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Compact stars in alternative theories of gravity. Einstein-Dilaton-Gauss-Bonnet gravity

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We develop a theoretical framework to study slowly rotating compact stars in a rather general class of alternative theories of gravity, with the ultimate goal of investigating constraints on alternative theories from electromagnetic and gravitational-wave observations of compact stars. Our Lagrangian includes as special cases scalar-tensor theories (and indirectly $f(R)$ theories) as well as models with a scalar field coupled to quadratic curvature invariants. As a first application of the formalism, we discuss (for the first time in the literature) compact stars in Einstein-Dilaton-Gauss-Bonnet gravity. We show that compact objects with central densities typical of neutron stars cannot exist for certain values of the coupling constants of the theory. In fact, the existence and stability of compact stars sets more stringent constraints on the theory than the existence of black hole solutions. This work is a first step in a program to systematically rule out (possibly using Bayesian model selection) theories that are incompatible with astrophysical observations of compact stars.

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I. INTRODUCTION

Compact stars as nuclear physics laboratories. Studies of compact stars in general relativity have been textbook material for decades [1, 2]. Neutron stars can be considered "cold" by nuclear physics standards, so their mass-radius relation $M(R)$ is uniquely determined by the equation of state (EOS) of matter at high densities, i.e. by the relation between pressure and energy density $P(\rho)$. From an observational point of view, one usually assumes general relativity to be correct. Under this assumption (which of course is backed up by a wealth of observational evidence [3]), the Holy Grail of astronomical observations of neutron stars is the determination of the EOS from measurements of macroscopic properties such as masses, radii and moments of inertia.

Better observational estimates of neutron star masses and radii are progressively improving our understanding of the EOS. Lindblom [4] presented a concrete scheme for reconstructing $P(\rho)$ from observations of $M(R)$. More recently, Read *et al.* [5] approximated the high-density EOS by piecewise polytropic models, showing that current astrophysical measurements yield stringent constraints on the piecewise polytropic parameters. In the same spirit, Lindblom [6] proposed to replace piecewise polytropes by spectral expansions, that should give a more faithful representation of the EOS.

Our understanding of the functional form of the EOS from observed masses and radii has made impressive strides in the recent past [7–9] (see also [10, 11] for reviews). Demorest *et al.* [12] recently determined a value $M = 1.97 \pm 0.04\ M_\odot$ for the mass of PSR J1614-2230, a pulsar in a white dwarf-neutron star binary system. This precisely measured mass is large enough to rule out many candidate EOSs [13, 14]. Vice versa, theoretical progress in microscopic calculations based on chiral effective field theory is leading to a better understanding of neutron-rich matter below nuclear densities, and hence to more stringent constraints on the mass-radius relationship [15].

Compact stars as strong gravity laboratories. Most studies of the possibility of reconstructing the EOS from compact star observations assume that general relativity is the correct theory of gravity. General relativity passed all observation tests so far [3], but the "real" theory of gravity may well differ significantly from it in strong field regions. In fact, cosmological observations and conceptual difficulties in quantizing Einstein's theory suggest that general relativity may require modifications.

Compact stars are an ideal natural laboratory to look for possible modifications of Einstein's theory and their observational signatures [16]. Besides ruling out specific models for the EOS, experiments may (and should) try to rule out also alternative theories of gravity that are unable to explain observations. A comprehensive study of how EOS models and alternative theories affect macroscopic observable quantities of compact stars requires a Bayesian model selection framework, where one compares the predictions of any specific theory of gravity (and of different EOS models) against the growing body of observational data. Of course, an important prerequisite of any such analysis is the construction of stellar models in the largest possible family of alternative theories of gravity that are not ruled out by weak-field experiments, cosmological constraints or observations of compact binary systems. The present work is a first step in this direction.

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The plan of the paper is as follows. In Sec. II, to put our work in context, we briefly review some studies of compact stars in alternative theories of gravity. In Sec. III we present the Lagrangian for what we call "extended scalar-tensor theories". In Sec. IV we write down the equations describing the structure of static and slowly-rotating stars in this class of theories, and we discuss our chosen models for the EOS. In Sec. V we present numerical results and discuss their implications. We conclude by discussing extensions of the present work to other theories and comparisons with observations. Unless stated otherwise, we use geometrical units ($G = c = 1$).

II. STARS IN ALTERNATIVE THEORIES: A BRIEF REVIEW

The study of compact stars in alternative theories of gravity has a long history. In this paper we begin a systematic exploration of stellar structure in a large class of modified gravity theories. Far from providing a comprehensive review, here we point to some relevant literature, mainly to put our work in context.

i) Scalar-tensor theories. Scalar-tensor gravity is one of the simplest and best-motivated modifications of general relativity, because scalar fields are predicted by almost all attempts to incorporate gravity into the standard model [17]. Therefore it should come as no surprise that most work on stellar structure concerns variants of scalar-tensor theory. The equations of hydrostatic equilibrium in the best-known variant of scalar-tensor theories (Brans-Dicke theory) were first studied by Salmona [18]. Soon after, Nutku [19] explored the radial stability of stellar models using a post-Newtonian treatment. Hillebrandt and Heintzmann [20] analyzed incompressible (constant density) configurations. Zaglauer [21] carried out a detailed calculation of the so-called "sensitivities" of neutron stars, which determine the amount of dipolar gravitational radiation emitted by compact binaries in scalar-tensor theories [22]. Most of these studies found that corrections to neutron star structure are suppressed by a factor $1/\omega_{\rm BD}$, where $\omega_{\rm BD}$ is the Brans-Dicke coupling constant. At present, the most stringent bound on this parameter ($\omega_{\rm BD} > 40,000$) comes from Cassini measurements of the Shapiro time delay [3].

As pointed out by Damour and Esposito-Farése [23], the coupling of the scalar with matter can produce a "spontaneous scalarization" phenomenon by which certain "generalized" scalar-tensor theories may pass all weak-field tests, and at the same time introduce macroscopically (and observationally) significant modifications to the structure of compact stars. More detailed studies of stellar structure [24, 25], numerical simulations of collapse [26–28] and a stability analysis [29] confirmed that "spontaneously scalarized" configurations would indeed be the end-state of stellar collapse in these theories. In fact, spontaneously scalarized configurations may arise as a result of semiclassical vacuum instabilities [30]. Tsuchida *et al.* [31] extended the Buchdahl inequality ($M/R \leq 4/9$ for incompressible stars) to generalized scalar-tensor theories. For a comprehensive study of analytic solutions and an extensive bibliography, see [32].

ii) f(R) theories. Theories that replace the Ricci scalar R by a generic function $f(R)$ in the Einstein-Hilbert action[1] can always, at least in principle, be mapped into scalar-tensor theories [35, 36] (see also [37]). The existence of compact stars in metric $f(R)$ models that have been proposed to explain cosmological observations, such as the Starobinsky model [38], was studied by many authors [39–44] with controversial results. One possible explanation of the partial disagreement between different authors is that the mapping between $f(R)$ theories and scalar-tensor theories is in general multivalued, and therefore one should be careful when considering the scalar-tensor "equivalent" of an $f(R)$ theory [45]. A perturbative approach to stellar structure in $f(R)$ gravity is also possible [46].

iii) Higher-curvature gravity. Besides theories where the Lagrangian is a generic function of R, it is of interest to consider theories where the Lagrangian is built out of quadratic [47] (or even higher-order) contractions of the Riemann and Ricci tensors. As we explain below, the requirement that the field equations should be second-order means that quadratic corrections must appear in the Gauss-Bonnet (GB) combination

$$\mathcal{R}^2_{\rm GB} = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}\,, \tag{1}$$

where R_{abcd} is the Riemann tensor and R_{ab} is the Ricci tensor. Since the GB term in four dimensions is a topological invariant, the GB combination introduces modifications to general relativity only when coupled to a nonzero scalar field or other forms of matter. The simplest and better motivated case[2] is Einstein-Dilaton-Gauss-Bonnet (EDGB) gravity [48], where the GB term is coupled to a dynamical scalar field, the dilaton. The EDGB correction to the Einstein-Hilbert action appears in low-energy, tree-level effective string theory [49].

The study of EDGB gravity in relativistic astrophysics has been limited to a mathematical analysis of black hole solutions [48, 50–52] and, more recently, to their possible observational signatures [47, 53, 54]. To our knowledge, the present study is the first investigation of compact stars in the theory (see Refs. [55, 56] for other nonrotating solutions in EDGB theory).

iv) Parity-violating theories. Chern-Simons gravity is the simplest theory that allows for parity-violating corrections to general relativity [57]. Due to the nature of

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[1] Here and below we refer to $f(R)$ theories in the metric formalism. Theories of the Palatini type have conceptual problems: for example, spherically symmetric polytropic "stars" present curvature singularities [33, 34].

[2] In analogy with $f(R)$ models, $f(\mathcal{R}^2_{\rm GB})$ models have been studied in a cosmological context. Observational constraints on $f(\mathcal{R}^2_{\rm GB})$ models are quite tight (see e.g. Sections 12.3 and 12.4 of [36]) and we will not include them in our analysis.

the Chern-Simons corrections, all spherically symmetric solutions of Einstein's theory are also solutions of Chern-Simons gravity. However, spinning objects in the non-dynamical [58] and dynamical [59] versions of the theory are affected by the Chern-Simons coupling. Future observations of the moment of inertia of compact stars may strongly constrain the parameters of the theory [59].

v) Lorentz-violating theories. Einstein-aether theory introduces a dynamical unit timelike vector coupled to gravity as a natural way to implement Lorentz violation in Einstein's theory. In the parameter space compatible with Solar System constraints, spherically symmetric neutron stars in Einstein-aether theory have a lower maximum mass than in general relativity [60, 61]. Another popular Lorentz-violating theory is Hořava gravity. The matching conditions necessary to obtain stellar solutions in this theory were considered in [62], but (to our knowledge) there are no phenomenological studies of compact stars using realistic EOS models.

vi) Massive gravity. Recently, Damour *et al.* [63] reconsidered the discontinuity problem of massive gravity and its possible resolution through Vainshtein's nonlinear resummation of nonlinear effects. As part of this study, the authors investigated the viability of spherically symmetric stars in the theory. They showed that some solutions show physical singularities, but also that there exist regular solutions interpolating between a modified general relativistic interior and a de Sitter exterior, with curvature proportional to the square of the putative graviton mass. A more phenomenological study of observational constraints (including stellar rotation) is still lacking.

vii) Eddington inspired gravity. Bañados and Ferreira [64] recently proposed a theory that is equivalent to general relativity in vacuum, but differs from it in the coupling with matter. An interesting aspect of this theory is that singularities cannot form in early cosmology and during gravitational collapse [64, 65]. The maximum mass of compact stars in the observationally viable sector of Eddington-inspired gravity may be larger than in general relativity, even for "ordinary" EOS models [65].

viii) Gravitational aether, f(T), TeVeS and other theories. Some alternatives to general relativity that were proposed to explain cosmological observations have also been analyzed, at least to some extent, in the context of compact stars. Among these theories we can list "gravitational-aether" theory [66], $f(T)$ gravity [67] and Bekenstein's TeVeS [68]. In higher-dimensional braneworld models, the embedding of four-dimensional stellar solutions "on the brane" within acceptable higher-dimensional solution is a nontrivial problem [69] (but see [70] for related work in a slightly different context).

III. EXTENDED SCALAR-TENSOR THEORIES

From the previous summary it should be clear that it is nearly impossible to discuss *all* strong-field modifications of general relativity in a unified framework. However, in this section we show that on the basis of some rather general arguments we can easily write down a Lagrangian encompassing the first four classes of theories reviewed above (namely general scalar-tensor theories, $f(R)$ theories, EDGB gravity and Chern-Simons gravity).

Our starting point is a Lagrangian in which gravity is coupled to a single (generically charged) scalar field ϕ in all possible ways, including all linearly independent quadratic curvature corrections to general relativity. We call these models "extended scalar-tensor theories". The most general Lagrangian of such a theory contains several functions of the scalar field in the combination

$$\begin{aligned}\mathcal{L}= & f_0(|\phi|)R-\gamma(|\phi|)\partial_a\phi^*\partial^a\phi-V(|\phi|)\\ & +f_1(|\phi|)R^2+f_2(|\phi|)R_{ab}R^{ab}+f_3(|\phi|)R_{abcd}R^{abcd}\\ & +f_4(|\phi|)R_{abcd}{}^*R^{abcd}+\mathcal{L}_{\rm mat}\left[\Psi,A^2(|\phi|)g_{ab}\right]\,, \end{aligned} \tag{2}$$

where ${}^*R^{abcd}$ is the dual of the Riemann tensor, which introduces possible parity-violating corrections [57]. From the Lagrangian above, the equations of motion read:

$$G_{ab}+\frac{1}{f_0}\left[\mathcal{H}_{ab}+\mathcal{I}_{ab}+\mathcal{J}_{ab}+\mathcal{K}_{ab}\right]=\frac{1}{2f_0}\left[A^2T^{\rm mat}_{ab}+T^{(\phi)}_{ab}\right]\,, \tag{3}$$

where $T^{\rm mat}_{ab}=2(-g)^{-1/2}\delta S_m/\delta g_{ab}$ is the matter stress-energy tensor in the Jordan frame,

$$\begin{aligned}T^{(\phi)}_{ab}= & \gamma\left[2\partial_{(a}\phi^*\partial_{b)}\phi-g_{ab}\partial_c\phi^*\partial^c\phi\right]-g_{ab}V\\ & +2\nabla_a\nabla_b f_0-2g_{ab}\nabla^2 f_0\,,\end{aligned} \tag{4}$$

and, following the notation of Ref. [47], we have defined

$$\begin{aligned}\mathcal{H}_{ab}\equiv & -4v^{(1)}_{(a}\nabla_{b)}R-2R\nabla_{(a}v^{(1)}_{b)}+g_{ab}\left[2R\nabla^c v^{(1)}_c+4v^{(1)}_c\nabla^c R\right]+f_1\left[2R_{ab}R-2\nabla_{ab}R-\frac{1}{2}g_{ab}\left[R^2-4\Box R\right]\right]\,,\\ \mathcal{I}_{ab}\equiv & -v^{(2)}_{(a}\nabla_{b)}R-2v^{(2)}_c\left[\nabla_{(a}R_{b)}{}^c-\nabla^c R_{ab}\right]+\nabla^c v^{(2)}_c R_{ab}-2R_{c(a}\nabla^c v^{(2)}_{b)}+g_{ab}\left[v^{(2)}_c\nabla^c R+R^{cd}\nabla_c v^{(2)}_d\right]\\ & +f_2\left[2R^{cd}R_{acbd}-\nabla_{ab}R+\Box R_{ab}+\tfrac{1}{2}g_{ab}\left[\Box R-R_{cd}R^{cd}\right]\right]\,,\\ \mathcal{J}_{ab}\equiv & -8v^{(3)}_c\left[\nabla_{(a}R_{b)}{}^c-\nabla^c R_{ab}\right]+4R^d_{acb}\nabla^c v^{(3)}_d-f_3\left[2\left[R_{ab}R-4R^{cd}R_{acbd}+\nabla_{ab}R-2\Box R_{ab}\right]-\tfrac{1}{2}g_{ab}\left[R^2-4R_{cd}R^{cd}\right]\right]\,,\\ \mathcal{K}_{ab}\equiv & 4v^{(4)}_c\epsilon^c{}_{de(a}\nabla^e R_{b)}{}^d+4\nabla_d v^{(4)}_c{}^*R_{(a}{}^c{}_{b)}{}^d\,,\end{aligned}$$

with $v^{(i)}_a\equiv\nabla_a f_i(|\phi|)$, $\nabla_{ab}=\nabla_a\nabla_b$, $\Box=\nabla_a\nabla^a$ and ϵ^{abcd} the Levi-Civita tensor. The modified Klein-Gordon

TABLE I. Specific models obtained from the Lagrangian (2). Here $\kappa \equiv (16\pi G)^{-1}$.

	f_0	f_1	f_2	f_3	f_4	ω	V	γ	A	$\mathcal{L}_{\rm mat}$
General relativity	κ	0	0	0	0	0	0	1	1	perfect fluid
Scalar-tensor (Jordan frame) [24]	$F(\phi)$	0	0	0	0	0	$V(\phi)$	$\gamma(\phi)$	1	perfect fluid
Scalar-tensor (Einstein frame) [23]	κ	0	0	0	0	0	$V(\phi)$	2κ	$A(\phi)$	perfect fluid
$f(R)$ [36]	κ	0	0	0	0	0	$\kappa\frac{Rf_{,R}-f}{16\pi G f_{,R}^2}$	2κ	$f_0^{-1/2}=f_{,R}^{-1/2}$	perfect fluid
Quadratic gravity [47]	κ	$\alpha_1\phi$	$\alpha_2\phi$	$\alpha_3\phi$	$\alpha_4\phi$	0	0	1	1	perfect fluid
EDGB [48]	κ	$e^{\beta\phi}$	$-4f_1$	f_1	0	0	0	1	1	perfect fluid
Dynamical Chern-Simons [59]	κ	0	0	0	$\beta\phi$	0	0	1	1	perfect fluid
Boson stars [71]	κ	0	0	0	0	ω	$\frac{m^2}{2}\vert\phi\vert^2$	1	1	0

equation reads

$$\Box\phi= \frac{\phi}{2|\phi|\gamma}\left[V'-\gamma'\partial_a\phi^*\partial^a\phi-f_0'R-f_1'R^2-f_2'R_{ab}R^{ab}\right.$$
$$\left.-f_3'R_{abcd}R^{abcd}-f_4'R_{abcd}{}^*R^{abcd}-A'A^3T^{\rm mat}\right]\,, \qquad (6)$$

together with its complex conjugate. In the equations above, a prime denotes a derivative with respect to $|\phi|$.

As shown in Table I, this theory is sufficient to discuss stellar structure in many of the alternative theories listed in Sec. II (and it can also describe boson stars in general relativity, if we work in vacuum). As a matter of fact, some terms in the Lagrangian (2) are redundant. For example, in ordinary scalar-tensor theories the functions f_0 and γ can be removed via a conformal transformations of the metric and a redefinition of the scalar field; i.e., by reformulating the theory in the Einstein frame [24]. However, depending on the explicit form of f_0 and γ, these transformations can be hard (if not impossible) to write in a closed analytic form. For this reason we find it convenient to start from the general Lagrangian (2), which reduces to standard scalar-tensor theories in the Jordan frame if $A(|\phi|)\equiv 1$ and in the Einstein frame if $f_0(|\phi|)\equiv 1/(16\pi)$ and $\gamma(|\phi|)\equiv 1$. Another advantage of this approach is that, at least in principle, it should also encompass generic $f(R)$ theories, which are equivalent to particular scalar-tensor theories (but see [45] for possible issues with this point of view).

A. Simplifying the model

For generic coupling functions, the terms $\mathcal{H}_{ab}$, $\mathcal{I}_{ab}$, $\mathcal{J}_{ab}$ appearing on the left-hand side of the equations of motion introduce higher-order derivatives of the metric functions, unless quadratic terms in the curvature enter the action in the GB combination (1). The GB combination corresponds to setting $f_2=-4f_1$ and $f_3=f_1$ in our model. Thus, if we only want second-order equations of motion the Lagrangian (2) must reduce to

$$\mathcal{L}= f_0(|\phi|)R+f_1(|\phi|)\mathcal{R}^2_{\rm GB}+f_4(|\phi|)R_{abcd}{}^*R^{abcd}$$
$$-\gamma(|\phi|)\partial_a\phi^*\partial^a\phi-V(|\phi|)+\mathcal{L}_{\rm mat}\left[\Psi,A^2(|\phi|)g_{ab}\right]\,. \qquad (7)$$

In order to avoid the complications related to higher-order derivatives, from now on we will specialize to this Lagrangian.

IV. PERFECT FLUID COMPACT STARS IN EXTENDED SCALAR-TENSOR THEORIES

A. Static solutions

We begin by looking for static, spherically symmetric equilibrium solutions of the field equations with metric

$$ds_0^2=-B(r)dt^2+\frac{dr^2}{1-2m(r)/r}+r^2d\theta^2+r^2\sin^2\theta\,d\varphi^2$$

and a charged, spherically symmetric scalar field

$$\phi(t,r)=\Phi(r)e^{-i\omega t}\,. \qquad (8)$$

Our ansatz for the scalar field also implies that the Klein-Gordon equation (6) and its conjugate coincide. In general, the term $f_4(\phi)R_{abcd}{}^*R^{abcd}$ gives rise to third-order derivatives in the field equations [57]. However, because of the assumed spherical symmetry, the Pontryagin density vanishes identically ($R_{abcd}{}^*R^{abcd}\equiv 0$) and the equations of motion do not depend on f_4.

We consider perfect-fluid stars with energy density $\rho(r)$ and pressure $P(r)$ such that

$$T^{\mu\nu}_{\rm mat}\equiv T^{\mu\nu}_{\rm perfect\,fluid}=(\rho+P)\,u^\mu\,u^\nu+g^{\mu\nu}P\,, \qquad (9)$$

where the fluid four-velocity $u^\mu=(1/\sqrt{B},0,0,0)$. Note that the matter fields are defined in the Jordan frame. The stress-energy tensor in the Einstein frame reads $T^{(E)}_{\mu\nu}=A^2(|\phi|)T_{\mu\nu}$. To close the system, as usual, we must also specify an EOS $P=P(\rho)$.

The field equations for a static, spherically symmetric perfect-fluid star in extended scalar-tensor theories read

$$
\begin{aligned}
E_{00} = & -\frac{r^5\omega^2\gamma(\Phi)\Phi^2}{B} - r^5 V(\Phi) - r^5 A(\Phi)^2\rho + 4r^3 f_0(\Phi) m' - 4r^4 f_0'(\Phi)\Phi' + 6r^3 m f_0'(\Phi)\Phi' + 16 r m f_1'(\Phi)\Phi' \\
& -48 m^2 f_1'(\Phi)\Phi' + 2r^4 f_0'(\Phi) m'\Phi' - 16 r^2 f_1'(\Phi) m'\Phi' + 48 r m f_1'(\Phi) m'\Phi' - r^5\gamma(\Phi)\Phi'^2 + 2r^4 m\gamma(\Phi)\Phi'^2 - 2r^5\Phi'^2 f_0''(\Phi) \\
& +4r^4 m\Phi'^2 f_0''(\Phi) - 16 r^2 m\Phi'^2 f_1''(\Phi) + 32 r m^2 \Phi'^2 f_1''(\Phi) - 2r(r-2m)\left[r^3 f_0'(\Phi) + 8 m f_1'(\Phi)\right]\Phi'' = 0\,, \qquad (10) \\
E_{11} = & \; r^4\omega^2\gamma(\Phi)\Phi^2 - (r-2m)B'\left\{2r^2 f_0(\Phi) + \left[r^3 f_0'(\Phi) - 8(r-3m) f_1'(\Phi)\right]\Phi'\right\} + rB\left\{4 f_0(\Phi) m + r\left[r^2 A(\Phi)^2 P\right.\right. \\
& \left.\left. -r^2 V(\Phi) + (r-2m)\Phi'\left(-4f_0'(\Phi) + r\gamma(\Phi)\Phi'\right)\right]\right\} = 0\,, \qquad (11) \\
E_{\rm cons} = & \; 4r^5 A(\Phi) B^2 f_0(\Phi) P A'(\Phi)\Phi' + r^5 A(\Phi)^2 B\left[f_0(\Phi)\left((P+\rho)B' + 2BP'\right) - 2BP f_0'(\Phi)\Phi'\right] + \Phi' \times \\
& \left\{r f_0(\Phi)(r-2m)B'^2\left[r^3 f_0'(\Phi) + 8 m f_1'(\Phi)\right] + 2B\left\{r f_0'(\Phi)\left[-r^4\omega^2\gamma(\Phi)\Phi^2 + (r-2m)B'\left(r^3 f_0'(\Phi)\right.\right.\right.\right. \\
& \left. -8(r-3m) f_1'(\Phi)\right)\Phi'\big] + f_0(\Phi)\left[-r^3 m B' f_0'(\Phi) + 8 r m B' f_1'(\Phi) - 24 m^2 B' f_1'(\Phi) + r^4 B' f_0'(\Phi) m'\right. \\
& -8r^2 B' f_1'(\Phi) m' + 24 r m B' f_1'(\Phi) m' + r^5\omega^2\Phi^2\gamma'(\Phi) + r^4\gamma(\Phi)\left(2r\omega^2\Phi + (r-2m)B'\Phi'\right) \\
& \left. -r(r-2m)\left(r^3 f_0'(\Phi) + 8 m f_1'(\Phi)\right)B''\right]\big\} + 2r^2 B^2\left\{r f_0'(\Phi)\left[r^2 V(\Phi) + (r-2m)\Phi'\left(4 f_0'(\Phi) - r\gamma(\Phi)\Phi'\right)\right]\right. \\
& + f_0(\Phi)\left[r\left(4 f_0'(\Phi) m' + r\left(-rV'(\Phi) + r\gamma'(\Phi)\Phi'^2 + 2\gamma(\Phi)\left(-\left(-2+m'\right)\Phi' + r\Phi''\right)\right)\right) - 2m\left(2 f_0'(\Phi)\right.\right. \\
& \left.\left.\left. + r\left(r\gamma'(\Phi)\Phi'^2 + \gamma(\Phi)\left(3\Phi' + 2r\Phi''\right)\right)\right)\right]\big\}\big\} = 0\,, \qquad (12) \\
E_{\rm scal} = & \; r(r-2m)B'^2\left[r^3 f_0'(\Phi) + 8 m f_1'(\Phi)\right] + 2B\left\{B'\left[r^3 f_0'(\Phi)\left(3m + r\left(-2+m'\right)\right) + 8(r-3m) f_1'(\Phi)\left(m - r m'\right)\right]\right. \\
& \left. -r^5\omega^2\Phi^2\gamma'(\Phi) + r^4\gamma(\Phi)\left[2r\omega^2\Phi + (r-2m)B'\Phi'\right] - r(r-2m)\left[r^3 f_0'(\Phi) + 8 m f_1'(\Phi)\right]B''\right\} \\
& +2r^3 B^2\left\{r^2 A(\Phi)^3(3P-\rho)A'(\Phi) + 4 f_0'(\Phi) m' - r^2 V'(\Phi) + \Phi'\left[-2\gamma(\Phi)\left(3m + r\left(-2+m'\right)\right) + r(r-2m)\gamma'(\Phi)\Phi'\right]\right. \\
& \left. +2r(r-2m)\gamma(\Phi)\Phi''\right\} = 0\,, \qquad (13)
\end{aligned}
$$

where E_{00} and E_{11} are the $\{0,0\}$ and $\{1,1\}$ components of the modified Einstein equations, $E_{\rm cond} = \nabla_a T^{a2} \equiv 0$ and $E_{\rm scal}$ denotes the field equation for the scalar field.

To construct spherically symmetric and static stellar configurations, we must solve the system above imposing regularity conditions at the center of the star, i.e.

$$
m(0) = 0\,, \quad \rho(0) = \rho_c\,, \quad \Phi(0) = \Phi_c\,, \quad \Phi'(0) = 0\,. \qquad (14)
$$

More in general, any field can be expanded close to the center as

$$
X(r) = X^{(0)} + X^{(1)} r + X^{(2)} r^2 + \mathcal{O}(r^3)\,, \qquad (15)
$$

where X schematically denotes any of the variables ρ, P, Φ, B and m. By using the field equations, all coefficients $X^{(i)}$ ultimately depend on two parameters only, say $\rho^{(0)} = \rho_c$ and $\Phi^{(0)} = \Phi_c$. Finally, the value of Φ_c is fixed through a shooting method in order to obtain an asymptotically flat solution[3]: $\Phi \to 0$ as $r \to \infty$.

The outcome of this shooting method is a one-parameter family of solutions characterized only by the central density ρ_c. For any value of ρ_c, we can compute the mass M and the radius R_s of the star. As usual, the mass is obtained from the asymptotic behavior at infinity

$$
B(r) \to 1 - \frac{2M}{r}\,, \qquad (16)
$$

whereas the radius is computed by imposing the usual matching condition at the stellar surface, $P(R_s) = 0$. In the exterior of the star, $P = \rho = 0$. Finally, the baryonic mass, corresponding to the energy that the system would have if all baryons were dispersed to infinity, is defined as

$$
\bar{m} = m_b \int d^3x \sqrt{-g}\, u^0 n(r)\,, \qquad (17)
$$

where n denotes the baryonic number. The normalized binding energy $\bar{m}/M - 1$ is positive for bound (but not necessarily stable) configurations.

B. Slowly rotating models

Once a static stellar model is known, it is easy to construct the corresponding slowly rotating model by generalizing the classic work by Hartle [72]. For this purpose, we consider the metric ansatz

$$
\begin{aligned}
ds^2 &= -B(r)dt^2 + \left[1 - 2m(r)/r\right]^{-1} dr^2 + r^2 d\theta^2 \\
&\quad + r^2\sin^2\theta\left(d\varphi - \frac{\zeta(r)}{2}dt\right)^2 \\
&= ds_0^2 - \zeta(r) r^2 \sin^2\theta\, dt\, d\varphi + \mathcal{O}(\zeta^2)\,. \qquad (18)
\end{aligned}
$$

The stress-energy tensor for a rotating fluid can be easily constructed from Eq. (9) and the four-velocity

$$
u^\mu = \left\{u^t, 0, 0, \Omega u^t\right\}\,, \quad u^t = \frac{1}{\sqrt{-(g_{tt} + 2\Omega g_{t\varphi} + \Omega^2 g_{\varphi\varphi})}}\,, \qquad (19)
$$

[3] This condition can be easily generalized to allow for nonvanishing values of the scalar field ($\Phi \to \Phi_\infty \neq 0$ as $r \to \infty$), like those arising in $f(R)$ theories and in other modified gravity theories that want to reproduce cosmological dynamics.

where Ω is the angular velocity of the fluid.

Note that, for any nonconstant $f_4(|\phi|)$, the gravitomagnetic part $g_{t\varphi}$ of the metric would source scalar perturbations through first-order parity-violating terms in Eq. (6). If f_0, f_1, A and γ are constant and $V \equiv 0$, the field equations admit the same spherically symmetric solutions as in general relativity, and the background scalar field vanishes. In this case, the only first-order corrections arise from the $\{t,\varphi\}$ component of the Einstein equations and from the (perturbed) scalar equation, since the stress-energy tensor is quadratic in the scalar field. These two equations can be solved for $\zeta(r)$ and for the scalar field perturbation for slow rotation and small scalar fields. This was done in Ref. [59], where the authors studied slowly rotating neutron stars in dynamical Chern-Simons gravity ($f_4(|\phi|) = \alpha|\phi|$). When (as in our case) the background scalar field is nonvanishing, all nontrivial Einstein equations would acquire first-order corrections. Hence, if $f_4(|\phi|)$ is not constant, Hartle's formalism cannot be applied. In particular, other first-order metric corrections (in addition to the gravitomagnetic part $g_{t\varphi}$) must be included in Eq. (18). This would result in a system of equations which is very difficult to decouple. To avoid these issues, from now on we will simply *assume* that $f_4(|\phi|)$ =const. In this case the Pontryagin density $R_{abcd}{}^*R^{abcd}$ is a total derivative and the field equations do not depend on the coupling f_4. The generalization of Hartle's approach to deal with a generic coupling $f_4(\phi)$ would be an interesting extension of our work.

Using the definition of the four-velocity (19) and linearizing in the angular velocity Ω, we find that the solution to the field equations corresponds to the background (nonrotating) solution plus the solution $\zeta(r)$ of an ordinary differential equation coming from the $\{t,\varphi\}$ component of the Einstein equations, namely:

$$\begin{aligned}\zeta''(r) + C_1(r)\zeta'(r) = [\zeta(r) - \Omega] \times \qquad (20)\\ \times \frac{r^3 A(\Phi)^2 (P+\rho)}{(r-2M)\left[r^2 f_0(\Phi) - 4(r-2M) f_1'(\Phi)\Phi'\right]}\,,\end{aligned}$$

where

$$\begin{aligned}C_1 =& \frac{1}{2rB(r-2M)\left(r^2 f_0(\Phi) - 4(r-2M) f_1'(\Phi)\Phi'\right)} \times \\ &\big[r(r-2M)B'\left(4(r-2M)f_1'(\Phi)\Phi' - r^2 f_0(\Phi)\right) \\ &+2B\left(-r^2 f_0(\Phi)\left(7M + r\left(M'-4\right)\right) + (r-2M)\times\right. \\ &\left(\Phi'\left(r^3 f_0'(\Phi) + 12 f_1'(\Phi)\left(M + r\left(M'-1\right)\right)\right)\right. \\ &\left.\left.-4r(r-2M)\Phi' f_1''(\Phi)\right) - 4r(r-2M) f_1'(\Phi)\Phi''\right)\big]\,.\end{aligned}$$

Equation (20) must be solved by imposing regularity conditions at the center of the star: $\zeta(0) = \zeta_c$, $\zeta'(0) = 0$. We must also require continuity of $\zeta(r)$ at the stellar radius. The asymptotic behavior at infinity reads

$$\zeta \to \zeta_\infty + \frac{2J}{r^3}\,, \qquad (21)$$

where J denotes the angular momentum. For the solution to be asymptotically flat, we must impose $\zeta_\infty = 0$. This can be easily achieved by noting that Eq. (20) is invariant under the transformation

$$\zeta \to \zeta - \eta\,, \quad \Omega \to \Omega - \eta\,, \qquad (22)$$

where η is some constant. Therefore we can proceed as follows [72]: (1) integrate Eq. (20) imposing regularity at the center and extract ζ_∞ and ζ'_∞ at some large (but finite) radius r_∞; (2) find the physical value of the angular velocity, i.e., $\Omega - \zeta_\infty$. After this translation, $\zeta \to 2J/r^3$ at infinity; (3) compute

$$J = -\lim_{r\to\infty} \frac{r^4 \zeta'_\infty}{6}\,. \qquad (23)$$

As we vary $\Omega - \zeta_c$ we obtain models with different specific angular momentum J/M^2. As long as $\Omega \ll \sqrt{M/R_s^3}$, the slow-rotation approximation is consistent. Ignoring terms of $\mathcal{O}(\Omega^2)$, the moment of inertia is given by $I = J/\Omega$ and it does not depend on ζ_c, but only on the stellar mass. Therefore we need to integrate Eq. (20) only once in order to obtain I for a given mass.

With the slowly rotating solution at hand, we can also study the possibility of ergoregion formation [73]. The ergoregion can be found by computing the surface at which g_{tt} vanishes, i.e., from Eq. (18):

$$-B(r) + \zeta^2 r^2 \sin^2\theta = 0\,. \qquad (24)$$

On the equatorial plane we simply have

$$r\zeta(r) = \sqrt{B(r)}\,, \qquad (25)$$

and, due to the linearity of the field equations, ζ will scale linearly with Ω. Thus, one needs only a single integration in order to compute the zeros of Eq. (25) as functions of Ω. For a given value of Ω, there can be no zeros (i.e. no ergoregion), two distinct zeros (with the ergoregion located between them) or two coincident zeros. The "critical frequency" at which we have two coincident zeros, say Ω_c, is the minimum rotation frequency for which an ergoregion exists. The slow-rotation approximation imposes $\Omega \lesssim \Omega_{\rm ms}$, where the mass shedding frequency is defined as $\Omega_{\rm ms} \equiv \sqrt{M/R_s^3}$, following Hartle's conventions. The existence of an ergoregion requires $\Omega > \Omega_c$, so an ergoregion can exist (in the slow-rotation approximation) only if

$$\Omega_c < \Omega_{ms}\,. \qquad (26)$$

C. Equation of state

We wish to establish some limits on the parameters of allowed theories of gravity, given a set of EOS models compatible with our present knowledge of nuclear physics. Instead of general relativity being assumed, the parameters characterizing any specific theory will be constrained based on astrophysical observations. These constraints will be sensitive to our assumptions on the EOS, that we describe in this section.

TABLE II. List of EOS used in this work.

EOS	Reference
Polytropic	[23]
FPS	[74]
APR	[75]
Causal limit	[15, 76]

A list of the EOS models used in this work is given in Table II. For code testing purposes, we have considered the same polytropic model used by Damour and Esposito-Farése [23]:

$$\rho = n m_b + K \frac{n_0 m_b}{\Gamma - 1}\left(\frac{n}{n_0}\right)^{\Gamma}, \quad P = K n_0 m_b \left(\frac{n}{n_0}\right)^{\Gamma}, \tag{27}$$

with $m_b = 1.66 \times 10^{-24}\,\mathrm{g}$, $n_0 = 0.1\,\mathrm{fm}^{-3}$, $\Gamma = 2.34$ and $K = 0.0195$.

We also considered two nuclear-physics motivated models (FPS [74] and APR [75], in the standard nomenclature), which are respectively a soft EOS and a more standard realistic EOS, as well as for the stiffest possible EOS constructed by combining the upper limit in the crust-core transition region of Hebeler et al. [15] with a causal limit EOS as in [76]. The polytropic model (27) gives results which are quantitatively very similar to those for FPS EOS.

We must remark that the FPS EOS seems to be ruled out by the recent observation of a neutron star with $M = (1.97 \pm 0.04)M_\odot$ [12], at least if we limit consideration to nonrotating models[4] within general relativity. However, these observations could be explained in terms of modified gravity at high density, rather than by invoking a different EOS. In fact, in some alternative theories the maximum mass of a neutron star can be sensibly *larger* than in general relativity [30, 65]. Another important motivation to use the FPS EOS is to make direct comparison with previous work. We explicitly checked that our two independent codes (written in MATHEMATICA and C++) are in excellent agreement with Refs. [5, 78, 79] in the general relativistic limit[5].

[4] Rapidly rotating neutron stars have a larger maximum mass which, using the FPS EOS, is still marginally compatible with the observational errors in a small region of the parameter space: some rapidly rotating models in Table 3 of [77] have a gravitational mass compatible with the value measured in [12].

[5] Incidentally, the moment of inertia shown in Fig. 3 of [59] is not computed using the FPS EOS, as erroneously written in the caption of that figure. This explains why our results for the FPS EOS do not agree with those in [59].

V. COMPACT STARS IN GAUSS-BONNET GRAVITY

As a first application of the formalism discussed above, in the remainder of this paper we study neutron stars in EDGB gravity. We defer a more general study of the full theory derived from the Lagrangian (7) to future work.

EDGB gravity is obtained from the Lagrangian (7) by considering a real scalar field $\phi = \Phi$ (or $\omega = 0$), $f_0 \equiv \kappa = (16\pi)^{-1}$, $V \equiv 0$ and

$$f_1 \equiv \frac{\alpha}{16\pi} e^{\beta\Phi}, \tag{28}$$

where α and β are coupling constants. When $\beta = \sqrt{2}$, this theory arises as a low-energy correction to the tree-level action in heterotic string theory [49]. Here we adopt a phenomenological point of view and consider α and β as free (real) parameters. We will show by an explicit calculation that, under reasonable assumptions for the nuclear EOS, the observation of compact stars with certain observed properties (such as mass, radius or moment of inertia) leads to the existence of rather stringent exclusion regions in the two-dimensional (α, β) parameter space.

Some results are shown in Figs. 1-3 for different EOS models and different values of α and β. In each figure, the top two panels show the mass-density relation and the mass-radius relation for static (nonrotating) stars. The bottom-left panel shows the binding energy as a function of the central density. In the bottom-right panel we display the moment of inertia as a function of (gravitational) mass.

In our numerical calculations, we observed that the scalar field in the interior of the star is always small: in special cases it can be as large as $\Phi \sim 10^{-2}$, but more typically $\Phi \lesssim 10^{-4}$ in most of the parameter space. In the small-field limit, $\Phi \ll 1$, the coupling f_1 in Eq. (28) can be Taylor-expanded:

$$16\pi f_1(\Phi) \sim \alpha + \alpha\beta\Phi\,. \tag{29}$$

Since the first term is constant and the GB term is a topological invariant, the first nonvanishing corrections arise from the second term. Therefore, in the small-field limit the equilibrium structure depends only on the product $\alpha\beta$ of the coupling constants. This is confirmed by our numerical results in Figs. 1-3: for instance the lines corresponding to $\alpha = 20M_\odot^2$, $\beta^2 = 1$ and $\alpha = 10M_\odot^2$, $\beta^2 = 4$ both correspond to the same $\alpha\beta = 20M_\odot^2$, and indeed they lie almost exactly on top of each other.

A similar degeneracy will occur for any other functional form of $f_1(\Phi)$, provided that the scalar field remains small everywhere, so that a Taylor expansion similar to Eq. (29) holds. In this sense, most of our results remain valid for a generic function $f_1(\Phi)$, and not only for EDGB gravity.

Furthermore, since only the second term on the right hand side of Eq. (29) contributes to the dynamics, it

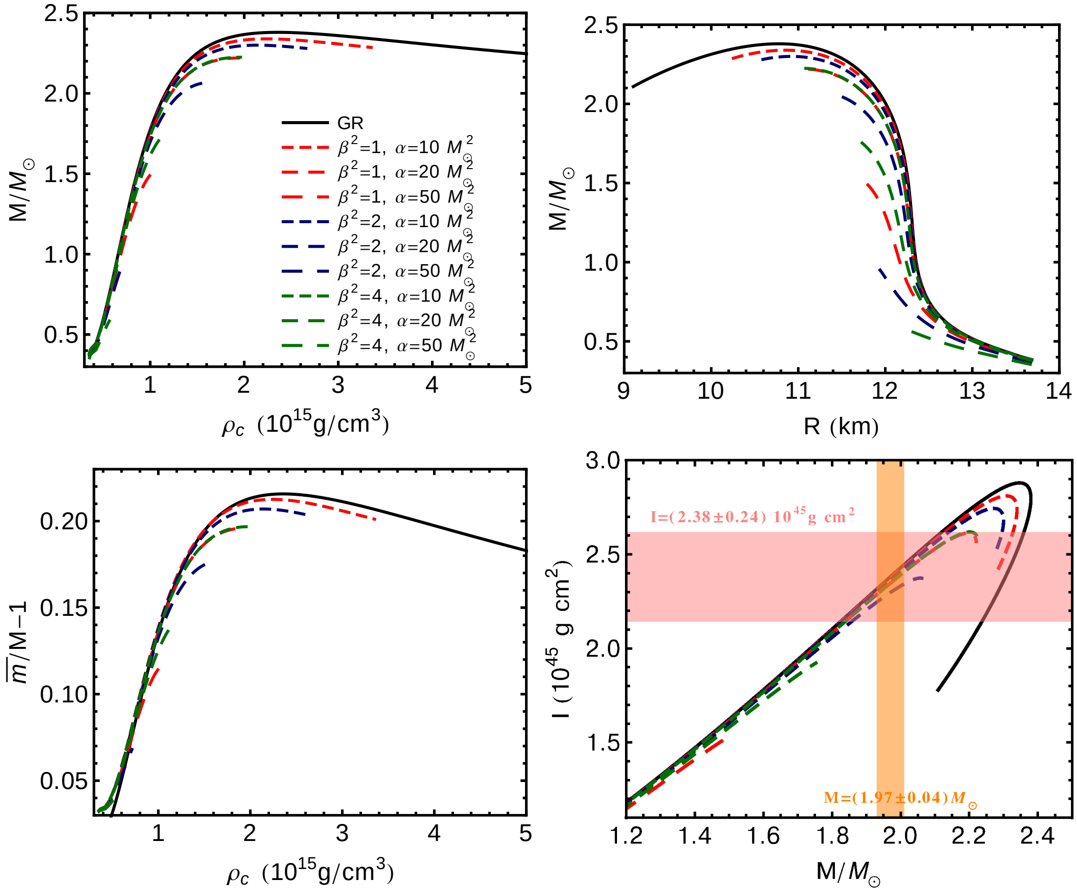

FIG. 1. Compact star models in EDGB gravity for different values of the parameters α and β, using the APR EOS. In the bottom right panel we show the recent observation of a neutron star with $M \approx 2M_\odot$ and a possible future observation of the moment of inertia confirming general relativity within 10% [80]. Curves terminate when the condition (31) is not fulfilled (cf. also the exclusion plot in Fig. 5).

follows that the field equations are (approximately) symmetric under the transformation

$$\alpha \to -\alpha\,, \qquad \Phi \to -\Phi\,. \tag{30}$$

Taking advantage of this symmetry, we present results only for the case $\alpha > 0$. The solutions for $\alpha < 0$ can be (approximately) obtained by simply inverting the sign of the scalar field while leaving other physical quantities (such as the mass, the radius or the moment of inertia) unchanged. This argument is confirmed by a numerical integration of the field equations. We have explicitly checked that the results shown in Figs. 1-3 differ by only 0.1% or less from the corresponding quantities computed when $\alpha < 0$.

We note that, in the small α limit, black hole solutions can be found analytically [50] and they share the same symmetry (30), which is exact in this case. However, numerical black hole solutions found for generic values of α may be coupled to a large scalar field (cf. Table I in Ref. [48]), so that the expansion (29) does not hold and the degeneracy between α and $-\alpha$ is broken. As we discussed above, this is not the case for neutron stars, for which the scalar field is typically small.

It is clear from Figs. 1-3 that, regardless of the EOS and for any value of α, the coupling to the dilaton tends to *reduce* the importance of relativistic effects. Indeed, as shown in Fig. 4, the maximum gravitational mass $M_{\rm max}$ monotonically decreases as a function of the product $\alpha\beta$ of the EDGB coupling parameters. Thus in EDGB gravity (as well as in general relativity) soft EOS models, like FPS, should be ruled out by observations of high-mass neutron stars. This is similar to what happens in gravitational-aether theory [66] and in Einstein-aether theory [61].

For small values of the product $\alpha\beta$, the maximum mass in Fig. 4 corresponds to a local maximum in the mass-density relation (cf. the upper left panels of Figs. 1-3). In general relativity these local maxima (or, equivalently, inversion points in the mass-radius diagram) correspond to marginally stable equilibrium configurations, and solutions to the right of the first maximum are unstable to radial perturbations (see e.g. [2]). We conjecture that the same property should hold also for extended scalar-tensor

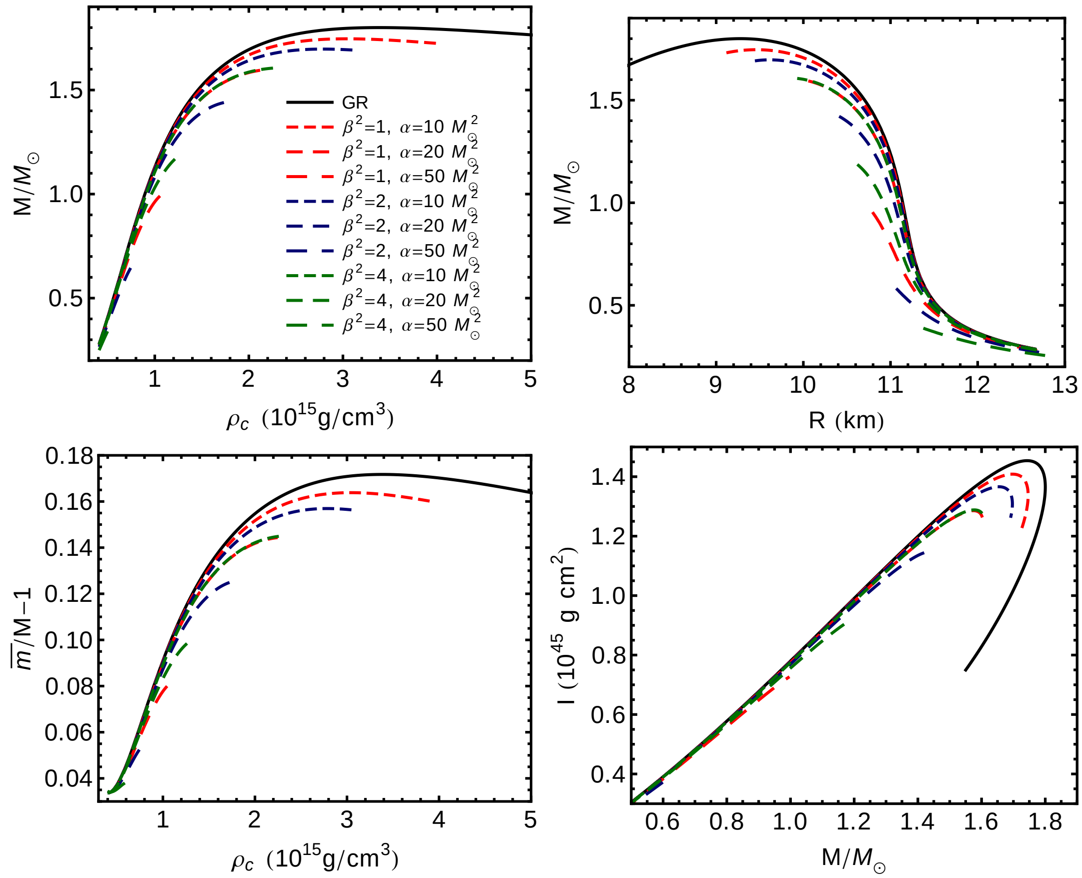

FIG. 2. Compact star models in EDGB gravity for different values of the parameters α and β, using the FPS EOS. Curves terminate when the condition (31) is not fulfilled (cf. also the exclusion plot in Fig. 5).

theories. This was proved for particular self-gravitating configurations involving scalar fields [29, 81, 82], but a more detailed stability analysis would be desirable (see also the discussion in [32]). In EDGB theory, spherically symmetric solutions can be constructed only up to a maximum central density $\rho_c^{\max}(\alpha, \beta)$, for reasons explained below: see in particular the discussion around Eq. (31). For large $\alpha\beta$ this maximum central density is such that the first local maximum in the mass-density curve is never reached. In Fig. 4, all values to the left of the solid circles correspond to a maximum mass obtained from the radial stability criterion. Values to the right of the solid circles correspond instead to the mass $M_{\max}$ obtained at the critical value of ρ_c beyond which we cannot find spherically symmetric perfect fluid solutions anymore.

A. Constraints on the EDGB couplings

In the near future, observations of the double pulsar may provide measurements of the moment of inertia to an accuracy of $\sim 10\%$ [80] (but see Ref. [83] for some criticism). Furthermore, precise observations of the mass-radius relation could be obtained from thermonuclear X-ray burst [84, 85]. These observations could be used in the context of a Bayesian model-selection framework to place strong constraints on EDGB gravity and, more generally, to remove the degeneracy between different EOS models and different proposed modifications of general relativity.

Nevertheless, even without assuming any particular EOS, we can set rather stringent theoretical constraints on the EDGB parameters. Indeed, as shown in Figs. 1-3, depending on α and β, there is a maximum value of the central density ρ_c, above which no compact star models can be constructed. For a given central density, the critical value of $\alpha\beta$ can be computed analytically in the small Φ limit, as follows. We first compute the series expansion (15) up to $\mathcal{O}(r^2)$. The resulting expressions are not very illuminating, but in general the series coefficients contain square roots, whose argument must be positive to ensure the existence of physical (real-valued) solutions. When $\Phi_c \ll 1$, by imposing this "reality condition" we

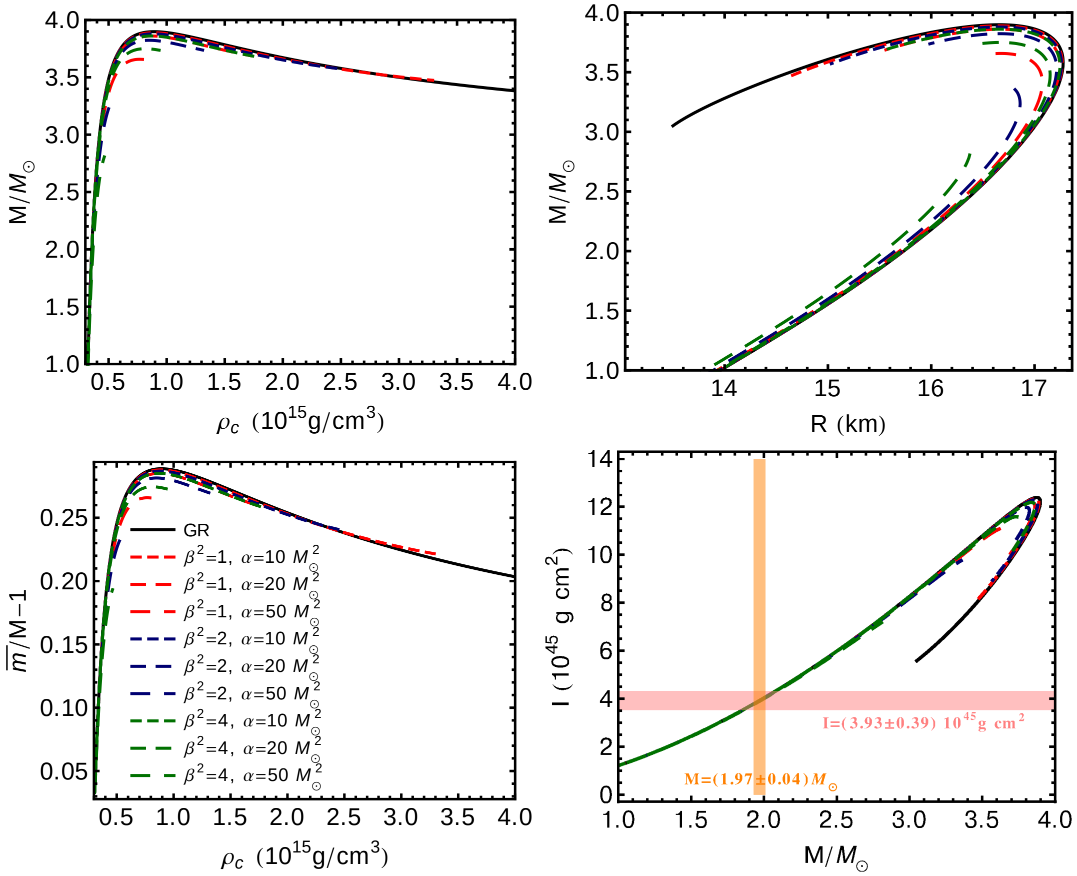

FIG. 3. Compact star models in EDGB gravity for different values of the parameters α and β, using a causal EOS. Curves terminate when the condition (31) is not fulfilled (cf. also the exclusion plot in Fig. 5).

find

$$\alpha^2\beta^2 < \frac{1}{7776\pi P_c^4\rho_c}\Big[128\rho_c^3 - 27P_c^2\rho_c + 288P_c\rho_c^2 + 54P_c^3 \\ -2\sqrt{(3P_c+\rho_c)\,(3P_c-8\rho_c)^2\,(3P_c+4\rho_c)^3}\Big]\,. \quad (31)$$

For a given value of $\alpha\beta$, the condition above implies that a maximum central density, $\rho_c^{\max}$, exists.

Equation (31) is in good agreement with numerical results, as shown in Fig. 5. This figure is basically an exclusion plot: it shows the maximum allowed values of $\alpha\beta$ as a function of the maximum central density $\rho_c^{\max}$ for different EOS models and nonrotating stars. For small values of $\alpha\beta$ (i.e., on the right of the figure), a local maximum in the mass-density relation is reached and the maximum central density $\rho_c^{\max}$ corresponds to the local maximum of $M(\rho_c)$, i.e. to the first inversion point in the mass-radius relation. If our stability conjecture is correct, no stable static configurations can be constructed in the region *above* these lines. The local maximum is never reached to the *left* of the points marked by filled circles. In this case, the $M(\rho_c)$ curves terminate before reaching a local maximum, and the maximum central density $\rho_c^{\max}$ is simply the point where the equilibrium sequence terminates. Dotted lines correspond to the analytical prediction (31), which agrees very well with the numerical value at which we cannot compute equilibrium models anymore. The bottom line is that no static models (either stable or unstable) can be constructed in the shadowed regions above these exclusion lines.

The quadratic EDGB corrections are expected to be stronger in high-density (high-curvature) regions, so the most stringent bounds should come from the stiffest EOS models. Indeed, among the models we consider, the strongest and weakest constraints come from the Causal EOS and from the FPS EOS, respectively.

The bounds on the central density can be translated into constraints on the maximum mass, which is an observable quantity. As shown in Fig. 4, for a given EOS the maximum mass is a monotonically decreasing function of $\alpha\beta$.

The requirement that the maximum mass $M_{\max}$ supported by the theory should be larger than some trusted observed value, can place a direct upper bound on $\alpha\beta$. In Table III, we consider $M_{\max} \gtrsim 1.4M_\odot$, $M_{\max} \gtrsim 1.7M_\odot$ and $M_{\max} \gtrsim 1.93M_\odot$ (which is the lower bound of the recent observation in [12]) and we translate them into upper bounds on $\alpha\beta$ using the data plotted in Fig. 4.

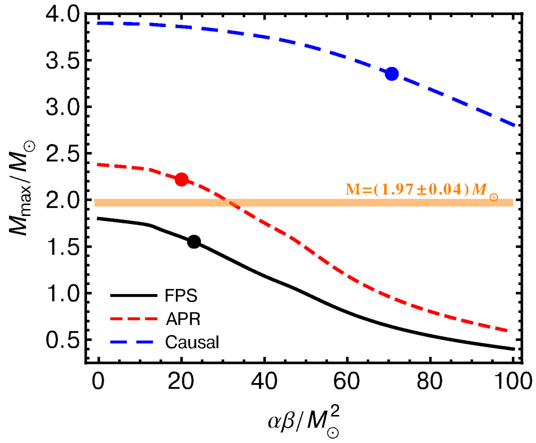

FIG. 4. Maximum mass as a function of the product $\alpha\beta$ of the EDGB coupling parameters, for different EOS models and in the nonrotating case (cf. the main text for details). To the left of the filled circle, this maximum mass corresponds to the radial stability criterion; to the right, it corresponds to the maximum central density for which we can construct static equilibrium models. The recent measurement [12] of a neutron star with $M \approx 2M_\odot$ is marked by a horizontal line. Only the combination $\alpha\beta$ is bounded, due to the approximation $\Phi \ll 1$ (cf. Eq. (29)).

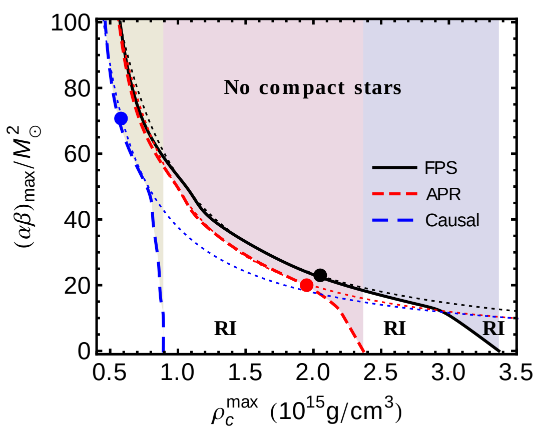

FIG. 5. Exclusion plot for the parameters $\alpha\beta$ in the small Φ_c limit and for nonrotating models. Only the combination $\alpha\beta$ is bounded, due to the approximation $\Phi \ll 1$ (cf. Eq. (29)). In the region above the dotted lines no compact star solution can be constructed (cf. Eq. (31)). In the region above the thick lines (marked as "RI", Radial Instability), static configurations are unstable against radial perturbations (cf. the main text for details). Markers indicate the maximum central density of radially stable stars in general relativity.

The Causal EOS models predict an unrealistically large maximum mass, so all neutron stars with $M_{\rm max} \lesssim 2.8M_\odot$ would place very mild constraints on alternative theories, and we omit this EOS from Table III.

For the "standard" value of the EDGB coupling ($\beta = \sqrt{2}$), if we consider the APR EOS as our "best candidate" EOS for a (nonexotic) neutron star interior within general relativity, the results in Table III imply $\alpha \lesssim 23.8M_\odot^2$.

TABLE III. Constraints on the EDGB parameters from an observation of a nonrotating neutron star with mass M. For the values of M we consider, constraints using the Causal EOS would allow values of $\alpha\beta$ larger than $100M_\odot^2$.

EOS	$M_{\rm max} \gtrsim 1.4M_\odot$	$M_{\rm max} \gtrsim 1.7M_\odot$	$M_{\rm max} \gtrsim 1.93M_\odot$
FPS	$\alpha\beta \lesssim 30.1M_\odot^2$	$\alpha\beta \lesssim 13.9M_\odot^2$	no models
APR	$\alpha\beta \lesssim 50.3M_\odot^2$	$\alpha\beta \lesssim 41.9M_\odot^2$	$\alpha\beta \lesssim 33.6M_\odot^2$

This bound should be compared to the bound on α that comes from requiring the existence of black hole solutions in the theory [48, 53]. For $\beta = \sqrt{2}$, this requirement implies

$$\frac{\alpha}{M_\odot^2} \lesssim 70 \left[\frac{M_{BH}}{10M_\odot}\right]^2 , \qquad (32)$$

where M_{BH} is the black hole mass. The observation of black holes with $M_{BH} \approx 8M_\odot$ (such as Cyg X1) constrains $\alpha \lesssim 44M_\odot^2$. The constraints on α coming from observations of compact stars (cf. Table III) are already smaller by a factor ~ 2 than those coming from the existence of stellar black holes, and they could become even more stringent in the near future.

We may hope that future observations of the moment of inertia could place even tighter bounds on the theory. Unfortunately this seems unlikely. To understand why, we can either look at the bottom right panel of Fig. 1 or at Fig. 6, where we show the moment of inertia for the APR EOS (normalized by its value in general relativity) as a function of $\alpha\beta$ for fixed values of the stellar mass. As it turns out, for values of $\alpha\beta$ smaller than those listed in Table III, the moment of inertia can deviate from the general relativistic value by 5% at most. The precision of future observations is expected to be $\sim$ 10% in optimistic scenarios [80]. Therefore, at least for EDGB gravity, the most stringent constraints on $\alpha\beta$ should come from mass measurements, rather than from measurements of the moment of inertia.

Let us mention, for completeness, that we also studied the possibility of the formation of an ergoregion for slowly rotating stars in EDGB gravity. For our 'realistic" EOS models the condition (26) is never met, and therefore no ergoregion can form outside the star. This is because, for phenomenologically viable parameters, the relativistic effects in this particular theory are actually *smaller* than in general relativity. We can anticipate that other sectors of the general theory described by the Lagrangian (7) could enhance relativistic effects and favor the existence of the ergoregion, with important implication for the stability of these solutions [73]. A more detailed analysis will be presented elsewhere.

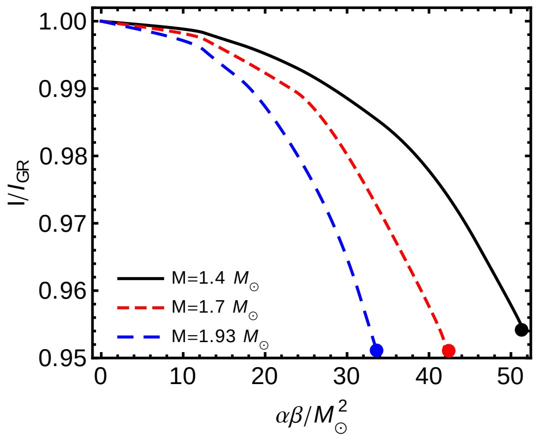

FIG. 6. Moment of inertia normalized by its value in general relativity for the APR EOS at fixed values of the gravitational mass. Curves terminate at the bounds listed in Table III (corresponding to the filled circles). The deviations from general relativity are always smaller than 5%.

VI. CONCLUSIONS AND OUTLOOK

Neutron stars are very promising laboratories to constrain strong-curvature corrections to general relativity. New proposed theories of gravity are usually tested against weak field observations and cosmological data, or by studying the existence and nature of black hole solutions. Our main goal in this paper was to develop a formalism for a comprehensive study of stellar structure in a broad class of alternatives to Einstein's general relativity. We focused on a class of theories ("extended scalar-tensor theories") where quadratic curvature corrections, non-minimal couplings and parity-violating terms are coupled to standard gravity through a single scalar field. Particular cases of this model include, but are not limited to, quadratic gravity, EDGB gravity, generic scalar-tensor theories and $f(R)$ theories (via their correspondence with scalar-tensor theories). We wrote down the field equations for static and spherically symmetric perfect-fluid stars in the general case, as well as the leading-order corrections in a slow-rotation expansion. For a given model and a given central density, the formalism allows us to obtain the mass, radius, binding energy and moment of inertia of compact stars. In future work we will show how these theoretical predictions can be compared to observations in order to constrain the parameter space of "extended scalar-tensor" and other alternative theories.

As a first application, in the second part of the paper we studied stellar structure in EDGB gravity. We found that, in general, the GB coupling tends to *reduce* relativistic effects in compact stars. We also showed that there is an exclusion region in the two-dimensional plane of the GB coupling parameters beyond which no compact star solutions can be constructed: cf. Eq. (31), Fig. 4 and Fig. 5.

Stability requirements for static models and future observational data could constrain the theory even further. The existence of high-mass neutron stars put the most stringent constraints on EDGB gravity (cf. Table III). As it turns out, these bound are tighter (by a factor of a few) than the bound coming from the existence of black hole solutions in EDGB theory, given in Eq. (32). They are also tighter than the bounds that could come from future precision measurements of the moment of inertia.

In this sense, the existence of large-mass neutron stars provides the best constraint on the EDGB coupling parameters obtained so far. Further explorations of stellar structure and better observational data on the mass-radius relation (see e.g. [84, 85]) have the potential to exclude a larger region of the parameter space of alternative theories. There is of course the possibility that theoretical and observational work may give us hints on how to modify general relativity to make it compatible with the standard model, which would be even more exciting.

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