figure 3), i.e. s-channel RR exchange. This diagram appears only in the flavor singlet channel and explicitly breaks axial $U(1)$ symmetry. But there is also a 4-quark contact term generated by RR exchange in the t-channel of the quark-antiquark amplitude (Figure 4). This interaction is the same for flavor singlet and nonsinglet channels. It is a $U(N_f) \times U(N_f)$ conserving Nambu-Jona Lasinio-type interaction, suggesting that RR exchange may be the driving force behind $S\chi SB$. Finding that topological fluctuations play a role in the formation of the chiral condensate would not be that surprising. An analogous mechanism in the framework of the instanton liquid model has been discussed [21]. In that model, the 'tHooft near-zero modes of the instantons provide the eigenstates occupied by the quarks in the chiral condensate. Similarly, in a vacuum of topological charge membranes (D6 branes), the chiral condensate forms from the surface modes of the quarks on the codimension one brane surfaces. RR exchange in the t channel provides an attractive interaction between a D6 brane and a neighboring anti-D6 brane, and hence an attractive quark-antiquark interaction.

The holographic framework and the connection between QCD topological charge and Ramond-Ramond charge highlights an old and long-discussed dichotomy between two rather different scenarios for the origin of topological susceptibility in QCD [22], one based on instantons and one based on large-N_c chiral Lagrangian arguments. In the large-N_c framework, topological gauge excitations are associated with the existence of quasistable "k-vacua", where the effective local value of the θ parameter is shifted by $2\pi k$ with k an integer. These quasivacua are separated by codimension one membranes (domain walls), where the local value of the θ parameter changes by $\pm 2\pi$ across a domain wall. Witten has argued [22] that for large N_c, the contribution of instantons is exponentially suppressed and that the leading $O(1/N_C)$ contribution to χ_t must come from gauge field fluctuations associated with these domain walls, which appear in the holographic framework as the intersection (I2 brane) of the color D4 branes with a D6 brane [1]. In the 4-dimensional Yang-Mills theory, the step in θ is equivalent to a codimension one sheet of Chern-Simons 3-form flux of the color gauge field. The topological charge excitation that arises from the presence of a θ domain wall is a dipole layer with opposite sign sheets of topological charge juxtaposed on opposite sides of the Chern-Simons sheet. (Such a structure closely resembles what is seen in Monte Carlo studies [9, 12].) The appearance of codimension one topological charge membranes separating discrete vacua was originally suggested by large N_c chiral Lagrangian arguments. In terms of the chiral field U for a theory with one or more light quarks, the correct form of the axial U(1) anomaly is a pure η' mass term (1) [18–20]. The existence of multiple discrete vacua in QCD is a direct reflection of the multibranch structure of the log in the η' mass term. [This is in contrast to the 'tHooft vertex induced by an instanton, which is proportional to Det U itself and not multivalued.] It was later shown [1] that the codimension one membranes expected from large N_c arguments had a natural holographic interpretation as $D6$ branes wrapped around a compact S_4, which appear as 2+1 dimensional membranes in spacetime. From the holographic view, the two scenarios for generating topological susceptibility correspond to an "electric" condensate of instantons or $D0$ branes (which are electric sources of RR field), and a "magnetic" condensate of $D6$ branes.

Both the instanton and codimension one membrane scenarios are realized in well-studied two-dimensional models [22]. For example, the topological susceptibility in the U(1) Higgs model can be analyzed in terms of instantons. On the other hand, the massive Schwinger model has finite topological susceptibility but instantons do not appear anywhere in the theory. The mechanism for generating χ_t in the Schwinger model is a 2-dimensional analog of the multiple discrete k-vacua in 4-dimensional QCD. In the 2-dimensional model, the θ parameter may be interpreted as a background electric field [23]. In one spatial dimension, a pointlike charged particle plays the role of a domain wall separating vacua that are characterized by different values of background electric flux on opposite sides of the charge. The flux in these adjacent vacua differs by one unit by Gauss's law. The unit step in electric field across a charged particle corresponds to a 2π step in θ. To see this consider a θ term in 2D Euclidean space where $\theta = \theta_0$ inside a closed region R bounded by a contour $\mathcal{C} = \partial R$, and zero outside $\mathcal{C}$. In 2D, the Chern-Simons current is $K_\mu = \epsilon_{\mu\nu}A^\nu$, and the topological charge is $\partial^\mu K_\mu = \frac{1}{2}\epsilon_{\mu\nu}F^{\mu\nu}$. Thus a θ term with a nonzero constant θ_0 inside the contour $\mathcal{C}$ and zero outside can be written as a 1D surface integral of the Chern-Simons current, which is just an ordinary Wilson loop operator around the boundary $\mathcal{C} = \partial R$,

$$\frac{\theta_0}{2\pi}\int_R d^2x\epsilon_{\mu\nu}F^{\mu\nu} = \frac{\theta_0}{2\pi}\oint_{\mathcal{C}} A^\mu dx_\mu \tag{2}$$

The charge that appears in the exponentiated Wilson loop is $\theta_0/2\pi$, so a Wilson loop with unit charge corresponds to a step in θ of $\pm 2\pi$ across the boundary. For the Schwinger model, the $q^2 = 0$ pole in the Chern-Simons correlator which leads to nonzero χ_t is just the photon propagator. Photon exchange produces a linear confining potential between charges on opposite sides of the Wilson loop. The difference in energy between the $k = 0$ vacuum outside the loop and the $k = \pm 1$ vacuum with a unit of flux inside the loop determines the topological susceptibility, which is just given by the strength of the linear potential. Of course there is no propagating photon in 2D. In a covariant treatment, the photon pole cancels against another unphysical $q^2 = 0$ pole corresponding to a decoupled Goldstone boson. What remains is a physical massive η' meson. This is the

"Kogut-Susskind dipole" mechanism. The η' mass term is the physical remnant of the long range Coulomb interaction between charges, as can be seen by bosonizing the fermion current, writing $\bar{\psi}\gamma_\mu\psi = \epsilon_{\mu\nu}\partial^\nu\eta'$. The Coulomb interaction reduces to a local η' mass term when integrated by parts,

$$\int dx dx' \partial_x \eta' |x - x'| \partial'_x \eta' = 2 \int dx \eta'^2 \tag{3}$$

In 4-dimensional Yang-Mills theory, the analog of the Wilson loop (2) in 2D U(1) gauge theory is the "Wilson bag" operator [5], which is the integral of the 3-index Chern-Simons tensor over a 3-dimensonal surface $\mathcal{C}$. In the holographic description, this 3-dimensional surface is an I2-brane, i.e. the intersection of a D6 brane and the color D4 branes. The $q^2 = 0$ pole in the Chern-Simons correlator represents a linear potential between sides of the bag and gives a "volume law" for Wilson bags which is equivalent to finite χ_t. Witten has argued [1, 22] that for 4D $SU(N_c)$ gauge theory in the large N_c limit, instantons are exponentially suppressed and topological structure is dominated by multiple discrete k-vacua and domain walls between them.

In the holographic framework, the contrast between the instanton (D0 brane) condensate and the codimension one membrane (D6 brane) condensate serves to emphasize the very different role played by the electric (C_1) and magnetic (C_7) Ramond-Ramond potentials in the two cases. The θ term in QCD arises from a 5-dimensional Chern-Simons term which couples the QCD topological charge to the C_1 potential,

$$\mathcal{L}_\theta = \int_{D4} C_1 \wedge \mathrm{Tr}(F \wedge F) \tag{4}$$

Instantons or D0 brane excitations are localized in 4-dimensional spacetime, and they generate a contact term (not a pole) in the $C1$ correlator. In fact even in the D6 brane condensate, one does not expect a $q^2 = 0$ pole in the C_1 correlator, because it couples to the gauge invariant topological charge $\mathrm{Tr}(F \wedge F)$. In the presence of D6 branes, the contact term in the topological charge correlator derives from a $q^2 = 0$ pole in the Chern-Simons correlator. The $q^2 = 0$ Ramond-Ramond pole that cancels the pole of the Chern-Simons correlator in gauge invariant amplitudes arises not from C_1 but from the magnetic Ramond-Ramond potential C_7. It is this potential which is minimally coupled to D6 branes. On the 3-dimensional I2 brane defined by the intersection of the D6 brane with the color D4 branes, the anomaly inflow constraint requires that the gauge variation of the bulk RR field strength dC_7 on the I2 brane surface under a Yang-Mills transformation must cancel the gauge variation of the Chern-Simons tensor. This equates the gauge variation of dC_7 to that of an 8-form $\delta_{D4} \wedge \mathcal{K}_3$. Here δ_{D4} is a 5-form delta function in the coordinates transverse to the

D4 branes, and $\mathcal{K}$ is the Chern-Simons 3-form. The $q^2 = 0$ pole in the dC_7 correlator plays the same role as the unphysical massless Goldstone pole in the Kogut-Susskind mechanism, i.e. in gauge invariant amplitudes it cancels the pole in the Chern-Simons correlator. Note that in the presence of D6 branes (magnetic RR sources), the RR field strengths dC_1 and $dC_7 = *(dC_1)$ are simply related, but there is no simple local relationship between potentials C_7 and C_1, so the C_7 pole does not contradict our expectation that C_1 will have no $q^2 = 0$ pole, since it couples directly to topological charge rather than to the Chern-Simons form.

We note that the analysis of the topological susceptibility and η' mass generation in Ref. [2] in terms of the coupling of C_1 to $\mathrm{Tr}(F \wedge F)$ applies to either mechanism for generating topological excitations, i.e. whether they are generated by D0 branes or D6 branes. In either case, the topological charge correlator is dominated by a positive contact term, and as discussed in [6], this is all that is necessary to give the Witten-Veneziano relation between χ_t and $m_{\eta'}$.

II. INSTANTONS AND TOPOLOGICAL CHARGE MEMBRANES AS ELECTRIC AND MAGNETIC SOURCES OF RAMOND-RAMOND FIELD

The origin of the QCD theta term was first discussed in the holographic framework in Ref. [1]. In Witten's construction, the color D4 branes are compactified around an S_1 with SUSY breaking boundary conditions. The QCD theta term arises from a 5-dimensional Chern-Simons term which couples the Wilson line of the RR gauge potential C_1 around the compact S_1 to the topological charge density of the 4-dimensional gauge field,

$$S_{CS} = \int_{D4} C_1 \wedge Tr\,(F \wedge F) \quad . \tag{5}$$

Here, Tr is a trace over color indices. As shown in Ref. [4] and discussed below, the form of the Chern-Simons term is dictated by anomaly inflow arguments. In the full IIA string theory, this term represents the fact that fluctuations of the color gauge fields on the D4 brane can absorb and emit closed RR string states which propagate in the bulk. In the field theory limit, after compactification around the S_1 (with the circumference of the S_1 playing a role analogous to lattice spacing in lattice QCD), the surviving part of this Chern-Simons term is proportional to the RR Wilson line around the compact direction,

$$\int_{S_1} C_1 \equiv \theta(x) \tag{6}$$

Here we denote 4-dimensional spacetime coordinates by x. If θ is a spacetime constant θ_0, the CS term reduces to a QCD theta term,

$$S_{CS} \to \theta_0 \int_{R_4} Tr(F \wedge F) \tag{7}$$

where R_4 is 4-dimensional spacetime. As shown by Witten [1, 20], large N_c chiral Lagrangian arguments suggest that the physical vacuum in 4-dimensional gauge theory should exhibit domain walls between discrete θ-vacua, i.e. codimension one surfaces where the effective local value of the θ parameter jumps by $\pm 2\pi$. In the Witten D4 brane construction of holographic QCD, these domain walls are identified as D6 branes wrapped around S4. The anomaly inflow picture discussed here shows that the 4D Yang-Mills topological charge should be localized on the codimension one membranes represented by the intersection of D4 and D6 branes. This can be contrasted with an instanton liquid model, which would correspond to D0 brane excitations [24]. With respect to the 10-dimensional RR U(1) gauge field, D0 branes and D6 branes can be regarded as electric and magnetic sources, respectively. In the D6 brane condensate model we consider here, the transition from the perturbative to the physical, confining vacuum is caused by a condensation of magnetic Ramond-Ramond charge [13]. In the large-N_c physical vacuum, populated by D6 branes, the role of the Chern-Simons term (5) is crucial even when the overall QCD theta parameter is set to zero. In fact, in the presence of magnetic D6 brane sources, the RR potential C_1 cannot be uniquely specified globally, but must be defined in terms of overlapping sections, as with Dirac monopoles in 4D Maxwell theory. Because of this nonuniquenes of C_1, it is more meaningful to write the Chern-Simons term in the form obtained by integrating by parts,

$$S_{CS} \to -\int_{D4} dC_1 \wedge \mathcal{K} \tag{8}$$

where $\mathcal{K}$ is the Chern-Simons 3-form satisfying $d\mathcal{K} = Tr(F \wedge F)$. Specifically,

$$\mathcal{K} = Tr\left(A \wedge F - \frac{1}{3} A \wedge A \wedge A\right) \tag{9}$$

Here and elsewhere the color gauge field potential on the D4 branes is denoted by A. The D8 brane gauge potential will be called $\mathcal{A}$. We associate the 3-form (9) with the 3-dimensional intersection between the D4 branes and the D6 brane. We will choose axes in 4D spacetime such that the D4-D6 "I-brane" intersection spans the x_0, x_1, and x_2 coordinates, with x_3 being the spacetime direction transverse to the I-brane (and hence transverse to the topological charge membranes of the color gauge field). (From detailed Monte Carlo studies [9, 11, 12, 25], the topological charge membranes are expected to be locally flat over a distance scale comparable to the confinemenet

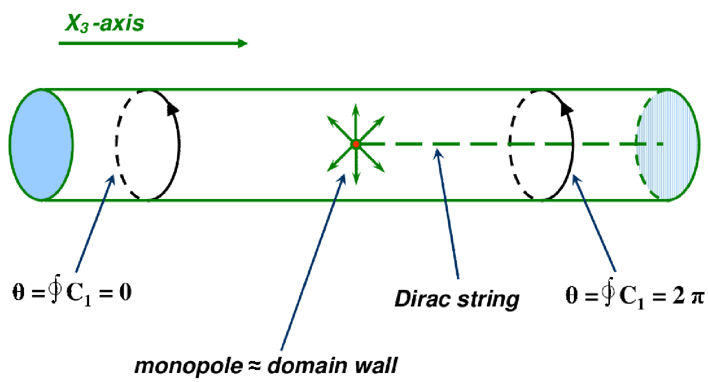


FIG. 2: View of a D6 brane as a magnetic monopole of the Ramond-Ramond gauge field in the 3 dimensions transverse to its worldvolume.

scale. Over larger distances, the branes bend and fold and their orientation decorrelates.) In the construction of Ref. [1], the three dimensions transverse to the D6 brane consist of the holographic (radial) direction x_5, the compact S_1 (angular) direction x_4, and one spacetime direction, which we take to be x_3. This 3D space can be represented as a solid cylider, as depicted in Fig. 2. The D6 brane is a point magnetic RR charge in this space, and the construction of a nonsingular gauge potential which is continuous away from the source requires C_1 to be defined separately on two disks or hemispheres on opposite sides of the D6 brane and matched together by a topologically nontrivial $U(1)$ gauge transformation around the equator, leading to the quantization of D6 brane charge. By Eq. (6), this quantization enforces the fact that the value of $\theta(x)$ on opposite sides of the D6 brane must differ by an integer multiple of 2π. So, rather than reducing to a constant theta parameter in the field theory limit, $\theta(x)/2\pi$ becomes an integer valued field which jumps by ±1 at the location of a D6 brane. With the Chern-Simons action written in the form (8), the codimension one topological charge membrane in the color gauge field appears at the intersection of the D6 brane and the D4 branes, where low-mass open D4-D6 strings reside. Open-string/closed-string duality implies a coupling due to massless RR exchange between D4 and D6 brane. Just as an instanton in the gauge theory can be interpreted as a D0 brane bound to the D4 color branes [24, 26], the topological charge membrane can be seen as a D6 brane (wrapped on S_4) bound to the color branes.

III. ANOMALY INFLOW AND RAMOND-RAMOND COUPLINGS

The Chern-Simons 3-form $\mathcal{K}$ in (8) is not invariant under a color gauge transformation. To maintain overall gauge invariance, we must assume that a color gauge transformation is accompanied by a $U(1)$ transformation of the RR gauge field C_1. The RR field strength G_2 satisfies an anomalous Bianchi identity, $G_2 \neq dC_1$, which includes a magnetic source term arising from the color gauge field,

$$G_2 = dC_1 - \mu\left[*(\mathcal{K} \wedge \delta_{D_4})\right] \tag{10}$$

Here, δ_{D_4} is a 5-form given by a product of delta functions in the coordinates transverse to the D4 brane, so the second term in (10) is nonvanishing only on the D4 brane surface. In this term the $*$ represents the Hodge dual in 10-dimensional space of the 8-form $\mathcal{K} \wedge \delta_{D_4}$. It is a 2-form with indices that are both on the D4 branes and transverse to the 0-1-2 I-brane. Thus the source term from the D6 brane contributes only to the 3-4 component of the Ramond-Ramond field. μ is fixed by the quantization condition. In terms of the D4 brane tension T_4 it is given by $\mu = T_4/16\pi^2$. Under a Yang-Mills transformation $A \rightarrow g^{-1}Ag + g^{-1}dg$, the RR field transforms in a manner dictated by the descent equations [4]. This is most easily expressed in terms of the dual magnetic RR field strength $G_8 = *(G_2)$ and associated potential C_7,

$$G_8 = dC_7 - \mu(\mathcal{K} \wedge \delta_{D_4}) \tag{11}$$

It is the potential C_7 that couples directly (i.e. minimally) to the 7-dimensional world volume of the magnetically charged D6 branes. Under a Yang-Mills transformation g, this transforms as

$$\delta(dC_7) = \mu(\delta\mathcal{K} \wedge \delta_{D_4}) \tag{12}$$

The explicit transformation of the Chern-Simons 3-form is given by

$$\delta\mathcal{K} = \mu\left[d\,Tr(dg\,g^{-1} \wedge A) \;+\; \frac{1}{3}Tr(g^{-1}dg \wedge g^{-1}dg \wedge g^{-1}dg)\right] \tag{13}$$

The second term in (13) is proportional to the winding number density of the 3-dimensional gauge transformation on the I-brane,

$$w(x) = \frac{1}{24\pi^2}Tr(g^{-1}dg \wedge g^{-1}dg \wedge g^{-1}dg) \tag{14}$$

With appropriate boundary conditions on g, $w(x)$ integrates to an integer over the 3-dimensional I-brane. The quantization of this Chern-Simons winding number term incorporates the 2π step function discontinuity in $\theta(x_3)$ associated with the presence of a D6 brane. A Yang-Mills transformation g with nonzero winding number on the 3-dimensional I-brane describes the matching of gauges on the two sides of the brane needed to implement the anomalous Bianchi identity for the RR U(1) field.

The gauge invariance of the RR field strength G_2, Eq.(10), ties the behavior of the bulk field dC_1 to fluctuations of the Wilson bag operator $\mathcal{K}$ on the D4 brane world volume. To see the implications of this connection for the 4-dimensional gauge theory, let us consider the D4 brane gauge theory in the presence of a D6 brane source. In the field theory limit, the relevant component of the RR field strength is $(G_2)_{34}$ where x_3 is the direction in 4-dimensional spacetime transverse to the D6 brane, and x_4 is the compact S_1 direction. Integrating (10) around S_1, defining $\theta(x)$ by (6), and writing indices explicitly, we have

$$\oint_{S_1} (G_2)_{34} = \partial_3\theta - K_3 \tag{15}$$

where we define the Chern-Simons current of the 4D Yang-Mills field ($\mu = 0, 1, 2, 3$)

$$K_\mu \equiv \epsilon_{\mu\nu\lambda\tau} Tr\left(A^\nu F^{\lambda\tau} - \frac{1}{3}A^\nu A^\lambda A^\tau\right) \tag{16}$$

satisfying

$$\partial^\mu K_\mu = \epsilon_{\mu\nu\sigma\tau} Tr(F^{\mu\nu}F^{\sigma\tau}) \tag{17}$$

The first term on the right hand side of (15) is a delta-function at the location of the D6 brane (because θ is a step function). A Yang-Mills gauge transformation induces a $U(1)$ gauge transformation of the second term in (15) which must be cancelled by the variation of the first term. Roughly speaking, a gauge invariant excitation consists of the charged D6 brane and its attached color gauge field (a topological charge membrane), which must fluctuate together to form a physical propagating wave. The mechanism by which this generates a contact term in the topological charge correlator and nonzero susceptibility is essentially the same as the Kogut-Susskind dipole mechanism in the 2-dimensional massive Schwinger model [14]. It is useful to recall the essential ingredients of this simple example of anomaly inflow to point out the parallels with the D6-brane/topological-charge system. In the 2D massive Schwinger model one has a choice of defining a conserved axial vector current j^5_μ which is not gauge invariant, or a gauge invariant current $\hat{j}^5_\mu$ which is not conserved:

$$\partial^\mu j^5_\mu = 0 \tag{18}$$

$$\partial^\mu \hat{j}^5_\mu = \epsilon_{\mu\nu}F^{\mu\nu} \tag{19}$$

The difference between the two can be obtained by a point-splitting regularization, where the gauge invariant current acquires an extra term from the gauge link between the fermion operators. This gives the correct form of the anomaly in 2D $U(1)$ gauge theory,

$$\hat{j}^5_\mu = j^5_\mu - \epsilon_{\mu\nu} A^\nu \tag{20}$$

In covariant Lorentz gauge quantization, the Hilbert space contains unphysical, negative metric states. In the chiral limit, the terms on the right hand side of (20) can be represented in terms of a pair of massless scalar fields ϕ_1 and ϕ_2, where ϕ_2 is a ghost field with negative norm. Physical states only couple to the gauge invariant combination $\phi_1 - \phi_2$. As a result, the massless poles in the ϕ_1 and ϕ_2 propagators always cancel in physical amplitudes. The physical meson has a finite mass due to the chiral anomaly (20). This is the Kogut-Susskind dipole mechanism. For 4-dimensional QCD, the anomaly inflow constraint on the RR field strength, Eq. (15), plays a role analogous to (20) in the Schwinger model. After introducing quark flavors (see Section IV), θ is identified with the U(1) chiral field η' via anomaly inflow on the D8 branes (c.f. Eq. (26)). The quantity $\partial_\mu\theta$ will be identified with the non gauge invariant flavor singlet axial vector current $\hat{j}^5_\mu - K_\mu$, so the first term on the right hand side of (15) is the analog of the first term in (20) for the Schwinger model. The form of the second term, which represents the gauge anomaly in these two equations, reflects the structure of a codimension one domain wall in the two cases. In 2D the discontinuity is represented by a Wilson line, while in 4D Yang-Mills, the D6 brane discontinuity is represented by a 3-dimensional Wilson bag integral [5].

By a generalization of the Kogut-Susskind pole cancellation mechanism, if we look at matrix elements of the gauge invariant field strength (15), the pole due to the massless RR gauge boson is cancelled by a "wrong sign" pole in the correlator of Chern-Simons currents K_μ,

$$\int d^4x e^{iqx} \langle (K_\mu(x) K_\nu(0) \rangle \overset{q\to 0}{\sim} \frac{q_\mu q_\nu}{(q^2)^2}\chi_t \tag{21}$$

Since $\partial^\mu K_\mu$ is the gauge invariant topological charge, the topological susceptibility χ_t is given exactly by the residue of the massless pole in the K_μ correlator (21). By the anomaly inflow cancellation, this residue must be equal in magnitude to the residue of the RR gauge boson pole. (Note that the "wrong sign" of the K_μ correlator that allows it to play the role of the Kogut-Susskind ghost is the same sign that allows it to contribute a positive contact term to the otherwise negative topological charge correlator.)

IV. THE RAMOND-RAMOND FIELD AND CHIRAL QUARK DYNAMICS

So far we have only considered the anomaly inflow on the D4 color branes due to a D6 brane source. The inclusion of D8 flavor branes into the system [2] allows us to consider the implications of the Ramond-Ramond gauge field for the chiral dynamics of quarks. This is most easily formulated in terms of the $U(N_f) \times U(N_f)$ chiral field constructed from quark bilinears,

$$U \propto \overline{q}(1 + \gamma_5)q \tag{22}$$

As discussed in Ref. [2], the chiral field is obtained from the Wilson line of the D8 brane gauge field taken along a U-shaped path on the 2-dimensional disk D starting on the D8 brane and ending on the $\overline{\text{D8}}$ brane at holographic infinity. Following Ref. [2] we choose coordinates y and z on the disk which are, respectively, transverse and parallel to the D8 brane. The chiral field is then give by the Wilson line of the D8 brane gauge field along the holographic direction,

$$U(x) = P \exp\left(-\int_{-\infty}^{\infty} dz' \mathcal{A}_z(x, z') \right) \tag{23}$$

Of particular interest is the flavor singlet η' meson field, which is given by the U(1) phase of the chiral field,

$$\eta'(x) = \frac{-i f_\pi}{\sqrt{2N_f}} \log \mathrm{Det} U = \frac{i f_\pi}{\sqrt{2N_f}} \int_{-\infty}^{\infty} dz' tr(\mathcal{A}_z) \tag{24}$$

(Throughout this paper, we use upper case Tr to denote color traces and lower case tr to denote a trace over flavor indices.) We have seen that an anomaly inflow requirement on the color branes led to the result that only the combination of operators in (10) is invariant under a color gauge transformation. A similar anomaly inflow argument on the flavor-brane end of the quark string leads to the usual identification of the chiral $U(1)$ phase and the theta parameter, which can be seen as follows: The gauge invariant Ramond-Ramond field strength in the presence of D8 branes is given by [2]

$$G_2 = dC_1 + i\, tr(\mathcal{A}) \wedge \delta_{D_8} \equiv dC_1 + i\, tr(\mathcal{A}) \wedge \delta(y) dy \tag{25}$$

(The factor $tr(\mathcal{A})$ here comes from the leading term in the expansion of the general Chern-Simons interaction of the form $dC_1 \wedge \mathrm{tr}\left(\mathcal{A} \wedge e^{i\mathcal{F}/2\pi}\right)$.) Note that here a quark is an electric source of G_2, while in (11), the topological charge membrane is a magnetic source of G_8. If we consider the component $(G_2)_{45}$ and integrate it over the 2-dimensional disk, we get

$$\int_D (G_2)_{45} = \theta + \frac{\sqrt{2N_f}}{f_\pi} \eta' \tag{26}$$

Thus the invariance of the RR field G_2 under a chiral gauge transformation on the D8 branes leads to the usual identification of θ with a chiral phase rotation. The Witten-Veneziano formula, relating the anomalous mass of the η' meson to the topological susceptibility χ_t of pure glue QCD is obtained in a straightforward way by the cancellation of massless poles required by anomaly inflow. As we have seen, the gauge invariance of the RR field strength on the color branes required a cancellation between the massless RR gauge boson pole in the dC_1 correlator and the massless pole in the $\langle K_\mu K_\nu \rangle$ correlator. The residue of the latter is the topological susceptibility χ_t. The expression (26) for the gauge invariant RR field strength on the flavor branes imposes an anomaly inflow cancellation between the massless RR pole in dC_1 and the Goldstone pole in the j_5^μ correlator. Working to leading order in $1/N_c$, we assume $f_{\eta'} = f_\pi$ and write

$$\langle 0|\hat{j}^5_\mu|\eta'\rangle = f_\pi p_\mu \tag{27}$$

so the residue of the Goldstone pole is $f_\pi^2 m_{\eta'}^2$. Equating the two residues gives the Witten-Veneziano relation

$$\chi_t = \frac{f_\pi^2 m_{\eta'}^2}{4N_f} \tag{28}$$

More directly, we may construct the effective action term that represents the effect of Ramond-Ramond exchange between quarks. The massless RR propagator in 4D momentum space has the same form as the Chern-Simons current correlator (c.f. Eq. (21))

$$\tilde{G}_{\mu\nu} = -\frac{q_\mu q_\nu}{(q^2)^2}\chi_t \tag{29}$$

In coordinate space, this satisfies

$$\partial^\mu \partial^\nu G_{\mu\nu}(x-y) = \chi_t \delta^4(x-y) \tag{30}$$

Combining the anomaly inflow constraints of (15) and (26) we see that the RR gauge boson couples to the η' field with an effective action

$$S_{int} = \frac{2N_f}{f_\pi^2}\int d^4x d^4y \partial^\mu \eta'(x) G_{\mu\nu}(x-y)\partial^\nu \eta'(y) = -\frac{2N_f \chi_t}{f_\pi^2}\int d^4x\, \eta'^2(x) \tag{31}$$

Thus, the RR exchange model reproduces the chiral Lagrangian form of the axial anomaly that was originally deduced from large N_c and OZI rule arguments [18–20], namely, a pure η' mass term,

$$\mathcal{L}_{int} = \frac{\chi_t}{4}\left(\log \mathrm{Det}U - \log \mathrm{Det}U^\dagger\right)^2 \tag{32}$$

The result (31) is the same as that obtained in Ref. [2], and in fact our derivation can be understood as a reformulation of the calculation in [2]. In [2] the η' mass term is obtained directly from the supergravity kinetic term $\propto (G_{\mu\nu})^2$ for the Ramond-Ramond field strength G_2, using (26), which gives a term in the effective action of the form

$$S_{C_1} = \frac{1}{2}\chi_t \int d^4x \left(\theta + \frac{\sqrt{2N_f}}{f_\pi}\eta'\right)^2 \tag{33}$$

With the anomaly inflow connection (26) between the θ parameter and the η' field, the derivation of the η' mass follows from the θ dependence of the vacuum energy. The correct form of the η' mass term thus follows from the quadratic θ dependence of the vacuum energy density

$$\mathcal{E}(\theta) = \frac{1}{2}\chi_t\theta^2 \tag{34}$$

But we can also obtain the θ-dependence of the vacuum energy by considerng the energy density inside a Wilson bag obtained by integrating the Chern-Simons 3-form over a closed codimension one surface $\mathcal{C}$ enclosing a 4-volume V. The massless pole in the Chern-Simons correlator produces a linear potential between opposite sides of the bag (i.e. a volume law $E \propto V$ for the Wilson bag energy) [5]. If the charge on the Wilson bag is taken to be $\theta/2\pi$, then the volume law for the Wilson bag gives $\mathcal{E}(\theta)$, which is proportional to the square of the bag charge and hence has the form (34). Thus the connection between our derivation of the result (31) and that of Ref. [2] can be understood as analogous to two equivalent ways of calculating the energy stored in a capacitor, either by integrating the Coulomb interaction between charges on opposite plates, or directly from the Maxwell action for the energy density of the electric field between the plates. This comparison also emphasizes the fact that the RR field strength provides a 4-dimensional generalization of the "background electric field" interpretation of the θ parameter that is intuitively useful in the 2D Schwinger model. That is, the domain walls carry RR charge and separate vacua which differ by one unit of RR flux.

Further information about the nature of quark-antiquark annihilation has been obtained from lattice studies of the spin-parity structure of the OZI rule [27]. It was found that, while the pseudoscalar hairpin correlator was easily measured and gave a reasonably accurate determination of the η' mass, the hairpin correlators in the vector and axial vector channels (e.g. $\langle \overline{q}\gamma^\mu q(x)\, \overline{q}\gamma_\mu q(y)\rangle$) are zero within errors, more than an order of magnitude smaller than the pseudoscalar case. This is also what is required by phenomenology, e.g. by the very small ρ-ω splitting compared to the large π-η' splitting. In Ref [27] the scalar hairpin correlator was also found to be large and comparable in magnitude to the pseudoscalar one. The s-channel RR exchange diagram, Fig. 3, provides a nice explanation for all of the observed properties of the quark-antiquark annihilation process in QCD. To a good approximation, a quark and antiquark will annihilate only if they are in a state

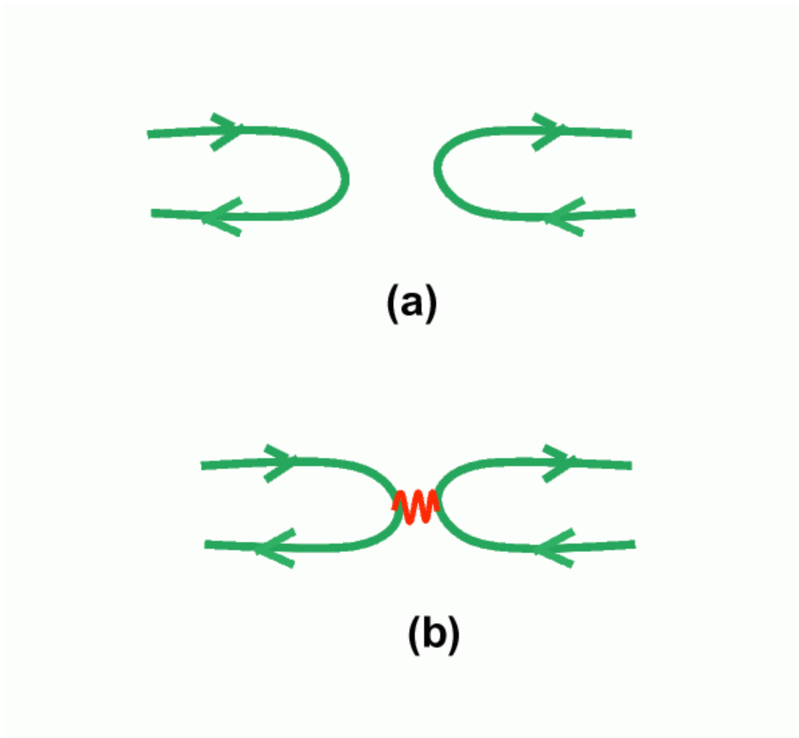
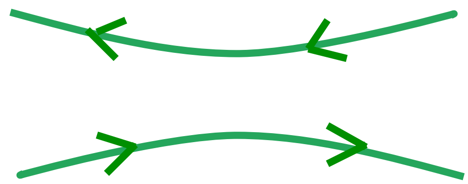


FIG. 3: (a) The quenched double hairpin correlator which measures the gluonic η' mass insertion. The indicated quark propagators are assumed to be summed over all gauge field configurations. (b) The s-channel RR exchange picture for the hairpin correlator. RR exchange results in a local 4-quark contact interaction due to its derivative coupling to the chiral field. These q_μ factors cancel the massless pole and convert it to a delta-function.

of zero total angular momentum, i.e. in either a scalar or pseudoscalar meson state.. In the RR exchange model of the quark-antiquark annihilation process, this property follows from the fact that the RR field couples directly to the chiral phase field $\propto \log \mathrm{Det} U$.

V. ORIGIN OF THE NAMBU-JONA LASINIO 4-QUARK INTERACTION

We have seen that RR exchange in the gluon sector explains the existence of a positive contact term in the $Tr(F \wedge F)$ correlator and thereby, the nonzero topological susceptibility of QCD. Extended to the quark sector, RR exchange in the s-channel of quark-antiquark scattering induces a 4-quark contact term which provides the annihilation vertex responsible for the η' mass insertion. We next consider the effect of t-channel RR exchange in the quark-antiquark amplitude, Fig. 4(b). To understand this effect, let us rewrite the anomaly-induced η' mass insertion as a 4-quark amplitude by expanding the log of the chiral field in (32) in small fluctuations around its vacuum



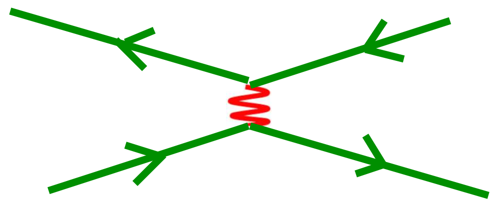


FIG. 4: (a) The valence diagram for quark-antiquark scattering in a meson. (b) The contribution of t-channel RR exchange to the valence diagram.

expectation value. Define

$$U_{ab} = \frac{1}{\langle \overline{q} q \rangle} \overline{q}_a (1 + \gamma_5) q_b \tag{35}$$

Then we can write $U = 1 + \delta U$ and expand to lowest order in δU,

$$\log \mathrm{Det} U = tr \log U \approx tr \delta U \tag{36}$$

The η' mass term can then be rewritten as a 4-quark interaction,

$$\left(\log \mathrm{Det} U - \log \mathrm{Det} U^\dagger \right)^2 \rightarrow \frac{1}{\langle \overline{q} q \rangle^2} \left(\sum_{a=1}^{N_f} \overline{q}_a \gamma_5 q_a \right)^2 \tag{37}$$

To determine the 4-quark interaction that is induced by t-channel RR exchange, we compare the string theoretic view of quark-antiquark scattering to the field theory description. In a field-theoretic calculation, the pseudoscalar, flavor singlet meson propagator is given by the sum of valence (Fig. 4) and hairpin (Fig. 3) correlators. The nonsinglet Goldstone pion propagators are given by the valence diagrams alone. Thus, the valence contribution to the correlator is invariant under $U(N_f) \times U(N_f)$ chiral transformations. The axial $U(1)$ anomaly comes entirely from the

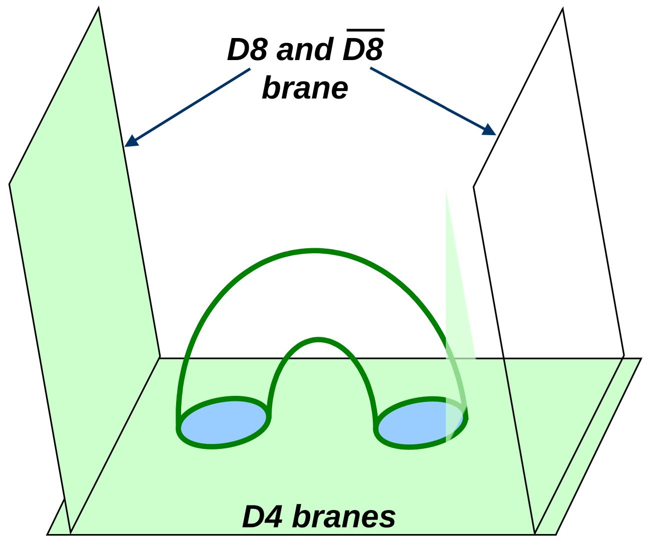


FIG. 5: String representation of the intermediate state in the double hairpin diagram for quark-antiquark scattering, which takes place by quark strings joining at the flavor ends. Here the cylinder represents the partition function of the D4-D4 intermediate string.

hairpin correlator. Now consider quark-antiquark scattering as the scattering of a D4-D8 and a D4-$\overline{\text{D8}}$ string. The tree level open string scattering amplitude is given by the sum of two diagrams, corresponding to joining and splitting apart at either the color end or the flavor end. If the flavor ends of the D4-D8 and D4-$\overline{\text{D8}}$ strings join and separate, the intermediate state is a pure glue D4-D4 string, Fig. 5, so this corresponds to the hairpin annihilation diagram, Fig. 3 in the field theoretic description. (In the Sakai-Sugimoto model, the joining of the flavor ends of the quark and antiquark string is allowed by the fact that the D8 and $\overline{\text{D8}}$ branes are joined together in the strong coupling region into a single U-shaped brane.) The valence Feynman diagram, Fig. 4, corresponds to the scattering of quark strings by joining together at the color end, on the D4-brane world volume, Fig. 6. This would be the only diagram for e. g. a charged pion, where the flavor ends of the two quark strings are on different D8 branes. The intermediate state is a quark-antiquark D8-$\overline{\text{D8}}$ string. Using open string/closed string duality, the D8-$\overline{\text{D8}}$ string can be described in the field theoretic limit by t-channel Ramond-Ramond exchange between quark and antiquark. We expect the Lorentz and flavor structure of the induced 4-quark interaction to be

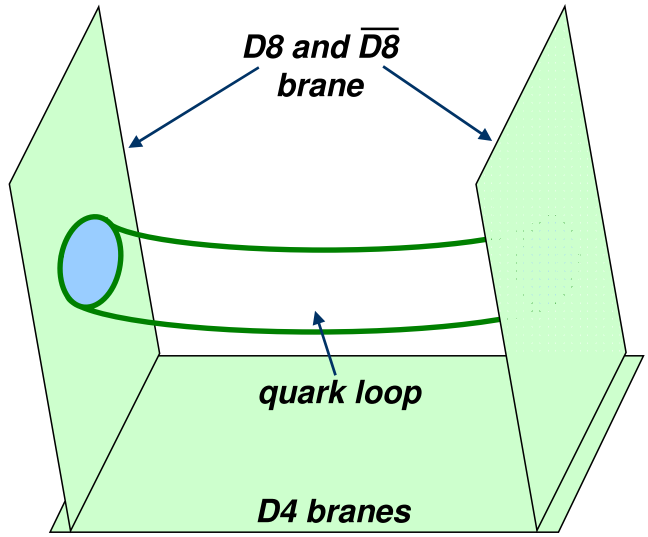


FIG. 6: String representation of the intermediate state in the valence diagram for quark-antiquark scattering, which takes place by quark strings joining at the color ends. Here the cylinder represents the partition function of the D8-$\overline{\text{D8}}$ intermediate string.

similar to the 4-quark anomaly term, Eq. (37). However, because the flavor string-ends don't participate, this diagram does not depend on whether the initial quark and antiquark were on the same or different flavor branes, i.e. whether the meson is a flavor singlet or nonsinglet. Thus, the effective interaction from t-channel RR exchange, Fig. 4(b) should be $U(N_f) \times U(N_f)$ invariant. Note that the $(\overline{q}\gamma_5 q)^2$ anomaly term includes both a $U(1)$ conserving term $2q_L^\dagger q_R q_R^\dagger q_L$ and a $U(1)$ violating term $(q_R^\dagger q_L)^2 + h.c.$. By adding scalar $(\overline{q}q)^2$ interactions to cancel the $U(1)$ violating terms, we are led to an effective action for t-channel RR exchange of the form

$$\mathcal{L}_t \propto \left(\sum_{i=1}^{N_f} \overline{q}_i(1+\gamma_5)q_i\right)\left(\sum_{j=1}^{N_f} \overline{q}_j(1-\gamma_5)q_j\right) \tag{38}$$

Here we defer a quantitative analysis to a subsequent paper and simply observe that the suggested form for t-channal RR exchange has the standard form of a $U(N_f)\times U(N_f)$ preserving Nambu-Jona Lasinio interaction [28]. (A similar mechanism for generating an NJL interaction in holographic QCD by integrating out the compactified component of the D4 brane gauge field has been discussed in [29].) NJL models have an extensive and highly successful phenomenology. Note that, although the interaction (38) is pure flavor singlet in the t-channel, the Fierz transformed expression describes a $U(N_f)\times U(N_f)$ invariant interaction in the s-channel which is equal for flavor singlet and nonsinglet channels. In the usual NJL Bethe-Salpeter analysis, this will produce massless pions in the nonsinglet channels. It is a longstanding idea in QCD that topological charge fluctuations are responsible not only for resolving the $U(1)$ problem but also for driving the spontaneous breaking of $SU(N_f)\times SU(N_f)$ and forming the chiral condensate [21, 30]. Previous discussions of topological charge driven $S\chi SB$ have been framed in the context of the instanton liquid picture, in which the approximate 'tHooft zero modes of the instantons form the chiral condensate [21]. From a slightly different viewpoint, Carlitz et al [30] argued that instantons would generate an attractive NJL quark-antiquark interaction which could drive $S\chi SB$. The RR exchange picture presented here suggests a reformulation of topological charge driven chiral symmetry breaking in the framework of holographic QCD.

VI. DISCUSSION

In many respects, the holographic framework for QCD provides an alternative to a lattice cutoff. The Monte Carlo evidence [9, 12] that the QCD vacuum is dominated by a laminated array of topological charge membranes fits very naturally into the holographic framework [13]. The lattice studies show clearly that the spacing between the alternating-sign membranes is fairly regular and of order a few lattice spacings, and remaining roughly constant in lattice units for a range of correlation lengths [9, 11, 25]. This implies that the scaling limit of QCD follows a non-Landau-Ginsburg paradigm, similar to that of antiferromagnetic systems, where alternating substructure at the lattice spacing scale can give rise to topological excitations in the scaling limit. In the holographic formulation, the Ramond-Ramond gauge field describes the collective fluctuations of the layered arrangement of D6 and $\overline{D6}$ branes. The Chern-Simons current K_μ of the color gauge field is related by anomaly inflow to $\partial_\mu\theta(x)$, i.e to the $\mu 4$ component of the RR field strength. When crossing a D6 brane, $\theta(x)$ jumps by 2π, representing the net outgoing RR flux from the charged D6 brane. By anomaly inflow, this discontinuity must coincide with the topological charge membrane of the color gauge field, represented in field theory by a "Wilson bag" integral of the Chern-Simons 3-form $\mathcal{K}$ over the 3-dimensional world volume of the bag surface. The $q^2=0$ pole in the K_μ correlator combines with the massless pole in the RR correlator in a Kogut-Susskind dipole mechanism. In the K-S dipole for the 2D Schwinger model, the gauge invariant axial vector current operator describes the motion of a charged fermion along with its attached gauge field. The massless pole in the conserved (non-gauge invariant) current matrix element is cancelled by the ghost pole describing the gauge field propagation. In holographic QCD the K-S dipole mechanism connects the motion of a charged D6 brane, represented by a 2π discontinuity in $\theta(x)$, with the topological charge membrane excitation in the color gauge field, represented by the Chern-Simons operator $\mathcal{K}$.

It may seem that the appearance of the Ramond-Ramond field, in the form of a spacetime dependent $\theta(x)$, represents an additional degree of freedom in the holographic framework compared to, say lattice QCD, where θ is assumed to be a constant θ_0 (usually zero). But in fact the spacetime dependent $\theta(x)$, which is constant between D6 branes with a $\pm 2\pi$ discontinuity at the brane surfaces, can be interpreted as a singular gauge transformation which smoothly connects the color fields on opposite sides of the brane. Note that the Wilson bag operator, given by the integral of $\mathcal{K}$ over a codimension one surface in spacetime is the gauge field operator which effectively inserts a discontinuity in θ. (For example, a Wilson bag operator over a closed surface is equivalent to a theta term in the interior of the bag, since $\partial^\mu K_\mu = TrF\tilde{F}$.). Thus, in a purely 4-dimensional Yang-Mills framework the RR field $\theta(x)$ can be regarded as an auxilary field which separates off the singular, sheet-like excitations in the Yang-Mills field and treats their dynamics separately. The requirement of overall gauge invariance of the RR field strength removes the redundancy of the auxiliary field and locks the motion of the $\theta(x)$ discontinuities to that of the topological charge membranes of the Yang-Mills field. It is very clear from Monte Carlo studies [9, 25] that these singular, sheet-like gauge excitations are responsible for the positive contact term that appears in the Euclidean topological charge correlator. In this paper, we have shown that such a contact term is the expected low-energy manifestation of Ramond-Ramond gauge boson exchange. This effect appears not only in the topological charge correlator, but also in the anomaly-induced 4-quark contact term that produces an η' "hairpin" mass insertion. We also discussed the possibility that RR exchange in the t-channel of quark-antiquark scattering generates a $U(N_f) \times U(N_f)$ symmetric Nambu-Jona Lasinio type interaction which could be responsible for the formation of the quark condensate. This provides an appealing physical picture of spontaneous $SU(N_f) \times SU(N_f)$ breaking and Goldstone boson propagation and its relation to vacuum topological charge structure in gauge theory. The picture that emerges is of a left handed chiral condensate of $\overline{q}(1-\gamma_5)q$ living on the surfaces of the D6 branes interleaved with sheets of right-handed $\overline{q}(1+\gamma_5)q$ on the surfaces of the anti-D6 branes. This is reminiscent of the instanton liquid model, in which left and right condensate modes live on instantons and antiinstantons, respectively, averageing out to a $\langle \overline{q}q \rangle \neq 0$ condensate. But a crucial difference for the D6 brane condensate is that the quarks modes in the condensate are delocalized along codimension one sheets instead of being 'tHooft zero modes confined to localized lumps. This provides a much more natural mechanism for Goldstone boson propagation than the mode-mixing or "hopping" that must be invoked in instanton liquid models.

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